Optimization in Power Grid Analysis

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Part I: Analyzing Vulnerability of a Power Grid

Part II: Detecting Faults from PMU Information

Part I: Analyzing Vulnerability of a Power Grid

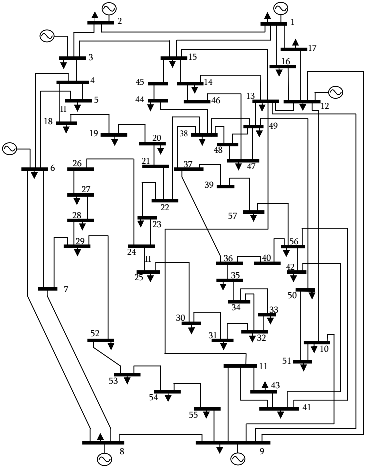
We analyze the vulnerability of a power grid by finding the "attack" (in the form of impedance increases on lines) that maximizes disruption.

- Uses AC power flow model.
- Vulnerabilities revealed by the lines involved in the attack.
- Two grid disruption measures.
  - **Voltage drops**
  - **Amount of load that cannot be served**
- Nonsmooth optimization and bilevel optimization formulations.
- Uses a first-order method with heuristics.

Deliberate impedance increase is not necessarily a realistic attack model, but

- Impedance changes due to high temperatures can degrade the grid (as in 2003 northeast US blackout)
- This model gives a continuous formulation that is a good proxy for a transmission-line-disconnection model, as we'll see.

A Power System: A Network



A power system consists of

- Set of nodes ("buses") $\mathcal{N}$
  - $\mathcal{G} \in \mathcal{N}$ (generators).
  - $\mathcal{D} \in \mathcal{N}$ (demand/load buses).
- Set of transmission lines: $\mathcal{L} \subset \mathcal{N} \times \mathcal{N}$.

Quantities:

- For node $i \in \mathcal{N}$:
  - Complex voltage: $V_i e^{\jmath \theta_i}$.
  - Complex power: $P_i + jQ_i$
- For transmission line $(i, k) \in \mathcal{L}$:
  - Complex admittances: $Y_{ik} = G_{ik} + jB_{ik}$

Power Balance Equations:

$$P_i = V_i \sum_{k \in \mathcal{N}} V_k (G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)),$$

$$Q_i = V_i \sum_{k \in \mathcal{N}} V_k (G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)).$$

AC Power Flow Problem

Power mismatch at each node i:

$$F_i^P(V,\theta) := V_i \sum_{k\in\mathcal{N}} V_k(G_{ik}\cos(\theta_i-\theta_k) + B_{ik}\sin(\theta_i-\theta_k)) - P_i,$$
$$F_i^Q(V,\theta) := V_i \sum_{k\in\mathcal{N}} V_k(G_{ik}\sin(\theta_i-\theta_k) - B_{ik}\cos(\theta_i-\theta_k)) - Q_i.$$

All F_i^P and F_i^Q should be zero at a solution, an operating point.

AC power flow unknowns are V_i for $i \in \mathcal{D}$, and θ_i for $i \in \mathcal{G} \cup \mathcal{D}$:

$$F(V,\theta) := \begin{bmatrix} F_{\mathcal{G}}^P(V,\theta) \\ F_{\mathcal{D}}^P(V,\theta) \\ F_{\mathcal{D}}^Q(V,\theta) \end{bmatrix}$$

- $F(V,\theta) = 0$ is a square nonlinear system.
- Can be solved using Newton's method, or enhancements.

Bad Cases:

- Low voltages: A solution is found, but the voltage V_i is too low at some buses.
- No solution: $F(V,\theta) = 0$ is infeasible.

Analyzing Vulnerability

Assuming the grid is operationally stable, which are the lines whose degradation leads to these bad (low voltages, no solution) cases?

The answer to this question could be used to defend lines or make them more robust, or to plan grid upgrades to improve robustness.

Goal: Identify the attack which would produce maximum disruption to the grid.

- A purported "attack" takes the form of increasing impedances on some lines. (Deliberate agent? Weather? Proxy for full disconnection of lines?)
- The attack is not all-powerful: We assume some limit to the attacker's resources. Their goal is to allocate this resource so as to maximize disruption.

How to measure disruption caused by an attack? Two alternatives:

- Deviation of voltage magnitude V from 1.
- Amount of power adjustment required to keep the voltages in a viable range.

Earlier* works mostly use simplified DC or lossless models.

*Salmeron et al. (2004), Arroyo and Galiana (2005), Donde et al. (2008), Pinar et al. (2010), Bienstock and Verma (2010)

Impedance Attack

Attack specified by a Line Perturbation Vector $\gamma \in \mathbb{R}_+^{|\mathcal{L}|}$.

When r is an index of the line connecting buses i and k,
γ_r defines change in admittances $Y_{ik} = G_{ik} + \jmath B_{ik}$.

$$\text{Conductance:}\quad G_{ik}(\gamma) = \begin{cases} \dfrac{G_{ik}}{1+\gamma_r} & \text{if } i \neq k \text{ and } r \equiv (i,k), \\ -\displaystyle\sum_{s:s\neq i} G_{is}(\gamma) & \text{if } i = k. \end{cases}$$

$$\text{Susceptance:}\quad B_{ik}(\gamma) = \begin{cases} \dfrac{B_{ik}}{1+\gamma_r} & \text{if } i \neq k \text{ and } r \equiv (i,k), \\ -\displaystyle\sum_{s:s\neq i} \left(B_{is}(\gamma) - \frac{1}{2} B_{is}^{sh} \right) & \text{if } i = k. \end{cases}$$

- $\gamma = 0 \Rightarrow$ No change to admittances.

AC power flow equations are now parameterized with γ:

$$F_i^P(V,\theta;\gamma) := V_i \sum_{k\in\mathcal{N}} V_k \left(G_{ik}(\gamma) \cos(\theta_i - \theta_k) + B_{ik}(\gamma) \sin(\theta_i - \theta_k) \right) - P_i,$$

$$F_i^Q(V,\theta;\gamma) := V_i \sum_{k\in\mathcal{N}} V_k \left(G_{ik}(\gamma) \sin(\theta_i - \theta_k) - B_{ik}(\gamma) \cos(\theta_i - \theta_k) \right) - Q_i.$$

Effect of Line Perturbation

After line perturbation, we still want to have

$$\begin{aligned} V_i \sum_{k \in \mathcal{N}} V_k (G_{ik}(\gamma) \cos(\theta_i - \theta_k) + B_{ik}(\gamma) \sin(\theta_i - \theta_k)) - P_i &= 0, \\ V_i \sum_{k \in \mathcal{N}} V_k (G_{ik}(\gamma) \sin(\theta_i - \theta_k) - B_{ik}(\gamma) \cos(\theta_i - \theta_k)) - Q_i &= 0. \end{aligned}$$

When there is no response from a system operator, i.e. P_i's and Q_i's are fixed, the effect of an attack will appear as voltage changes.

If a system operator tries to keep voltage levels in a viable range, then P_i's and Q_i's might need to be adjusted.

We use these two cases as disruption measures.

Disruption Measures

Aim to find a line perturbation vector γ that maximizes one of the following objectives:

Voltage Disturbance ($\mathcal{F}_V$): The sum-of-squares deviation of the voltages from 1.

$$\mathcal{F}_V(\gamma) := \begin{cases} \frac{1}{2}\sum_{i \in \mathcal{D}} (V_i - 1)^2, & \text{where } V_i\text{'s are from the solution of } F(V, \theta; \gamma) = 0. \\ +\infty, & \text{when } F(V, \theta; \gamma) = 0 \text{ has no solution.} \end{cases}$$

Here $F(V, \theta; \gamma)$ are the AC power flow equations for the grid in which impedances are changed according to γ.

Power-Adjustment ($\mathcal{F}_L$): The sum of changes of (real) power injection on buses.

$$\begin{aligned} \mathcal{F}_L(\gamma) \;:=\; \min_x \quad & p^T x \\ \text{subject to} \quad & c(x; \gamma) = 0, \\ & \underline{x} \le x \le \overline{x}. \end{aligned}$$

(If infeasible, define $\mathcal{F}_L(\gamma) := +\infty$.)

More on Power Adjustment Measure...

Minimize a weighted sum of demand and generation adjustments needed to restore feasibility to the AC equations.

$$\begin{aligned}
\mathcal{F}_L(\gamma) \;=\; \min_{\substack{V_\mathcal{D},\theta_{\mathcal{D}\cup\mathcal{G}},\\ \sigma^\pm_\mathcal{G},\rho_\mathcal{D}}} \quad & \sum_{i\in\mathcal{G}} |P_i|(\sigma_i^+ + \sigma_i^-) + \sum_{i\in\mathcal{D}} |P_i|\rho_i \\
\text{subject to} \quad & F^P_\mathcal{G}(V,\theta;\gamma) - |P_\mathcal{G}| \odot (\sigma^+_\mathcal{G} - \sigma^-_\mathcal{G}) = 0, \\
& F^P_\mathcal{D}(V,\theta;\gamma) - |P_\mathcal{D}| \odot \rho_\mathcal{D} = 0, \\
& F^Q_\mathcal{D}(V,\theta;\gamma) - |Q_\mathcal{D}| \odot \rho_\mathcal{D} = 0, \\
& \underline{V}_\mathcal{D} \le V_\mathcal{D} \le \overline{V}_\mathcal{D}, \\
& 0 \le \sigma^+_\mathcal{G} \le \overline{\sigma}^+_\mathcal{G}, \\
& 0 \le \sigma^-_\mathcal{G} \le \overline{\sigma}^-_\mathcal{G}, \\
& 0 \le \rho_\mathcal{D} \le \overline{\rho}_\mathcal{D}.
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_L(\gamma) \;=\; \min_x \quad & p^T x \\
\text{subject to} \quad & c(x;\gamma) = 0, \\
& \underline{x} \le x \le \overline{x}.
\end{aligned}$$

- Voltage magnitudes $V_\mathcal{D}$ constrained e.g to $[0.93, 1.07]$.

Note: When a system is feasible, the objective $\mathcal{F}_L(\gamma) = 0$.

Vulnerability Analysis Model

Restrictions on the attack:

- Maximum perturbation on a single line : $\overline{\gamma}$.
- Total amount of perturbation : $\kappa\overline{\gamma}$ (Roughly speaking, attack at most κ lines.)

Maximal attack problem:

$$\mathcal{H}(\kappa, \overline{\gamma}) := \max_{\gamma} \mathcal{F}(\gamma) \quad \text{subject to } e^T\gamma \le \kappa\overline{\gamma}, \;\; 0 \le \gamma \le \overline{\gamma}e.$$

where $\mathcal{F}(\gamma)$ is the disruption measure ($\mathcal{F}_V$ or $\mathcal{F}_L$).

When $\mathcal{F} = \mathcal{F}_V$ (Voltage Disturbance model), it is a nonconvex nonlinear program.

When $\mathcal{F} = \mathcal{F}_L$ (Power-Adjustment model), $\mathcal{H}(\kappa, \overline{\gamma})$ is a bilevel optimization problem.

$$\begin{aligned} \mathcal{H}(\kappa, \overline{\gamma}) := \max_{\gamma} \min_{x} \;\; & p^T x \\ \text{subject to } \;\; & c(x; \gamma) = 0, \\ & \underline{x} \le x \le \overline{x}, \\ & e^T\gamma \le \kappa\overline{\gamma}, \;\; 0 \le \gamma \le \overline{\gamma}e. \end{aligned}$$

Vulnerability Analysis Model: Plausible, but Invalid Reformulations

May consider an alternative (re)formulation :

Voltage Disturbance model : $\max_{\gamma, V, \theta} \quad \frac{1}{2} \sum_{i \in \mathcal{D}} (V_i - 1)^2$

$$\text{subject to} \quad \boldsymbol{F(V, \theta; \gamma) = 0}, \; e^T \gamma \le \kappa \bar{\gamma}, \; 0 \le \gamma \le \bar{\gamma} e.$$

Excludes **the most disruptive attack**, which makes $F(V, \theta; \gamma) = 0$ infeasible!

Power-Adjustment model : $\max_{x, \gamma} \; p^T x$

where x satisfies the **KKT conditions of** $\mathcal{F}_L(\gamma)$,

$$e^T \gamma \le \kappa \bar{\gamma}, \;\; 0 \le \gamma \le \bar{\gamma} e.$$

This reformulation is widely used in existing literature. But, again, it excludes **the most disruptive attack**, which makes $\mathcal{F}_L$ infeasible.

Additionally, since the KKT conditions are only necessary, saddle points and local maximizers of the lower-level problem are feasible!

Conditional Gradient (Frank and Wolfe, 1956)

$$\mathcal{H}(\kappa, \bar{\gamma}) \; := \; \max_{\gamma} \mathcal{F}(\gamma) \quad \text{subject to } e^T \gamma \le \kappa \bar{\gamma}, \;\; 0 \le \gamma \le \bar{\gamma} e.$$

At iteration k, solve this linearized subproblem:

$$w^k \; = \; \arg\max_{w} \quad \nabla \mathcal{F}(\gamma^k)^T (w - \gamma^k)$$
$$\text{subject to } \; e^T w \le \kappa \bar{\gamma}, \;\; 0 \le w \le \bar{\gamma} e.$$

Closed-form solution:

$$w_i^k \; = \; \begin{cases} \bar{\gamma} & \text{if } \nabla \mathcal{F}(\gamma^k)_i \text{ is one of the } \kappa \text{ largest positive entries in } \nabla \mathcal{F}(\gamma^k), \\ 0 & \text{otherwise.} \end{cases}$$

New iterate is $\gamma^{k+1} := \gamma^k + \alpha_k (w^k - \gamma^k)$ for some steplength $\alpha_k \in (0, 1]$.

(Use a backtracking strategy to choose α_k.)

We need to calculate the gradient $\nabla \mathcal{F}$.

Gradient Calculation

For gradient calculation, we use **implicit function theorem** for both models.

Voltage Disturbance objective $\mathcal{F}_V$ is smooth over most of its domain. Calculate $\nabla \mathcal{F}_V(\gamma)$ by applying implicit function theorem to $F(V, \theta; \gamma) = 0$.

(This requires nonsingularity of the Jacobian of $F(V, \theta; \gamma) = 0$ with respect to (V, θ).)

Power-Adjustment objective $\mathcal{F}_L$ is nonconvex and nonsmooth, with points of nonsmoothness occurring as components of x and associated Lagrange multipliers reach their bounds.

In practice, $\mathcal{F}_L$ seems to be smooth "almost everywhere" on its effective domain, so we can use the implicit function theorem applied to the optimality conditions for the lower-level problem in the bilevel formulation, to find $\nabla \mathcal{F}_L(\gamma)$.

Algorithm

Algorithm 1 Vulnerability Analysis

Require:

$\overline{\gamma}$: Upper bound for impedance increases γ_i, $i \in \mathcal{L}$;

κ: Number of lines to attack;

Ensure:

γ^*: Impedance vector that optimizes the attack;

$k \leftarrow 0$;

$\gamma^0 \leftarrow \mathbf{0}$;

while $k \le$ MaxIter **do**

  Find the gradient $\nabla \mathcal{F}(\gamma^k)$ of the objective $\mathcal{F}$ at γ^k; ▶ Implicit function theorem.

  Find linearized optimum w^k from linearized subproblem; ▶ Closed-form solution.

  Use the backtracking to find step size $\alpha_k \in [0, 1]$;

  $\gamma^{k+1} \leftarrow \gamma^k + \alpha_k (w^k - \gamma^k)$;

  $k \leftarrow k + 1$;

  Stop if termination conditions are satisfied, and set $\gamma^* \leftarrow \gamma^k$;

end while

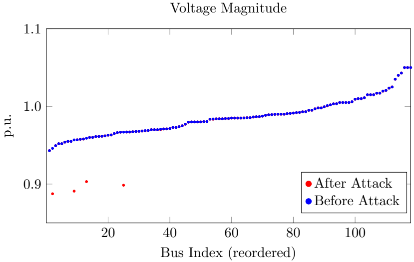
Voltage Disturbance Model: IEEE 118. ($|\mathcal{N}| = 118$, $|\mathcal{L}| = 186$, $\overline{\gamma} = 3$)

Line No.	Connected Buses		γ_i
	From	To	
71	49	51	3.00
74	53	54	3.00
82	56	58	3.00

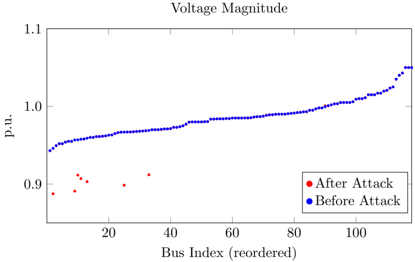
Optimal 3-Line Attack. (by our algorithm)

Line No.	Connected Buses		γ_i
	From	To	
25	19	20	3.00
29	22	23	3.00
71	49	51	3.00
74	53	54	3.00
82	56	58	3.00

Optimal 5-Line Attack. (by our algorithm)

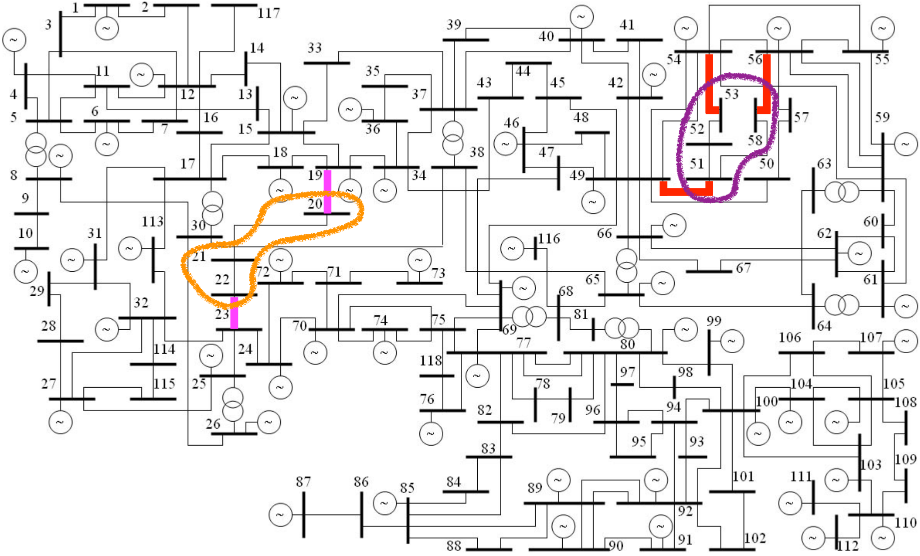


Distribution of Voltage Magnitudes.



Distribution of Voltage Magnitudes.

IEEE 118: Attack Creates an "Island" in the Grid

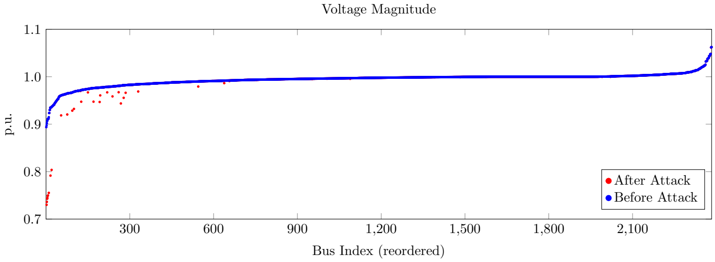


IEEE 118-Bus System

Voltage Disturbance Model: Polish 2383. ($|\mathcal{N}| = 2383$, $|\mathcal{L}| = 2896$, $\overline{\gamma} = 2$)

Line No.	Connected Buses		γ_i
	From	To	
5	10	3	1.04
404	434	188	0.25
405	437	188	2.00
467	340	218	2.00
501	340	240	0.71

Optimal 3-Line Attack. (by our algorithm)

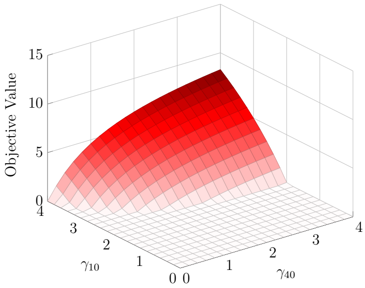


Distribution of Voltage Magnitudes.

Note: When $\kappa = 5$, our algorithm finds a solution with $\mathcal{F}_V(\gamma) = +\infty$.

Returning to the Power-Adjustment Model

Power grids often have some robustness to perturbations. The load-shedding objective $\mathcal{F}_L$ is zero over most of the feasible region $e^T\gamma \leq \kappa\overline{\gamma}$, $0 \leq \gamma \leq \overline{\gamma}e$.

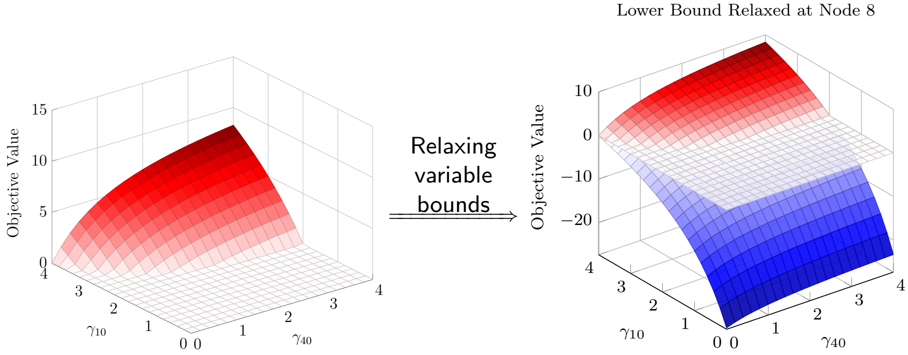


30-Bus System (`case30.m`): Attacking line 10 and 40.

Thus, it's difficult to identify a starting point with nonzero objective or gradient — the algorithm usually gets stuck at γ^0 when applied naively.

Power-Adjustment Model Initialization

To overcome this complication, we extend the range of $\mathcal{F}_L(\gamma)$ by relaxing the zero lower bounds on load-shedding variables at some buses.



The extend function value $\mathcal{F}_L(\gamma)$ itself might not be very interesting, but we can obtain good information for a search direction.

Power-Adjustment Model: IEEE 118: 3-Line Attacked ($\overline{\gamma} = 3$)

Line No.	Connected Buses		γ_i
	From	To	
71	49	51	3.00
72	51	52	0.26
74	53	54	3.00
82	56	58	2.74
Objective (MW)			22.39

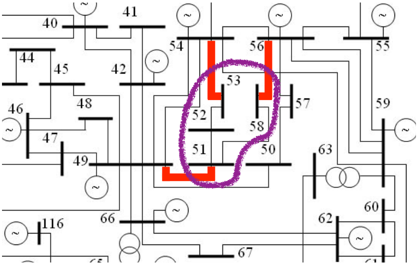
Optimal Attack (our algorithm)

Lines Selected			Load-Shedding (MW)
71	74	82	22.13
71	72	74	19.07
71	74	83	17.21
71	74	184	15.61
71	74	97	13.27

Worst Attacks (per enumeration)

Same solution as for the voltage disturbance model.
(Reminder: The attack is creating a small "island" in the grid.)

Load must be shed on buses 51, 52, 53.



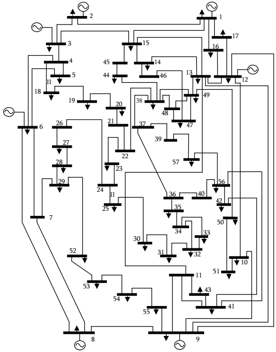
Power-Adjustment Model: Polish 2383: 5-Line Attacked ($\overline{\gamma} = 2$)

Line No.	Buses		From Bilevel Formulation			From $N-1$	
	From	To	Cont. (γ_i)	Top-5	Best-5	Top-5	Best-5
5	10	3	-	-	-	2.00	-
268	126	118	2.00	2.00	2.00	2.00	2.00
269	142	118	2.00	2.00	2.00	-	-
289	135	125	2.00	2.00	2.00	2.00	2.00
296	145	128	2.00	2.00	-	2.00	2.00
317	142	135	1.54	2.00	2.00	-	-
405	437	188	-	-	-	2.00	2.00
467	340	218	-	-	-	-	2.00
2142	1693	1658	0.46	-	2.00	-	-
Objective (MW)			1109.72	1086.67	1460.25	594.09	597.71
# of Buses with Power Adjustment			77	78	71	53	53

Power-Adjustment Model: Polish 2383-Bus System with $\kappa = 5$.

$*$ **From** $N-1$: Enumerate all $N-1$ instances, then pick Top-5 and Best-5 combinations from the lines causing load-shedding.

Part II: Fault Detection from PMU Information



Goal: Fast and accurate identification of power line outages. Important for preventing faults, routine monitoring, control.

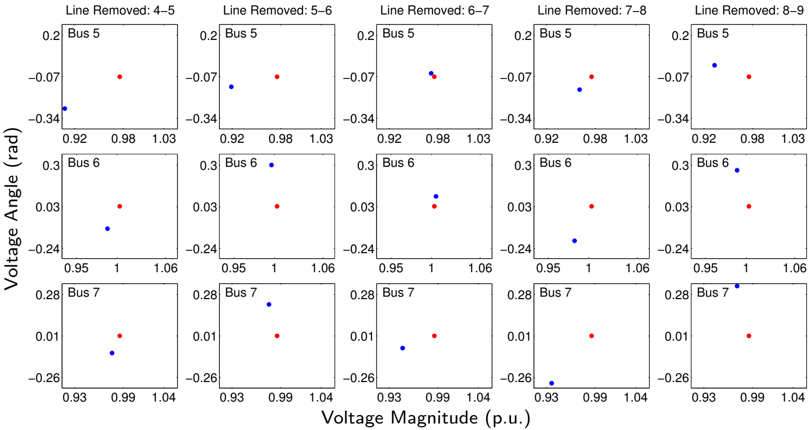
Tool: Phasor Measurement Units (PMU) can be placed on lines in the grid (typically near buses) to provide streaming data about voltage phasors and currents. Sample at 30 Hz.

Key Observation: Each outage event has a distinctive "signature" of changes to voltage phasor measurements.

Approach: Use machine learning / optimization to recognize and classify each signature, thus detecting outages rapidly.

Design: Use regularized optimization frameworks to decide where to place a limited number of PMUs in the grid, to maximize detection performance.

Example: 9-Bus System



- Red: Voltages before line outages.
- Blue: Voltages after line outages.

Shift in complex voltage phasor (before vs after) has a distinctive signature.

(We wait for transient dynamics in the grid to settle down before capturing the shift.)

Ingredients

- System network topology and transmission line parameters (admittance).
- SCADA data (active and reactive powers, voltage magnitudes) during a day.
- AC power flow model to simulate each outage under different demand conditions.

$\Rightarrow$ Train a multiclass classifier on simulated data to recognize each outage signature.

- PMU: Real-time phasor measurements (voltage magnitudes and angles)

$\Rightarrow$ Real-time line outage identification using the trained classifier.

This data analysis technique is fairly standard.

Real-time identification does not require model inversion in real time - just a bunch of inner products.

Multiclass Logistic Regression (MLR)

Data for the MLR learning problem are vectors $X_1, X_2, \ldots, X_M$ and outage indicators $Y_1, Y_2, \ldots, Y_M$.

- Each observation vector X has the form

$$X = \begin{bmatrix} \Delta V_1 & \ldots & \Delta V_n & \Delta\theta_1 & \ldots & \Delta\theta_n & G & 1 \end{bmatrix}^T,$$

  where ΔV_l and $\Delta\theta_l$ are voltage phasor shifts at buses $i = 1, 2, \ldots, n$ resulting from the outage indicated by label Y, G is the total generation level at the moment (observable from SCADA).
- The vectors X are simulated by the AC power flow model, using a typical demand patterns for some time during the day.
- Each outcome Y is a label from the set $\{1, 2, \ldots, K\}$, where K is the number of possible outages.
- There are multiple observation vectors X_i for which $Y_i = k$, for each outage scenario $k \in \{1, 2, \ldots, K\}$, each corresponding to a different demand pattern.
- In the training process, we learn coefficient vectors β_k, $k = 1, 2, \ldots, K$ such that

$$Y_i = p \;\Rightarrow \langle \beta_p, X_i \rangle \gg \langle \beta_k, X_i \rangle\,, \quad \text{for } k \neq p.$$

Multiclass Logistic Regression (MLR)

Having learnt the coefficient vectors β_k, $k = 1, 2, \ldots, K$, we classify an unknown observation vector X by assigning a likelihood for each of the K possibilities:

$$\Pr(Y = p|X) := \frac{e^{\langle \beta_p, X \rangle}}{\sum_{k=1}^{K} e^{\langle \beta_k, X \rangle}}.$$

The index p^* for which this probability is maximized is the predicted class of X.

(Also pay attention to the other indices in $1, 2, \ldots, K$ that nearly achieve the max, e.g. the "top 3," as these are also possible outcomes.)

Data Analysis: Use training data $X_1, X_2, \ldots, X_M$ and $Y_1, Y_2, \ldots, Y_M$ to determine suitable values of the coefficient vectors $\beta_1, \beta_2, \ldots, \beta_K$.

Learning the Coefficient Vectors $\beta_1, \beta_2, \ldots, \beta_K$

- Joint probability of observing $(Y_1, Y_2, \ldots, Y_M)$, given $(X_1, X_2, \ldots, X_M)$:

$$\prod_{i=1}^{M} \Pr(Y = Y_i | X_i) = \prod_{i=1}^{M} \frac{e^{\langle \beta_{Y_i}, X_i \rangle}}{\sum_{k=1}^{K} e^{\langle \beta_k, X_i \rangle}}$$

- Log-likelihood function $f(\beta)$, where $\beta := \begin{bmatrix} \beta_1 & \beta_2 & \ldots & \beta_K \end{bmatrix}$:

$$f(\beta) := \sum_{i=1}^{M} \left(\langle \beta_{Y_i}, X_i \rangle - \log \sum_{k=1}^{K} e^{\langle \beta_k, X_i \rangle} \right) \quad \Leftarrow \text{Concave function!}$$

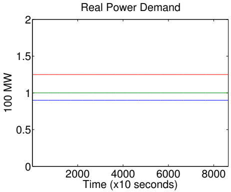
- Maximize log-likelihood function f to estimate $(2n+2) \times K$ coefficient matrix β:

$$\beta^* = \arg\max_{\beta} \quad f(\beta) \quad \Leftarrow \text{Convex Problem!}$$

- Can use standard optimization tools e.g. LBFGS.

Experiment Setup: Synthetic Data

1. Generate 24-hour demand data. (additive Ornstein-Uhlenbeck process) †



IEEE Test Case Demands

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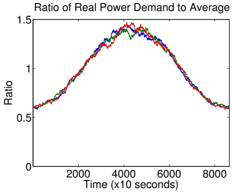

Ratios by Stochastic Process

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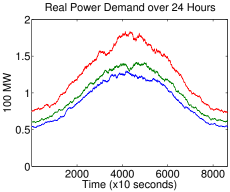


24 Hour Demands

2. For demand scenarios at selected time points, solve the AC model to find the voltage phasors for (a) the full grid, and (b) each of the K single-line outage scenarios. (Omit those outages that cause the grid to become infeasible.)
3. Obtain training data $X_1, X_2, \ldots, X_N$ and $Y_1, Y_2, \ldots, Y_M$ for the MLR classifier from phasor shifts for these outages. Specifically, compare pre-outage phasors at time $t-1$ to post-outage phasors at time t.

† Provided by Prof. Chris DeMarco and Mr. Jong-Min Lim (UW-Madison)

Experiment Setup: Training Sets and Test Sets

- Multiple lines that connects the same two buses are considered as a single line.
- Training Set: 5 equally-spaced samples from the first half of the 24-hour period, for each line outage.

  $\Rightarrow$ Voltage phasors calculated using SCADA measurements.
- Test Set: 50 random samples from the second half of the 24-hour period, for each line outage.

  $\Rightarrow$ Real-time measurements from PMUs.

System	MATPOWER test case	# of Lines		# of Samples	
		Feas.	Infeas./Dup.	Training	Test
14-Bus	case14	18	2	90	900
30-Bus	case_ieee30	37	4	185	1850
57-Bus	case57	67	13	335	3350
118-Bus	case118	170	16	850	8500

Results: PMUs Measurements from All Buses

System	Probability			Ranking		
	≥ 0.9	≥ 0.7	≥ 0.5	1	≤ 2	≤ 3
14-Bus	100%	100%	100%	100%	100%	100%
30-Bus	100%	100%	100%	100%	100%	100%
57-Bus	99.5%	99.7%	99.8%	99.8%	100%	100%
118-Bus	99.8%	99.8%	99.8%	99.8%	99.9%	100%

Percentage of Test Cases

- Probability : Probability of correct answer.
  (e.g. For 99.8% of test cases, the probability of correct answer is ≥ 0.9.)
- Ranking : Ranking of correct answer by probability.
  (e.g. For 99.8% of test cases, the probability of correct answer is the highest.)

Line outages can be identified very well.

Double-Line Outage Identification

Model is extended easily to multiple line outage scenarios.

Double-line outages for 14-Bus system and 30-Bus system.

- 14-Bus system: 18 single-line outages and 143 double-line outages.
- 30-Bus system: 37 single-line outages and 632 double-line outages

The number of columns in the classifier β increases linear in number of scenarios. The number of rows in the classifier β remains the same.

System	Probability			Ranking		
	≥ 0.9	≥ 0.7	≥ 0.5	1	≤ 2	≤ 3
14-Bus	99.5%	99.6%	99.6%	99.6%	99.9%	100%
30-Bus	98.6%	98.8%	98.8%	98.8%	99.7%	99.8%

Double-Line Outage Detection Accuracy on Test Set with PMUs on All Buses.

The number of scenarios has increased hugely, but the performance is only slightly degraded.

Line Outage Or Not?

The MLR model is effective at discriminating between different line outage scenarios, but it should be prefixed by a screening procedure to detect whether a line outage has happened at all, or if the grid is in a "normal" state. ("Outage" vs "No Outage")

In contrast to line outages, normal operation is characterized by small fluctuations in voltage phasor. Easily classified using RBF-Kernel Support Vector Machine (SVM) classifier.

System	# of Outage Scenarios	Train Set	Test Set	Outage Event Detection Accuracy
14-Bus	18	180	720	100%
30-Bus	37	370	1480	99.6%
57-Bus	67	670	2680	99.3%
118-Bus	170	1700	6800	99.4%

Outage Event Detection Using RBF-Kernel Support Vector Machine Classifier ($\gamma = 5 \times 10^4$)

PMU Location

Do we really need phasor measurements from **all** buses in the system?

- It is probably not economical or practical to install PMUs at all buses.
- If PMUs were installed everywhere, we could monitor lines *directly* — no need to use the indirect evidence of phasor changes.

Goal: Finding a subset of buses at which the PMU measurements can give satisfactory identification of line outage.

This is an Optimal PMU Placement Problem with correct line outage identification as criterion for optimality.

(There are other possible criteria: Network observability, state estimation, etc.)

Approach: Use techniques from sparse optimization to decide where to place the PMUs. In particular, use a group sparse heuristic (GroupLASSO), later modified to a more effective greedy group sparse heuristic.

Group Sparse Heuristic

- Each bus is associated with 2 entries of the observation vector X.
- Bus j not observed $\Leftrightarrow$ entries j and $(j+n)$ of X are ignored.
  $\Leftrightarrow$ rows j and $(j+n)$ of $\beta = \begin{bmatrix}\beta_1 & \dots & \beta_K\end{bmatrix}$ are fixed at **0**.

Reminder:

$$X = \begin{bmatrix}\Delta V_1 & \dots & \Delta V_n & \Delta\theta_1 & \dots & \Delta\theta_n & G & 1\end{bmatrix}^T, \quad \Pr(Y=p|X) := \frac{e^{\langle \beta_p, X\rangle}}{\sum_{k=1}^K e^{\langle \beta_k, X\rangle}}$$

Add regularization functions to the max-likelihood objective, to suppress the entries j and $n+j$ (for each $j=1,2,\dots,n$) unless they are "really needed." Solve for $\tau > 0$:

$$\beta^* = \arg\max_{\beta} \quad f(\beta) - \tau \sum_{j=1}^{n} q_j(\beta)$$

where $q_j(\beta) := \left\| \beta_{\{j,j+n\},\cdot} \right\|_F$ for $j=1,2,\dots,n$ (Frobenius norm of a $2\times K$ submatrix).

Define the best r placement locations for PMUs, denoted by $\mathcal{R}^r \subset \{1,2,\dots,n\}$:

$$\mathcal{R}^r := \arg\max_{\mathcal{R}:|\mathcal{R}|=r, \mathcal{R}\subseteq\{1,\dots,n\}} \quad \sum_{j\in\mathcal{R}} q_j(\beta^*).$$

(Could also manipulate τ until exactly r values of $q_j(\beta^*)$ are nonzero.)

Greedy Group Sparse Heuristic

- GroupLASSO is solved using `SpaRSA`[‡].
- In the greedy variation, only one bus is selected at each "outer" iteration.
- Then solve a modified version of GroupLASSO to select the next bus — removing regularization terms (and hence bias in β) for the buses already selected.

Let $\mathcal{S} = \{1, 2, \ldots, n\}$, the set of all buses.

Group Sparse

1: $\beta^* = \arg\max_{\beta} \quad f(\beta) - \tau \sum_{i \in \mathcal{S}} q_i(\beta)$

2: $\mathcal{R}_r = \arg\max_{\mathcal{R}:|\mathcal{R}|=r, \mathcal{R} \subseteq \mathcal{S}} \quad \sum_{j \in \mathcal{R}} q_j(\beta^*)$

Greedy Variant

1: $\mathcal{R}^0 = \emptyset$

2: **for** $l = 1, 2, \ldots, r$ **do**

3: $\quad \beta^* = \arg\max_{\beta} \; f(\beta) - \tau \sum_{j \notin \mathcal{S} \setminus \mathcal{R}^{l-1}} q_j(\beta)$

4: $\quad s^l \leftarrow \arg\max_{j \notin \mathcal{R}^{l-1}} \quad q_j(\beta^*)$

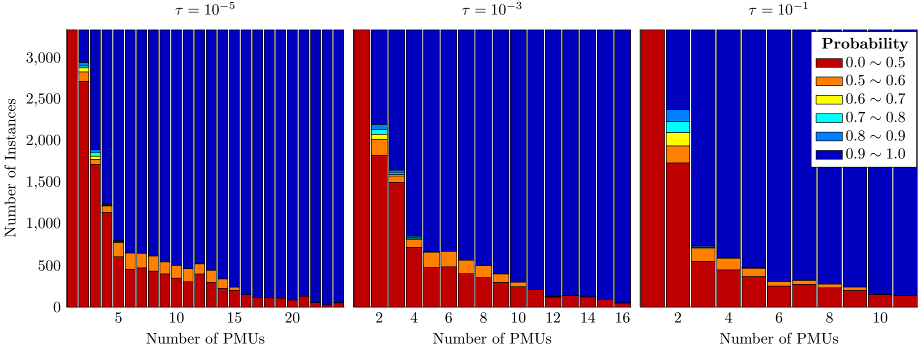
5: $\quad \mathcal{R}^l \leftarrow \mathcal{R}^{l-1} \cup \{s^l\}$

6: **end for**

Greedy Advantage: Selection of redundant PMU locations is suppressed by already-selected, non-penalized locations.

[‡] Wright et al. (2009)

Results: Greedy Heuristic: 57-Bus System

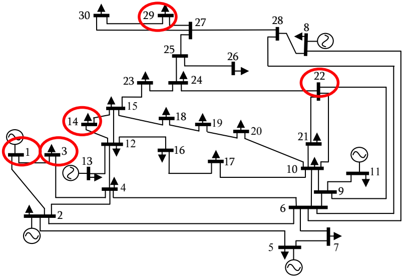


Highly reliable detection obtained with just 15 PMUs (with $\tau = 10^{-3}$).

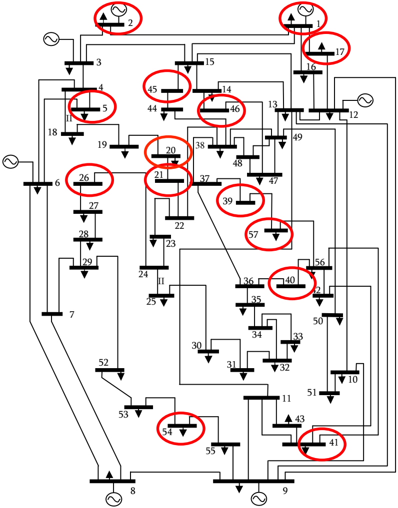
For our test networks, classification performance with 25% PMUs is as good as fully instrumented grid.

The greedy heuristic avoids selecting PMU locations that give redundant information.

PMU Locations on 30-Bus and 57-Bus Systems



IEEE 30 Bus System (5 PMUs)



IEEE 57 Bus System (14 PMUs)

Conclusions

Vulnerability Analysis of Power Systems

- Devise a framework for vulnerability analysis based on impedance attacks for two different measures.
- Can solve problems of tractable scale using conditional gradient in combination with heuristics.

Line Outage Identification using PMU data

- Real time line outage identification can be done via MLR.
- PMU locations can be decided using the greedy group sparse heuristic.
- PMUs on $< 25\%$ of buses are enough for the line outage identification.
- Direct obsevation information can be incorporated to improve the identification performance.

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