Endogenous Matching: Adverse Selection & Moral Hazard On-Demand

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Economic Motivation

Technology (computers, internet, smartphones . . .) has made revolution in provision/delivery of

- goods to consumers: Amazon, etc.
- services to consumers: Uber, TaskRabbit, etc.

Also making revolution in provision/delivery of *services to firms*

- surveys: Amazon Mechanical Turk
- software: TopCoder
- consulting: Capgemini, Eden McCallum
- outsourcing

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On-demand economy

Issues

"On-demand companies . . . have difficulties training, managing and motivating workers." (*Economist* Jan 3, 2015)

Two-sided Matching with Transferable Utility

$w \in W \leftrightarrow t \in T$

- buyers $\leftrightarrow$ sellers
- workers $\leftrightarrow$ firms/jobs
- men $\leftrightarrow$ women

Output/Value $Y(w, t)$

Objective: match W with T to maximize total output

$$\sum_{\text{match}} Y(w, t)$$

Linear Program: Primal

Let

- $\omega =$ counting measure on W
- $\tau =$ counting measure on T

Choose (positive) measure θ on $W \times T$ to maximize

$$\int Y(w,t)d\theta$$

subject to

$$\begin{aligned} \theta_W &\leq \omega \\ \theta_T &\leq \tau \end{aligned}$$

Linear Program: Dual

Choose functions $\varphi : W \to [0, \infty)$, $\psi : T \to [0, \infty)$ to minimize

$$\int \varphi(w) d\omega + \int \psi(t) d\tau$$

subject to

$$\varphi(w) + \psi(t) \geq Y(w, t)$$

Interpretation: Housing Market

- θ matching of buyers and sellers
- $\psi(t)$ is the price paid for house of seller t
- $\varphi(w)$ is the utility surplus obtained by buyer w

Interpretation: Job Market

- θ matching of workers and jobs
- $\varphi(w)$ is the wage paid to worker w
- $\psi(t)$ is the utility surplus remaining to owner of job t

Competitive Formulation

Competitive environment

- Many buyers, many sellers
- Many workers, many jobs/firms

Competition drives matching and prices/wages

W, T compact metric spaces; ω, τ non-atomic measures
$Y : W \times T \to [0, \infty)$ is continuous

- Many near-perfect substitutes for each buyer (worker)
- Many near-perfect substitutes for each seller (job/firm)

Example

$W = T = [0, 1]$

$\omega = \tau =$ Lebesgue measure

$Y(w, t) = wt$ *supermodular*

Example: Optimal Matching

Supermodularity $\Rightarrow$ Matching is *assortative*
– better buyers are matched to better sellers

$$w_1 \leftrightarrow t_1, w_2 \leftrightarrow t_2, w_1 < w_2 \Rightarrow t_1 \leq t_2$$

Optimality $\Rightarrow$

$$\begin{aligned} w_1 t_1 + w_2 t_2 &\geq w_1 t_2 + w_2 t_1 \\ w_1 t_1 - w_1 t_2 &\geq w_2 t_1 - w_2 t_2 \\ w_1(t_1 - t_2) &\geq w_2(t_1 - t_2) \\ t_1 &\leq t_2 \end{aligned}$$

$\Rightarrow$ unique optimal matching
$\theta =$ normalized Lebesgue measure on diagonal

Example: Prices – Solving for $\psi(t)$

- Suppose buyer w buys house from seller t
  - pays price $\psi(t)$
  - obtains net utility $f(t) = wt - \psi(t)$
- Optimal matching = diagonal
  $\rightarrow$ w buys house from seller w
  $\rightarrow$ $f(t)$ maximized when $t = w$
- First Order Condition

$$\begin{aligned} f'(t)|_{t=w} &= 0 \\ \psi'(t) &= t \\ \psi(t) &= (1/2)t^2 + \text{const} \\ \text{initial condition } \psi(0) = 0 &\rightarrow \text{const} = 0 \end{aligned}$$

More generally: $W = [w_0, 1]$, $T = [t_0, 1]$

- $w_0 < t_0$: more buyers than houses
  buyers in $[w_0, t_0)$ do not buy house, get 0 utility
  determines initial condition
  prices determinate
- $w_0 > t_0$: more houses than buyers
  houses in $[t_0, w_0)$ do not sell, priced at 0
  determines initial condition
  prices determinate
- $w_0 = t_0$
  price of house t_0 indeterminate
  price of house t_0 determines initial condition
  competition + price of house t_0 determines all prices

What is left out in this model (and other matching models)?

- Housing market: characteristics may not be observable
  $\rightarrow$ adverse selection

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- Job market/marriage market: output must be produced
  Production requires effort
  Effort is unobservable & costly
  $\rightarrow$ moral hazard

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- Job market/marriage market: output must be produced
  Production requires effort
  Effort is unobservable & costly
  $\rightarrow$ moral hazard
- Job market: Interaction is repeated/ongoing

Setting I (future work)

Platform (Uber, TaskRabbit, etc.)

- one platform
- many clients arrive each period, each with a task and a payment schedule
- clients matched to workers
- workers choose effort, perform task, get paid
- platorm seeks to maximize commission
- ongoing

Setting II (this paper)

Firm outsourcing

- one firm
- firm has many tasks to be performed each period
- outsourced to many workers
- firm sets payment schedule matches workers to tasks
- workers choose effort, perform task
- firm seeks to maximize profit = total output - payments
- ongoing

Who knows/observes what?

Firm knows/observes

- Task types
- History of output

Firm does not know/observe

- Characteristics of workers
- Effort

Firm cannot disentangle
"what happened?" from "why did it happen?"

Who knows/observes what? (cont)

Workers know/observe

- Task types
- Own characteristics
- Own history
- Output distribution

Workers do not know/observe

- Other workers' characteristics

Essence of the Problem

Adverse selection

- Firm does not observe worker characteristics

Moral hazard

- Firm does not observe worker effort

Repeated interaction

Endogenous matching

Match better producers to better tasks

- assortative matching $\rightarrow$ optimal matching
  eliminate adverse selection
- mitigate moral hazard
- profit comparisons
  - random matching, fixed payment schedule
  - our solution
  - incentive optimum with assortative matching (2nd best)

Model

Workers

- $W = [0, 1]$
- uniformly distributed (without essential loss of generality) workers identified by location in distribution
- Cost $C(e, w) : [0, \infty) \times W \to [0, \infty)$ smooth

$$
\begin{aligned}
C(0, w) &= 0 \\
C_2 &\leq 0 \\
C_1 &> 0 \quad \text{if } e > 0, w > 0 \\
C_1 &= 0 \quad \text{if } e = 0 \\
C_{11} &> 0 \\
C_{12} &\leq 0
\end{aligned}
$$

Model (cont)

Tasks

- $T = [0, 1]$
- uniformly distributed (without essential loss of generality) tasks identified by location in distribution

Model (cont)

Output Y smooth

$$Y(e, w, t) : [0, \infty) \times W \times T \to [0, \infty)$$

- $ewt = 0 \Rightarrow Y(e, w, t) = 0$
- $ewt \neq 0 \Rightarrow Y_1, Y_2, Y_3 > 0$
- $Y(e, w, t) \to \infty$ as $e \to \infty$ if $(w, t) > (0, 0)$
- Y is strictly supermodular in each pair of variables
- $Y_{11} \leq 0$
- messy technical conditions (multiplicatively separable in task enough)

Model (cont)

Payment schedule $P : [0, \infty) \to [0, \infty)$ smooth

$$\begin{aligned} P(y) &\leq y \\ P'(y) &> 0 \\ P''(y) &\leq 0 \end{aligned}$$

Utility of worker w matched to task t, exerts effort e:

$$U(e, w, t) = P[Y(e, w, t)] - C(e, w)$$

Canonical Example

- Output

$$Y(e, w, t) = ewt$$

- Payment rule

$$P(y) = \lambda y$$

- Cost

$$C(e) = e^2$$

- Worker utility

$$U(e, w, t) = \lambda ewt - e^2$$

Firm Objective

Maximize

profit = total output - total payment to workers

Stationary Assortative Equilibrium

Equilibrium notion:

- Stationary (steady-state)
- Assortative: match better workers to better tasks
- Matching rule: rank in output distribution
- Worker strategy: effort g, output G

$$g : W \times T \times \text{history} \to [0, \infty)$$

$$G : W \times T \times \text{history} \to [0, \infty)$$

- Workers discount future utility at constant rate $\delta \in (0, 1]$
- Workers optimize (given that others play equilibrium)
- Stationarity $\to$ G depends only on w *on equilibrium path*

$$G : W \to [0, \infty)$$

Main Results - 1

Theorem 1

- Stationary Assortative Equilibrium exists and is unique.
- The output distribution G is continuously differentiable.
- Better workers are matched with better tasks, produce higher output, receive higher utility (but may not exert greater effort).

Notes:

- No adverse selection at equilibrium.
- Wages $= P(G(w))$ endogenous

Main Results - 2

Theorem 2 Payment schedule linear $\Rightarrow$
Rankings of firm profit (output net of payments)

Profit(random matching, workers exert optimal effort given fixed payment schedule)

$<$ Profit(stationary assortative equilibrium, fixed payment schedule)

$<$ Profit(assortative matching, firm uses incentive-optimal payment schedule)

Firm has *two* instruments to provide incentives

- pays for current output
- conditions future assignments on current output

Canonical example

$$\begin{aligned} Y(e,w,t) &= ewt \\ C(e) &= e^2 \\ P(y) &= \lambda y \\ U(Y(e,w,t)) &= \lambda ewt - e^2 \end{aligned}$$

Solve for G ?

Stationary deviation: worker w pretends to be worker $\hat{w}$

- today: w matched to task w, produces output $G(\hat{w})$
- future: w matched to task $\hat{w}$, produces output $G(\hat{w})$

$$
\begin{aligned}
U(\hat{w}|w) &= \lambda G(\hat{w}) - [G(\hat{w})/w^2]^2 \\
&\quad + \delta/(1-\delta)\left\{\lambda G(\hat{w}) - [G(\hat{w})/w\hat{w}]^2]\right\}
\end{aligned}
$$

Equilibrium $\rightarrow$ optimum occurs when $\hat{w} = w$
FOC for equilibrium: $dU/d\hat{w} = 0$ when $\hat{w} = w$

$$
G' = \frac{2\delta G^2}{[2G - \lambda w^4]w} = \Phi(w, G)
$$

Initial condition: $G(0) = 0$

This is an ODE that we have never seen before but . . .

- it is just a first order ODE
- how bad can it be?

Guess $G(w) = Aw^4$

Plug in and equate

$$4Aw^3 = \frac{2\delta A^2 w^8}{w[2Aw^4 - \lambda w^4]}$$

Eliminate w

$$4A = \frac{2\delta A^2}{[2A - \lambda]}$$

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Two (!) solutions

$$A = \frac{2\lambda}{4 - \delta} \; ; \; A = 0$$

Both solutions satisfy the initial condition $G(0) = 0$

OOPS

Why not inconsistent with existence/uniqueness for ODE's?

$$G' = \frac{2\delta G^2}{[2G - \lambda w^4]w} = \Phi(w, G)$$

Φ is not well-behaved: denominator can be 0
For ODE's of this type:

- uniqueness does not obtain
- existence is in doubt

Workaround

- Solutions must be *constructed*
- “Correct” solution must be identified
  - Worker utility > 0

Constructing a Solution

Key Observations

- Denominator $2G - \lambda w^4$ ... and

$$2G - \lambda w^4 = 0$$

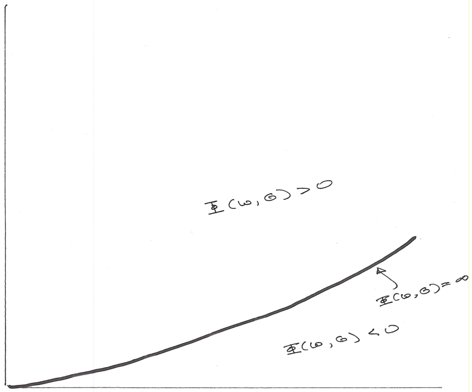
  is the FOC for optimality when workers are myopic $\delta = 0$
- One period utility is

$$(\lambda w^4 - G)(G/w^4)$$

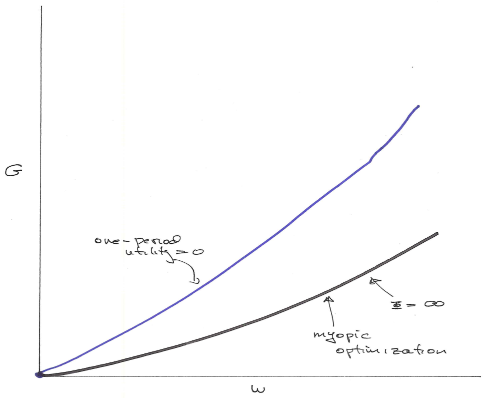
- worker utility increasing $\Rightarrow$ worker utility > 0

$$
\begin{aligned}
U(w|w) &= \lambda G(w) - [G(w)/w^2]^2 \\
\frac{dU(w|w)}{dw} &= \lambda G' - 2GG'/w^4 + 5G^2/w^5 \\
&= \lambda G' \frac{\lambda w^4 - 2G}{w^4} + 5G^2/w^5 \\
&= \left(-\frac{2\delta G^2}{[2G - \lambda w^4]w}\right)\left(\frac{\lambda w^4 - 2G}{w^4}\right) + 5G^2/w^5 \\
&= \frac{-2\delta G^2}{w^5} + 5G^2/w^5 \\
&> 0
\end{aligned}
$$

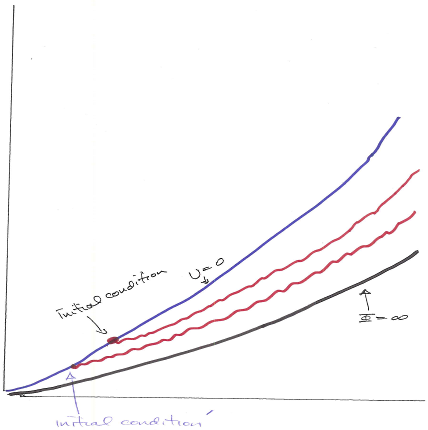
Proof picture - lower bound for solution



Proof picture - upper bound for solution



Proof picture - solving ODE



Steps in proof of Theorem 1

- **Step 1** Equilibrium $\rightarrow$ no deviations
  $\rightarrow$ no *stationary deviations*
- **Step 2** No stationary deviations $\rightarrow$ FOC $\rightarrow$ *ODE*

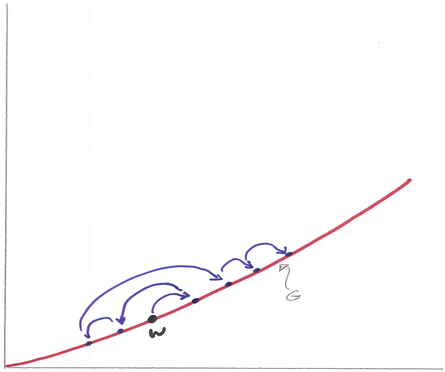
$$G' = \Phi(w, G)$$

- **Step 3** ODE has unique *"correct" solution* G_*

From "correct" solution G_* to SAE

- **Step 4** If there is a profitable deviation from G_* then there is a profitable *finite* deviation

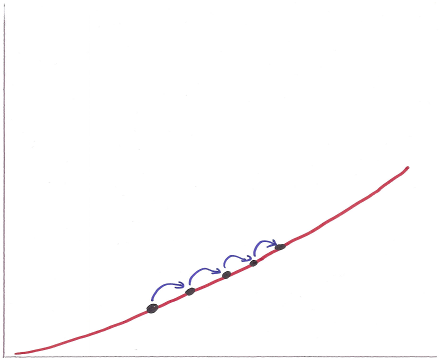
Proof picture - Finite deviation



From “correct” solution G_* to SAE

- **Step 5** If there is a profitable finite deviation from G_* then there is a profitable finite *monotone* deviation

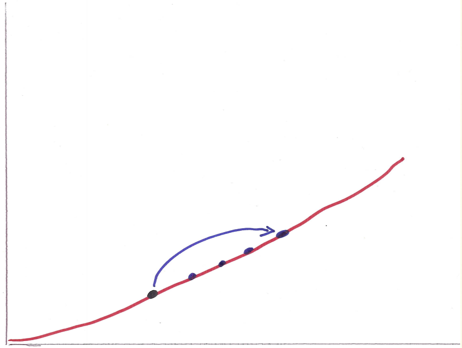
Proof picture - Finite Monotone deviation



From "correct" solution G_* to SAE

- **Step 6** If there is a profitable finite monotone deviation from G_* then there is a profitable *stationary* deviation

Proof picture - Stationary deviation



From "correct" solution G_* to SAE

- **Step 7** If there is a profitable stationary deviation from G_* then there is a profitable *infinitesimal* stationary deviation

$\rightarrow$ this contradicts ODE

Comparisons in the Canonical Example

Random matching, worker optimization

Worker w matched with task t exerts effort to maximize

$$\lambda ewt - e^2$$

Effort $e = \lambda wt/2$, output $= \lambda w^2 t^2/2$
Total net of payments

$$\int_0^1 \int_0^1 (1-\lambda)[\lambda w^2 t^2/2] dt dw = [(1-\lambda)\lambda][1/18]$$

Comparisons in the Canonical Example (cont)

Assortative matching, myopic worker optimization

Worker w matched with task w exerts effort to maximize

$$\lambda eww - e^2$$

Effort $e = \lambda w^2/2$, output $= \lambda w^4/2$
Total net of payments

$$\int_0^1 (1-\lambda)[\lambda w^4/2]dw = [(1-\lambda)\lambda][1/10]$$

Comparisons in the Canonical Example (cont)

Steady-State Assortative Equilibrium

Worker w matched with task w produces

$$G(w) = [2\lambda/(4-\delta)]w^4$$

Total net of payments

$$\int_0^1 (1-\lambda)\big(2\lambda/(4-\delta)]w^4\big)\,dw = [(1-\lambda)\lambda][2/5(4-\delta)]$$

Comparisons in the Canonical Example

2nd best: assortative + incentive-optimal payment

Worker w matched with task w is paid p to maximize

$$ew^2 - p \quad \text{subject to} \quad p \geq e^2$$

Effort $e = w^2/2$, output $= w^4/2$, net $= w^4/4$
Total net of payments

$$\int_0^1 [w^4/4]dw = 1/20$$

Comparisons in the Canonical Example (cont)

$$[(1-\lambda)\lambda][1/18] < [(1-\lambda)\lambda][1/10] < [(1-\lambda)\lambda][2/5(4-\delta)] < 1/20$$

Random < Myopic < Stationary Assortative EQ < 2nd best

Comparisons in the Canonical Example (cont)

First three cases: optimal $\lambda = 1/2$

$$1/72 < 1/40 < 1/[10(4 - \delta)] < 1/20$$

Random < Myopic < Stationary Assortative EQ < 2nd best

Wages

Payment schedule is chosen exogenously.
Wages are determined endogenously.
Wages are highly convex in worker type

- Better workers matched to better tasks
- To maintain high ranking, better workers must produce more output

What is the optimal payment schedule ?

- What does "optimal" mean? What does firm know?
- Linear payment schedules? (Carroll)

Setting I: many clients, each with task

- Payment schedules are task-specific
- Who sets payment schedules?
- Payment schedules determined as part of equilibrium
- Competition determines payment schedules?
  (Gretsky, Ostroy & Zame)

Many competing platforms

- Uber, Lyft, Carmel . . .
- Issues
  - exclusive dealing by clients?
  - exclusive dealing by workers?
  - who knows reputation of workers?
  - who knows reputation of platforms?

Unequal sides of the market

- random arrivals
- waiting, queues