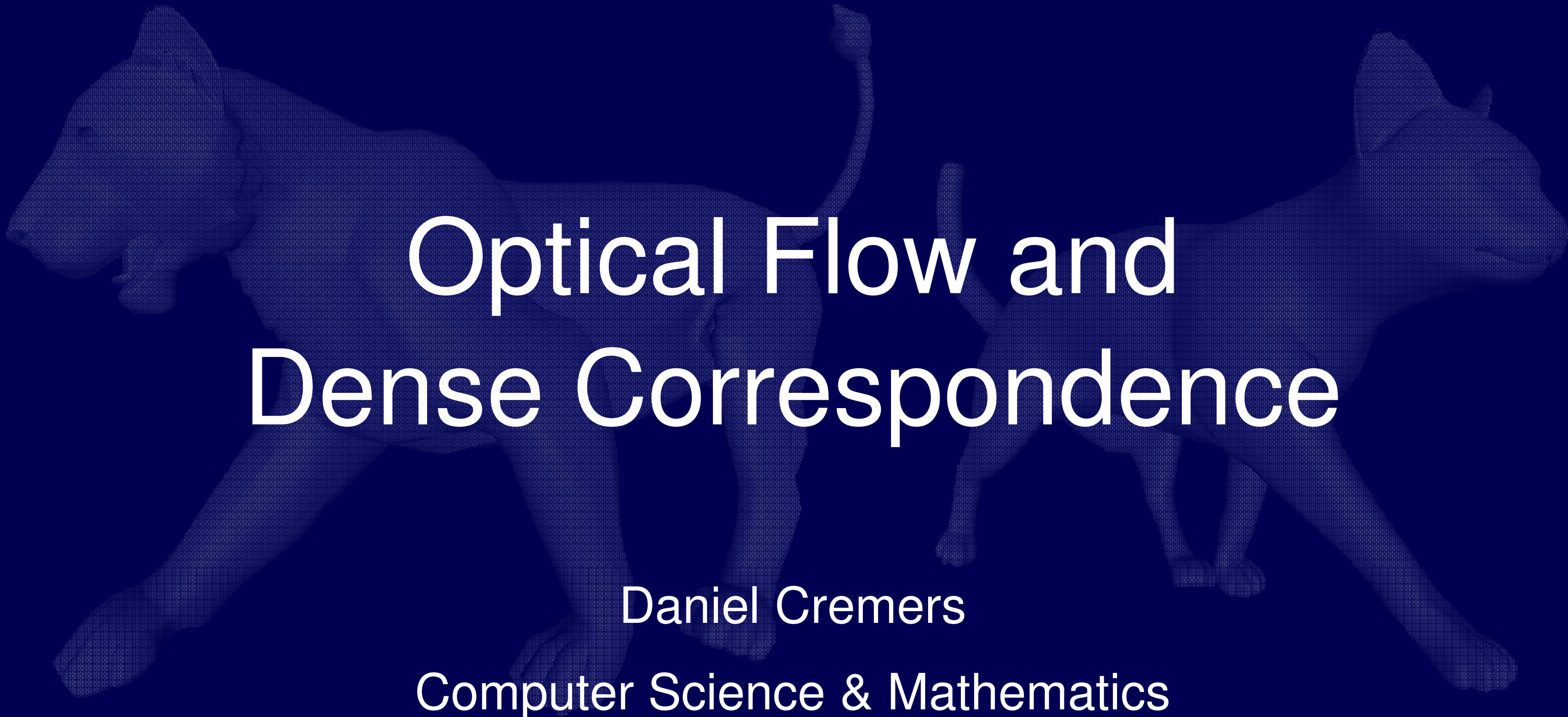
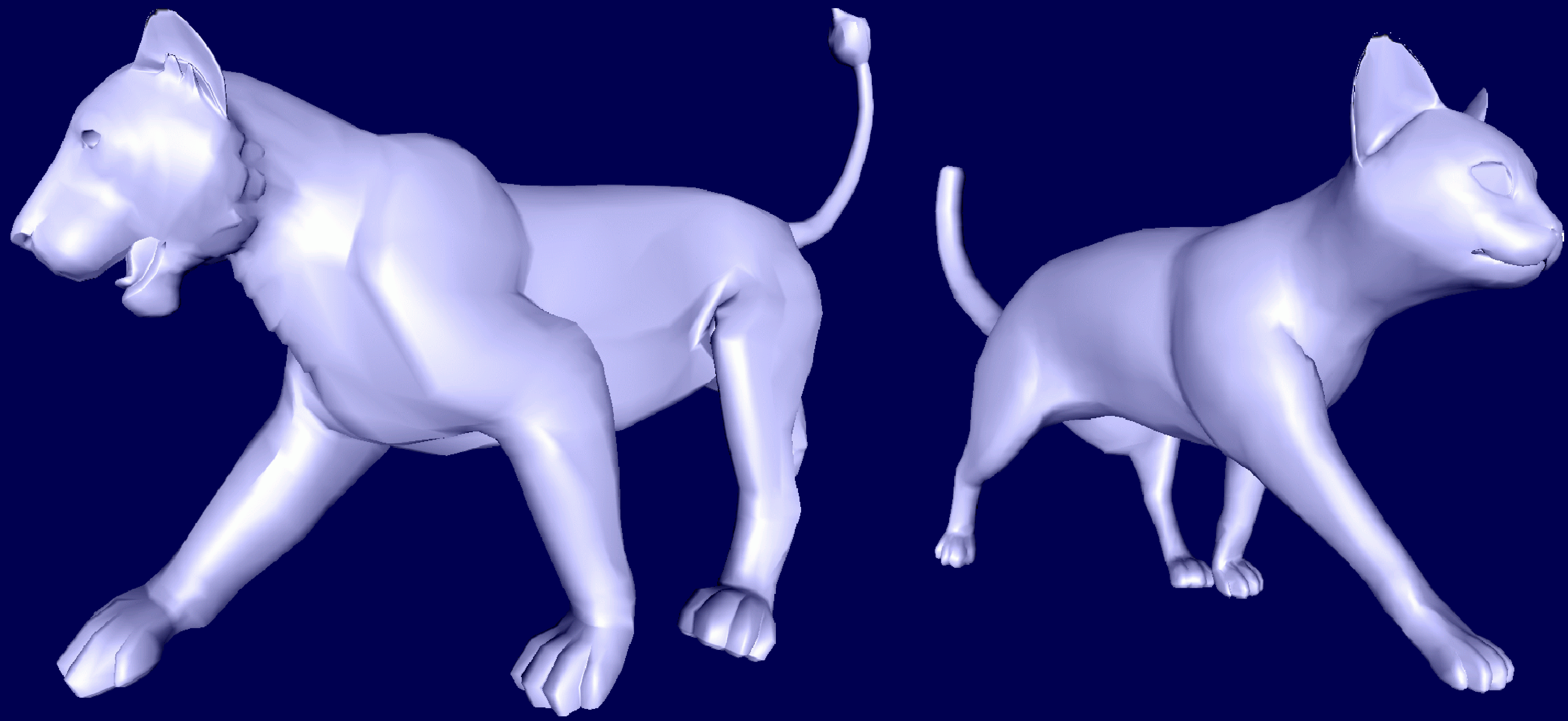
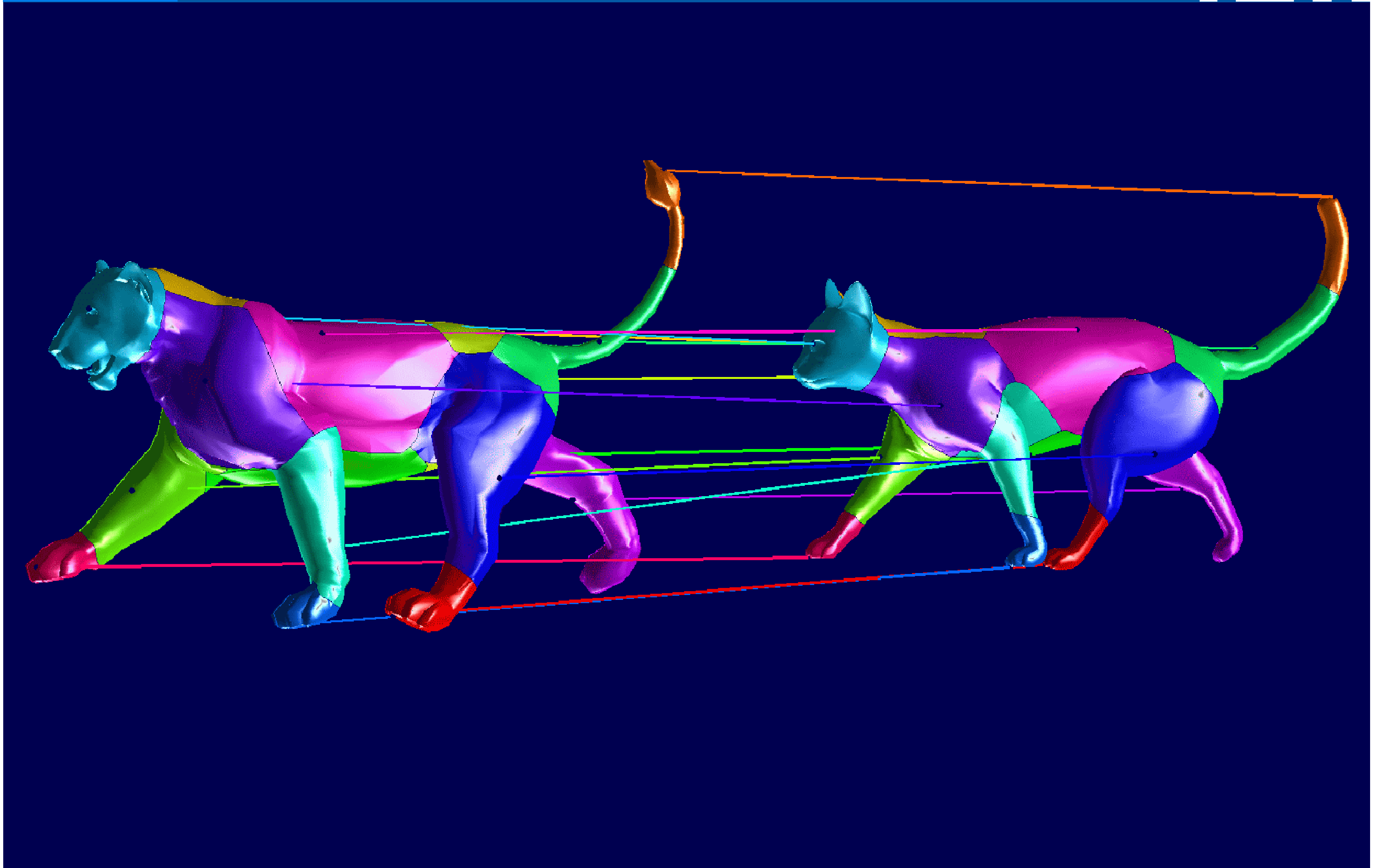


Optical Flow and Dense Correspondence

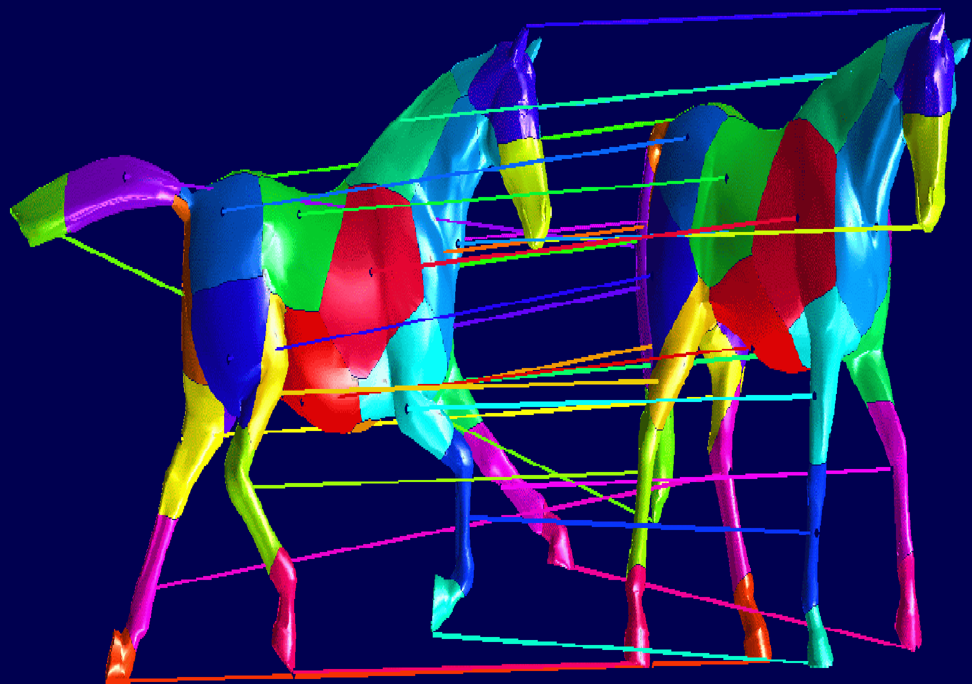
Daniel Cremers
Computer Science & Mathematics
TU Munich

Correspondence Problems in Vision



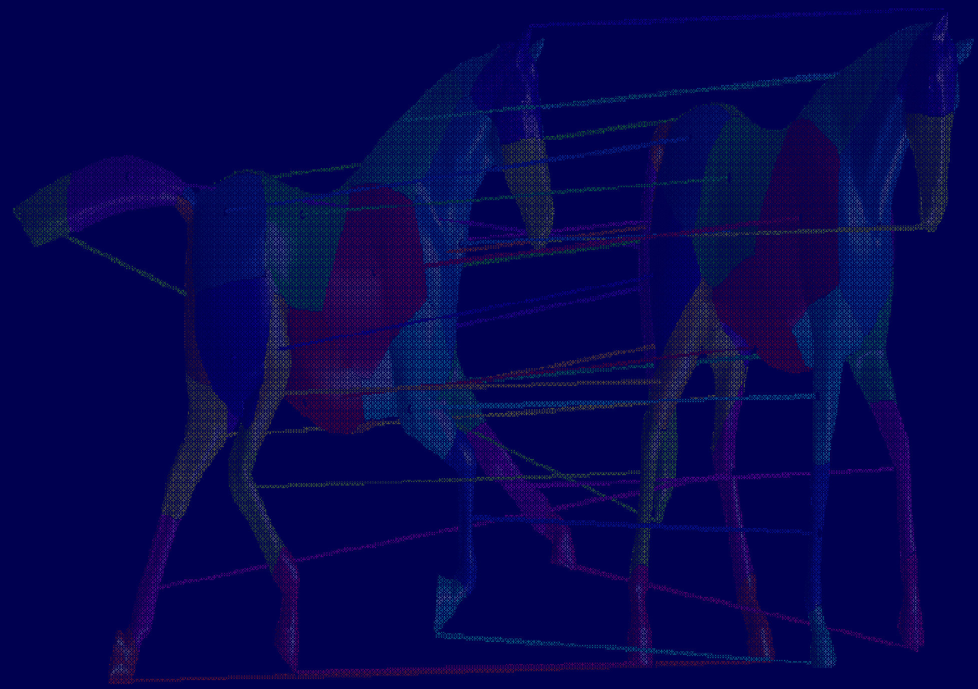


Optical Flow Estimation



Dense Elastic Shape Matching

Optical Flow Estimation



Dense Elastic Shape Matching

$$\min_{u:\Omega\rightarrow\mathbb{R}^2} \int_\Omega \left| f_1(x) - f_2(x+u) \right| dx + J(u)$$

Input video

Optical flow field

Variational Optical Flow

$$\min_{u:\Omega\to\mathbb{R}^2} \int_\Omega \left| f_1(x) - f_2(x+u) \right| dx + J(u)$$

Variational optical flow estimation

Horn, Schunck ‘81

Non-quadratic regularization

Nagel, Enkelmann ‘86, Herve, Shulman ‘89

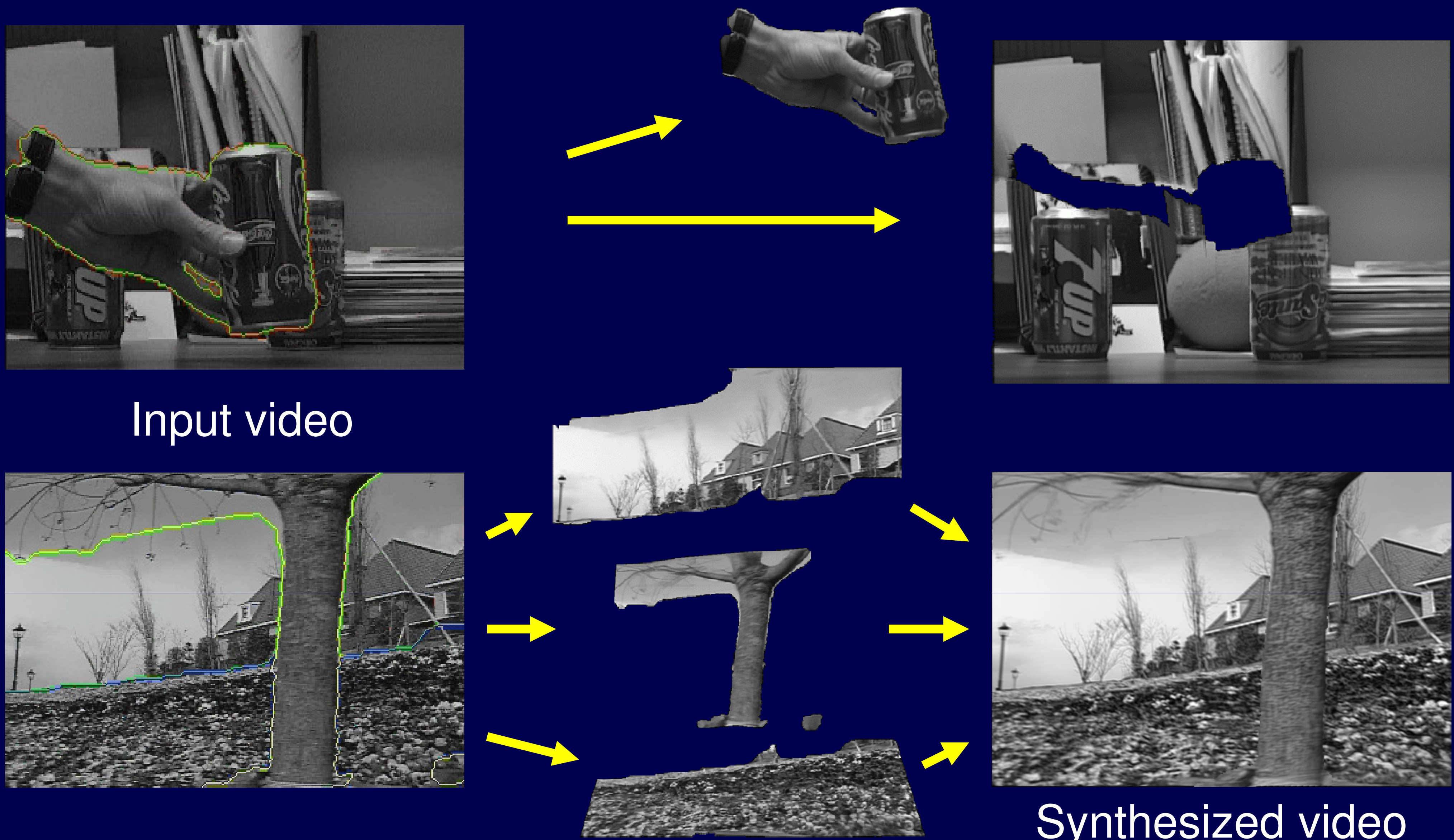
Black, Anandan ’93

Coarse to fine warping

Memin, Perez ’98, Brox et al. ‘04 , Zach et al. ’07

Adaptive regularization

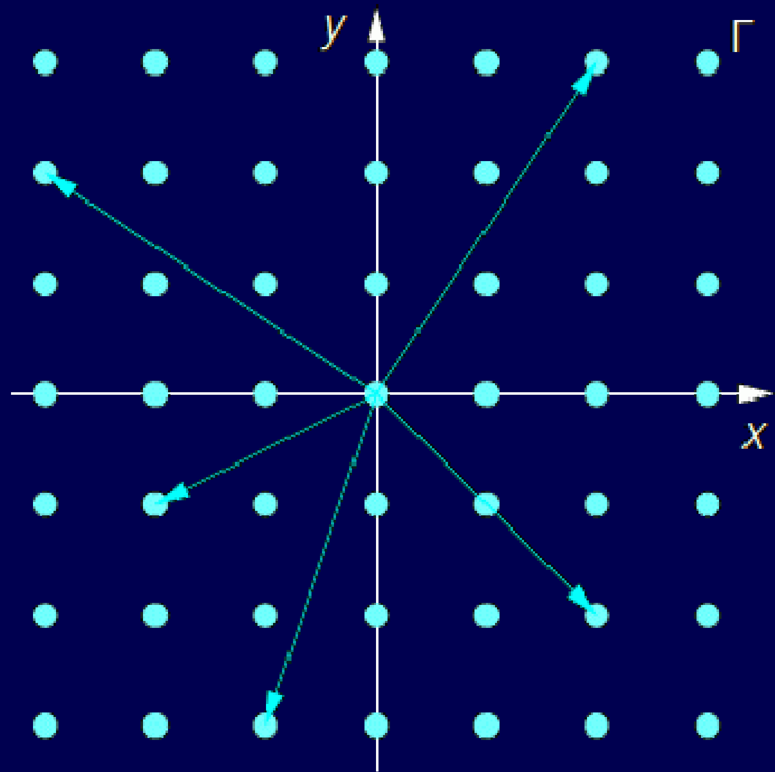
Wedel et al. ’09, Werlberger et al. ‘10, …



Schoenemann & Cremers TIP '12

Optical Flow Estimation

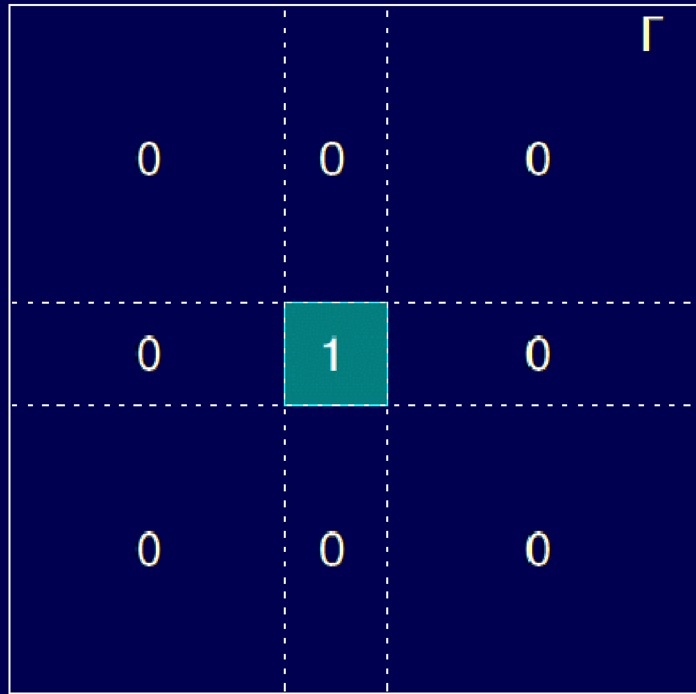
Solution by linearization & coarse-to-fine warping does not provide optimality guarantees.



Optical flow: $$\min_{u:\Omega\to\Gamma\subset\mathbb{R}^d}\int_\Omega \left|f_1(x)-f_2(x+u)\right|dx + J(u)$$

Challenge: Thousands of labels cannot be handled in previous relaxations.

Goldluecke, Cremers ECCV ’10, Strekalovskiy et al. ICCV ‘11



Optical flow: $$\min_{u:\Omega\to\Gamma\subset\mathbb{R}^d} \int_\Omega \left| f_1(x) - f_2(x+u) \right| dx + J(u)$$

Challenge: Thousands of labels cannot be handled in previous relaxations.

Goldluecke, Cremers ECCV '10, Strekalovskiy et al. ICCV '11

Convex Optical Flow

$$\min_{u:\Omega\to\Gamma} E_{data}(u) + E_{reg}(u) = \min_{u:\Omega\to\Gamma} \int_\Omega \rho(x,u)\,dx + \sum_{i=1}^{d} J(u_i)$$

Introduce: $v_i(x,\gamma_i) := \delta(u_i(x) - \gamma_i) \quad \forall\, i \in \{1,..,d\},\ \gamma_i \in \Lambda_i$

$$\min_{v_1,..,v_d} \int_{\Omega\times\Gamma} \rho(x,\gamma) \boxed{\prod_{i=1}^{d} v_i(x,\gamma_i)}\, dx\, d\gamma = \min_{v_1,..,v_d} \sup_{q\in Q} \left\{ \sum_{i=1}^{d} \int_{\Omega\times\Lambda_i} q_i\, v_i\, dx\, d\gamma_i \right\}$$

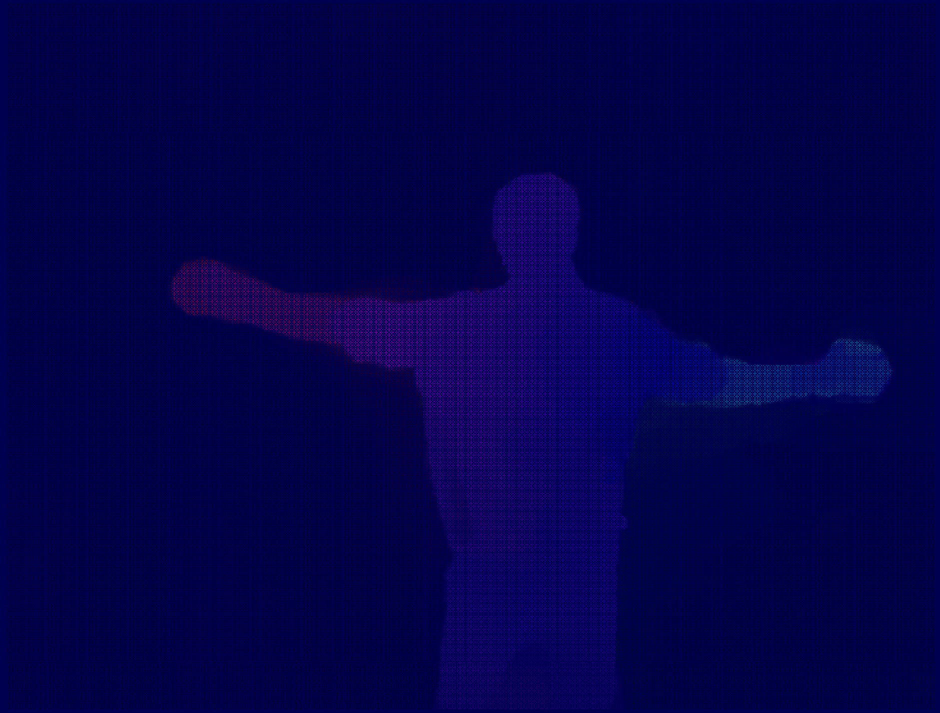
with: $Q = \left\{ (q_i : \Omega\times\Lambda_i \to \mathbb{R})_{i=1..d} \;\middle|\; \sum_{i=1}^{d} q_i(x,\gamma_i) \le \rho(x,\gamma)\ \forall x,\gamma \right\}$

Goldluecke, Cremers ECCV '10, Strekalovskiy et al. ICCV '11

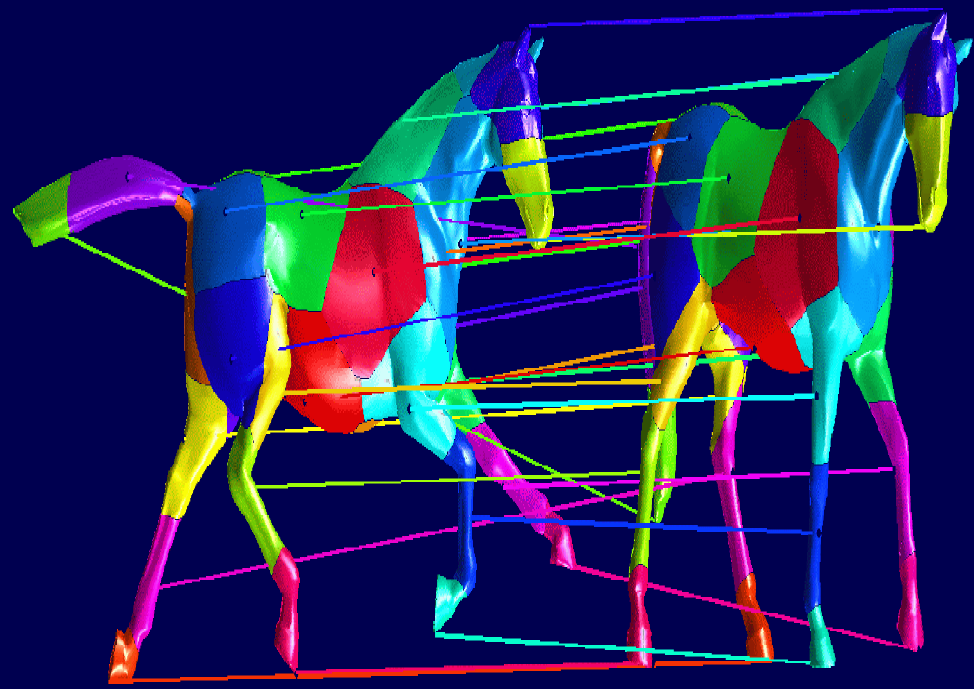
Convex Optical Flow

Experimental optimality bounds ~ 3% - 5%

Goldluecke, Cremers ECCV '10, Strekalovskiy et al. ICCV '11



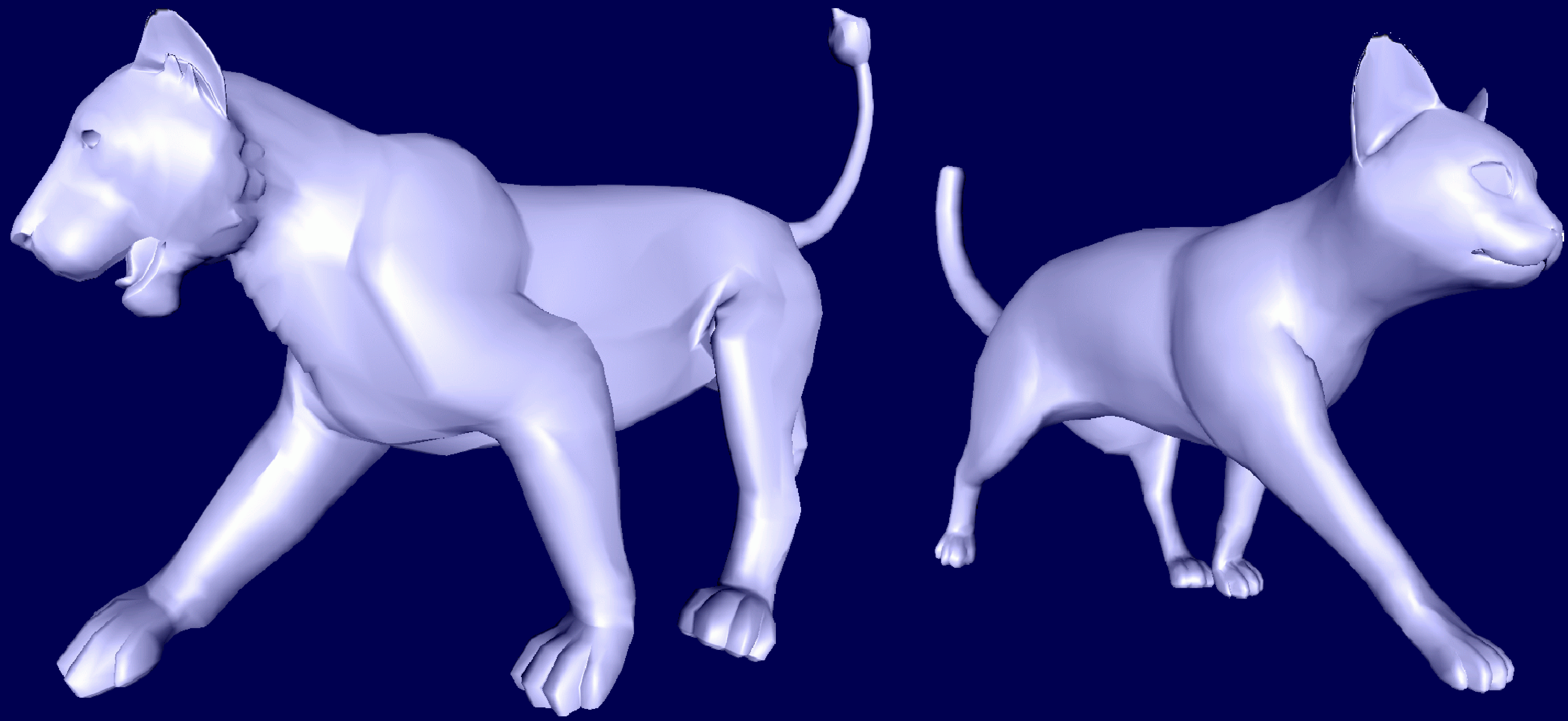
Optical Flow Estimation



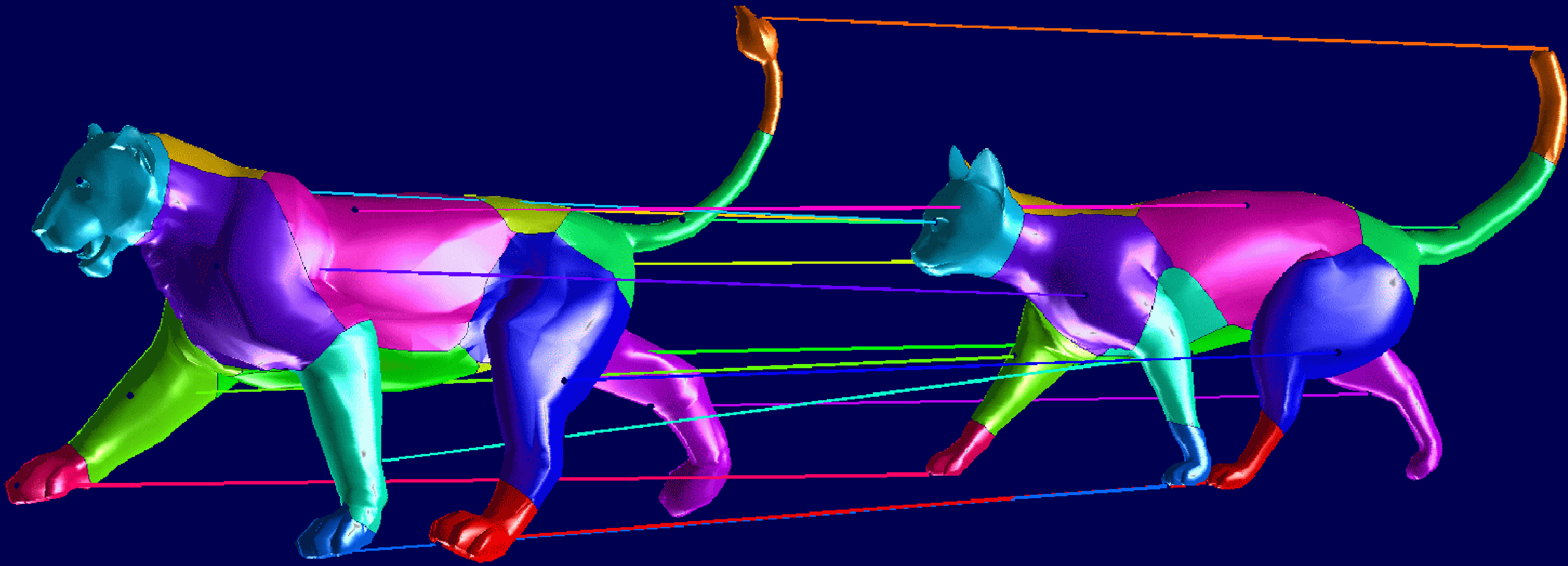
Dense Elastic Shape Matching

Windheuser et al., ICCV '11

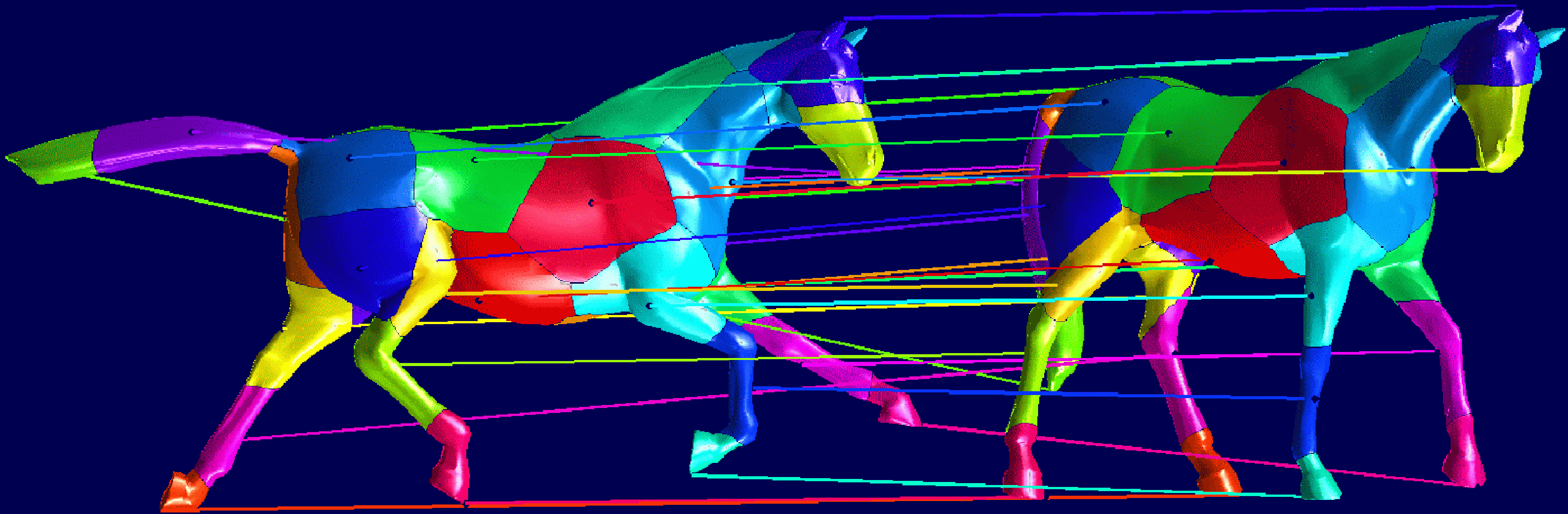
Shape Similarity and Shape Matching



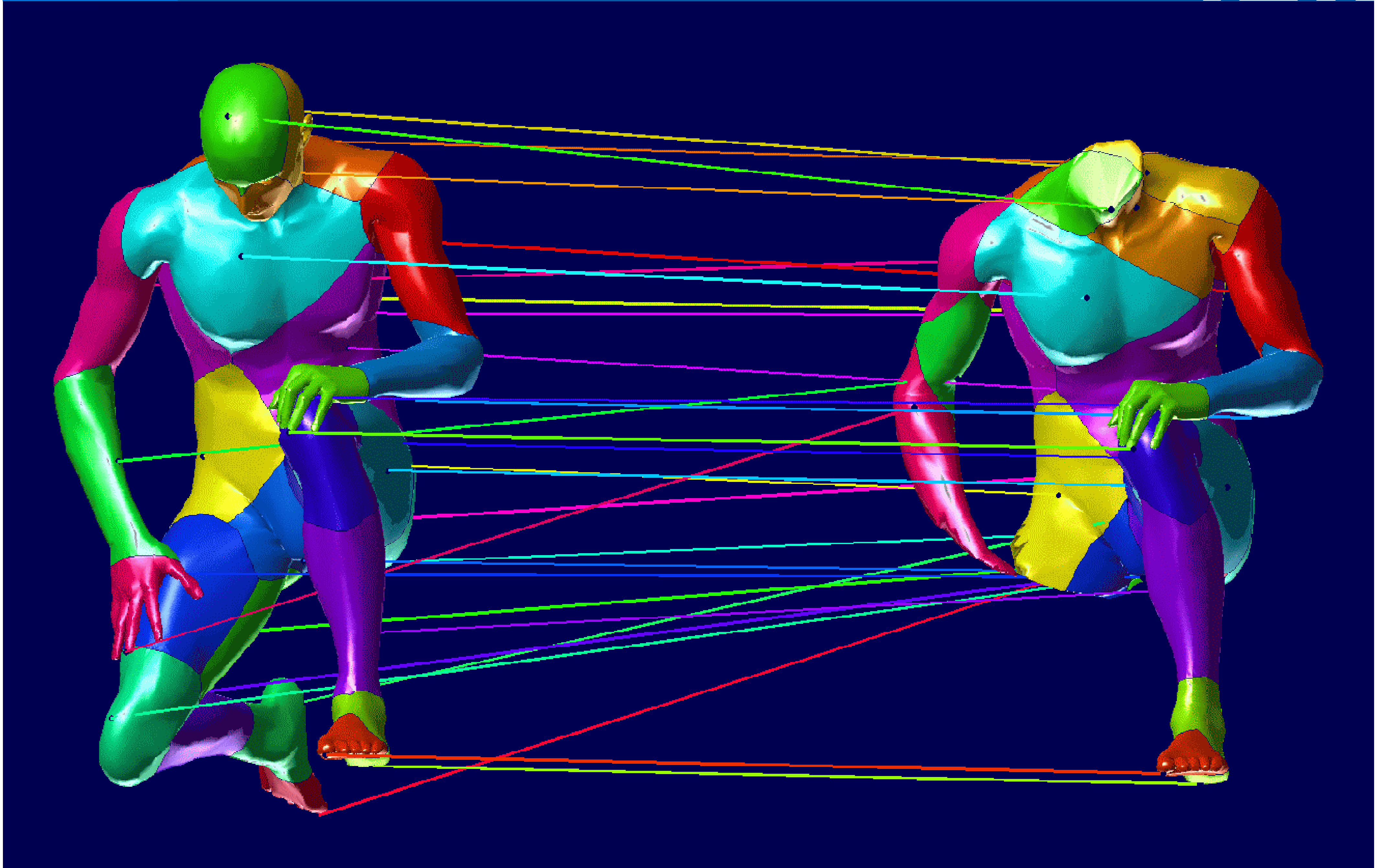
Dense Correspondence

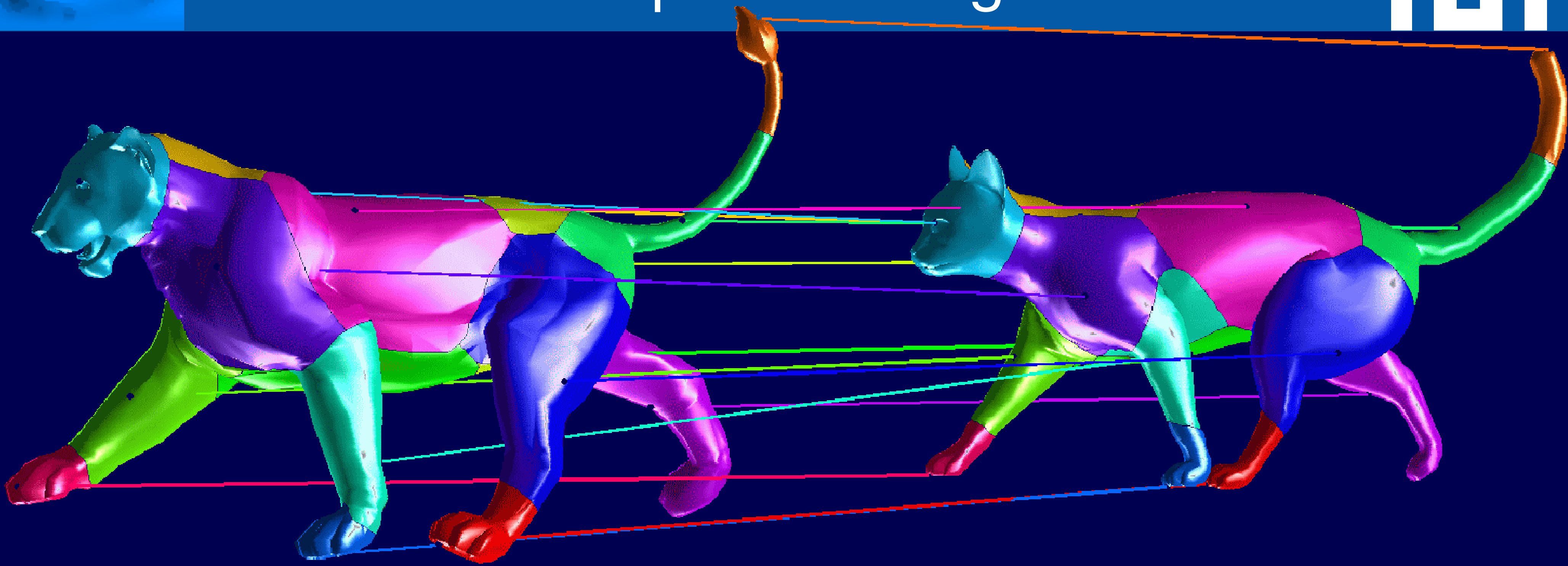


Robustness to Articulation / Deformation



Robustness to Missing Parts





- Favor meaningful correspondences
- Allow for stretching, shrinking and bending
- Assure geometric consistency
- Optimal or near-optimal solutions

Some Related Work

Modeling:

physics-based:	*Litke et al., SGP '05,*
	Wirth et al., EMMCVPR '09
Gromov-Hausdorff:	*Memoli, Sapiro, Found. Comp. Math. '05,*
	Bronstein et al., PNAS '06
Feature descriptors:	*Sun et al., SGP '09,*
	Aubry et al., 4DMOD '11

Diffeomorphic matching:

Kurtek et al., CVPR '10

Combinatorial point matching:

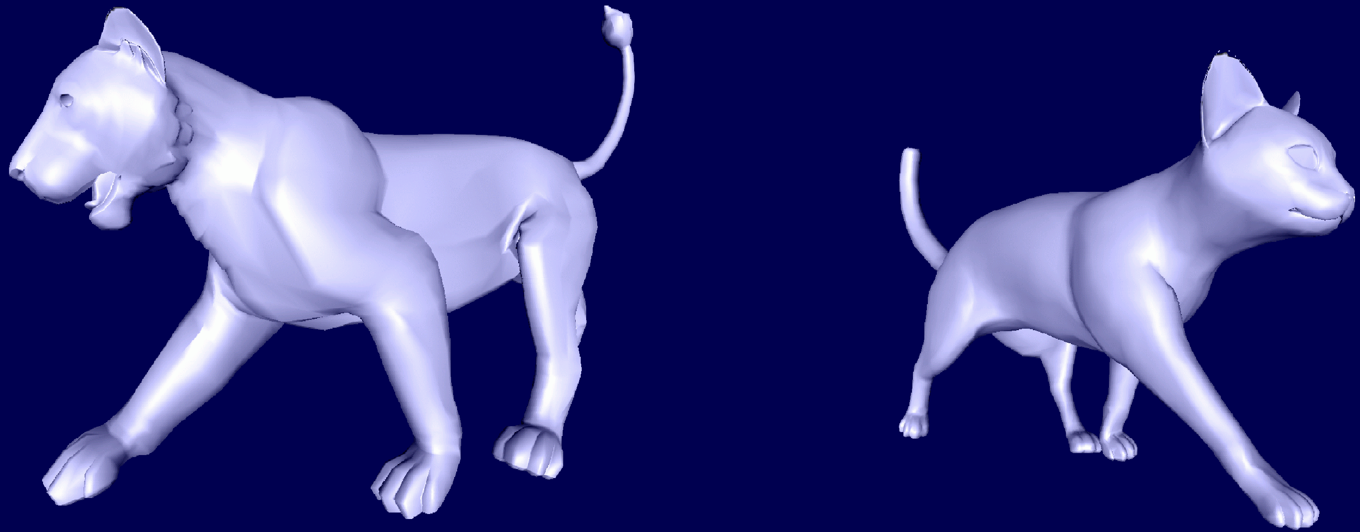
Torresani et al., ECCV '08

Zeng et al., CVPR '10

Lipman, Daubechies, Adv. Math. '11

3D shape benchmarks:

Sumner, Popovic '04, Vlasic et al. '08, Boyer et al. '11

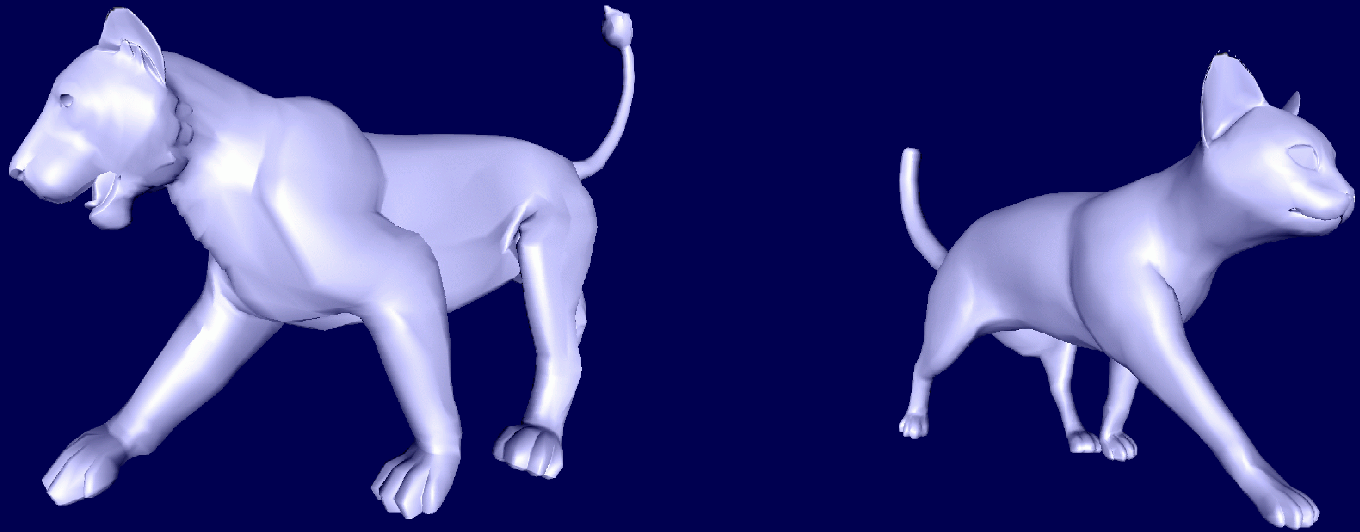


Matching $\varphi: \quad (X \subset \mathbb{R}^3) \longrightarrow (Y \subset \mathbb{R}^3)$

Optimum $\varphi^* = \arg \min_{\varphi \in \mathrm{Diff}^+(X,Y)} E(\varphi) + E(\varphi^{-1})$

Thin shell energy $E(\varphi) = E_{bend}(\varphi) + E_{stretch}(\varphi)$ *[Koiter 1966]*

Non-convex optimization problem!



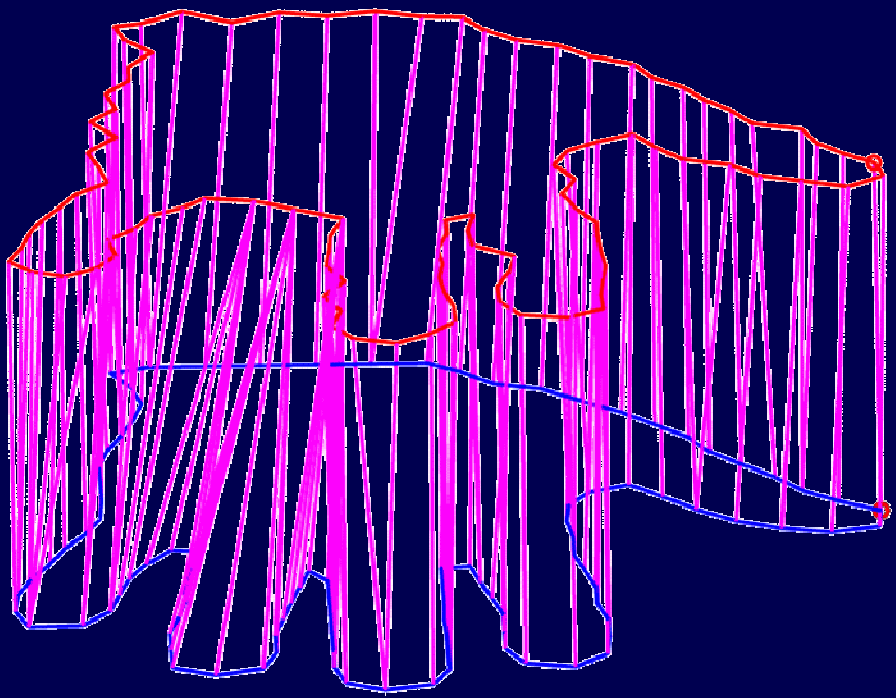
Matching $\varphi : \quad (X \subset \mathbb{R}^3) \longrightarrow (Y \subset \mathbb{R}^3)$

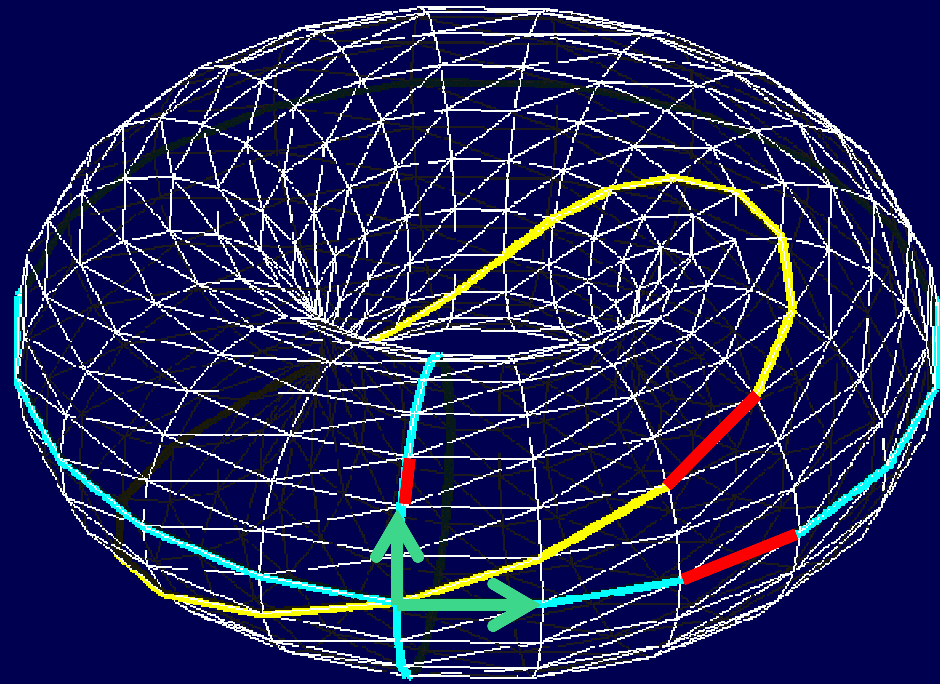
Graph $\Gamma_\varphi = \Big\{ \big(x, \varphi(x)\big) \,\Big|\, x \in X \Big\} \subset X \times Y,$ $\quad \pi_X : \Gamma_\varphi \to X$, $\quad \pi_Y : \Gamma_\varphi \to Y$

Optimum $\Gamma^* = \arg\min_\Gamma E(\Gamma)$ → 2D closed minimal surface in 4D

Polynomial-time solutions?

Planar Elastic Shape Matching



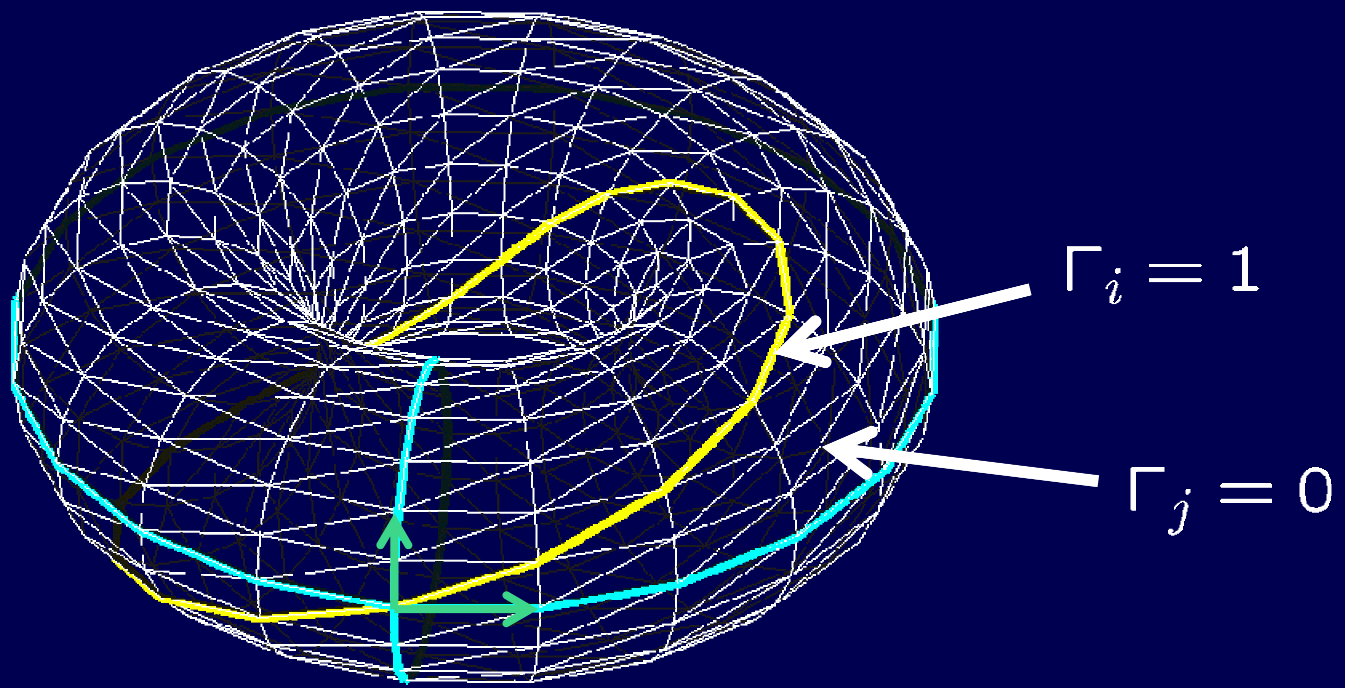


Matching $\varphi : X \to Y$

$$\Gamma^* = \arg\min_{\Gamma} E(\Gamma)$$

Solution: Shortest cyclic and monotonous path on a torus.

Schmidt, Farin, Cremers, ICCV 2007: Fast shape matching in sub-cubic runtime

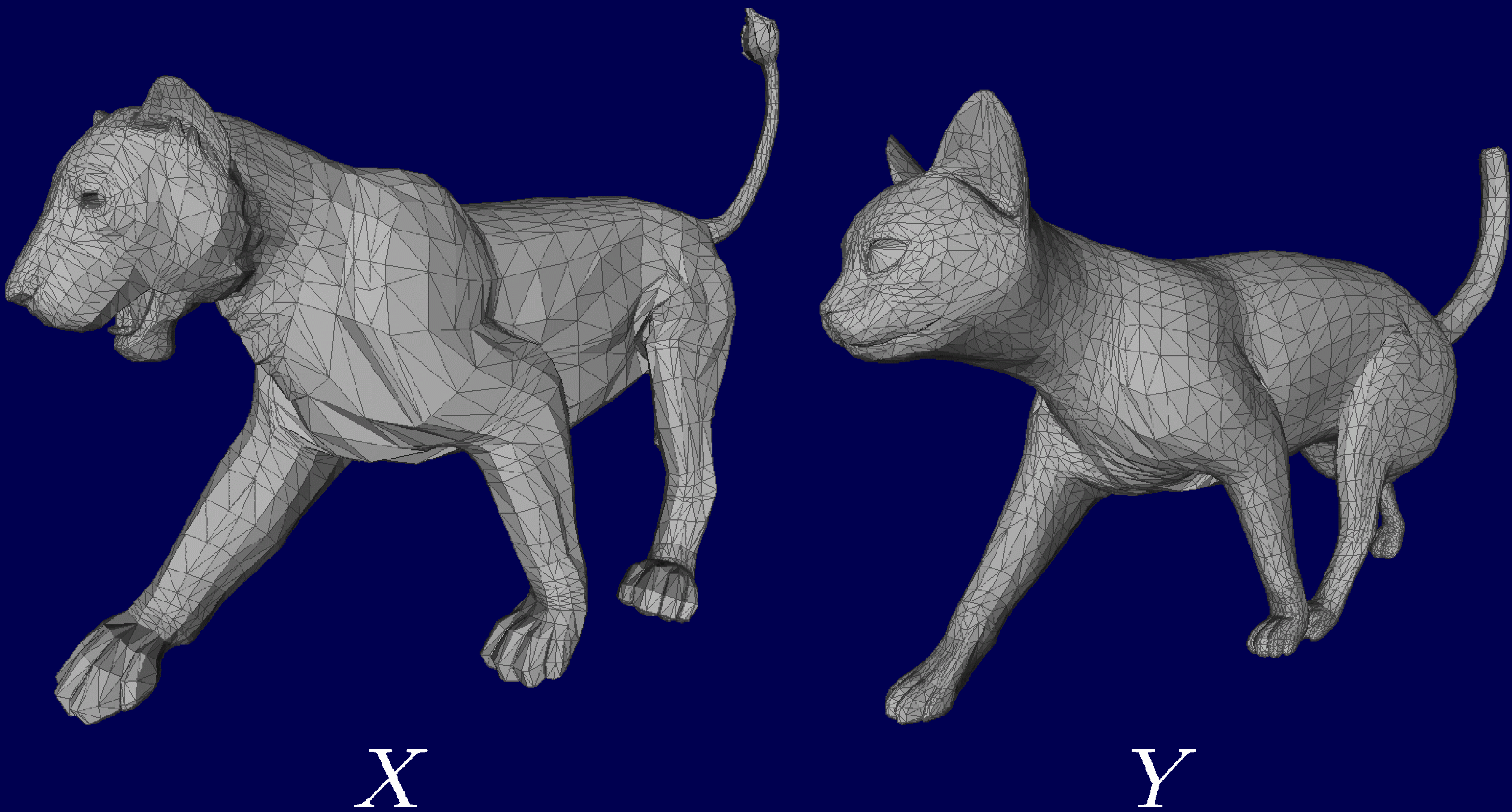


Integer Linear Program:

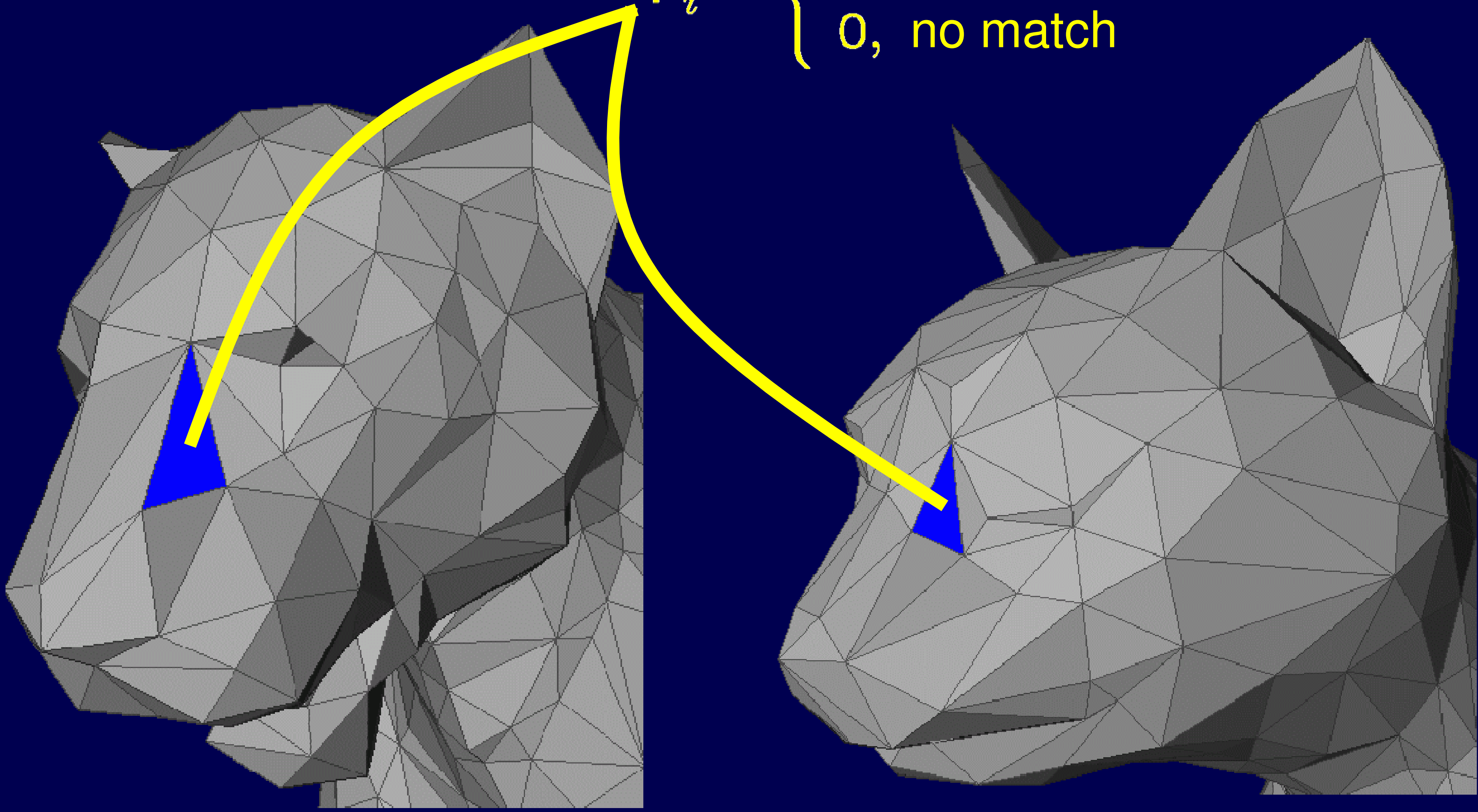
$$\min_{\Gamma \in \{0,1\}^N} E^\top \Gamma$$

$$\text{subject to} \begin{pmatrix} \partial \\ \pi_X \\ \pi_Y \end{pmatrix} \cdot \Gamma = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

Extension to 3D Shape Matching

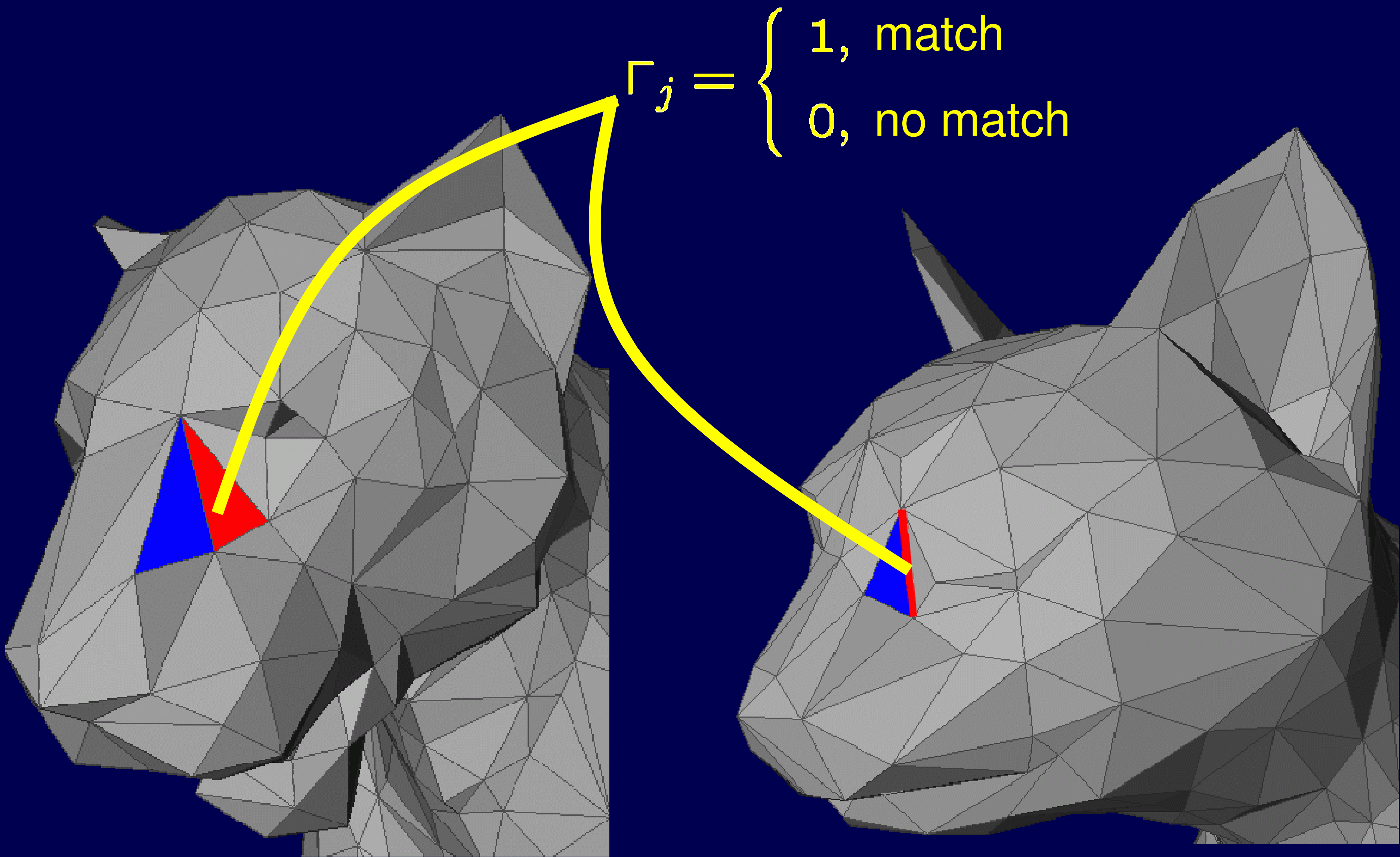


$$\Gamma_i = \begin{cases} 1, & \text{match} \\ 0, & \text{no match} \end{cases}$$



Windheuser et al., ICCV 2011

Allow for Shrinking / Expansion



Windheuser et al., ICCV 2011

Indicator variable $\Gamma_i \in \{0,1\}$ for each possible basic correspondence.

Assignment cost $E_i \in \mathbb{R}$ for each basic correspondence.

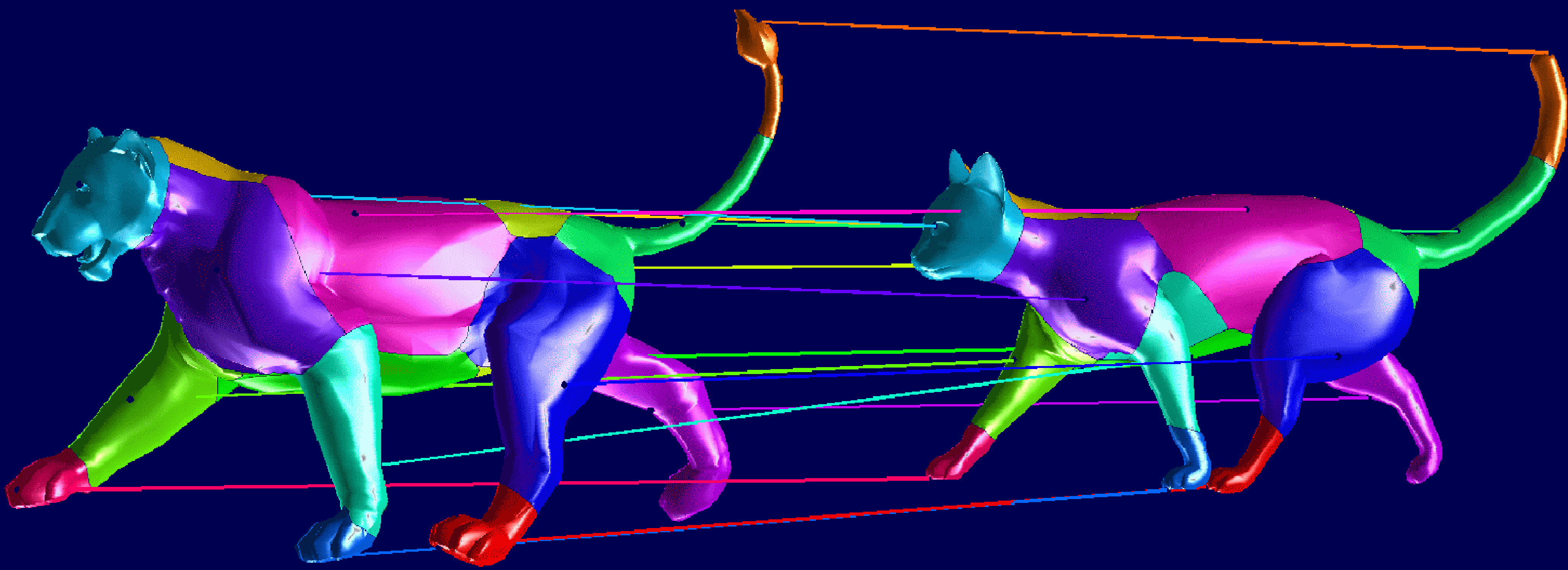
Determine best matching $\Gamma = (\Gamma_1, \ldots, \Gamma_N) \in \{0,1\}^N$ by solving

Integer Linear Program:

$$\boxed{\min_{\Gamma \in \{0,1\}^N} \sum_{i=1}^{N} \Gamma_i E_i \quad \text{subject to} \quad \begin{pmatrix} \partial \\ \pi_X \\ \pi_Y \end{pmatrix} \cdot \Gamma = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.}$$

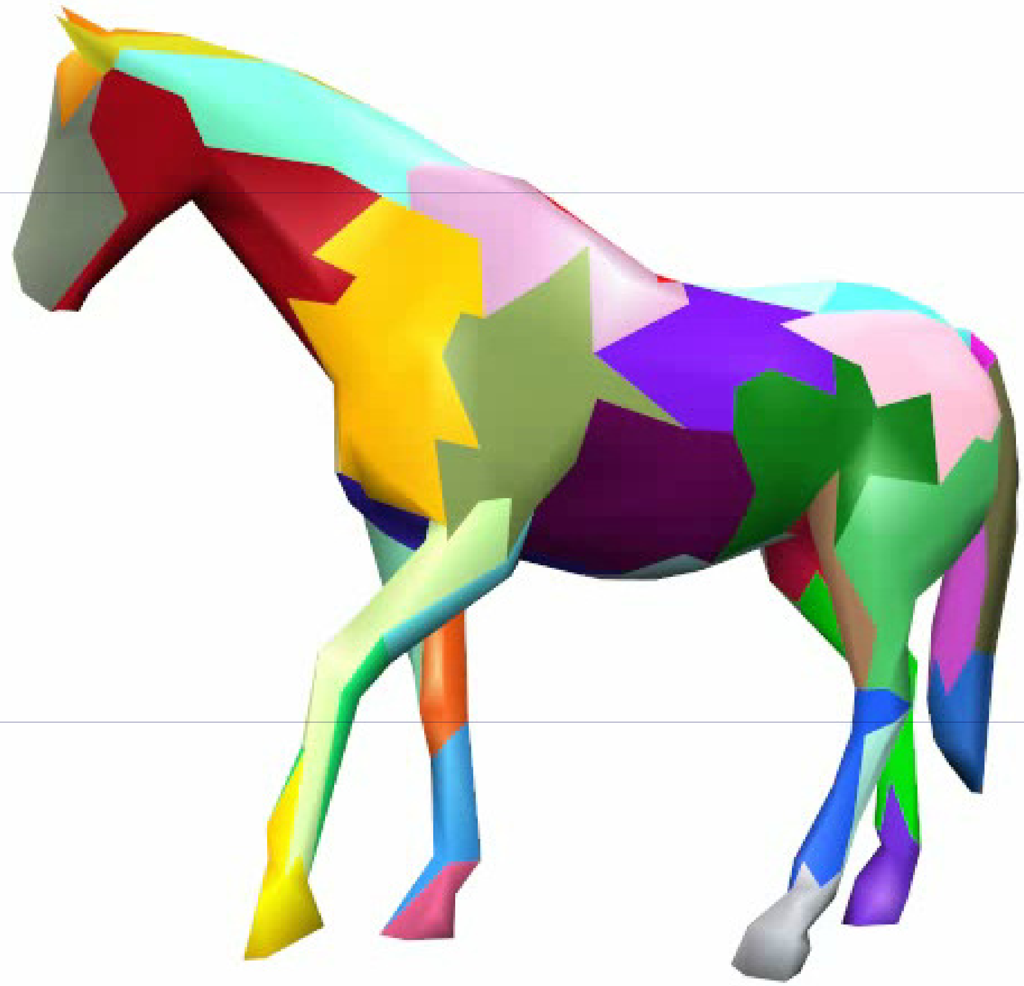
Solve relaxed problem & iteratively binarize variables until binary solution.

Computed Matching (3360 triangles)

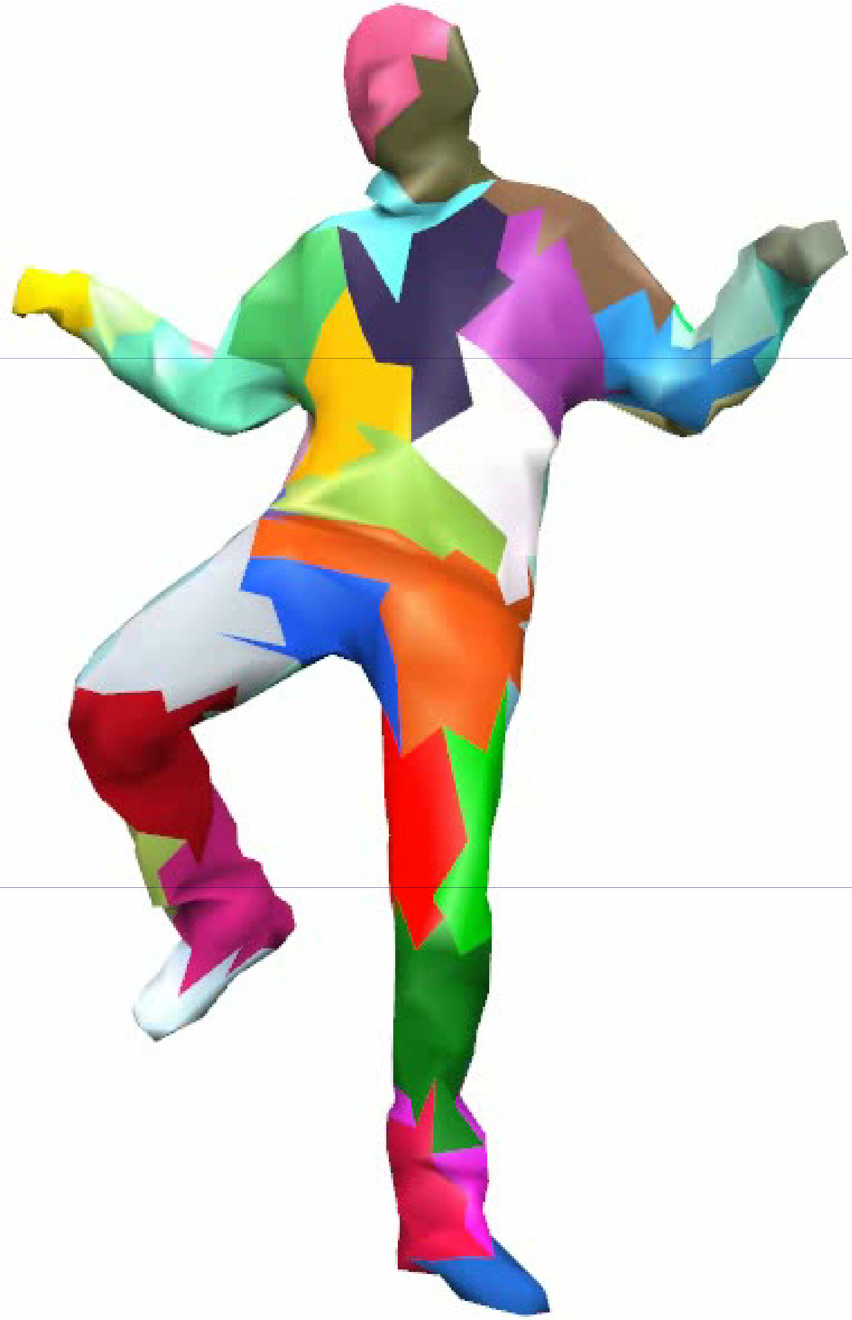


Shape Morphing by Linear Interpolation

Surface Matching (1504 triangles)

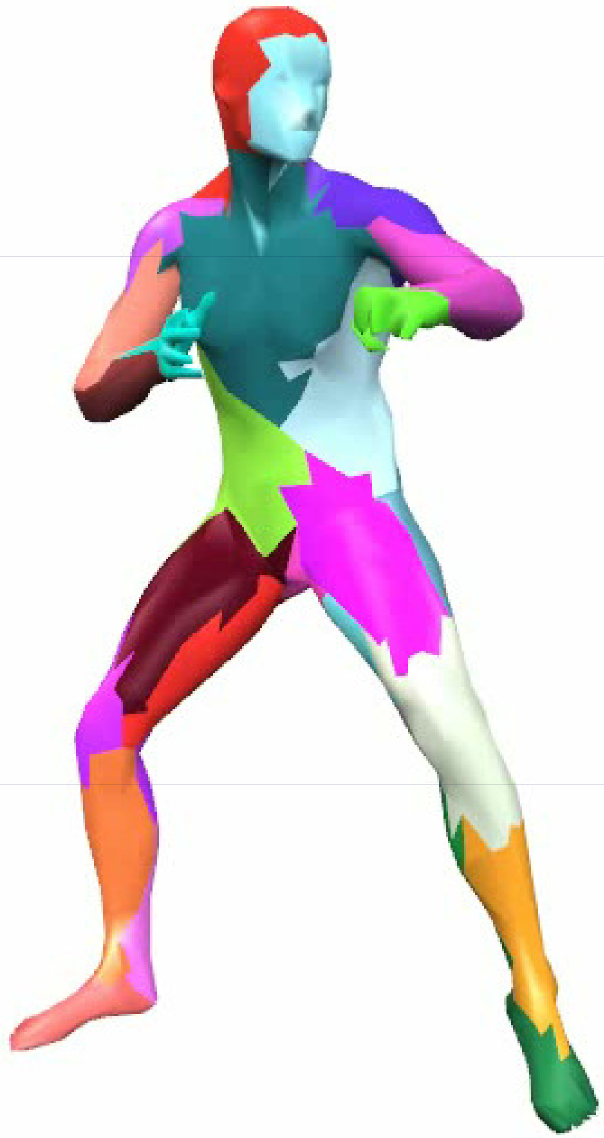


Surface Matching (2456 triangles)

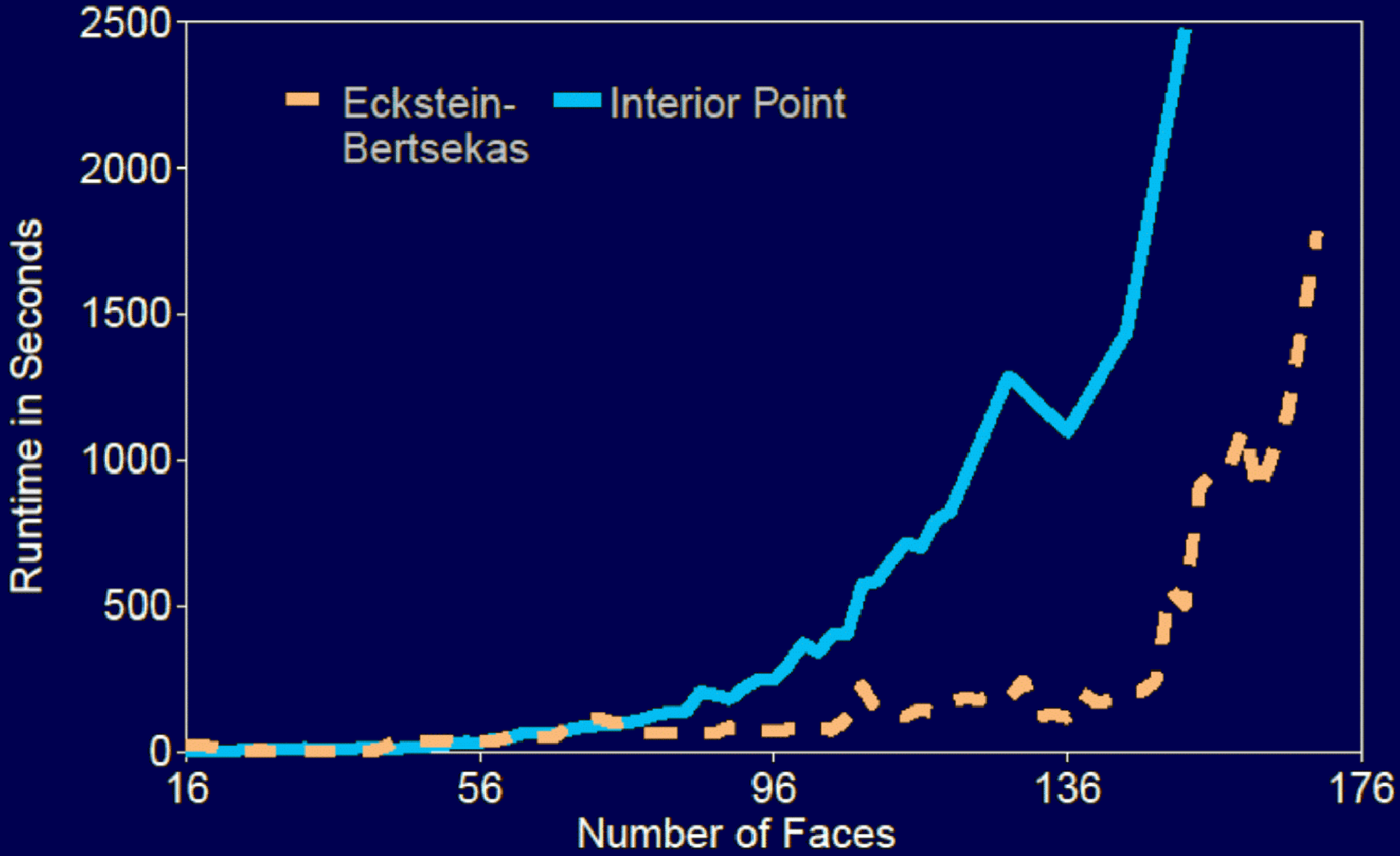


Surface Matching (2440 triangles)

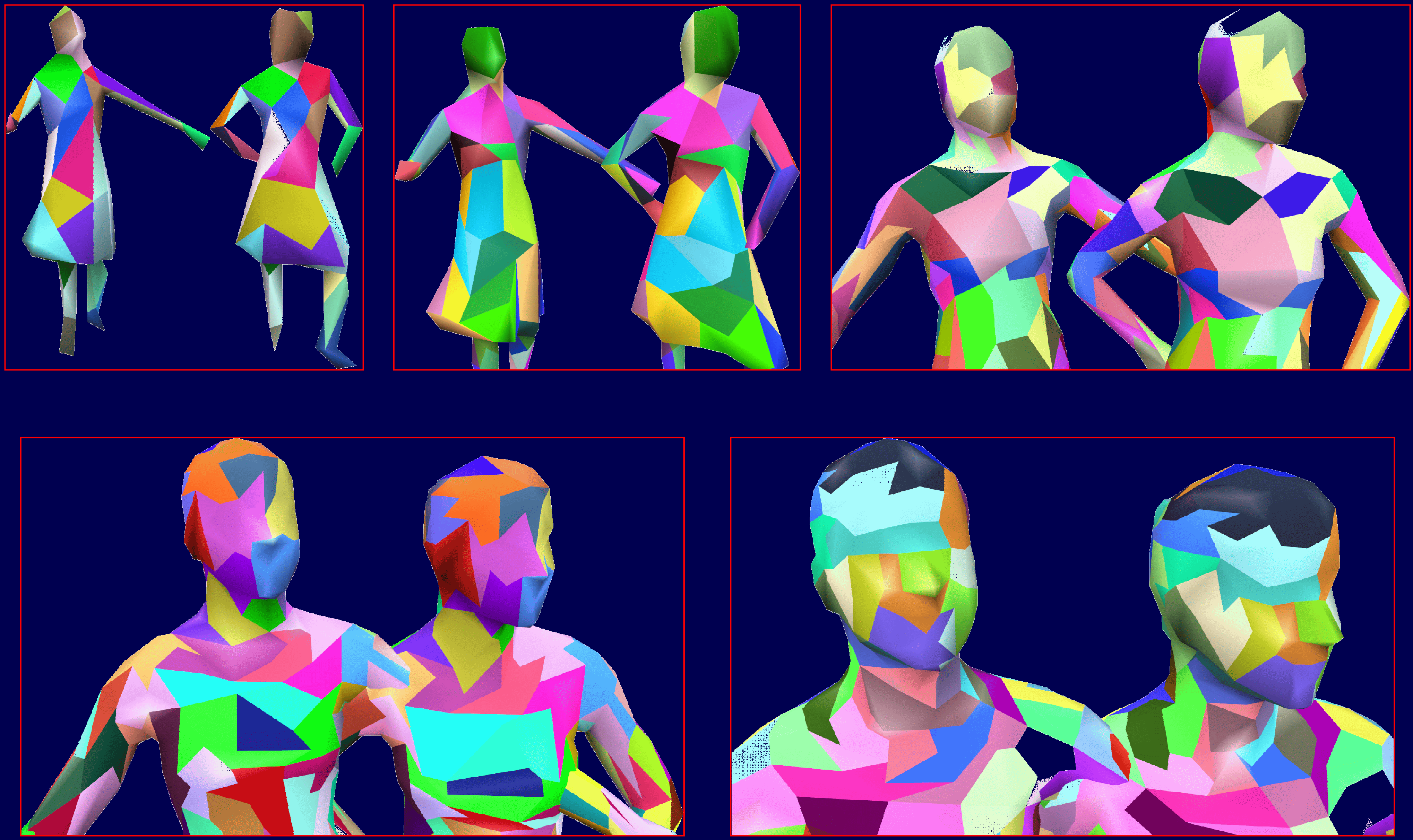
Surface Matching (3620 triangles)



Runtime Comparison



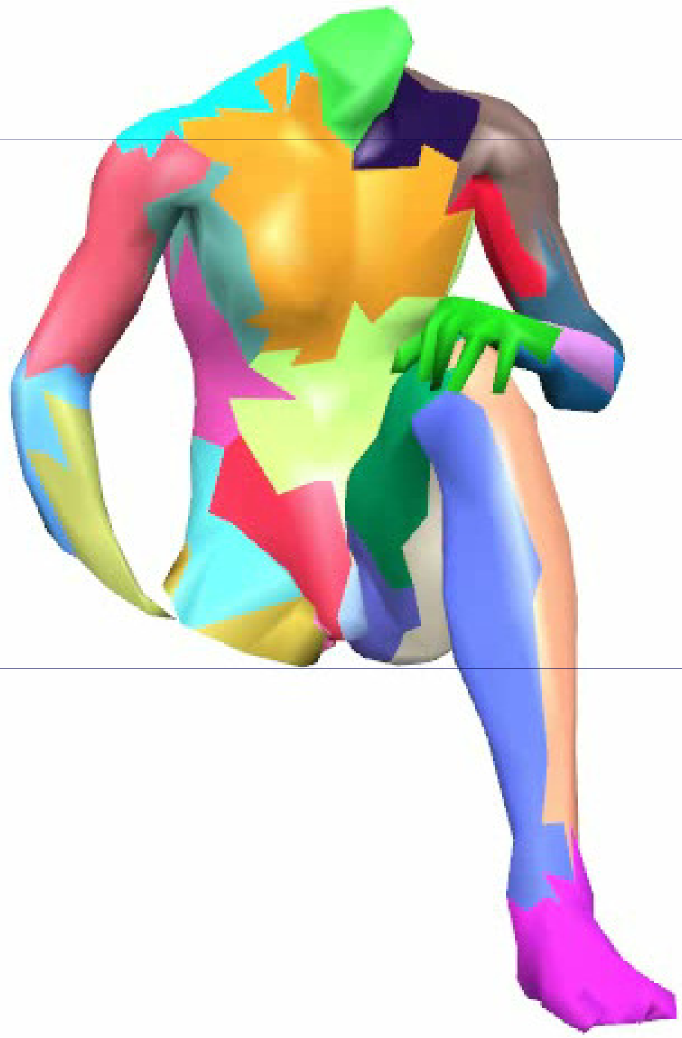
Coarse-to-fine Implementation



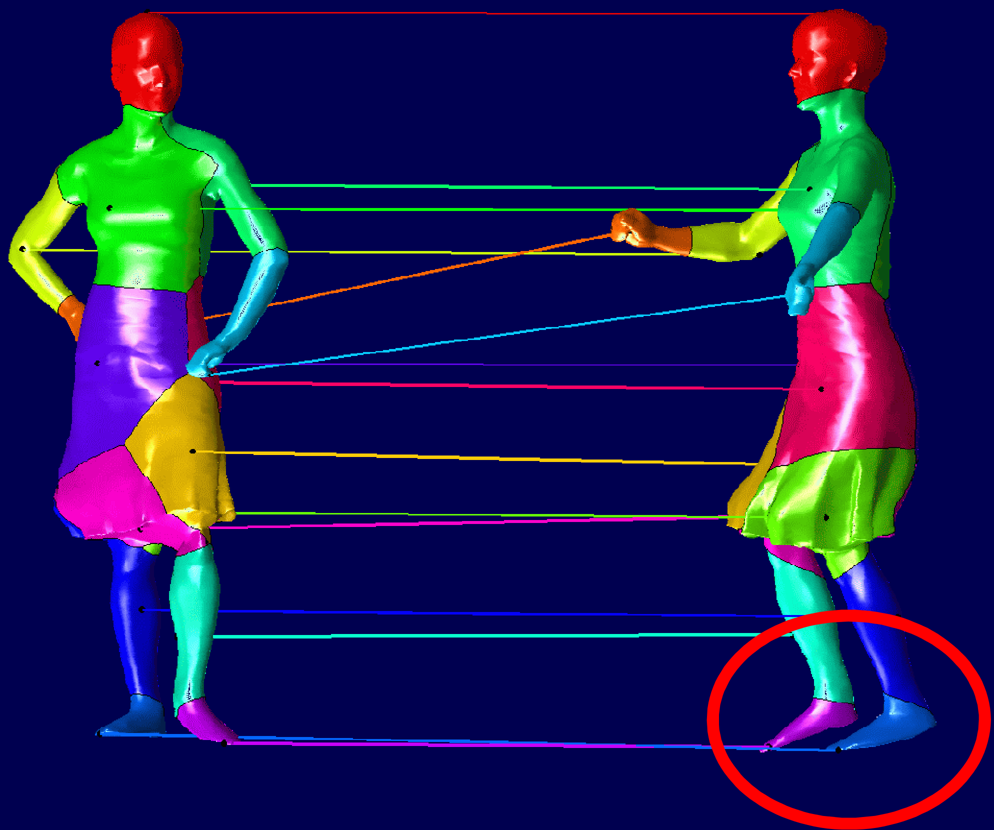
Surface Matching (6454 triangles)

Morphing a Woman into a Man

Matching with Missing Parts



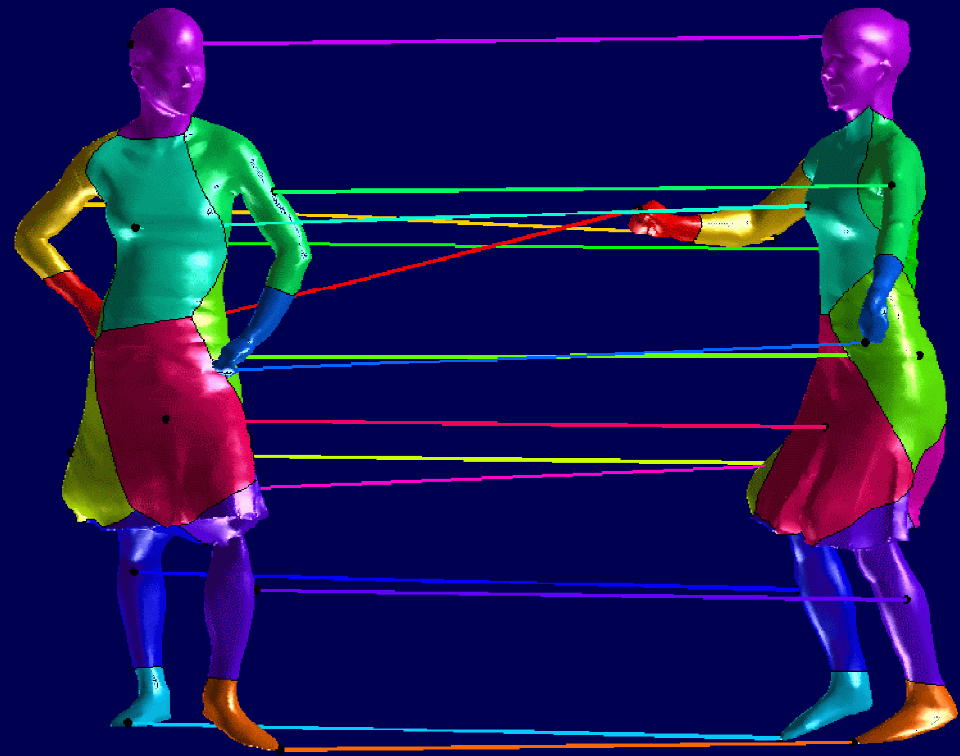
Comparison to Point Matching Methods



Bronstein et al. point matching

inversion: 12/30, partial inversion: 8/30

mean geodesic error: 0.079



Proposed surface matching

Preserves orientation

mean geodesic error: 0.03

Conclusion

Optical Flow Estimation

- variational methods
- fast coarse-to-fine algorithms
- near-optimal solutions by convex relaxation

Elastic 3D Shape Matching

- Dense matching via LP relaxation
- Stretching, shrinking and bending
- Requires no initialization