Detecting Nuclear Materials from Cosmic-Ray Muon Scattering Data

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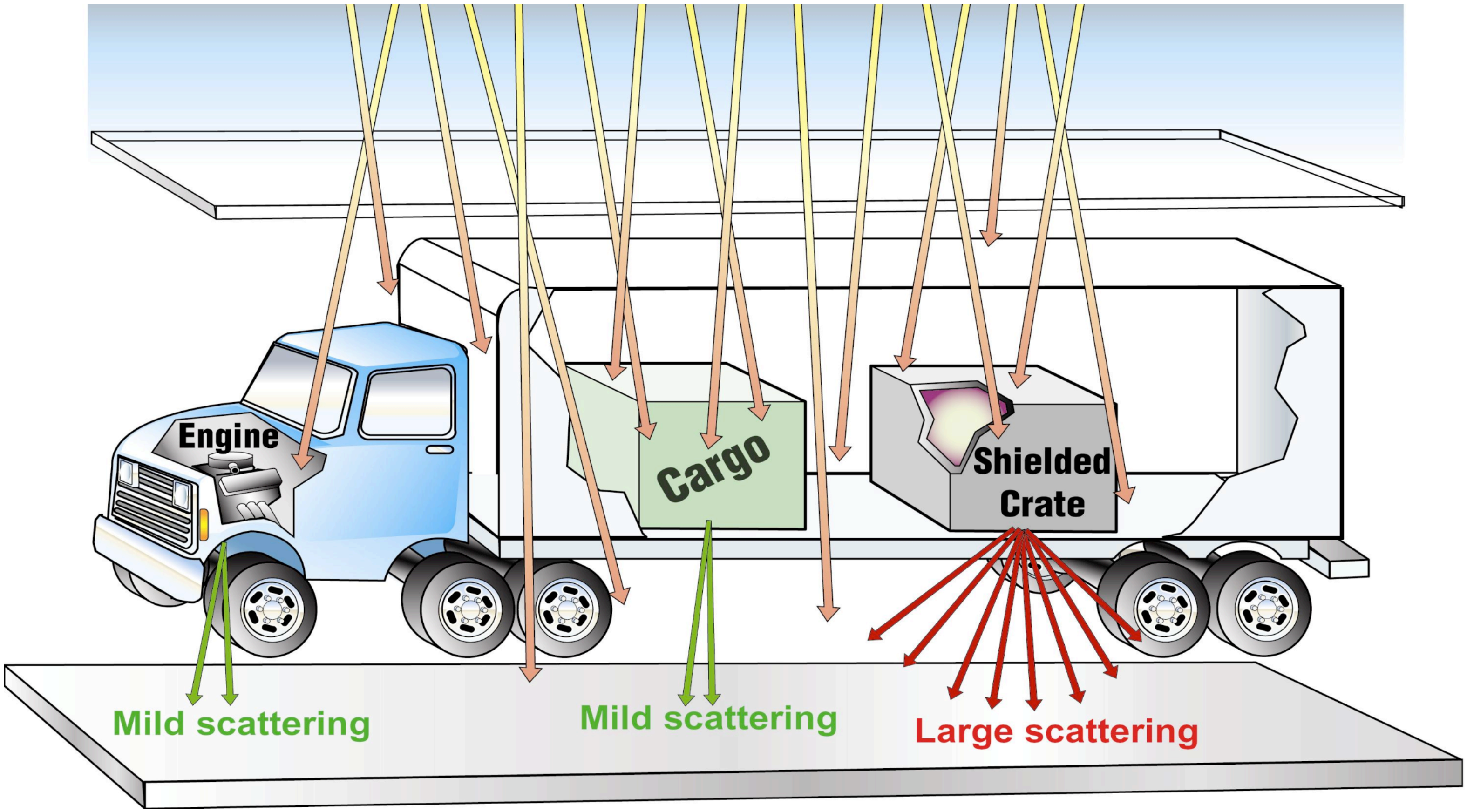
Muon Radiography Operational Concept



Expected muon scattering increases with increasing atomic number, due to the electric charge of muons.

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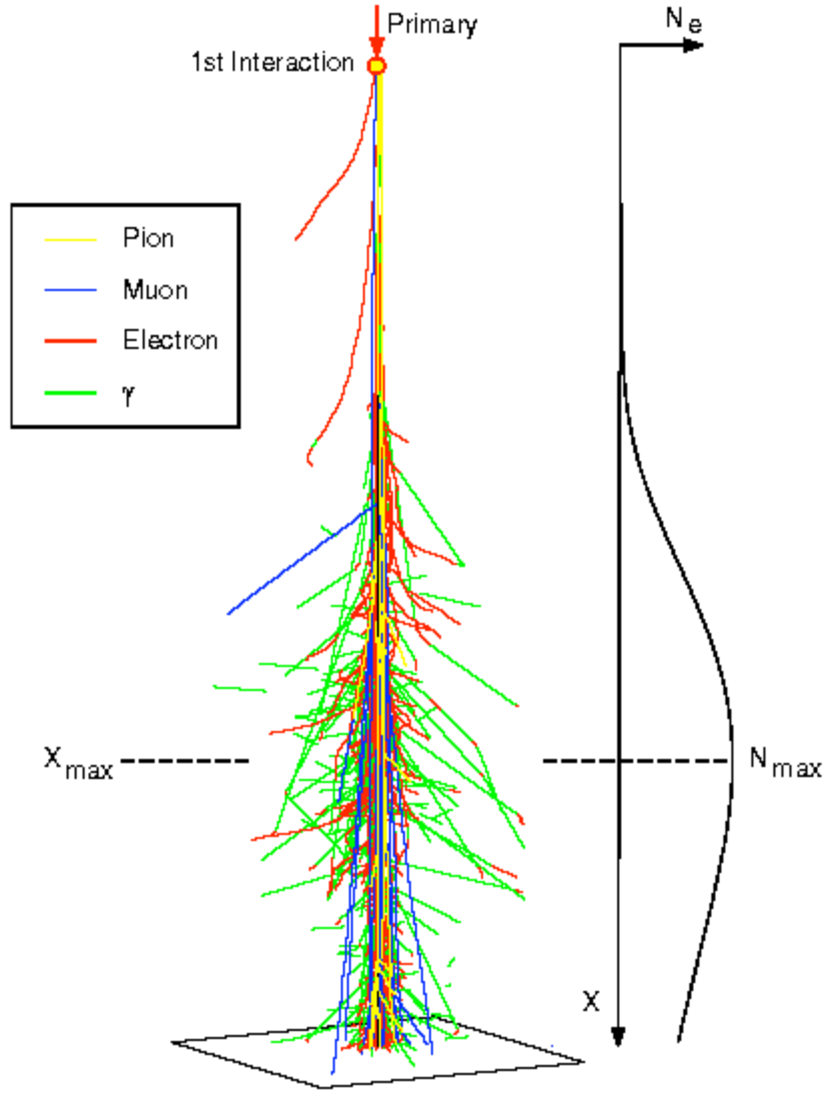
Cosmic-Ray Muons

- Primary cosmic rays strike the atmosphere and generate a cascade of secondary particles.
- Muons are the dominant particle at the Earth's surface.
- Most muons can penetrate tens of meters of rock or more.
- Muons arrive at a rate of 10,000 per square meter per minute (at sea level).
- The spectrum of cosmic ray muons (energy, arrival angle) is well documented. Peak at 3 GeV



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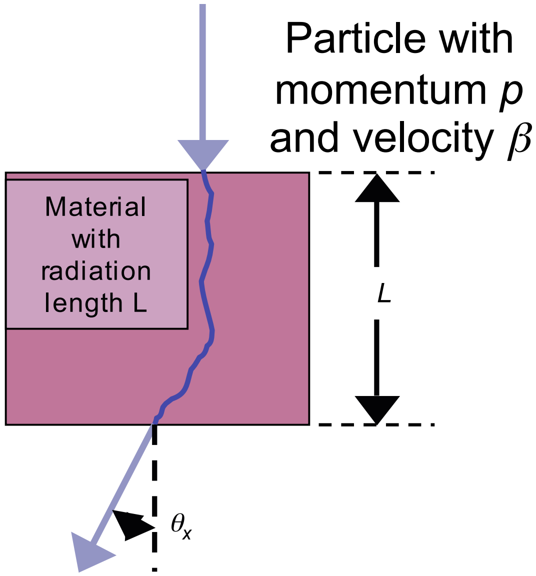
Physics of interaction: multiple scattering



Scattering distribution is approximately Gaussian

$$\frac{dN}{d\theta_x} = \frac{1}{\sqrt{2\pi}\theta_0} e^{-\frac{\theta_x^2}{2\theta_0^2}}$$

and the width of the distribution depends on material and muon properties (λ is a radiation length)

$$\theta_0 = \frac{13.6}{p\beta}\sqrt{\frac{L}{\lambda}} + H.O.T.$$

Scattered particles carry information from which material may be identified.

Material	λ , cm	θ_0, mrad*
Water	36	2.3
Iron	1.76	11.1
Lead	0.56	20.1
*10 cm of material, 3 Gev muons		

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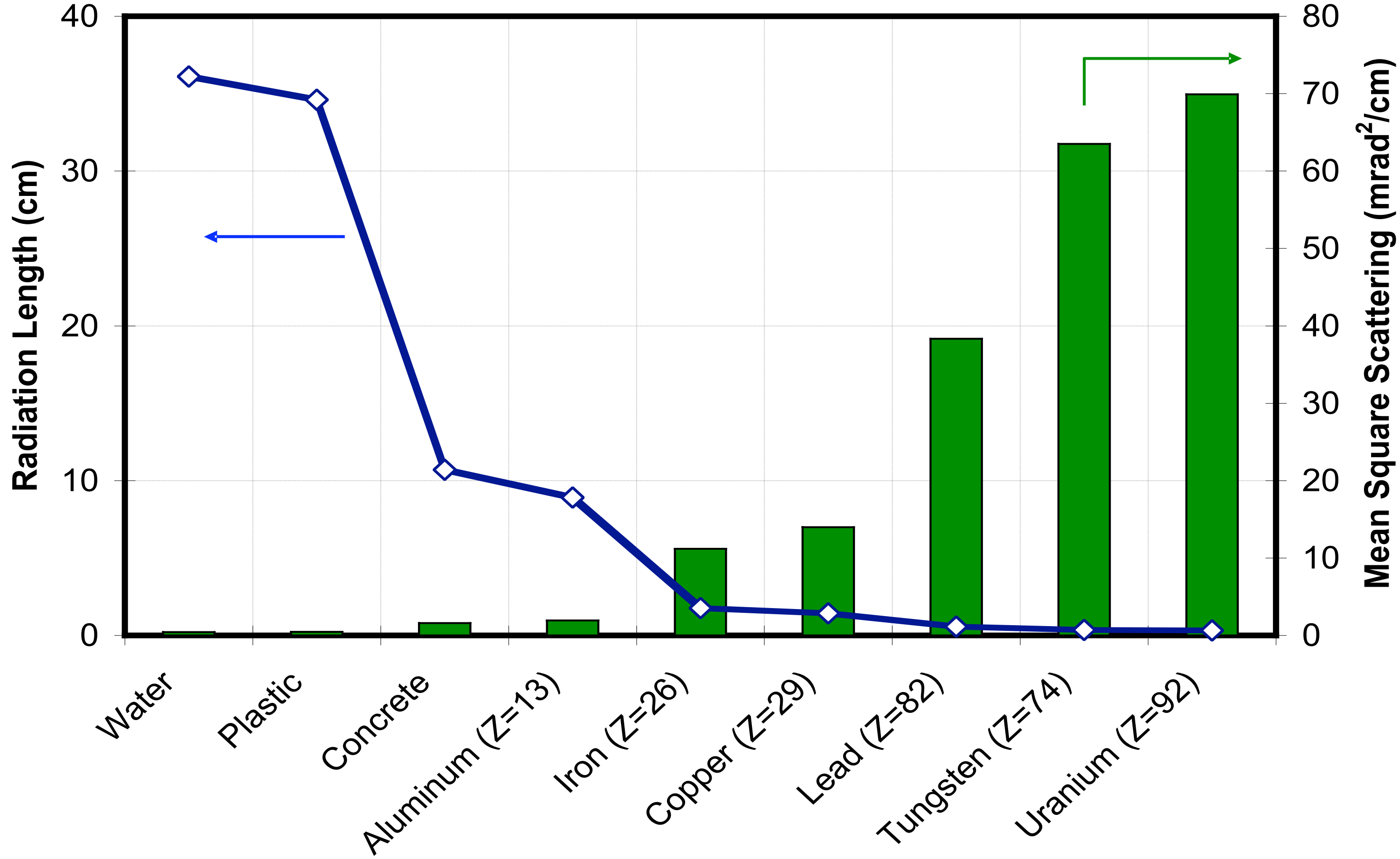
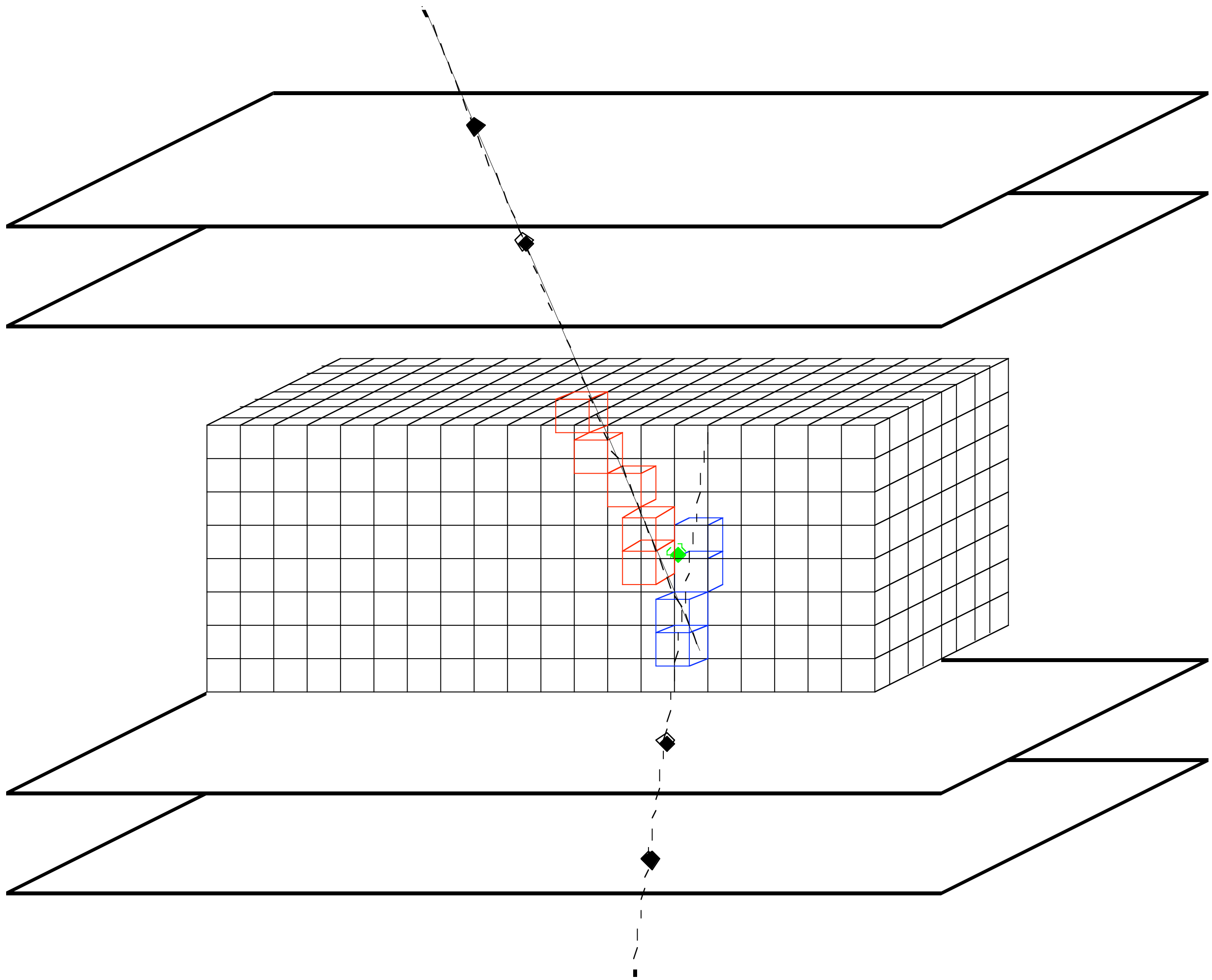
Radiation Lengths and Mean Square Scattering for Example Materials



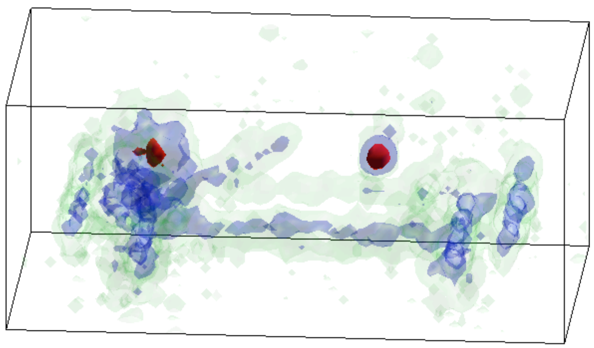
Dense, high-Z materials scatter muons more than other materials.

Voxels, Rays, and Point of Closest Approach



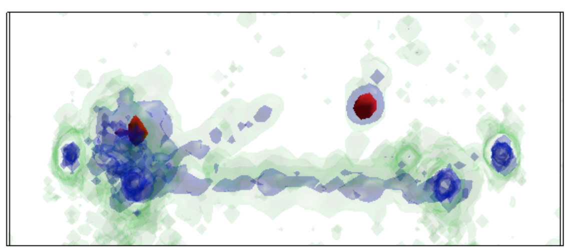
Maximum-Likelihood Reconstruction

Automobile, 60 seconds of muon exposure

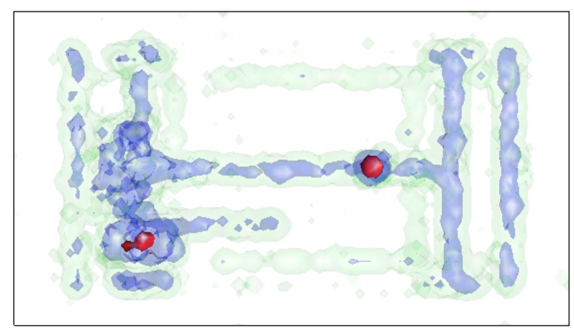


Two 20-kg U spheres

Side View



Top View



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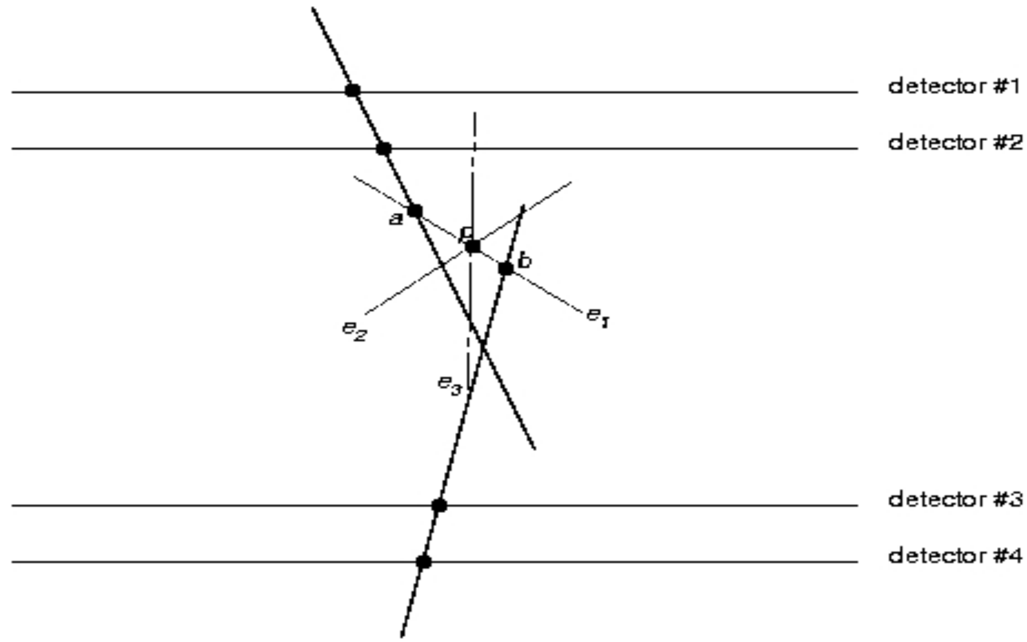
Voxel Histograms

- Several detection approaches begin with a distillation of the data into 3-D histograms of voxel scattering statistics.
- Each muon's scattering is measured, normalized by an estimate of the muon's energy, and then assigned to one or more voxel bins.
- This assignment can be done in many different ways:
  - Only to the voxel containing the PoCA:
    - over-localized, as the PoCA is a coarse estimate of scattering location
  - To each voxel containing the muon's ray tracks:
    - under-localized; doesn't take into account location information from PoCA
  - Weighted among voxels according to a measure of PoCA uncertainty

Muon Metric

- A distance function is constructed to quantify and orient uncertainty in the PoCA.
- Orthonormal coordinate vectors for each muon:
  - e_1: orthogonal to both paths
  - e_2: in direction of the deflection
  - e_3: follows tracks most closely
- Choice of weights quantifies the uncertainty in appropriate directions.

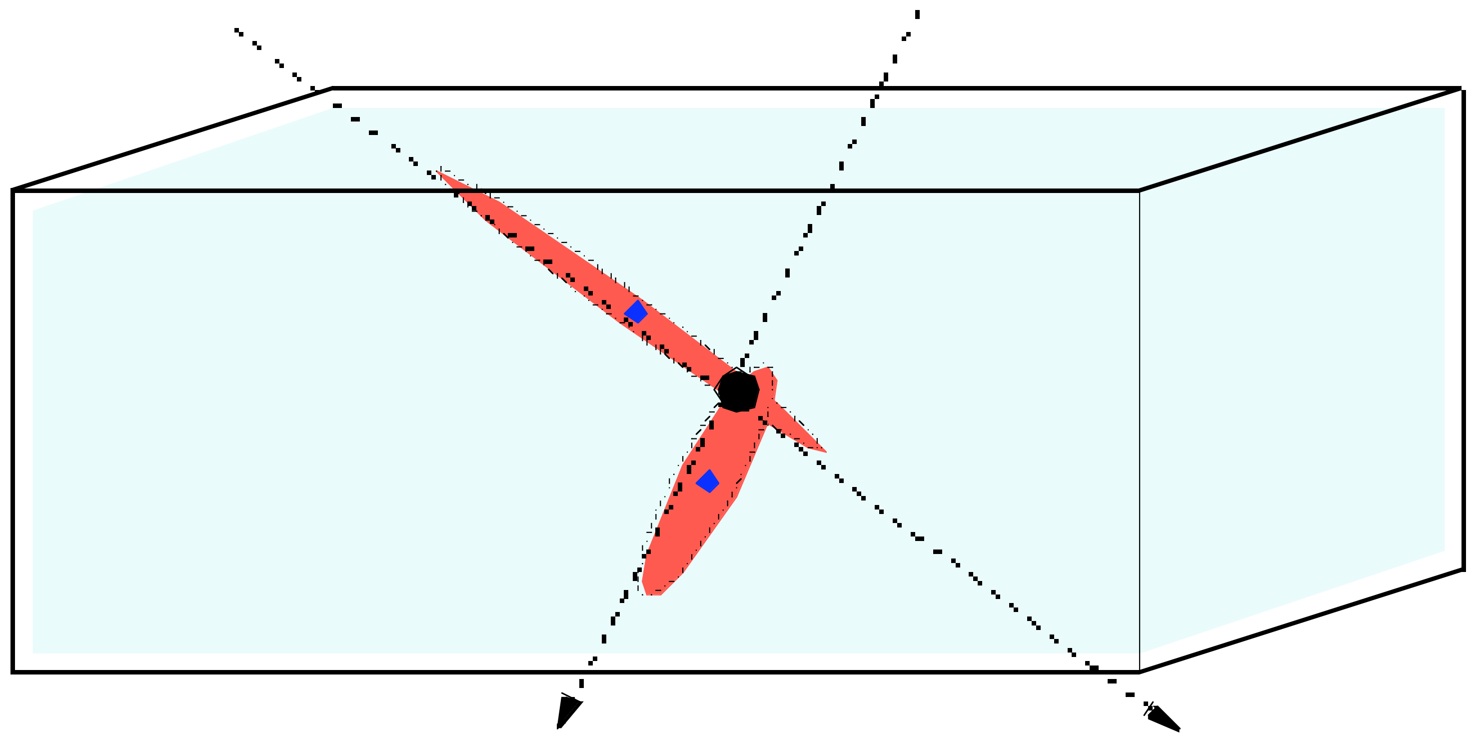


$$d(x) = \sqrt{\sum_{i=1}^{3} \left(\alpha_i (x - p) \cdot e_i\right)^2}$$

$$\alpha_1 = \alpha_2 \propto \left(|a - b| \cdot \Delta\theta\right)^{-1},$$

$$\alpha_3 \propto |a - b|^{-1}$$

Muon Metric Neighborhoods



Muons passing through a threat object in a cluttered container.

PoCAs in “wrong” place, but metric neighborhoods overlap at object.

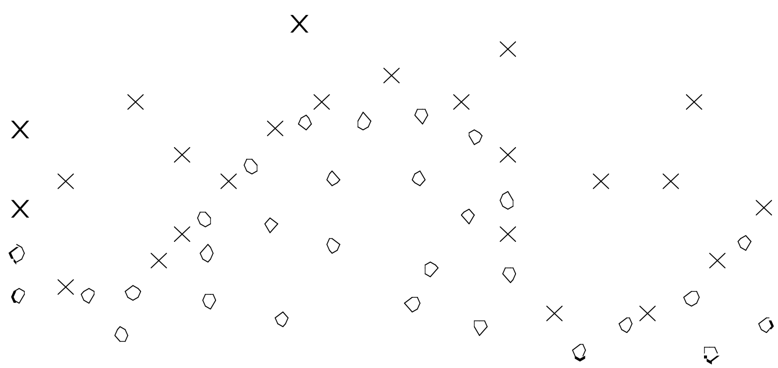
Classification

- Interpreting images can be difficult, and requires human involvement.
- Instead of using the data to create images, we sought to determine directly from the data whether a threat object is present.
- As an alternative to trying to determine how to get an answer from the voxel histograms, we used *machine learning* techniques.
- A computer algorithm is “trained” using many examples of datasets where the answer is known and provided to the algorithm.
- The algorithm “learns” how to discern the answer from the data.

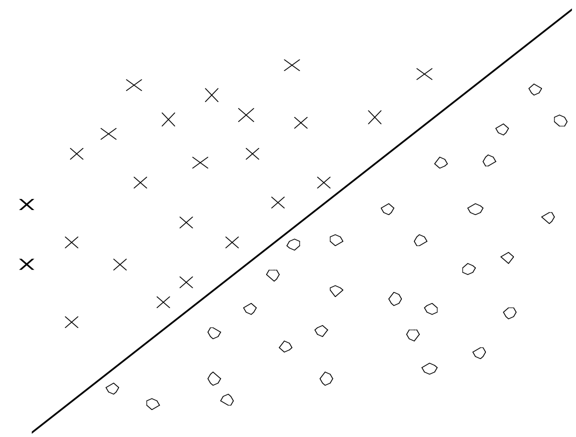
Machine Learning Implementation

- The classifier is trained using muon-scattering data generated by computer simulations of cargo containers or vehicles containing a variety of materials, some with high-Z material and some without.
- The features for each sample are the mean squared scattering (angle times energy) for each 10-cm voxel in a 3-voxel cube surrounding the sample location.
- We chose the Support-Vector Machine method for the classifier, as it is easy to implement and computationally efficient.

Support-Vector Machines



Thousands of points in 27-dimensional space



Thousands of points in infinite-dimensional space

- Linear classification is made possible by nonlinearly embedding the data into an infinite-dimensional function space.
- Given a kernel $k(x,y)$, the embedding maps y to $x \mapsto k(x,y)$.
- For $k(x,y) = e^{-\Gamma\|x-y\|^2}$, the data are mapped to a simplex; any two subsets can be separated by a hyperplane.

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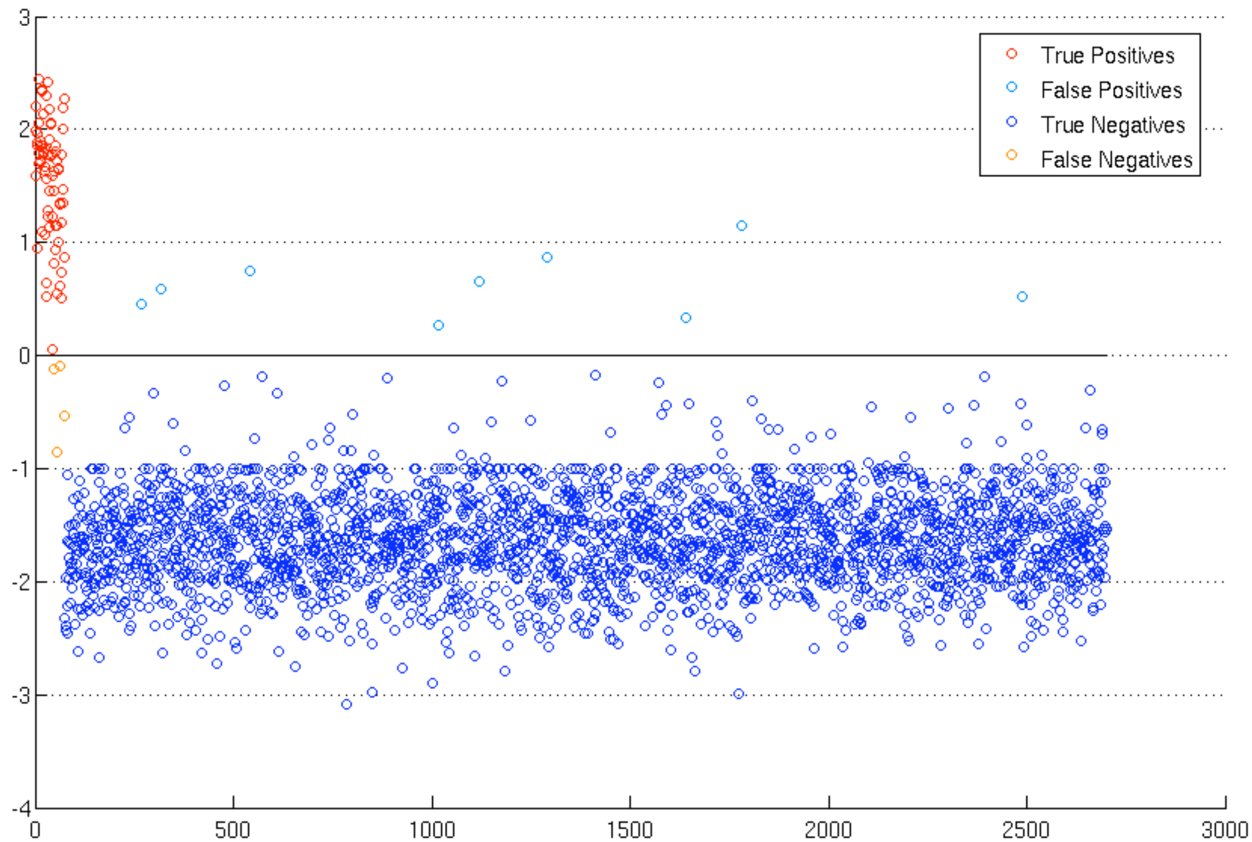
Results---Test Data

15 seconds of muon exposure



Positive samples have 20-kg U sphere centered on the central voxel.

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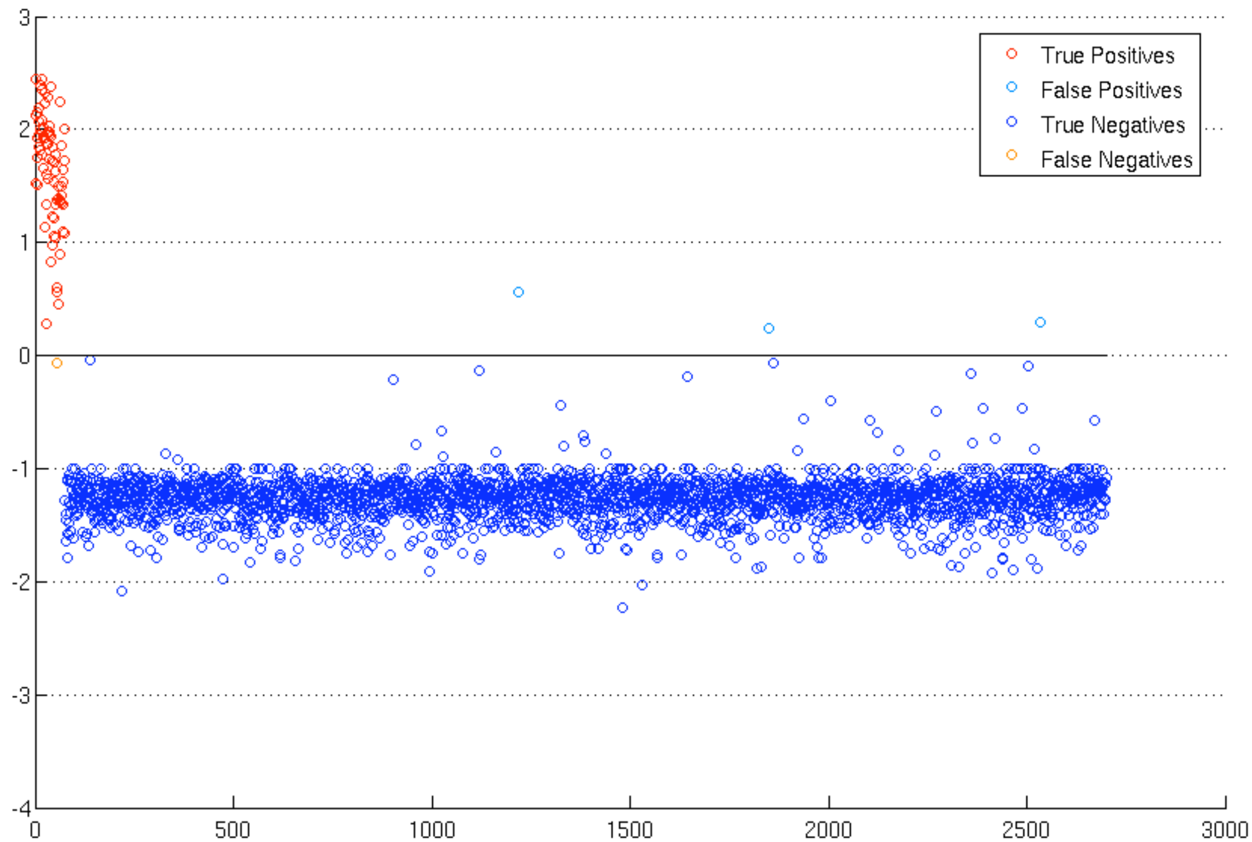
Results---Test Data

30 seconds of muon exposure



Positive samples have 20-kg U sphere centered on the central voxel.

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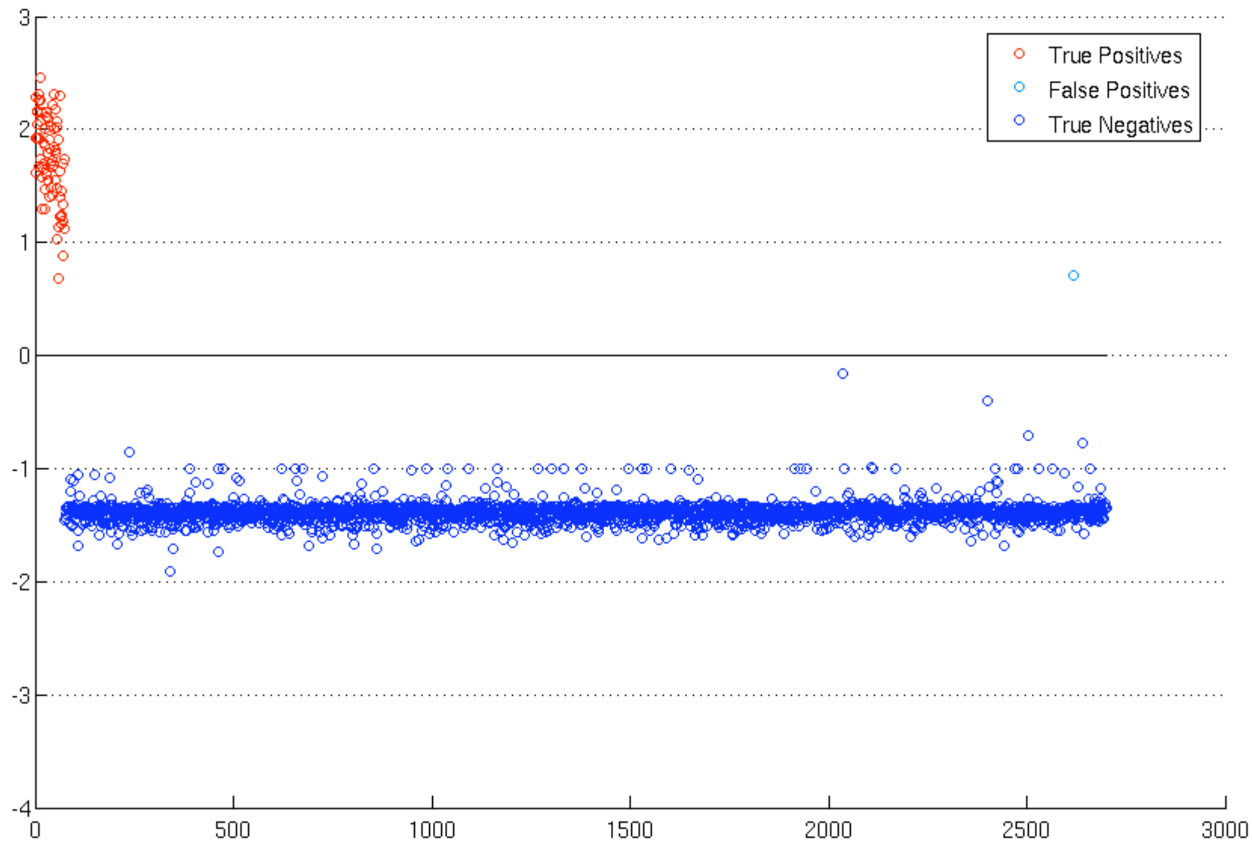
Results---Test Data

45 seconds of muon exposure



Positive samples have 20-kg U sphere centered on the central voxel.

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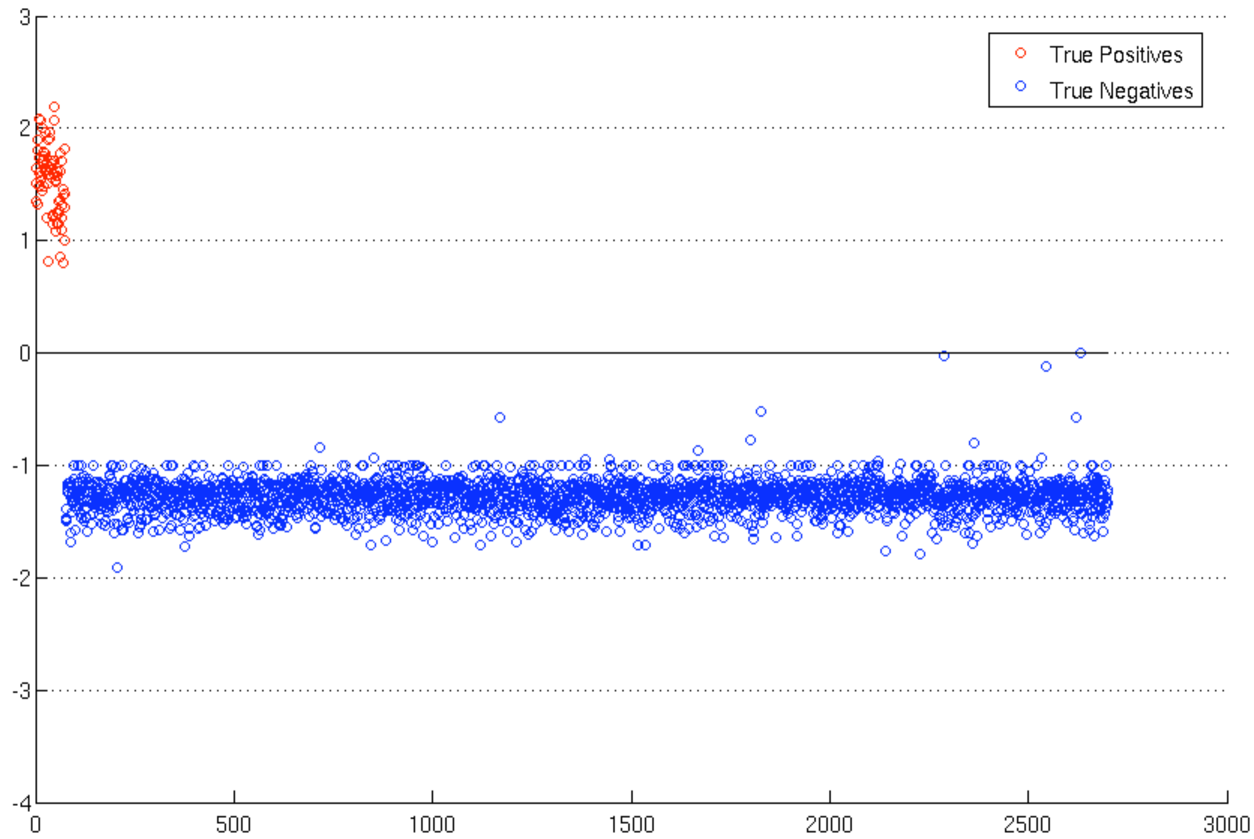
Results---Test Data

60 seconds of muon exposure



Positive samples have 20-kg U sphere centered on the central voxel.

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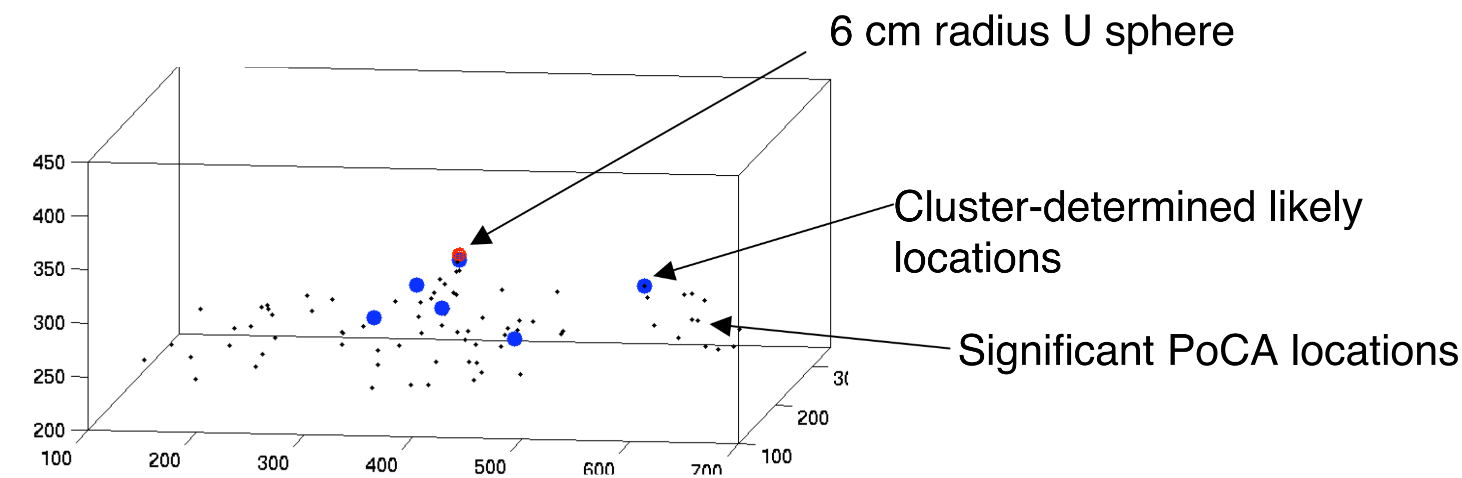
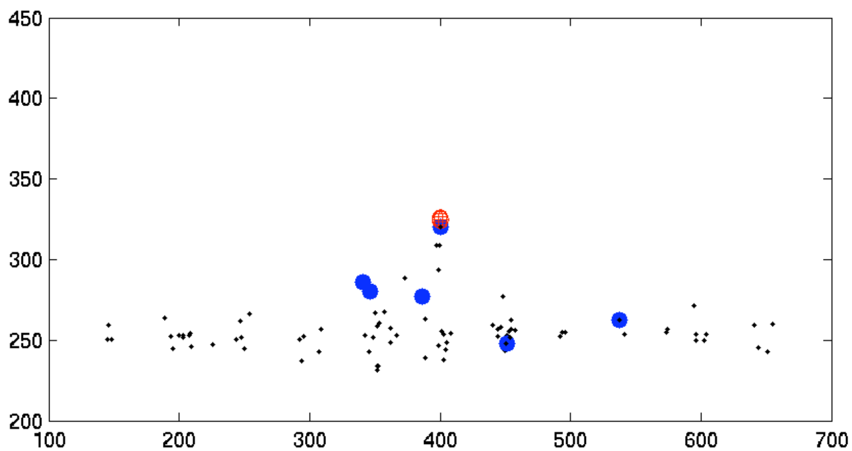
Clustering for Location Selection

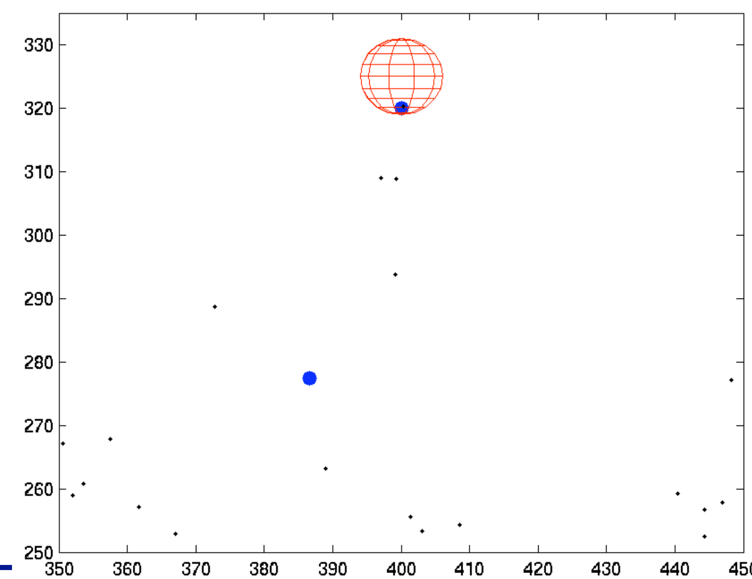
- The classifier works well for a given location; what about an entire vehicle or container?
  - Testing each voxel would lead to accumulation of test errors.
- A selection of the most scattered muons is used for a clustering algorithm.
  - The number of muons selected, ~ 100, can vary with the cargo weight, the duration of muon exposure, and the minimum size of threat object to be detected.
- A *k*-means clustering algorithm is used to find the clusters.
  - The largest cluster is iteratively checked for possible division, according to a criterion of the form $r_{p+} + r_{p-} < \gamma\, r_p$.
  - Stopping criteria involve the size of γ, the number of clusters, and the size of the smallest cluster.
- The cluster centroids are determined and passed to a classifier.
  - The centroid of each cluster is the point having the least total distance to each point of the cluster, as measured by the muon metric.

Clustering Results

20-kg U sphere, 14000 kg of Fe spheres







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Likelihood Function for Location Selection

- We identify a small number of locations to test with the classifier by finding local maxima of a likelihood function.
- The function is a sum over a selection of the most scattered muons of scattering times a function of the physics-based distance.
- More principled functions of distance will become feasible with optimization improvements (with John Dennis, Rice U.)

$$U(s) = \sum_j \frac{\Delta\theta_j E_j}{\max(d_j(s), a)}$$

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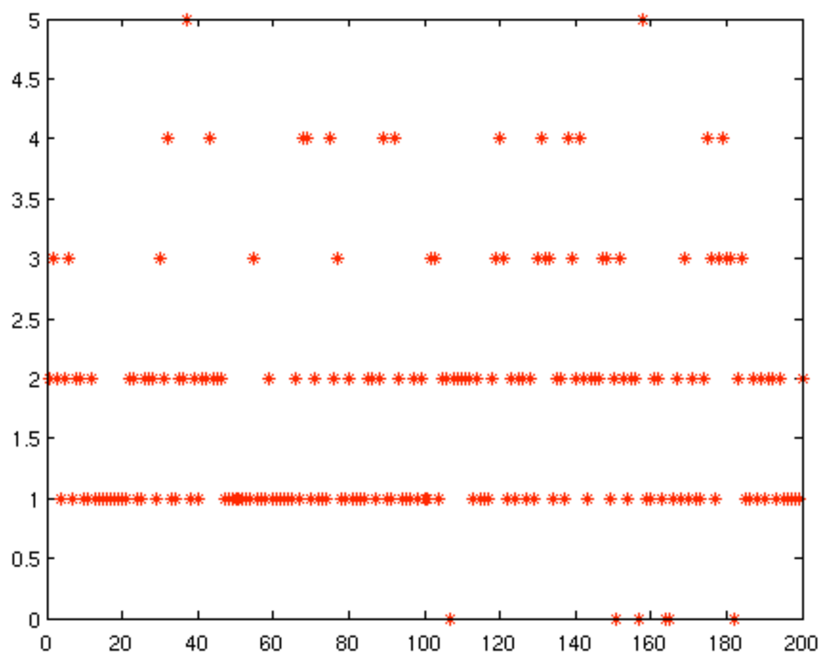
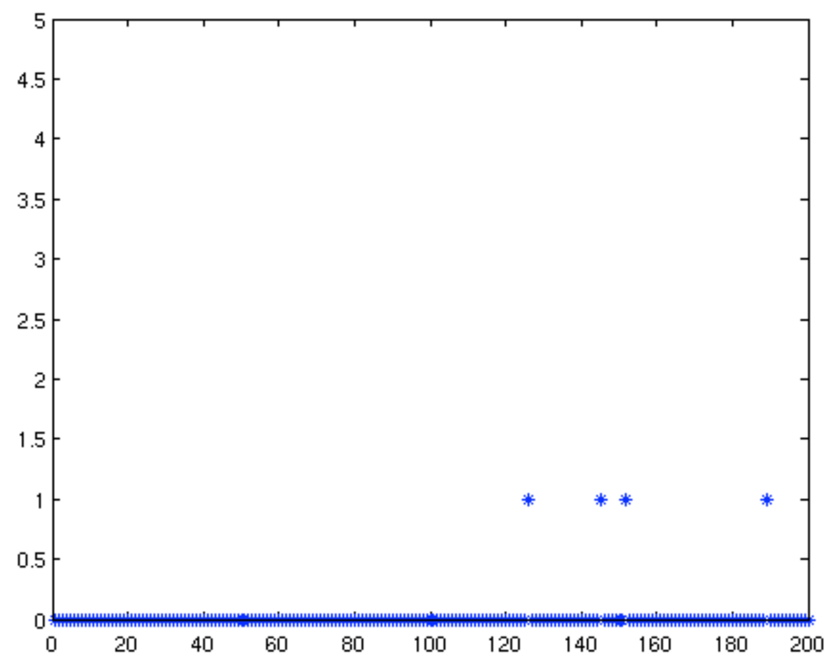
Preliminary Results---Classifying Candidate Locations

Number of positively-classified locations per container



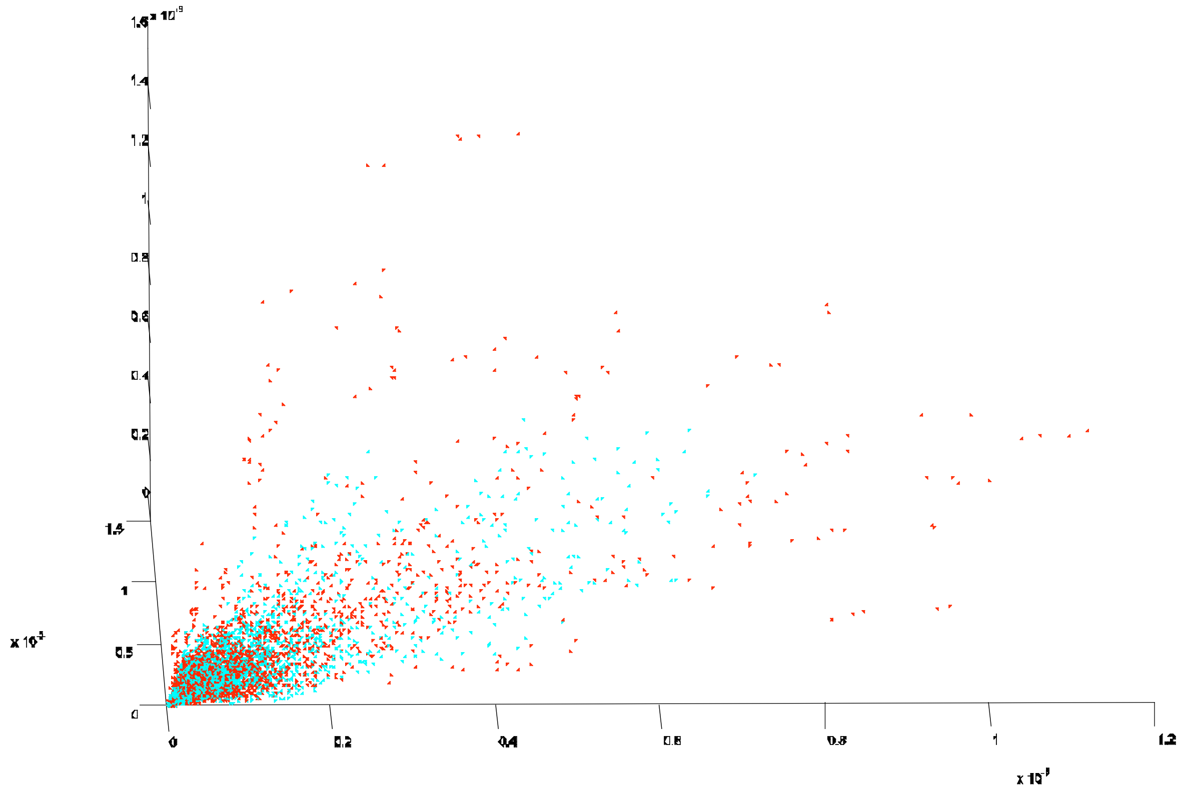


97% of containers with a 20-kg U sphere have at least one positive location

98% of containers with no high-Z material have no positive locations

Detection via Multiscale Geometry of the Data

Values of all 27-voxel cubes for each of two automobiles:
one with a threat object and one with no high-Z material.



Multiscale geometric analysis can identify and quantify outliers of the 27-dimensional data (with Gilad Lerman, UMN).

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