



Extraction of Coherent Structures in Turbulent Flows using Orthogonal Wavelets

Marie Farge,
LMD-CNRS, ENS, Paris,
Kai Schneider,
CMI, Université de Provence

& PhD students:
Alexandre Azzalini, Carsten Beta, Giulio Pellegrino

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Outline

- *Extraction of coherent structures using orthogonal wavelets*
 - *Definition, principle and algorithm*
- *Application in 1D*
 - *Ion density fluctuations from Tore-Supra*
- *Application in 2D*
 - *Rotating 2D turbulent flow measured by PIV*
 - *Transport of passive scalar and particles in 2D isotropic turbulent flow computed by DNS*
- *Applications in 3D*
 - *3D isotropic turbulent flow computed by DNS*
- *Perspectives for Coherent Vortex Simulation (CVS)*

What are coherent structures?

- They are **long-lived and long-range correlated structures**, which are produced by nonlinear instabilities,
- Their **time scales are much longer than the eddy-turnover time**, characteristic of turbulent fluctuations,
- They are **responsible of convective transport**.

Our motivation is to extract and characterize them, assuming that the **remaining diffusive transport corresponds to a Gaussian white noise**, whose effect will be easy to model.

Our prior is not on the coherent structures, but on the noise: **coherent structures are by definition what remains after denoising**, while the noise is supposed to be Gaussian and decorrelated.

Our model is: **one realization of a turbulent flow = coherent structures + Gaussian white noise**

What are wavelets?

Wavelets are functions well localized in both physical and spectral space. The wavelet representation is based on a set of dilated and translated wavelets ψ_{ji} indexed by scale j and position i .

Local information of a function f is kept by each wavelet coefficient \tilde{f}_{ji} indexed by scale j and position i .

But local information cannot be retrieved from few Fourier coefficients \hat{f}_k since it kept by the phase of all coefficients indexed by their wavenumber k .

In consequence, filtering the wavelet coefficients preserves the temporal or spatial structure of the function, which is not the case for Fourier filtering.

Special wavelets can generate orthogonal bases. Using them, a signal sampled on N points is represented by N wavelet coefficients and the computing cost is proportional to N operations.

Orthogonal wavelet transform

Wavelet analysis

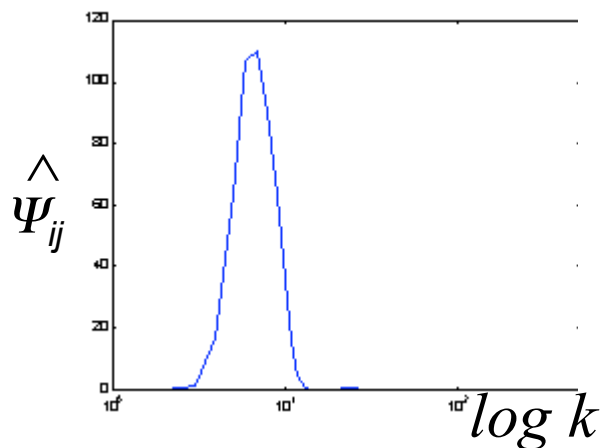
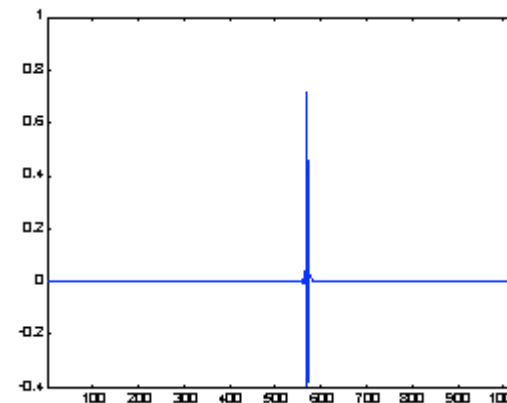
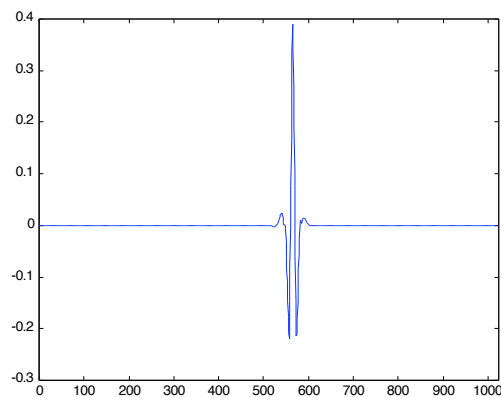
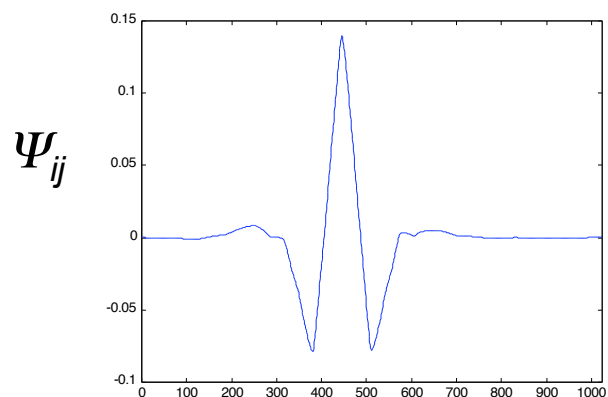
$$\tilde{f}_{ji} = \langle \psi_{ji} | f \rangle \quad \text{with} \quad \psi_{ji}(x) = 2^{j/2} \psi(2^j x - i)$$

Wavelet synthesis

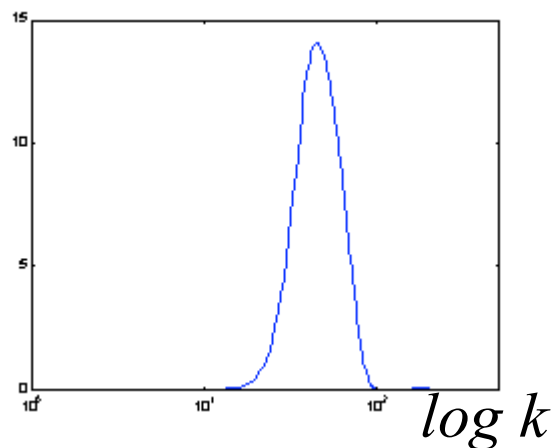
$$f(x) = \sum_{j=0}^{+\infty} \sum_{i=-\infty}^{+\infty} \langle \psi_{ji} | f \rangle \cdot \psi_{ji}(x).$$

*A signal sampled on N points
is analyzed and synthesized in CN operations,
e.g. C=12 for Coifman wavelets.*

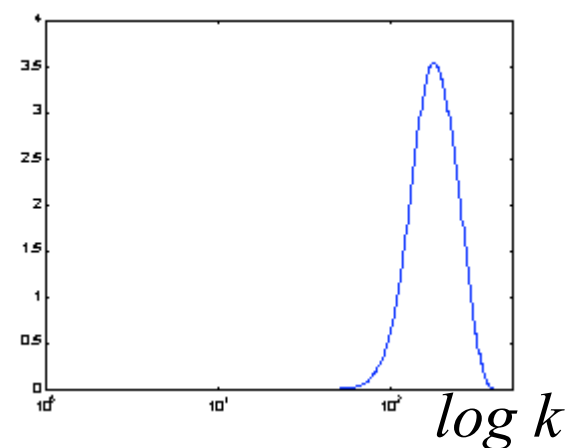
Wavelet basis (Coifman 12)



Large scale



Medium scale



Small scale

Wavelets are bandpass filters with $\Delta k/k = \text{constant}$

Algorithm principle

We want to **extract coherent structures** out of turbulent flows. We consider this as a **denoising problem**, not **requiring** any **hypotheses** on the coherent structures but **only on the noise to be eliminated**. We suppose we want to eliminate a **Gaussian white noise**, since it is the simplest to be modelled.

We **represent the signal in an orthogonal wavelet basis**, which is optimal (Donoho and Johnstone's theorem) for intermittent signals as those encountered in turbulence. We **retain only wavelet coefficients whose amplitude is larger than a given threshold**, which only depends on the noise variance and the number of samples.

We propose an **iterative method to estimate the noise variance** and we have proven (*cf. Appl. Comp. Harmonic Analysis, 2005*) that:

- the **algorithm converges**, the smaller the signal to noise ratio is, the faster the convergence,
- the corresponding filter is **idempotent** and is a **projector**,
- it **preserves the signal** if there is no noise,
- it **removes all the noise** if there is only signal.

Why wavelet denoising?

Gaussian white noise is by definition equidistributed among all modes and the amplitude of the coefficients is given by its r.m.s.

Therefore the coefficients of a noisy signal whose amplitudes are larger than the r.m.s. of the noise belong to the denoised signal. This procedure corresponds to nonlinear filtering.

The advantage of performing such a nonlinear filtering using the wavelet representation is that wavelet coefficients preserve the local information, in contrast to the Fourier representation.

Since we do not know a priori the r.m.s. of the noise, we have proposed an iterative procedure which assumes as first guess that it is equal to the r.m.s. of the noisy signal.

Iterative algorithm for wavelet denoising

Signal $f =$ Coherent signal + Incoherent noise

Iterative algorithm :

1. The signal f , sampled on N points, is projected on a wavelet basis to obtain the coefficients \tilde{f} .
2. Set $n=1$ and $\tilde{f}_I = \tilde{f}$.
3. Compute the threshold $T_n = \left(2 \langle \tilde{f}_I \rangle^2 \ln N\right)^{\frac{1}{2}}$.
4. The incoherent coefficients \tilde{f}_I are those for which $|\tilde{f}| < T_n$.
5. If $T_n \neq T_{n-1}$ go to 3. and set $n=n+1$, else go to 6.
6. The coherent signal f^C is reconstructed from $|\tilde{f}| \geq T_n$.
The incoherent noise f^I is reconstructed from $|\tilde{f}| < T_n$.

Application to 1D turbulent edge plasma signal

Ion density fluctuations measured in the scrape off layer (SOL) of Tore Supra Tokamak by :

Pascal Devynck
*Association CEA-Euratom,
CEA-DSM-DRFC, Cadarache*

The signal has been recorded at 1 MHz by a **Langmuir probe** located 2.8 cm away from the last closed flux surface, where the **edge safety factor is $q = 6.71$** .

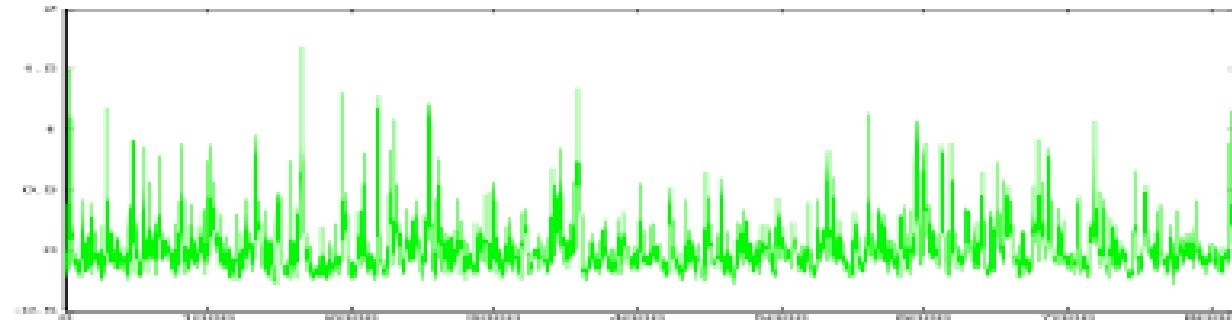
The shot lasted 18s and the signal corresponds to the middle of the plasma current plateau when the **plasma current was $I_p = 0.84$ MA**.

The **mean ion density** was **$\langle n_i \rangle = 1.37$ m⁻³**.

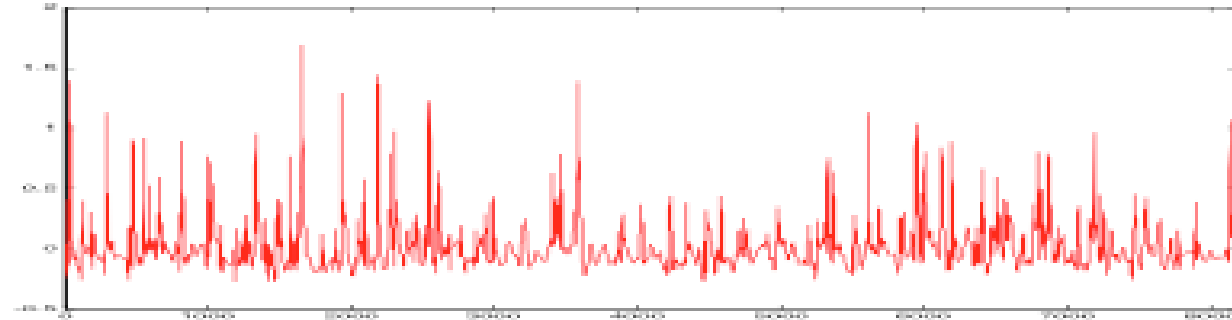
Signal TS 28338

Ion density fluctuations

Total
N=8192



Coherent
5.8% N



Incoherent
94.2% N

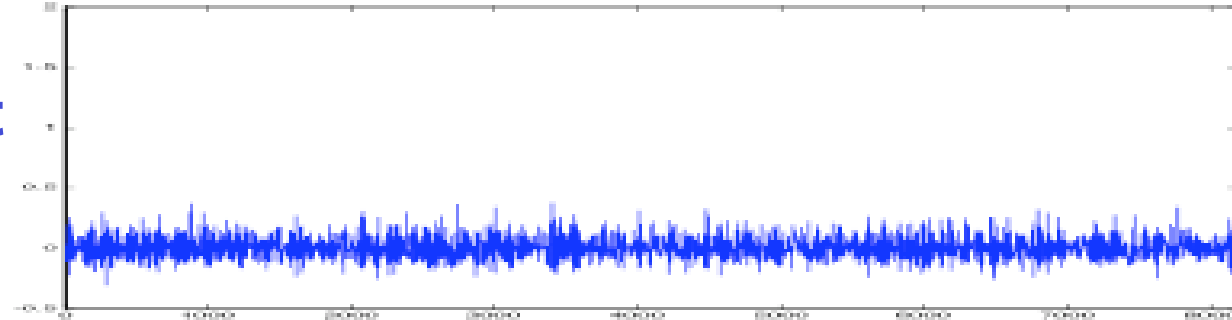


Figure 4: Signal Tore Supra, t28338. Top: total signal S . Middle: coherent part S^C . Bottom: incoherent part S^I .

Signal TS 28338

PDF of ion density

Total
N=8192

Coherent
5.8%N

Incoherent
94.2%N

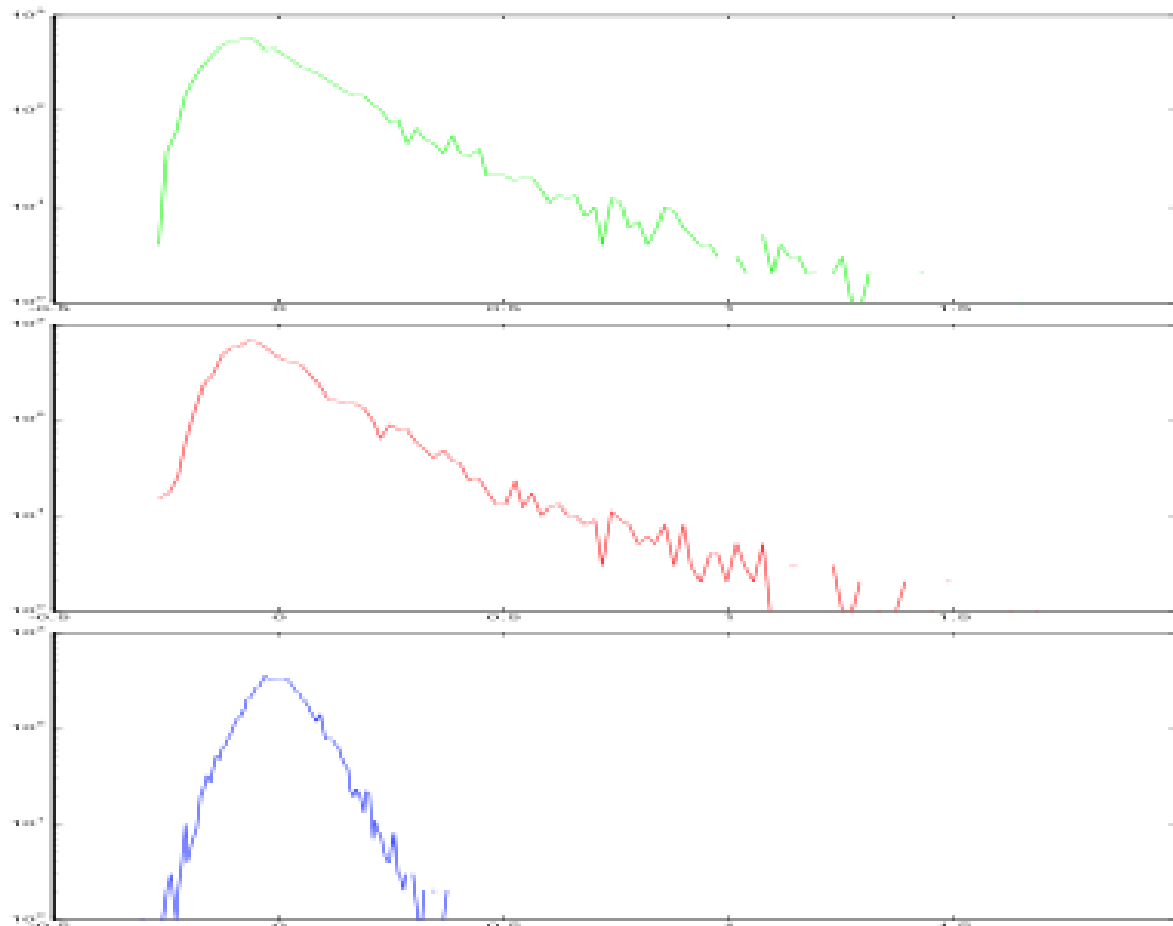
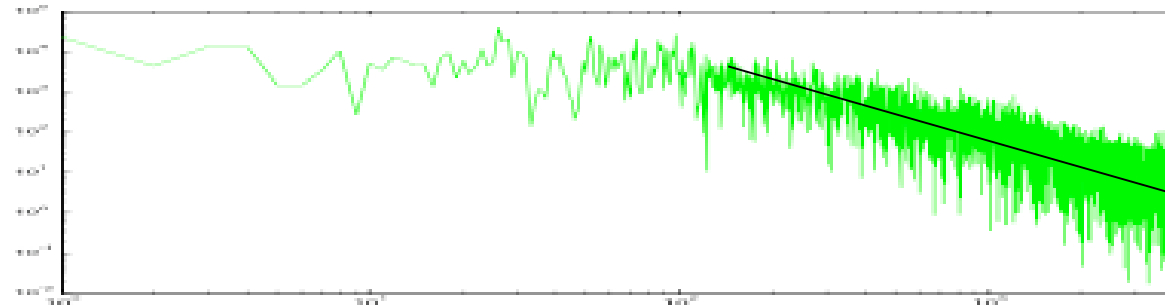


Figure 5: Probability density of signal Tore Supra, t28338. Top: pdf of S . Middle: p S^C . Bottom: pdf S^I .

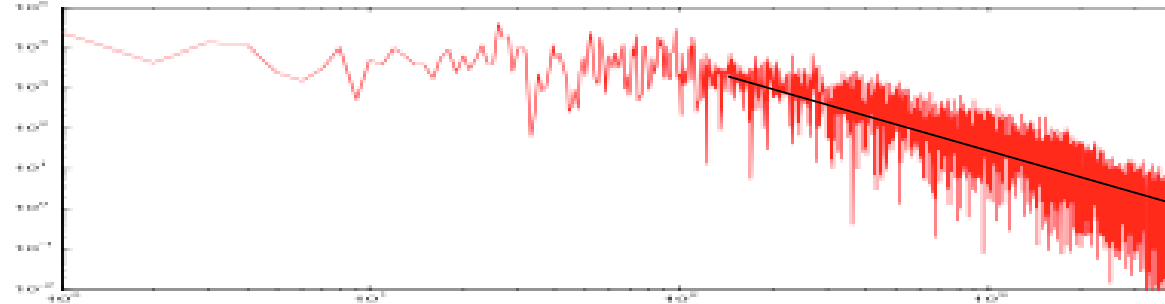
Signal TS 28338

Spectrum of ion density

Total
100% E



Coherent
87% E



Incoherent
13% E

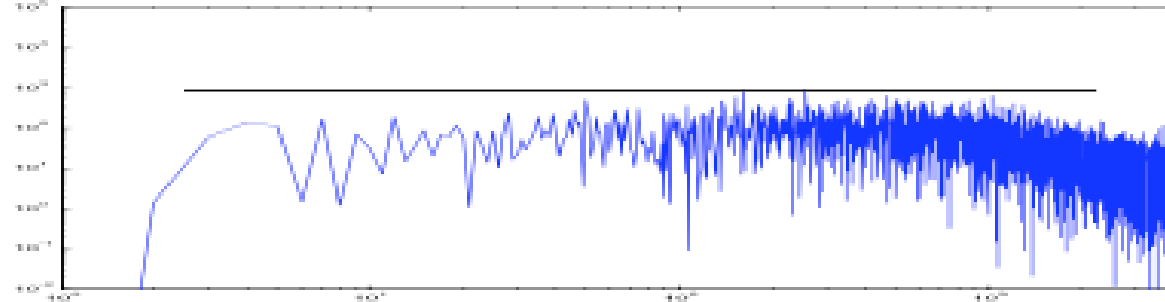


Figure 6: Energy density of signal Tore Supra, t28338. Top: pdf of S . Middle: pdf S' . Bottom: pdf S'' .

Coherent Vortex Extraction

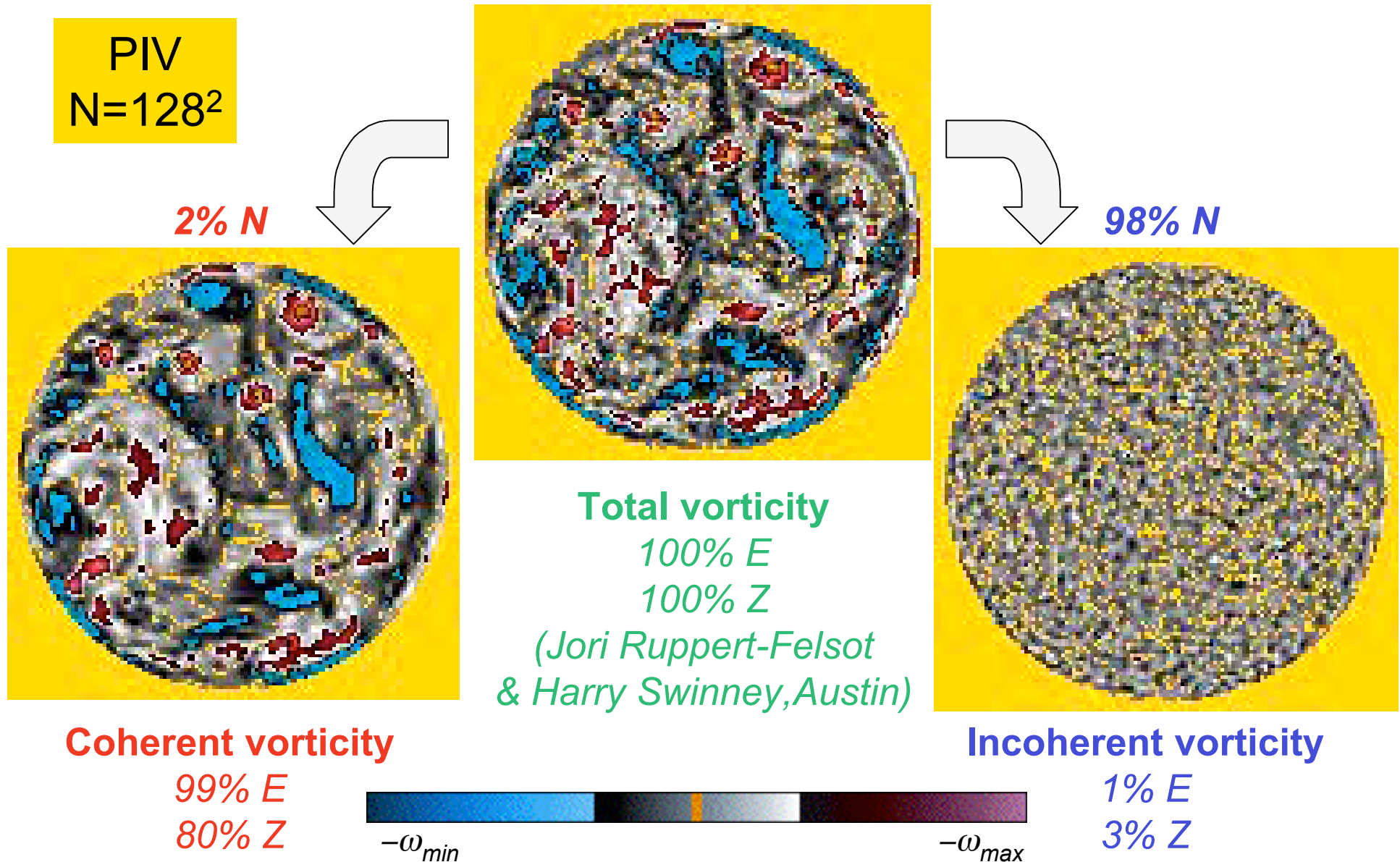
Farge, Schneider, Kevlahan, Phys. Fluids 11(8), 1999
Farge, Pellegrino, Schneider, Phys. Rev. Lett. 87(5), 2001

- Vorticity $\vec{\omega} = \nabla \times \vec{v}$ at resolution $N = 2^{3J}$
- Wavelet transform $\tilde{\omega} = \langle \vec{\omega}, \psi_\lambda \rangle$
- Thresholding: $T = (4/3Z \ln N)^{1/2}$

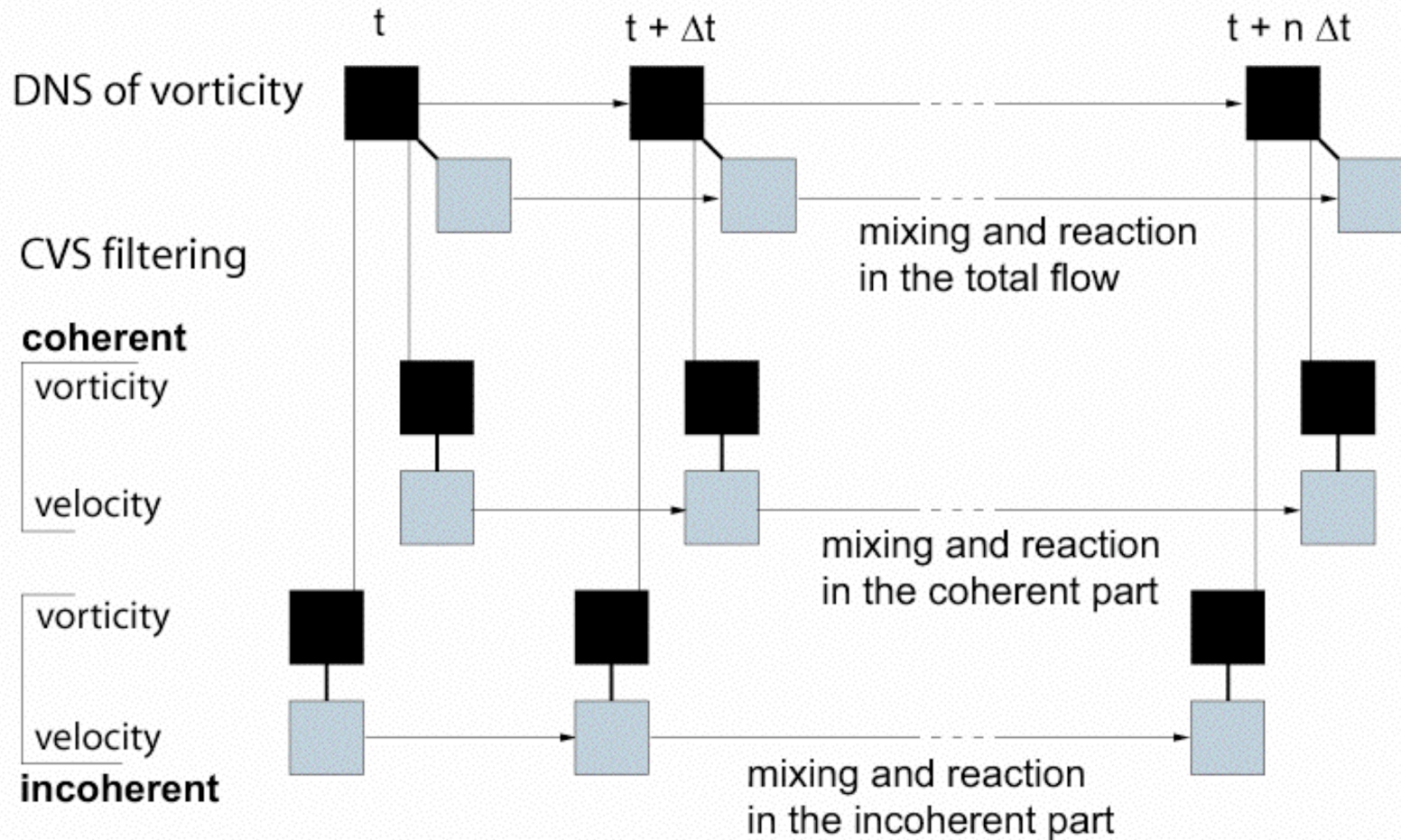
$$\tilde{\omega}_C = \begin{cases} \tilde{\omega} & \text{for } |\tilde{\omega}| \geq T, \\ 0 & \text{for } |\tilde{\omega}| < T \end{cases} \quad \tilde{\omega}_I = \begin{cases} \tilde{\omega} & \text{for } |\tilde{\omega}| < T, \\ 0 & \text{for } |\tilde{\omega}| \geq T \end{cases}$$

- Inverse wavelet transform to reconstruct $\vec{\omega}_C + \vec{\omega}_I = \vec{\omega}$
- Apply Biot-Savart operator to reconstruct $\vec{v}_C + \vec{v}_I = \vec{v}$
with $\vec{v} = \nabla \times \nabla^{-2} \vec{\omega}$
- Remark: $Z = Z_C + Z_I$ (orth. dec.) and $E \approx E_C + E_I$
- Linear complexity, $\mathcal{O}(N)$

2D vortex extraction using wavelet filter in laboratory experiment by PIV



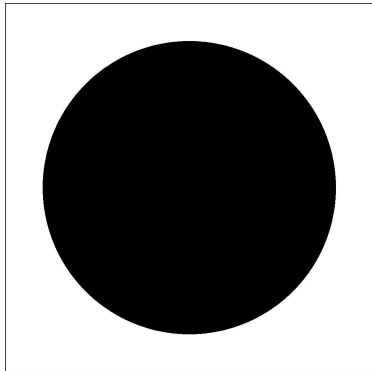
Wavelet filtering of the flow evolution



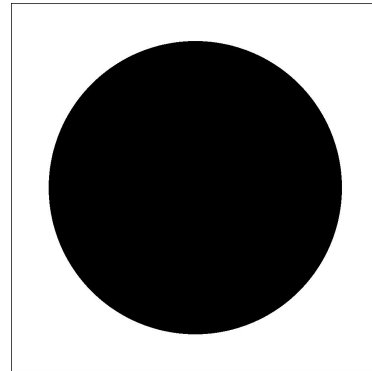
*Beta, Schneider and Farge, 2003,
Wavelet filtering to study mixing in 2D isotropic turbulence
Comm. Nonlin. Sci. Num. Sim., 8 (3-4), 537-545*

Passive scalar advection *in numerical experiment*

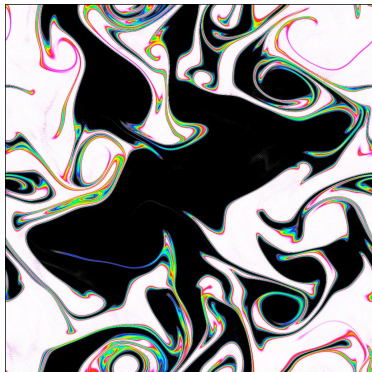
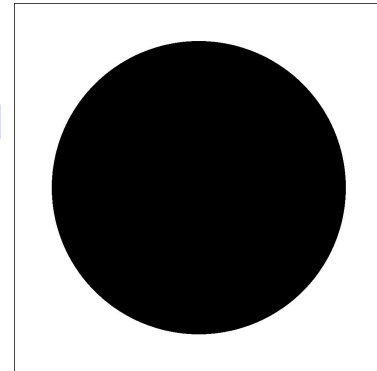
DNS
N=512²



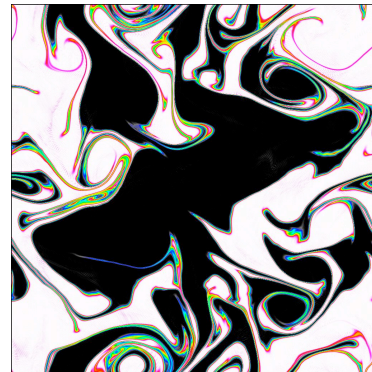
0.2%N
99.8%E
93.6%Z



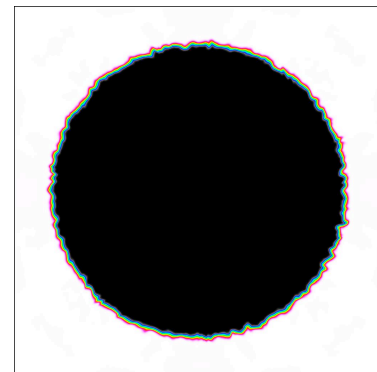
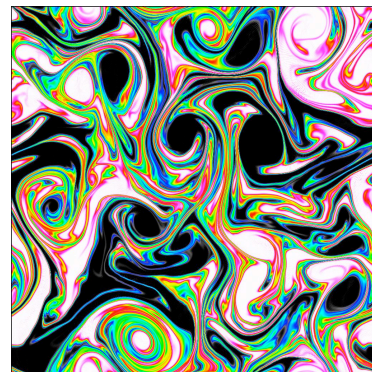
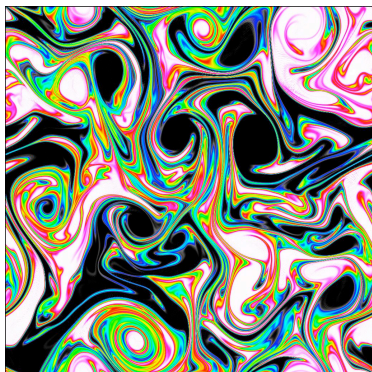
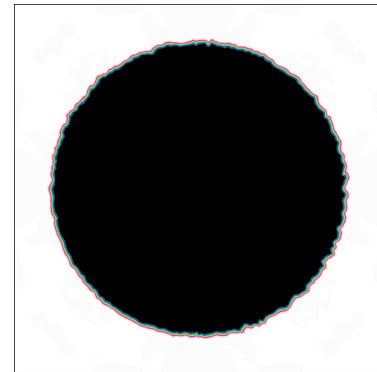
99.8%N
0.2%E
6.4%Z



=



+



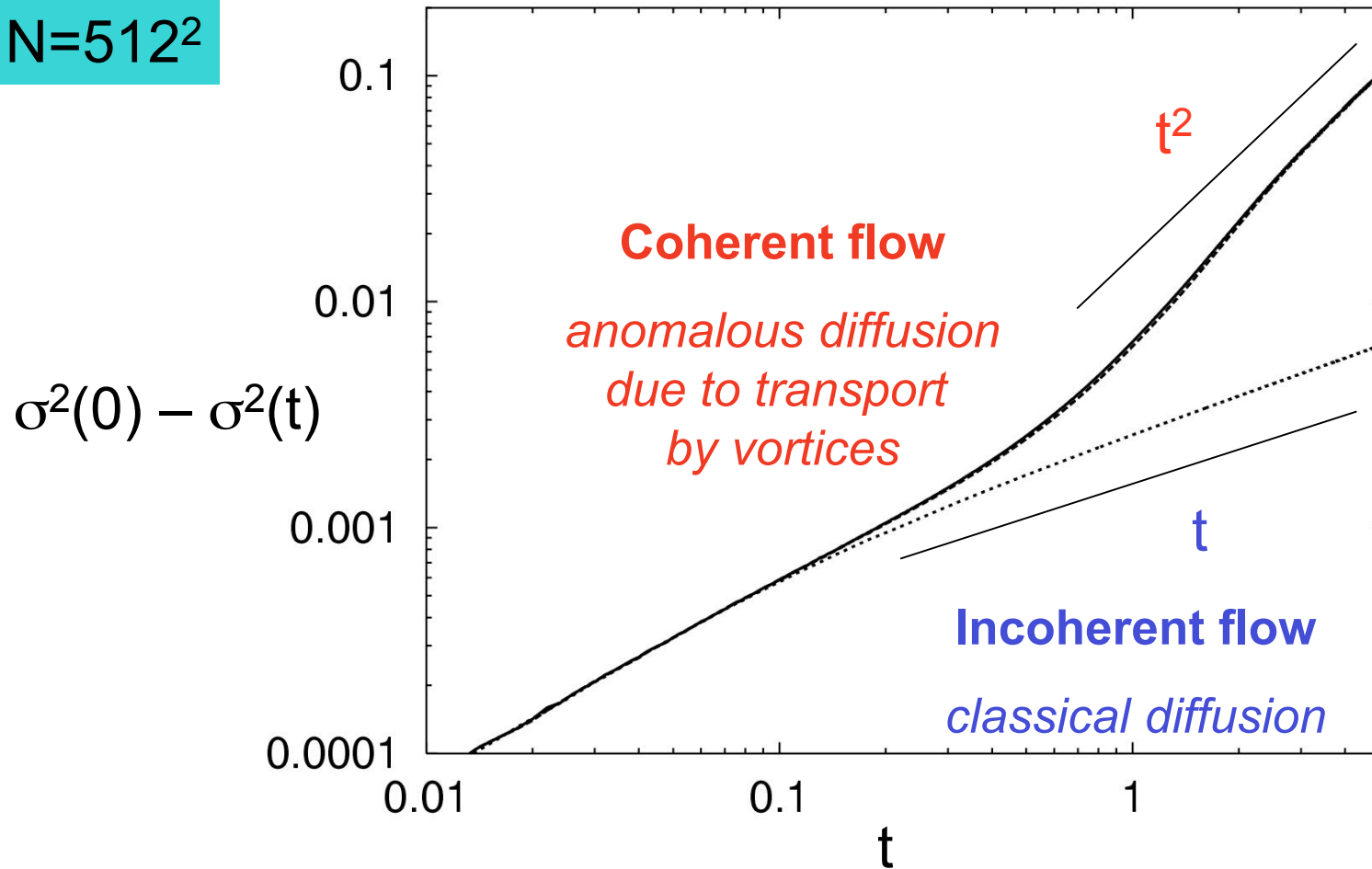
Total flow

Coherent flow

Incoherent flow

Time evolution of the concentration variance *in numerical experiment*

DNS
 $N=512^2$



Advection of tracer particles in numerical experiment

DNS
N=512²

0.2 % of coefficients
99.8 % of kinetic energy
93.6 % of enstrophy

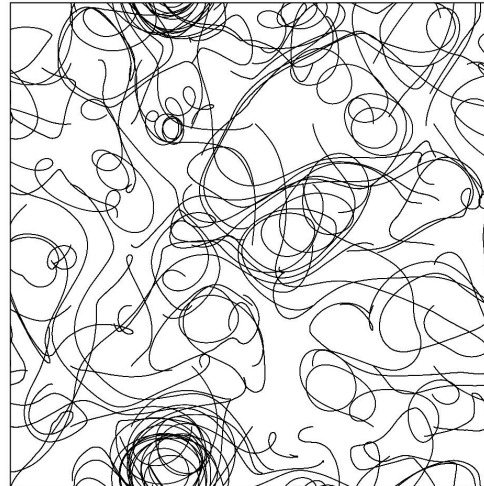
99.8 % of coefficients
0.2 % of kinetic energy
6.4 % of enstrophy

by the total flow



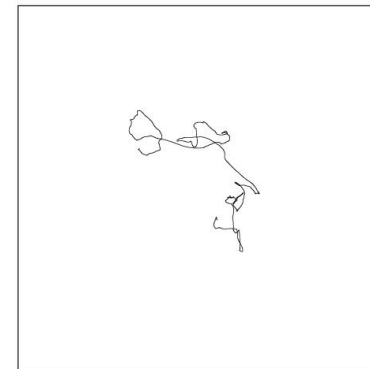
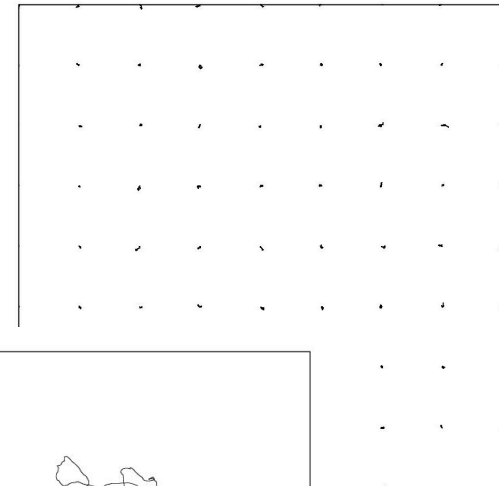
=

by the coherent flow



+

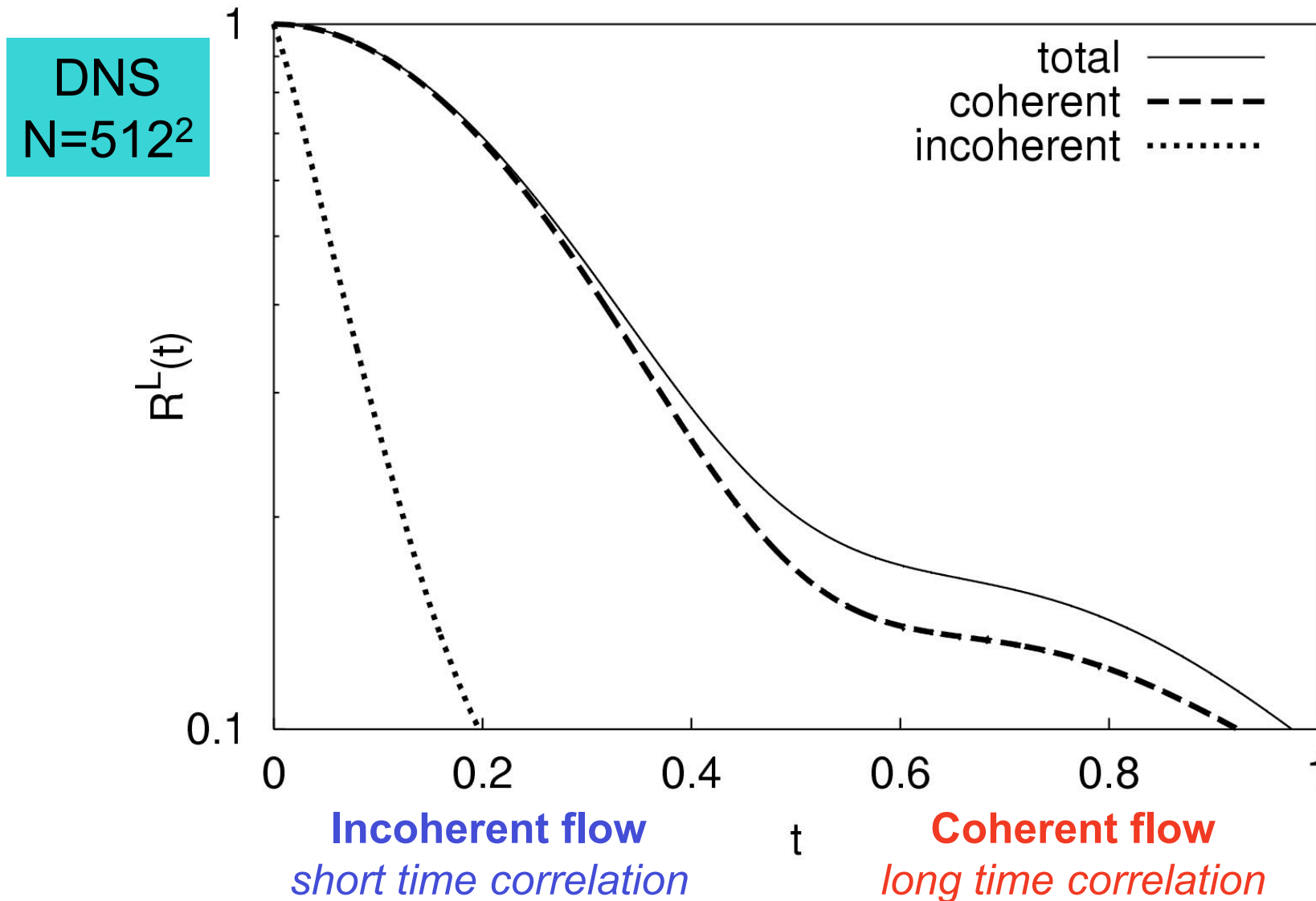
by the incoherent flow



Trapping and transport by the vortices

Diffusion by Brownian motion

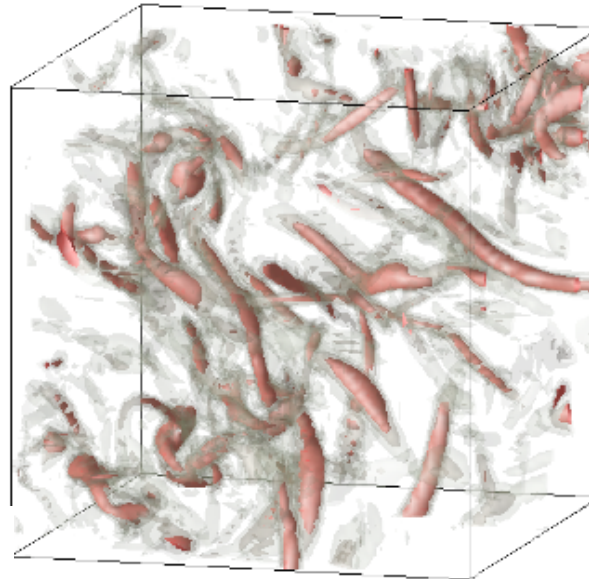
Time correlation of the Lagrangian velocity *in numerical experiment*



Extraction of coherent vortices with wavelet filter *in numerical experiment*

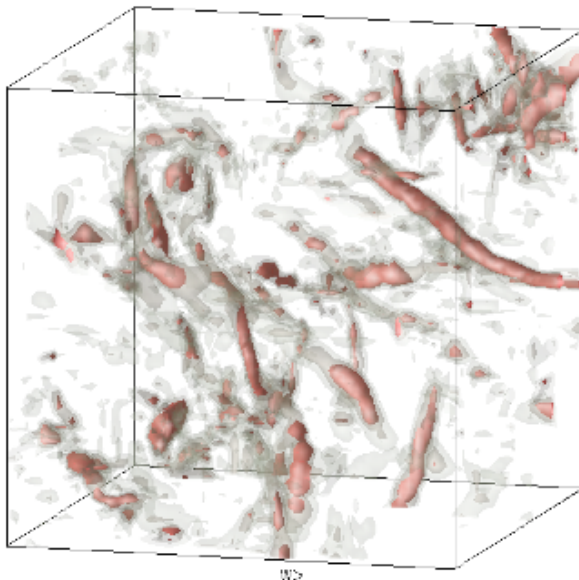
3 % of coefficients
99% of kinetic energy
75% of enstrophy

**Coherent
vorticity**



97 % of coefficients
0.6 % of kinetic energy
25 % of enstrophy

**Incoherent
vorticity**

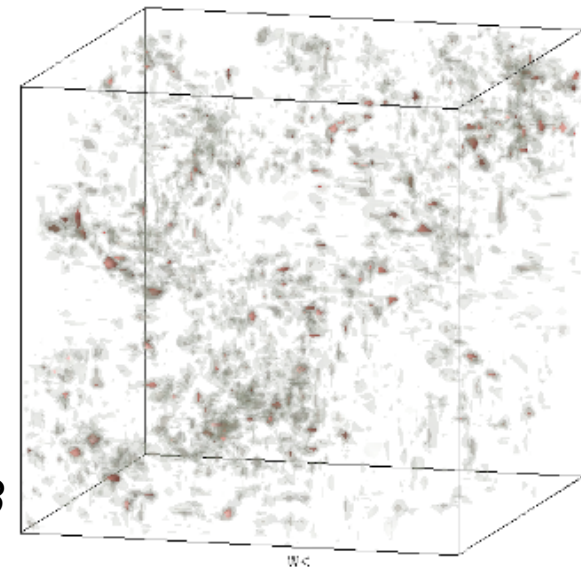


Total vorticity
(Wray, NASA)

$$R_\lambda = 168$$

$$N = 256^3$$

Zoom at 64^3

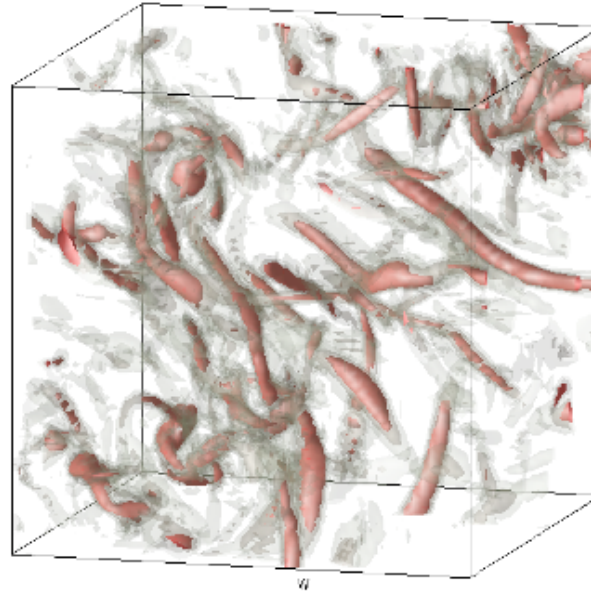
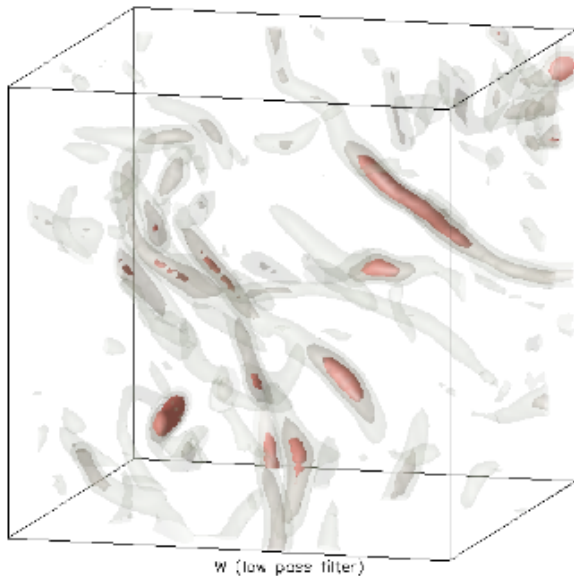


Farge, Schneider et al., 2003
Phys. Fluids, 15 (10), 2886

Extraction of coherent vortices with Fourier filter *in numerical experiment*

3 % of coefficients
99% of kinetic energy
64% of enstrophy

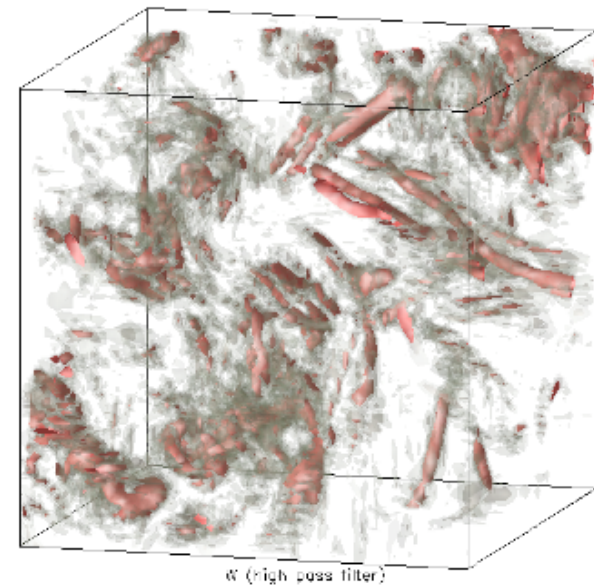
**Large scale
vorticity**



Total vorticity
(Wray, NASA)
 $R_\lambda = 168$
 $N = 256^3$
Zoom at 64^3

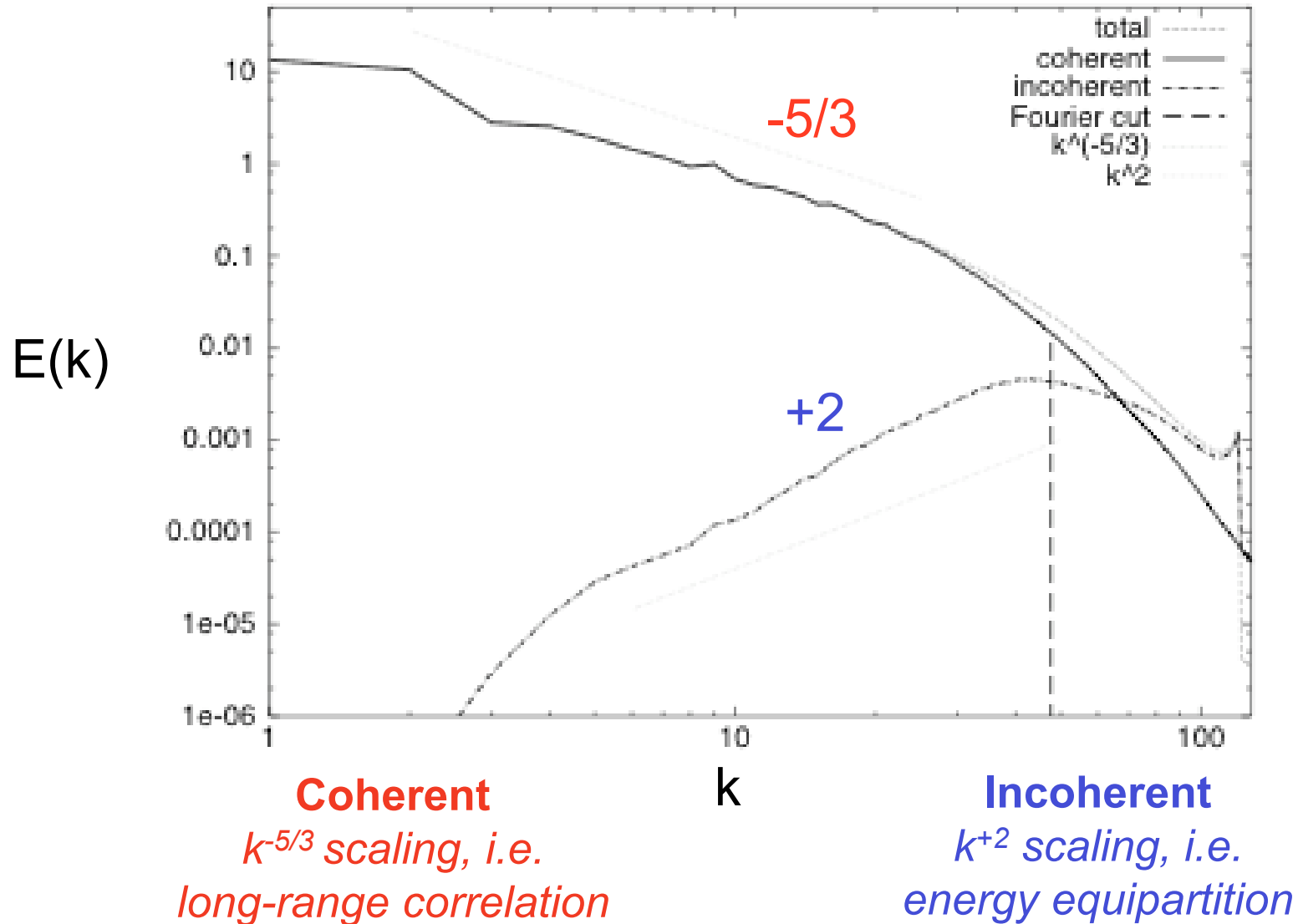
97 % of coefficients
0.9 % of kinetic energy
36 % of enstrophy

**Small scale
vorticity**

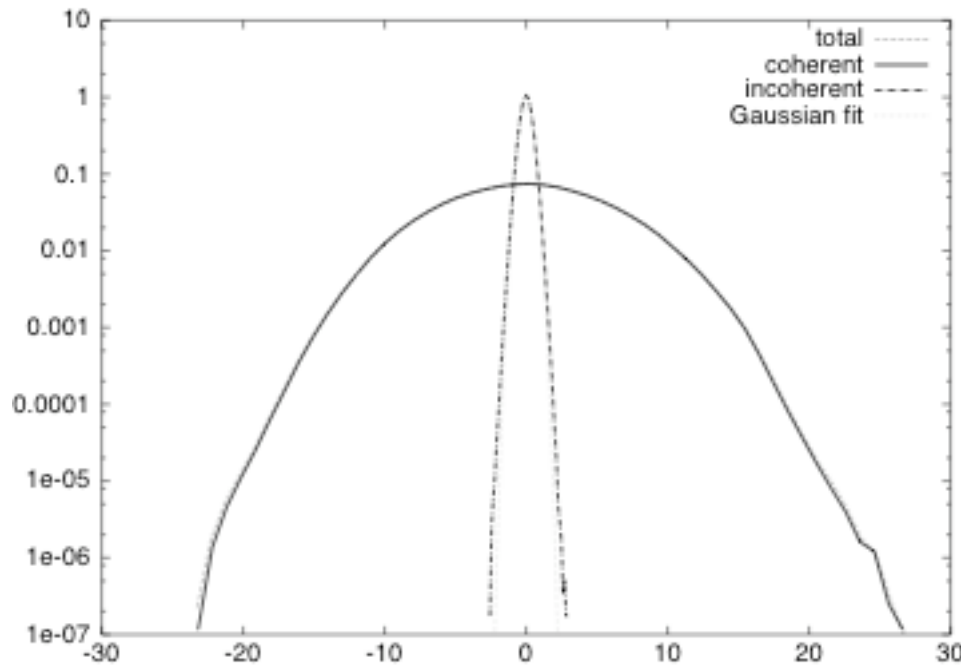


*Farge, Schneider et al., 2003
Phys. Fluids, 15 (10), 2886*

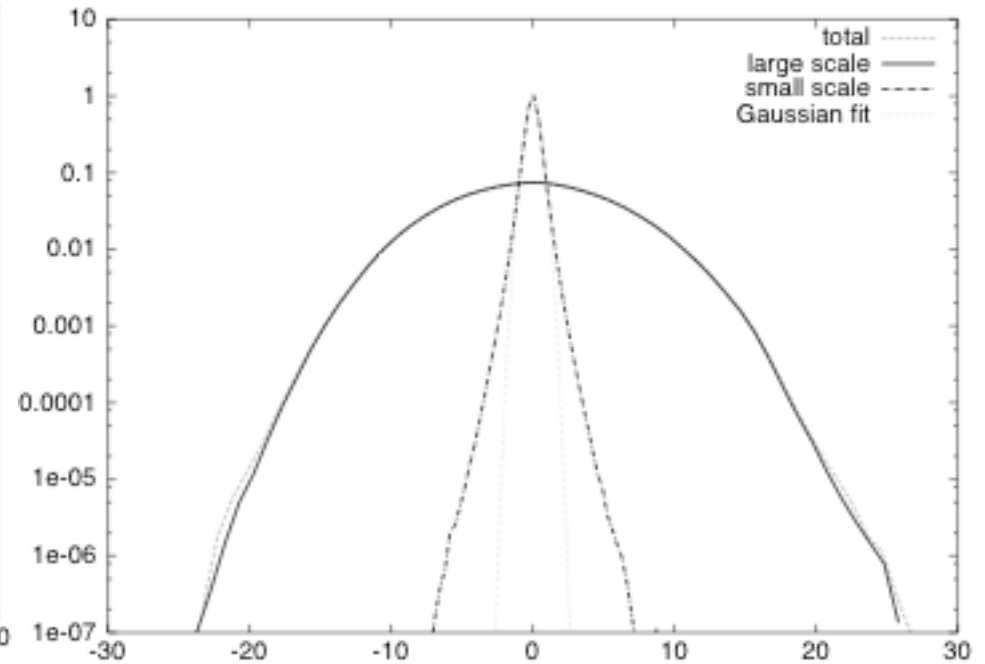
Energy spectrum



PDF of velocity



Wavelet filter
*Gaussian incoherent flow,
i.e. easy to model*

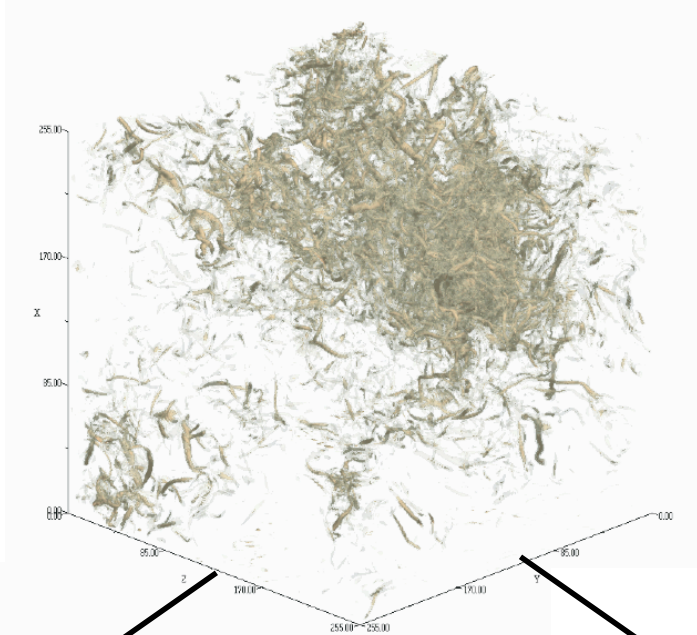


Fourier filter
*non-Gaussian small scales,
i.e. difficult to model*

Extraction of coherent vortices with wavelet filter *in numerical experiment*

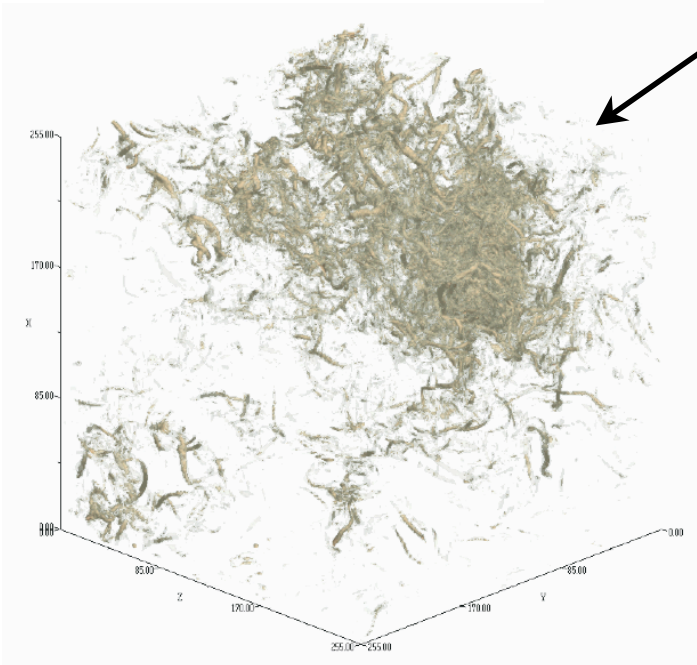
Coherent vorticity

*3.8 % N coefficients
81.7% enstrophy*



Incoherent vorticity

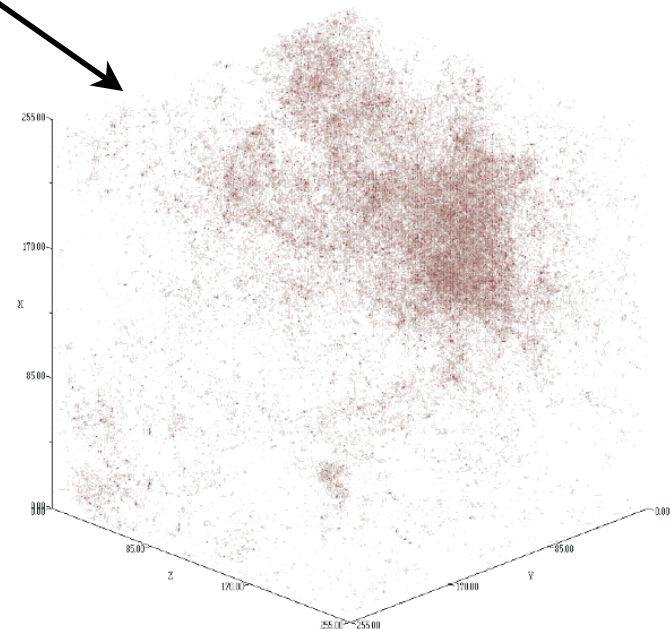
*96.2 % N coefficients
18.3 % enstrophy*



Total vorticity
*(Kaneda et al.,
Nagoya, Japan)*

$R_\lambda = 732$
 $N = 2048^3$
Zoom at 256^3

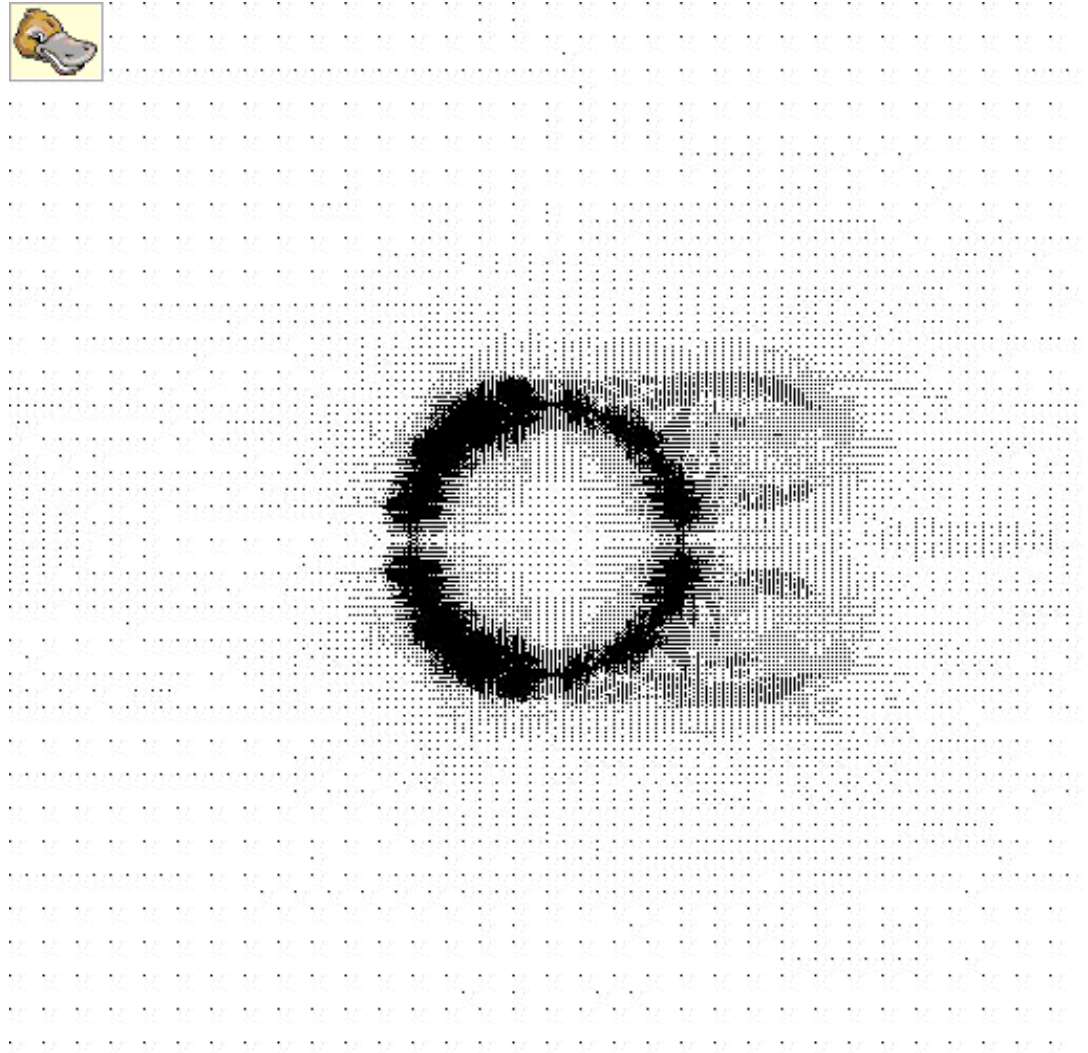
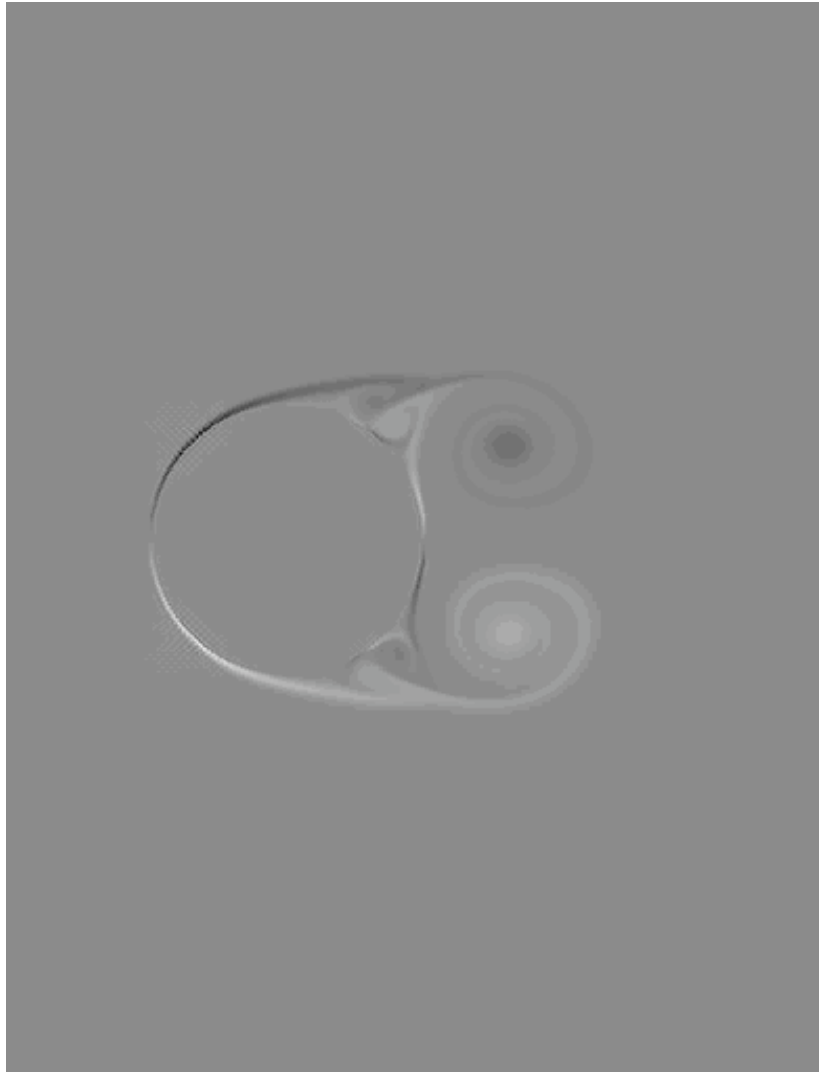
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Conclusion

- ◆ Very few wavelet coefficients retain :
 - ↪ most of energy,
 - ↪ all nonlinearly active scales,
 - ↪ all coherent structures whatever their scales.
- ◆ Orthogonal wavelet decomposition disentangles :
 - ↪ convective transport by the coherent flow,
 - ↪ diffusive transport by the incoherent flow.
- ◆ We conjecture that discarding the incoherent flow is sufficient to model turbulent dissipation
 - ↪ **Coherent Vortex Simulation (CVS)**
which only computes the coherent flow evolution
in a dynamically adapted wavelet basis,
which corresponds to a locally refined grid.

Coherent Vortex Simulation (CVS)



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Coherent Vortex Simulation (CVS), a semi-deterministic turbulence model using wavelets

Flow, Turbulence and Combustion, **66**, 393-426

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