Liability Concentration and Losses in Financial Networks: Comparisons via Majorization

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Systemic Risk in Financial Networks

- Risk that distress or failures of critical financial institutions destabilizes the overall financial system
- The intricate structure of linkages can be naturally captured via a network representation of the financial system
- Foundational work by Eisenberg and Noe (2001) develops a framework for determining payments in a cleared network
- Systemic risk is measured as the length of the domino chain triggered by failure of an entity

Related Literature

- Models of financial networks: Allen and Gale (2001), Eisenberg and Noe (2001), Elsinger et al. (2006)
- Impact of bankruptcy costs: Rogers and Veraart (2012), Glasserman and Young (2014)
- Impact of shocks: Gai and Kapadia (2010), Acemoglu et al. (2014), Elliott et al. (2013), Glasserman and Young (2014)
- Empirical studies: Craig and Von Peter (2014), Cont et al (2012)

Overview of Main Contributions

- Develop a new framework to compare systemic losses under different network topologies
    - Identify balancing and unbalancing systems to bring out the implications of liability concentration on the system's loss profile
    - Relate them to perfectly and imperfectly tiered networks identified by empirical research
    - Validate our framework using data from the European banking network.
- Informing policy making
    - Support regulatory policies of the Basel Committee limiting the size of gross exposures to individual counterparties.

Systemic Losses

- The clearing payment vector p^* is a solution to the fixed point equation (Eisenberg and Noe (2001)):

$$p^* = \ell \wedge (p^* \mathbf{\Pi} + c)$$

ℓ: vector of total liabilities, $\mathbf{\Pi}$: relative liability matrix, c: vector of outside assets.

- Loss vector under the network topology $(\mathbf{\Pi}, \ell, c)$:

$$s(\mathbf{\Pi}, \ell, c) := \ell - p^*(\mathbf{\Pi}, \ell, c)$$

- We can handle bankruptcy costs as in Glasserman and Young (2014), where

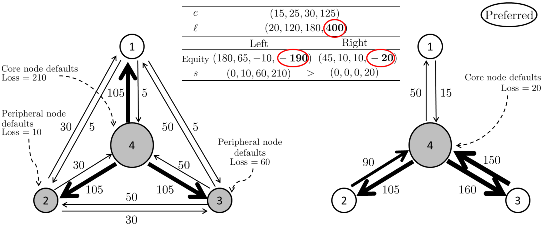
$$p^* = \left([\ell \wedge (p^* \mathbf{\Pi} + c)] - \gamma [\ell - (p^* \mathbf{\Pi} + c)]^+ \right)^+,$$

but assume $\gamma = 0$ for simplicity.

Objective of the Study

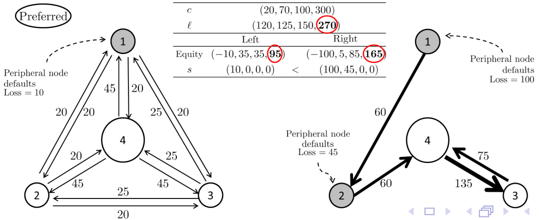
Less concentrated liabilities More concentrated liabilities

c	$(15, 25, 30, 125)$		
ℓ	$(20, 120, 180, \mathbf{400})$		
	Left		Right
Equity	$(180, 65, -10, \mathbf{-190})$		$(45, 10, 10, \mathbf{-20})$
s	$(0, 10, 60, 210)$	$>$	$(0, 0, 0, 20)$



Less concentrated liabilities More concentrated liabilities

c	$(20, 70, 100, 300)$		
ℓ	$(120, 125, 150, \mathbf{270})$		
	Left		Right
Equity	$(-10, 35, 35, \mathbf{95})$		$(-100, 5, 85, \mathbf{165})$
s	$(10, 0, 0, 0)$	$<$	$(100, 45, 0, 0)$



Salient Features

- The node with the smallest equity value generates the largest loss in the system.
- The network with the smallest net exposure to this node is always the most preferred in terms of losses.
- Distinguishing Feature:
  - In the top panels, the undesired system is the network whose liabilities are less concentrated.
  - In the bottom panels, the undesired system is the network with higher concentration of liabilities.
- Our goal:
  - Capture this behavior quantitatively through the concepts of balancing and unbalancing systems

A key insight

Definition

A 3-tuple $(\mathbf{\Pi}_\alpha, \ell, c_\alpha)$, $\alpha \in [0,1)$, is called the α-relaxed equivalent version of a financial system $(\mathbf{\Pi}, \ell, c)$, if $\mathbf{\Pi}_\alpha = (1-\alpha)\mathbf{\Pi} + \alpha\mathbf{I}$ and $c_\alpha = (1-\alpha)c$.

Lemma

Let $(\mathbf{\Pi}, \ell, c)$ be a financial system, then it holds that $p^*(\mathbf{\Pi}, \ell, c) = p^*(\mathbf{\Pi}_\alpha, \ell, c_\alpha)$ for $\alpha \in [0,1)$.

Equilibrium corresponding to a network is also the equilibrium to a whole family of networks

Vector Majorization I

- Let x, y be two vectors. x is *majorized* by y, denoted by $x \prec y$, if

$$\sum_{i=1}^{k} x_{[i]} \leq \sum_{i=1}^{k} y_{[i]} \text{ for } k = 1, \ldots, n-1, \qquad \sum_{i=1}^{n} x_{[i]} = \sum_{i=1}^{n} y_{[i]},$$

or equivalently,

$$\sum_{i=1}^{k} x_{(i)} \geq \sum_{i=1}^{k} y_{(i)} \text{ for } k = 1, \ldots, n-1, \qquad \sum_{i=1}^{n} x_{(i)} = \sum_{i=1}^{n} y_{(i)}.$$

Vector Majorization II

- x is *weakly submajorized* by y, denoted by $x \prec_w y$, if

$$\sum_{i=1}^{k} x_{[i]} \leq \sum_{i=1}^{k} y_{[i]} \text{ for } k = 1, \ldots, n.$$

- x is *weakly supermajorized* by y, denoted by $x \prec^w y$, if

$$\sum_{i=1}^{k} x_{(i)} \geq \sum_{i=1}^{k} y_{(i)} \text{ for } k = 1, \ldots, n.$$

Loss Preferences

Let $x := s(\mathbf{\Pi}^a, \ell, c)$ and $y := s(\mathbf{\Pi}^b, \ell, c)$ be the loss vectors associated with two network systems.

- x is preferred to y if $x \prec_w y$, i.e.

$$\sum_{i=1}^{k} x_{[i]} \leq \sum_{i=1}^{k} y_{[i]} \text{ for } k = 1, \ldots, n.$$

- $k = 1$: maximum loss in a smaller than in b.
- $1 < k < n$: sum of the k largest losses in a smaller than in b
- $k = n$: total loss in a smaller than in b.

Concentration of Liabilities

- Use matrix majorization to compare financial systems in terms of liability concentration
- Let $\mathbf{X}$ and $\mathbf{Y}$ be two matrices. $\mathbf{X}$ is *majorized* by $\mathbf{Y}$, $\mathbf{X} \prec \mathbf{Y}$, if there exists a doubly stochastic matrix $\mathbf{S}$ such that $\mathbf{X} = \mathbf{YS}$.

Definition

Given two financial systems $(\mathbf{\Pi}^a, \ell, c)$ and $(\mathbf{\Pi}^b, \ell, c)$, we say that b has higher liability concentration than a if there exists $\alpha \in [0, 1)$ such that $\mathbf{\Pi}^a_\alpha \prec \mathbf{\Pi}^b_\alpha$.

Concentration of Liabilities: Example I

Less concentrated liabilities

More concentrated liabilities

c	$(15, 25, 30, 125)$	
ℓ	$(20, 120, 180, \mathbf{400})$	
	Left	Right
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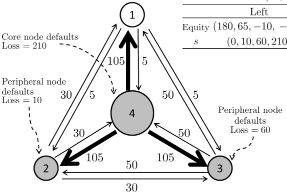
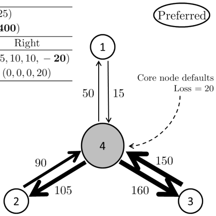
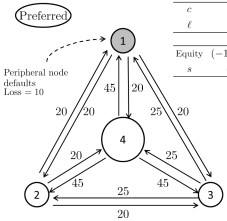




$$\underbrace{\begin{pmatrix} 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 \end{pmatrix}}_{\Pi^a_{0.2}} = \underbrace{\begin{pmatrix} 0.2 & 0 & 0 & 0.6 \\ 0 & 0.2 & 0 & 0.6 \\ 0 & 0 & 0.2 & 0.6 \\ 0.1 & 0.2 & 0.3 & 0.2 \end{pmatrix}}_{\Pi^b_{0.2}} \underbrace{\begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{pmatrix}}_{\mathbf{S}}$$

Concentration of Liabilities: Example II

Less concentrated liabilities



	Left		Right
c	$(20, 70, 100, 300)$		
ℓ	$(120, 125, 150, \mathbf{270})$		
Equity	$(-10, 35, 35, \mathbf{95})$		$(-100, 5, 85, \mathbf{165})$
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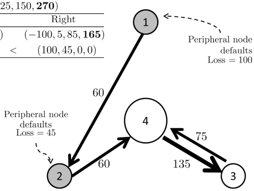
More concentrated liabilities



$$\underbrace{\begin{pmatrix} 0.14 & 0.14 & 0.14 & 0.14 \\ 0.14 & 0.14 & 0.14 & 0.14 \\ 0.14 & 0.14 & 0.14 & 0.14 \\ 0.14 & 0.14 & 0.14 & 0.14 \end{pmatrix}}_{\mathbf{\Pi}^a_{0.14}} = \underbrace{\begin{pmatrix} 0.14 & 0.42 & 0 & 0 \\ 0 & 0.14 & 0 & 0.42 \\ 0 & 0 & 0.14 & 0.42 \\ 0 & 0 & 0.42 & 0.14 \end{pmatrix}}_{\mathbf{\Pi}^b_{0.14}} \underbrace{\begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{pmatrix}}_{\mathbf{S}}$$

Network Setup

- Nodes are labeled so that $\ell_1 \leq \ell_2 \leq \cdots \leq \ell_n$.
- We assume that the outside asset vector c is similarly ordered to the liability vector ℓ. (empirically verified)

Definition

Two vectors x and y are similarly ordered if

$$(x_i - x_j)(y_i - y_j) \geq 0 \text{ for all } i, j.$$

Balancing and Unbalancing financial systems I

Definition

(I) $(\mathbf{\Pi}, \ell, c)$ is balancing if, for $j = 1, \ldots, n-1$,

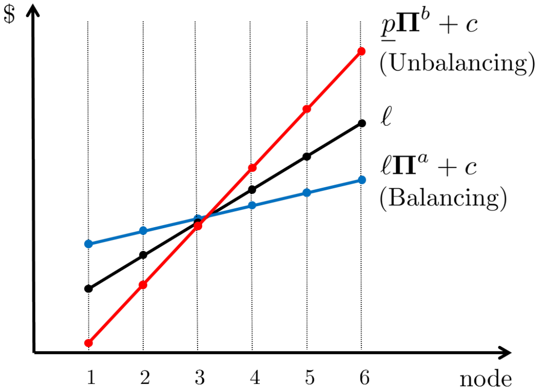
$$\underbrace{\left[\sum_{i=1}^{n} \ell_i \pi_{i,j+1} + c_{j+1}\right] - \ell_{j+1} \leq \left[\sum_{i=1}^{n} \ell_i \pi_{i,j} + c_j\right] - \ell_j}_{\text{equity of node } j+1 \,\leq\, \text{equity of node } j \text{ under the best-case scenario}} \quad .$$

(II) $(\mathbf{\Pi}, \ell, c)$ is unbalancing if, for $j = 1, \ldots, n-1$,

$$\underbrace{\left[\sum_{i=1}^{n} \underline{p}_i \pi_{i,j+1} + c_{j+1}\right] - \underline{p}_{j+1} \geq \left[\sum_{i=1}^{n} \underline{p}_i \pi_{i,j} + c_j\right] - \underline{p}_j}_{\text{equity of node } j+1 \,\geq\, \text{equity of node } j \text{ under the worst-case scenario}} \quad ,$$

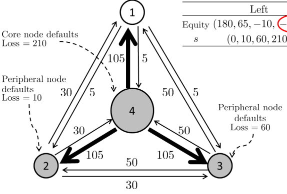
where $\underline{p}$ is a lower bound on the clearing payment vector in a class of unbalancing systems.

Balancing and Unbalancing financial systems II



Balancing system

Less concentrated liabilities



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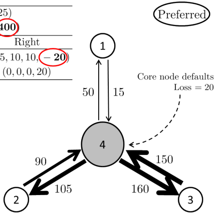
More concentrated liabilities

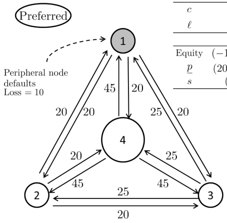


$$\underbrace{\left[\sum_{i=1}^{n} \ell_i \pi_{i,j+1} + c_{j+1}\right] - \ell_{j+1} \leq \left[\sum_{i=1}^{n} \ell_i \pi_{i,j} + c_j\right] - \ell_j}_{\text{equity of node } j+1 \,\leq\, \text{equity of node } j \text{ under the best-case scenario}} \quad .$$

- When every node repays its liabilities in full, a node with a larger liability (ℓ_{j+1}) will have a smaller equity after clearing

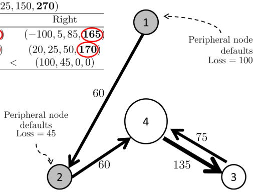
Unbalancing system

Less concentrated liabilities



More concentrated liabilities

c	(20, 70, 100, 300)		
ℓ	(120, 125, 150, **270**)		
	Left		Right
Equity	(−10, 35, 35, **95**)		(−100, 5, 85, **165**)
$\underline{p}$	(20, 25, 50, **170**)		(20, 25, 50, **170**)
s	(10, 0, 0, 0)	<	(100, 45, 0, 0)



$$\underbrace{\left[\sum_{i=1}^{n} \underline{p}_i \pi_{i,j+1} + c_{j+1}\right] - \underline{p}_{j+1} \geq \left[\sum_{i=1}^{n} \underline{p}_i \pi_{i,j} + c_j\right] - \underline{p}_j}_{\text{equity of node } j+1 \,\geq\, \text{equity of node } j \text{ under the worst-case scenario}} \quad .$$

- If a node makes a larger payment ($\underline{p}_{j+1}$), in the worst-case bankruptcy scenario, then it also has a larger equity after clearing

Order and majorization preserving relations

- $\mathbf{D}$ is order preserving w.r.t. $\mathcal{P}$ if $x\mathbf{D}$ is similarly ordered to x for any $x \in \mathcal{P}$.
- $\mathbf{D}$ is *weak submajorization preserving* w.r.t $\mathcal{P}$ if for $x, y \in \mathcal{P}$, $x \prec_w y$ implies $x\mathbf{D} \prec_w y\mathbf{D}$.
- $\mathbf{D}$ is *weak supermajorization preserving* w.r.t $\mathcal{P}$ if for $x, y \in \mathcal{P}$, $x \prec^w y$ implies $x\mathbf{D} \prec^w y\mathbf{D}$.

Define

$$\mathcal{P} = \{p \,|\, p \text{ is similarly ordered to } \ell, \mathbf{0} \leq p \leq \ell\},$$

which identifies a large class of payment vectors, where absolutely priority and limited liability can be violated.

Relaxation and order preserving

- The presence of zero elements on the diagonal of relative liability matrices drastically reduces the set of order-preserving matrices.
- But, ... the relaxed equivalent version enlarges the set.

$$\underbrace{\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}}_{x} \underbrace{\begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{pmatrix}}_{\mathbf{\Pi}} = \begin{pmatrix} 2.5 & 2 & 1.5 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}}_{x} \underbrace{\begin{pmatrix} 0.5 & 0.25 & 0.25 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}}_{\mathbf{\Pi}_{0.5}} = \begin{pmatrix} 1.75 & 2 & 2.25 \end{pmatrix}.$$

Equivalent Characterizations

Lemma

The following statements hold:

- $\mathbf{D} \in \mathbb{R}^{n\times n}$ *is weak submajorization preserving w.r.t.* $\mathcal{P}$ *iff*

$$\sum_{j=k}^{n} d_{i,j}^{\mu(p)} \leq \sum_{j=k}^{n} d_{i+1,j}^{\mu(p)} \qquad k = 1, \dots, n, i = 1, \dots, n-1, \text{ for all } p \in \mathcal{P}$$

- $\mathbf{D} \in \mathbb{R}^{n\times n}$ *is weak supermajorization preserving w.r.t.* $\mathcal{P}$ *iff*

$$\sum_{j=1}^{k} d_{i,j}^{\mu(p)} \geq \sum_{j=1}^{k} d_{i+1,j}^{\mu(p)} \qquad k = 1, \dots, n, i = 1, \dots, n-1, \text{ for all } p \in \mathcal{P}$$

$\mu(p)$ is defined as $\mu_k(p) = p_{(k)}$, and $d_{i,j}^{\mu(p)} := d_{\mu_i(p),\mu_j(p)}$. Set $D := \mathbf{\Pi}_\alpha$ and $p = \ell$

- $D := \mathbf{\Pi}_\alpha$ weak submajorization preserving: nodes with high liabilities are more liable to nodes with higher liabilities
- $D := \mathbf{\Pi}_\alpha$ weak supermajorization preserving: nodes with high liabilities are less liable to nodes with smaller liabilities

Clearing payments, Liabilities and Losses

Proposition

Suppose $\mathbf{\Pi}_\alpha$ is order preserving w.r.t. to $\mathcal{P}$ for some $\alpha \in [0,1)$. Then,

(I) *p^* is similarly ordered to ℓ.*

(II) *If $(\mathbf{\Pi}, \ell, c, \gamma)$ is balancing, then $\ell_n - p_n^* \geq \ell_{n-1} - p_{n-1}^* \geq \cdots \geq \ell_1 - p_1^*$.*

(III) *If $(\mathbf{\Pi}, \ell, c, \gamma)$ is unbalancing, then $\ell_1 - p_1^* \geq \ell_2 - p_2^* \geq \cdots \geq \ell_n - p_n^*$.*

- Nodes with larger liabilities make larger payments.
- Balancing: larger losses by nodes with higher liabilities.
- Unbalancing: larger losses by nodes with smaller liabilities.

Unbalancing Systems: Liability Concentration and Losses I

Theorem

Let $(\mathbf{\Pi}^a, \ell, c, \gamma), (\mathbf{\Pi}^b, \ell, c, \gamma)$ *be unbalancing. Suppose there exists* $\alpha \in [0, 1)$ *such that both* $\mathbf{\Pi}^a_\alpha$ *and* $\mathbf{\Pi}^b_\alpha$ *are order preserving w.r.t.* $\mathcal{P}$ *and*

(I) $\mathbf{\Pi}^a_\alpha$ *or* $\mathbf{\Pi}^b_\alpha$ *is weak supermajorization preserving w.r.t.* $\mathcal{P}$,

(II) $\mathbf{\Pi}^a_\alpha \prec \mathbf{\Pi}^b_\alpha$.

Then

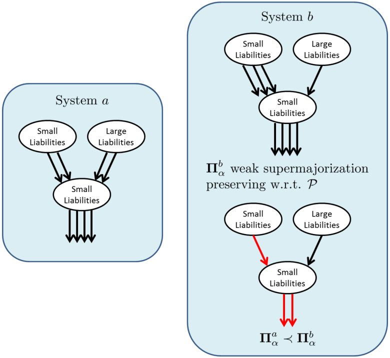
$$p^{a*}(\mathbf{\Pi}^a, \ell, c) \prec^w p^{b*}(\mathbf{\Pi}^b, \ell, c)$$

and

$$s(\mathbf{\Pi}^a, \ell, c) \prec_w s(\mathbf{\Pi}^b, \ell, c).$$

Unbalancing Systems: Liability Concentration and Losses II

- From the proposition, the largest losses occur at nodes with small liabilities in an unbalancing system
- Such losses are larger in the system b with higher liability concentration:



Balancing Systems: Liability Concentration and Losses I

Theorem

Let $(\mathbf{\Pi}^a, \ell, c)$ *and* $(\mathbf{\Pi}^b, \ell, c)$ *be balancing. Suppose there exists* $\alpha \in [0,1)$ *such that both* $\mathbf{\Pi}^a_\alpha$ *and* $\mathbf{\Pi}^b_\alpha$ *are order preserving w.r.t.* $\mathcal{P}$ *and*

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(II) $\mathbf{\Pi}^a_\alpha \prec \mathbf{\Pi}^b_\alpha$.

Then,

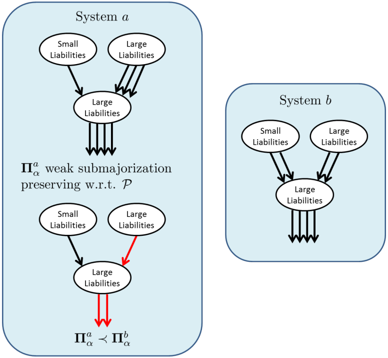
$$p^{a*}(\mathbf{\Pi}^a, \ell, c, \gamma) \prec_w p^{b*}(\mathbf{\Pi}^b, \ell, c, \gamma)$$

and

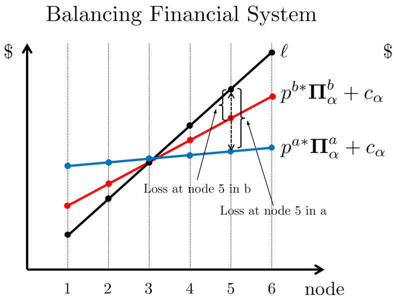
$$s(\mathbf{\Pi}^a, \ell, c, \gamma) \succ_w s(\mathbf{\Pi}^b, \ell, c, \gamma).$$

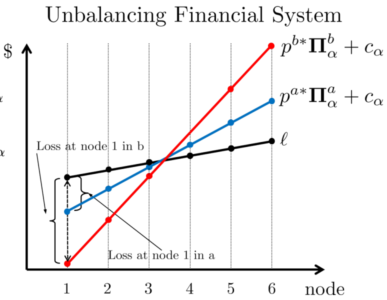
Balancing Systems: Liability Concentration and Losses II

- From the proposition, the largest losses occur at nodes with large liabilities in a balancing system
- Such losses are larger in the system *a* with lower liability concentration:



Liability Concentration and Losses





Tiered networks

Consider *core-periphery financial systems*: core nodes significantly larger than peripheral nodes. (Craig and Von Peter (2014))

$$\mathbf{\Pi}^a = \underbrace{\begin{pmatrix} 0 & \pi^a_{12} & \pi^a_{13} & \pi^a_{14} \\ \pi^a_{21} & 0 & \pi^a_{23} & \pi^a_{24} \\ \pi^a_{31} & \pi^a_{32} & 0 & \pi^a_{34} \\ \pi^a_{41} & \pi^a_{42} & \pi^a_{43} & 0 \end{pmatrix}}_{\text{imperfectly tiered}} \quad \mathbf{\Pi}^b = \underbrace{\begin{pmatrix} 0 & 0 & 0 & \pi^b_{14} \\ 0 & 0 & 0 & \pi^b_{24} \\ 0 & 0 & 0 & \pi^b_{34} \\ \pi^b_{41} & \pi^b_{42} & \pi^b_{43} & 0 \end{pmatrix}}_{\text{perfectly tiered}}$$

- Perfectly tiered tend to have higher liability concentration than imperfectly tiered systems.

Perfect Tiered v.s. Imperfectly Tiered

- When both are unbalancing, the imperfectly tiered is preferred
  - Losses occur at peripheral nodes.
  - Imperfectly tiered: both periphery and core pay to periphery.
  - Perfectly tiered: periphery only receives payments from core.
  - Larger losses in perfectly tiered networks.
- When both are balancing, perfectly tiered is preferred
  - Losses occur at core nodes.
  - Imperfectly tiered: periphery makes payments both to core and periphery.
  - Perfectly tiered: periphery only makes payments to core.
  - Larger losses in imperfectly tiered network.

Data Sources

- Consider financial system induced by the banking sectors of eight representative European countries
- These countries account for 80% of the total liabilities of the European banking sector
- Consolidated banking data released from the European Central Bank and foreign claims data from the BIS to estimate parameters of the financial system.

Banks' consolidated foreign claims (BIS)

December 2009	(UK)	(Germany)	(France)	(Spain)	(Netherland)	(Ireland)	(Belgium)	(Portugal)
(UK)	0.00	500.62	341.62	409.36	189.95	231.97	36.22	10.43
(Germany)	172.97	0.00	292.94	51.02	176.58	36.35	20.52	4.62
(France)	239.17	195.64	0.00	50.42	92.73	20.60	32.57	8.08
(Spain)	114.14	237.98	219.64	0.00	119.73	30.23	26.56	28.08
(Netherland)	96.69	155.65	150.57	22.82	0.00	15.47	28.11	11.39
(Ireland)	187.51	183.76	60.33	15.66	30.82	0.00	64.50	21.52
(Belgium)	30.72	40.68	301.37	9.42	131.55	6.11	0.00	1.17
(Portugal)	24.26	47.38	44.74	86.08	12.41	5.43	3.14	0.00

June 2010	(UK)	(Germany)	(France)	(Spain)	(Netherland)	(Ireland)	(Belgium)	(Portugal)
(UK)	0.00	462.07	327.72	386.37	135.37	208.97	43.14	7.72
(Germany)	172.18	0.00	255.00	39.08	149.82	32.11	20.93	3.93
(France)	257.11	196.84	0.00	26.26	80.84	18.11	29.70	8.21
(Spain)	110.85	181.65	162.44	0.00	72.67	25.34	18.75	23.09
(Netherland)	141.39	148.62	126.38	20.66	0.00	12.45	23.14	11.11
(Ireland)	148.51	138.57	50.08	13.98	21.20	0.00	53.99	19.38
(Belgium)	29.15	35.14	253.13	5.67	108.68	5.32	0.00	0.39
(Portugal)	22.39	37.24	41.90	78.29	5.13	5.15	2.57	0.00

Table : All values are in USD billion.

Consolidated banking sector data (ECB)

December 2009 (in USD billion)				
Country	Assets	Liabilities	Equity†	c
UK	13,833	13,204	674	12,849
Germany	12,366	11,901	504	10,557
France	9,053	8,616	472	7,155
Spain	5,350	5,024	374	4,545
Netherlands	3,795	3,632	213	2,747
Ireland	1,919	1,828	149	1,446
Belgium	1,706	1,629	133	1,427
Portugal	732	686	104	627

June 2010 (in USD billion)				
Country	Assets	Liabilities	Equity†	c
UK	13,956	13,258	736	12,982
Germany	11,533	11,126	443	9,936
France	8,485	8,077	439	6,864
Spain	4,765	4,482	325	4,052
Netherlands	3,506	3,366	184	2,715
Ireland	1,758	1,678	129	1,337
Belgium	1,530	1,464	116	1,276
Portugal	654	615	90	563

†The equity under the worst case payment scenario

Unbalancing states are persistent

- $(\mathbf{\Pi}, \ell, c)$ is unbalancing if, for $j = 1, \dots, n-1$,

$$\underbrace{\left[\sum_{i=1}^{n} \underline{p}_i \pi_{i,j+1} + c_{j+1}\right] - \underline{p}_{j+1} \geq \left[\sum_{i=1}^{n} \underline{p}_i \pi_{i,j} + c_j\right] - \underline{p}_j}_{\text{equity of node } j+1 \,\geq\, \text{equity of node } j \text{ under the worst-case scenario}} \quad ,$$

	Degree of unbalance
Dec-2008	71%
Dec-2009	100%
Jun-2010	100%
Dec-2010	86%
Jun-2011	86%
Dec-2011	86%
Jun-2012	71%
Dec-2012	71%
Jun-2013	86%
Jun-2014	86%

Policy Implications

- Empirical evidence suggest that real-world networks are most likely to be in an unbalancing state.
- Higher concentration of liabilities induce larger systemic losses in unbalancing systems
- Desirable for regulatory purposes to prevent high concentration of liabilities in the network.
- Support the supervisory framework put forward by the Basel Committee aiming at limiting the size of gross exposures to individual counterparties.

Conclusion

- New framework to quantify the impact of liability concentration on systemic losses.
- Loss preferences expressed via vector majorization. Liability concentration captured by matrix majorization.
- Balancing and unbalancing systems bring out the qualitatively different implication of liability concentration on systems's loss profile
- Empirical analysis suggests that real-world networks are unbalancing or close to it, persistently over time.
- Support regulatory policies of Basel Committee aiming at reducing gross exposures to individual counterparties.

Reference

Capponi, Agostino and Chen, Peng-Chu and Yao, David D. (2014) Liability Concentration and Losses in Financial Networks: Comparisons via Majorization. *Available at SSRN: http://ssrn.com/abstract=2517755.*