Energy & Commodity Markets III

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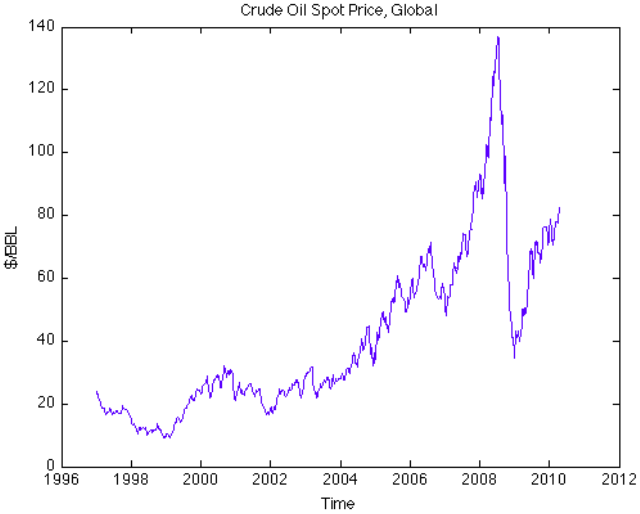
Overview

1. Traditional commodity markets issues and models. Electricity price models.
2. Energy production from exhaustible resources and renewables: game threoretic models.
3. Financialization of commodity markets.

Background

- Tang & Xiong (2009) report empirical evidence of commodity & fuel prices moving more in sync with financial markets throughout the 2000s, and in contrast to previously.
- They conjecture the financialization of commodity markets largely due to influx of external traders: commodity index funds, ETFs (e.g. on silver), etc.
- Commodities a significant part of hedge fund (and others') portfolios. Their trading is governed by portfolio diversification, speculation, momentum/mean-reversion strategies in contrast to supply/demand needs of "traditional" commodities traders.
- Example: hedge funds trading lean hogs to bet on a weak US $. Puts the oddities in commodities markets.

Oil Prices 1997-2011



Source: U.S. Energy Information Administration
http://tonto.eia.doe.gov/dnav/pet/hist/LeafHandler.ashx?n=PET&s=WTOTWORLD&f=W

Politicization: Bernie Sanders on CNN.com, 2/28/12

- *“Gas prices approaching $4 a gallon on average are causing severe economic pain for millions of Americans. Pump prices spiked 5% in the past month alone. Crude oil prices: $108”*
- *“What’s the cause? Forget what you may have read about the laws of supply and demand. Oil and gas prices have almost nothing to do with economic fundamentals.”*
- *“Is Big Oil to blame? Sure. Partly. Big oil companies have been gouging consumers for years.”*
- *“But there’s another reason for the wild rise in gas prices. The culprit is Wall Street. Speculators are raking in profits by gambling in the loosely regulated commodity markets”*
- *“A decade ago, speculators controlled only about 30% of the oil futures market. Today, Wall Street speculators control nearly 80% of this market.”*

Table: Correlations between the commodities and the S&P 500: 1990-2004 & 2004-11.

Commodity, correlated to S&P	non-indexed period	indexed period
CBOT Corn Futures	0.0099	0.2231
CSCE Cocoa Futures	-0.0182	0.1473
NYMEX Crude Oil Futures	-0.0546	0.3287
NYCE Cotton Futures	0.0116	0.1511
CSCE Coffee Futures	0.0290	0.1928
NYMEX Natural Gas Futures	-0.0092	0.1035
CSCE Sugar No. 11 Futures	-0.0082	0.1499
CBOT Soybean Futures	0.0124	0.2074
COMEX Silver Futures	-0.0516	0.1848
CBOT Wheat Futures	0.0005	0.1943

Some Recent Literature on Financialization of Commodities

- Tang and Xiong (2010): empirical evidence of increased exposure of commodities prices to shocks in other asset classes.
- Report by FTI UK Holdings Limited (2011) on the impact of speculation in commodity markets. Also Brunetti and Buyuksahin (2009), Buyuksahin and Robe (2013), Silvennoinen and Thorp (2013), Henderson, Pearson, and Wang (2012), Gilbert (2010), Irwin and Sanders (2012), Kaufmann (2011), Mayer (2012).
- Models of financialization: Basak & Pavlova (2013), Sockin & Xiong (2013), Singleton (2013), Leclercq and Praz (2013), Anthropelos, Kupper & Papapantaleon (2015).
- Discuss next work with P. Chan & M. Stein (2014).

Reference Model (no speculators)

- The commodity is bought and sold by market users (reference traders), who trade in it for direct industrial use or for hedging operational exposure.
- We assume a fixed constant supply A of the commodity available for trading each time period.
- The reference traders have a stochastic incomes process I_t, the aggregated amount of capital available for business investment.
- Given the commodity price Y_t, their demand is

$$D(Y_t, I_t, t) = \frac{I_t^{\lambda}}{Y_t}, \quad \lambda > 0.$$

Reference Model (*ctd.*)

- In the absence of financial traders, label the price $Y = Y^{(0)}$.
- Then DEMAND = SUPPLY gives

$$Y_t^{(0)} = \frac{I_t^{\lambda}}{A}.$$

- Take I to be a geometric Ornstein-Uhlenbeck process (*i.e.* logOU or expOU). Then $Y^{(0)}$ is one too, and we will write

$$dY_t^{(0)} = \left(a(m - \log(Y_t^{(0)}))\right) Y_t^{(0)}\, dt + bY_t^{(0)}\, dW_t^c,$$

where W^c is a Brownian motion. (Schwartz '97 model of mean-reverting commodity prices).

Adding Portfolio Optimizers

- Introduce *portfolio optimizers*, rational investors with no direct operational or hedging interest in the commodity, who trade to maximize expected utility.
- Assume power utility of wealth x at a fixed terminal horizon T:

$$U(x) = \frac{x^{(1-\gamma)}}{1-\gamma}, \quad \gamma > 0$$

  and portfolio split between
  - money market account with constant interest $r \geq 0$;
  - stock market index S:

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t^s,$$

    where W^s is a Brownian motion;
  - the commodity whose price Y is determined by supply & demand.

Fixed point characterization

A pair $(\widehat{\pi}^*, \widehat{\theta}^*)$, is an equilibrium solution if

1. The stock price S_t is GBM and the commodity price Y_t is determined by the market clearing condition

$$D(Y_t, I_t) + \tilde{\epsilon}\frac{\widehat{\theta}_t X_t}{Y_t} = A,$$

where X_t is the controlled wealth process:

$$\frac{dX_t}{X_t} = \frac{\widehat{\pi}_t^*}{S_t}\, dS_t + \frac{\widehat{\theta}_t^*}{Y_t}\, dY_t + r\left(1 - \widehat{\pi}_t^* - \widehat{\theta}_t^*\right)\, dt.$$

2. The pair $(\widehat{\pi}^*, \widehat{\theta}^*)$ maximizes the expected utility of terminal wealth Z_T

$$\sup_{\widehat{\pi}, \widehat{\theta}} \mathbb{E}\left[U(Z_T) | \mathcal{F}_t\right],$$

under the budget constraint

$$\frac{dZ_t}{Z_t} = \frac{\widehat{\pi}_t}{S_t}\, dS_t + \frac{\widehat{\theta}_t}{Y_t}\, dY_t + r\left(1 - \widehat{\pi}_t - \widehat{\theta}_t\right)\, dt.$$

Some Context

- Models of the form
  Reference traders + Noise traders $\rightarrow$ *equilibrium*!
  have long history.
- In continuous time/stochastic calculus: Frey-Stremme ('95); Schonbucher-Wilmott ('96,'00); Sircar-Papanicolaou ('98) [SP98]; Platen-Schweizer ('98); Mitton ('05); Jones ('07); Nayak-Papanicolaou ('08) [NP08].
- Related: illiquid markets, price impact, large agent: Jonsson-Keppo ('02); Cetin-Jarrow-Protter ('04); Bank-Kramkov ('10), ...
- In SP98 (for example), feedback comes from program traders who are hedging options. Main conclusion: program trading leads to increased volatility (*destabilizing*). ['87 crash, Brady report].

Some Context

- In NP08, feedback comes from portfolio optimizers of Merton type (long only). Main conclusion: rational trading lowers volatility (*stabilizing*).
- With hedging feedback, the noise trader demand is “easy”: the option Δ and the system closes “nicely” (nonlinear Black-Scholes PDE).
- With portfolio optimizers, the θ comes from optimally trading a stock whose dynamics you influence.
- Here: reference model is mean-reverting and feedback demand will be long and short. Interested in impact on correlation between commodity Y and stock S.

Feedback iteration

... to capture the successive improvement of trading strategies due to the increasing awareness of self-impact by the commodity traders.
Interlude - guessing 2/3 of the average: players pick an integer in [0, 100]; winner is the one closest to 2/3 times the average .

- *Stage-0*: A typical player ignores the other players and choose a random number between 0 and 100.
- *Stage-1*: He realizes that if the other players are following the *stage-0* strategy, the average is about 50; he can take advantage of this and update his guess to be $100/3$.
- *Stage-2*: He notices that if the other players are following the *stage-1* strategy, then the average is $100/3$; he updates to $200/9$.

As $k \to \infty$, the only rational guess is zero.
Nagel (1995) experiment: many students chose 33 and 22, which are 2/3 of the midpoint 50 and 2/3 of 2/3 of the midpoint: most students seemed to be doing between 0 and 3 rounds.

Stage-k iteration

- Each individual trader is too small to affect the market price, but their aggregate demand does have an impact, which is enforced by the market-clearing constraint.
- Suppose $Y^{(k)}$ follows

$$\frac{dY^{(k)}}{Y^{(k)}} = P^{(k)}\,dt + Q^{(k)}\,dW_t^c + R^{(k)}\,dW_t^s,$$

  for some coefficients $P^{(k)}, Q^{(k)}$, and $R^{(k)}$.
- The bulk of the portfolio optimizers employ the stage-$(k-1)$ strategy $(\pi^{(k-1)}, \theta^{(k-1)})$, wealth process is X_t.
- Suppose that $P^{(k)} = P^{(k)}(t, X_t, Y_t^{(k)})$ and similarly for $Q^{(k)}$ and $R^{(k)}$, which come from the solution of the stage-$(k-1)$ problem.

$$P^{(0)}(t,x,y) = a\left(\tilde{m} - \log y\right), \quad Q^{(0)}(t,x,y) = \lambda b, \quad R^{(0)} = 0.$$

Stage-k portfolio optimization problem

- At stage-k, all but one of the commodity traders follow the stage-$(k-1)$ strategy: a single "smart" trader seeks to outperform the others by taking into consideration the price impact of their stage-$(k-1)$ strategy.
- Strategy $(\pi^{(k)}, \theta^{(k)})$, wealth process Z_t for the "smart" trader:

$$
\begin{aligned}
dZ_t &= \frac{\pi_t^{(k)}}{S_t}\, dS_t + \frac{\theta_t^{(k)}}{Y_t^{(k)}}\, dY_t^{(k)} + r(Z_t - \pi_t^{(k)} - \theta_t^{(k)})\, dt \\
&= \left(rZ_t + \pi_t^{(k)}(\mu - r) + \theta_t^{(k)} \left(P^{(k)}(t, X_t, Y_t^{(k)}) - r \right) \right) dt \\
&\quad + \theta_t^{(k)} Q^{(k)}(t, X_t, Y_t^{(k)})\, dW_t^c \\
&\quad + \left(\pi_t^{(k)} \sigma + \theta_t^{(k)} R^{(k)}(t, X_t, Y_t^{(k)}) \right) dW_t^s.
\end{aligned}
$$

Goal is to maximize expected utility at the terminal time T: Merton problem to determine the stage-k optimal portfolio.

Deriving the stage-$(k+1)$ dynamics

- Given the stage-k optimal portfolio $(\pi^{(k)}, \theta^{(k)})$ of the "smart" trader, we determine the stage-$(k+1)$ commodity price process.
- Because of power utility, the optimal Merton strategies are of the form $\pi_t^{(k)} = \widehat{\pi}^{(k)}(t, X_t, Y_t^{(k)})Z_t$ and $\theta_t^{(k)} = \widehat{\theta}^{(k)}(t, X_t, Y_t^{(k)})Z_t$.
- The 'smart" trader realizes other traders will follow the same reasoning and trade according to the stage-k strategy.
- The aggregate position on the commodity is then $\widehat{\theta}^{(k)}(t, X_t, Y_t^{(k+1)})X_t$.
- The stage-$(k+1)$ market clearing constraint is

$$Y_t^{(k+1)} = \frac{I_t^\lambda}{A} + \epsilon\widehat{\theta}^{(k)}(t, X_t, Y_t^{(k+1)})X_t.$$

Dynamics of stage-$(k+1)$ commodity price process $Y^{(k+1)}$

Proposition

$$
\begin{aligned}
\frac{dY_t^{(k+1)}}{Y_t^{(k+1)}} &= P^{(k+1)}(t, X_t, Y^{(k+1)})\, dt + Q^{(k+1)}(t, X_t, Y^{(k+1)})\, dW_t^c \\
&\quad + R^{(k+1)}(t, X_t, Y^{(k+1)})\, dW_t^s,
\end{aligned}
$$

$$
\begin{aligned}
Q^{(k+1)}(t,x,y) &= \frac{\lambda b(y - \epsilon x \widehat{\theta}^{(k)})}{y - \epsilon x\big(y\partial_y \widehat{\theta}^{(k)} + \widehat{\theta}^{(k)} x \partial_x \widehat{\theta}^{(k)} + (\widehat{\theta}^{(k)})^2\big)}, \\
R^{(k+1)}(t,x,y) &= \frac{\epsilon x \sigma \widehat{\pi}^{(k)}\big(x\partial_x \widehat{\theta}^{(k)} + \widehat{\theta}^{(k)}\big)}{y - \epsilon x\big(y\partial_y \widehat{\theta}^{(k)} + \widehat{\theta}^{(k)} x \partial_x \widehat{\theta}^{(k)} + (\widehat{\theta}^{(k)})^2\big)}, \\
P^{(k+1)}(t,x,y) &= \cdots .
\end{aligned}
$$

Value function and HJB equation

$$V(t,x,y,z) = \sup_{(\pi^{(k)},\theta^{(k)})\in\mathcal{A}} \mathbb{E}\left[U(Z_T)|X_t = x, Y_t^{(k)} = y, Z_t = z\right],$$

solves the HJB PDE: $0 =$

$$V_t + \mathcal{L}_{\mathbf{x}}V + rzV_z + \sup_{\boldsymbol{\nu}\in\mathbb{R}^2}\left[\frac{1}{2}\boldsymbol{\nu}^T C_1 \boldsymbol{\nu} V_{zz} + \boldsymbol{\nu}^T(\mu_1 - r)V_z + \boldsymbol{\nu}^T \sigma_1 \sigma_2^T \nabla_{\mathbf{x}} V_z\right],$$

$$\mu_1 = \begin{pmatrix} \mu \\ P^{(k)} \end{pmatrix}, \qquad \sigma_1 = \begin{pmatrix} 0 & \sigma \\ Q^{(k)} & R^{(k)} \end{pmatrix}, \qquad C_1 = \sigma_1 \sigma_1^T.$$

Proposition

The value function is given by

$$V(t,\mathbf{x},z) = \frac{z^{1-\gamma}}{1-\gamma}\left(G(t,\mathbf{x})\right)^{\gamma},$$

where $G(t,\mathbf{x})$ solves the linear PDE problem

$$G_t + \mathcal{L}_{\mathbf{x}}G + \left(\frac{1-\gamma}{\gamma}\right)\left(\sigma_2\sigma_1^{-1}(\mu_1 - r)\right)^T \nabla_{\mathbf{x}}G + \frac{\zeta}{\gamma}G = 0,$$

with terminal condition $G(T,\mathbf{x}) = 1$, where

$$\zeta = \left(r(1-\gamma) + \frac{1-\gamma}{2\gamma}M\right), \qquad M = (\mu_1 - r)^T C_1^{-1}(\mu_1 - r).$$

The optimal portfolio $\boldsymbol{\nu}_t^* = (\pi^{(k)}, \theta^{(k)})^T$ is given by

$$\boldsymbol{\nu}_t^* = \left(\frac{1}{\gamma}C_1^{-1}(\mu_1 - r) + (\sigma_2\sigma_1^{-1})^T \frac{\nabla_{\mathbf{x}}G}{G}\right) Z_t.$$

The stage-0 value function is given by

$$V(t,y,z) = \frac{z^{1-\gamma}}{1-\gamma}\exp\left(f_0(t) + f_1(t)\log y + f_2(t)(\log y)^2\right),$$

where

$$f_2(t) = \frac{a(1-\gamma)}{2\lambda^2 b^2}\frac{\sinh\left(\frac{a}{\sqrt{\gamma}}(T-t)\right)}{\sinh\left(\frac{a}{\sqrt{\gamma}}(T-t)\right) + \sqrt{\gamma}\cosh\left(\frac{a}{\sqrt{\gamma}}(T-t)\right)},$$

$$f_1(t) = (1-\gamma)\frac{\frac{r-a\tilde{m}}{\lambda^2 b^2}\sinh\left(\frac{a}{\sqrt{\gamma}}(T-t)\right) + \sqrt{\gamma}\left(\frac{r}{\lambda^2 b^2} - \frac{1}{2}\right)\Bigl(\cosh\left(\frac{a}{\sqrt{\gamma}}(T-t)\right)}{\sinh\left(\frac{a}{\sqrt{\gamma}}(T-t)\right) + \sqrt{\gamma}\cosh\left(\frac{a}{\sqrt{\gamma}}(T-t)\right)}$$

$$f_0(t) = k(T-t) + \int_t^T \Biggl\{\left(\frac{a\tilde{m} - (1-\gamma)r}{\gamma} - \frac{\lambda^2 b^2}{2}\right)f_1(s) + \lambda^2 b^2 f_2(s) + \frac{\lambda^2 b}{2\gamma}$$

Implementation

- Stage-0 explicit, higher stages involves solving the linear PDE for G numerically.
- Solved by finite differences.
- Stage-k volatility $\eta^{(k)}$ of the commodity price and the stage-k correlation $\rho^{(k)}$ with the stock price are given by

$$\eta^{(k)} = \sqrt{(Q^{(k)})^2 + (R^{(k)})^2}, \qquad \rho^{(k)} = \frac{R^{(k)}}{\sqrt{(Q^{(k)})^2 + (R^{(k)})^2}}.$$

Stage-0

$$\eta^{(0)} = \lambda b \quad \text{and} \quad \rho^{(k)} = 0.$$

Stage-1 Explicit from the explicit solution to the stage-0 problem.

$$\eta^{(1)} = \lambda b \left(1 + \frac{\epsilon X_t}{Y_t}\left(Y_t \partial_y \widehat{\theta} + \widehat{\theta}^2 - \widehat{\theta}\right) + \mathcal{O}(\epsilon^2)\right)$$

$$\rho^{(1)} = \frac{\epsilon \sigma X_t \widehat{\pi}\widehat{\theta}}{\sqrt{\lambda^2 b^2 \left(Y_t - \epsilon X_t \widehat{\theta}_t\right)^2 + \left(\epsilon \sigma X_t \widehat{\pi}\widehat{\theta}\right)^2}}$$

Correlation Creation

Since $\widehat{\pi}^{(0)} > 0$ (long in stock whose $\mu > r$),

$$\text{sign}\left[\rho_t\right] = \text{sign}\left[\widehat{\theta}^{(0)}(t, Y_t^{(1)})\right].$$

- Commodity price "low", portfolio optimizers long and correlation positive.
- Commodity price "high", portfolio optimizers short and correlation negative.

Numerical Illustration

λ	demand from market users	1.0
a	mean-reversion rate	0.3
r	risk-free rate	0.02
m	mean of commodity log-price	3.0
b	volatility of commodity price	0.3
μ	drift of stock price	0.08
σ	volatility of stock price	0.2
γ	coefficient of risk aversion	1.5
A	market supply	1.0
ϵ	relative size of portfolio optimizers	0.5
T	Investment horizon	2.0

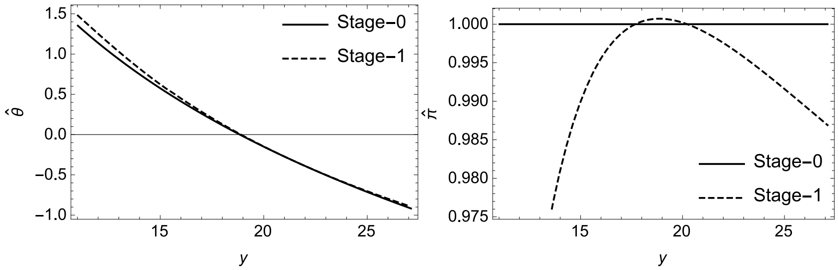


Figure: Stage-0 and 1 optimal portfolio as functions of the current commodity level y.

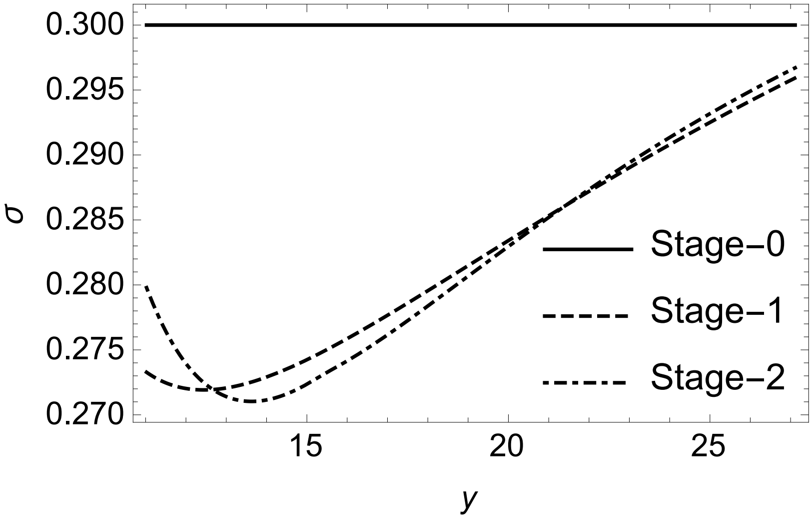


Figure: Volatility of the commodity price.

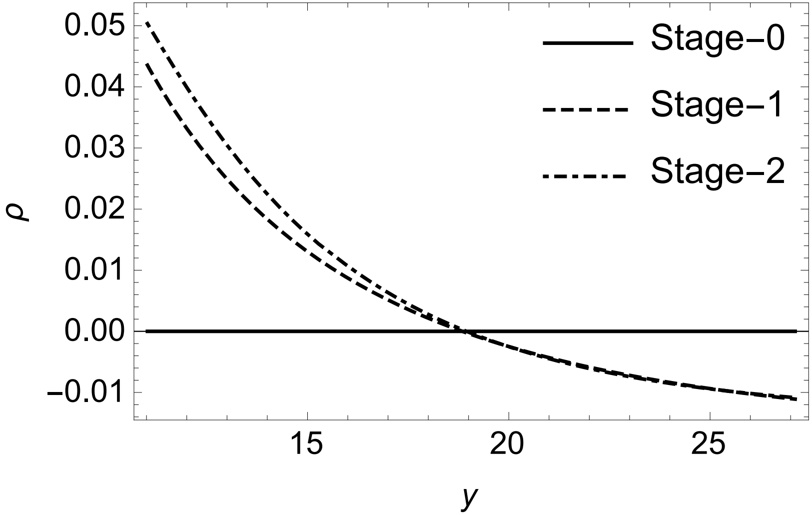


Figure: Correlation between the commodity and stock.

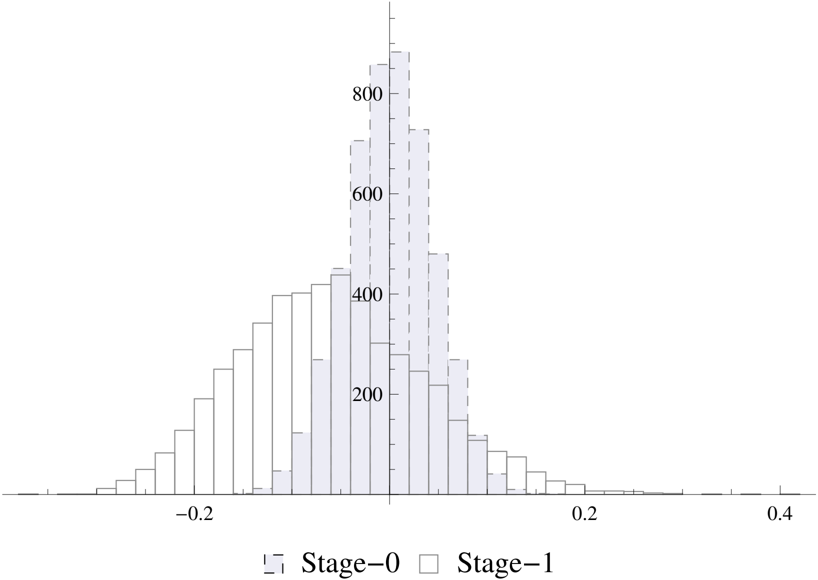


Figure: Low commodity volatility $b = 0.3$.

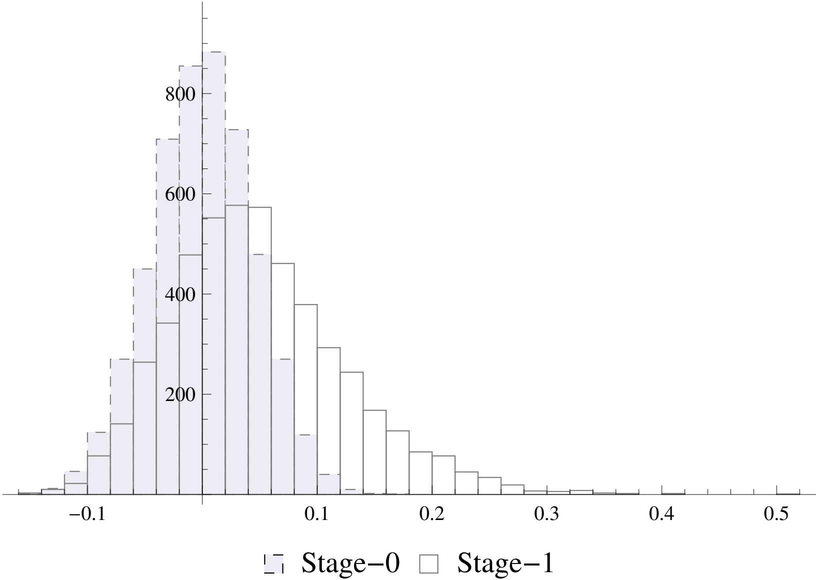


Figure: High commodity volatility $b = 1$.

Concluding Remarks

- Feedback iteration is tractable computationally.
- The model is simple but can demonstrate even in stage-1 the feedback effect of mean-reverting portfolio strategies on commodity-stock correlation.
- Would be interesting to infer back from data the extent ϵ of the recent financialization: calibrate on unperturbed data from the 1990s and get ϵ through the 2000s (and how it has changed).
- Exponential utility and arithmetic dynamics: no feeedback correlation.
- Commodity-commodity correlations, portfolios with baskets, ...