Emission Markets II. Allocation Mechanisms & Partial Equilibrium Models

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- **Free Allocation**
  - National Plan & Limit
  - Grandfathering (give away based on past usage)
  - Firms **DO NOT HAVE TO PAY** for their initial allocation
- **Auctioning**
  - Permits aren't just handed out
  - Permits are **auctioned off**
  - Major policy issue: what to do with the proceeds?
- **Distribution Proportional to Production**
  - Can we control the cap?
  - Impact on prices, social costs, windfall profits, ...
- **Straight Tax**
  - Again where should the proceeds go?

Introduction of **Taxes** / **Subsidies**

$$\begin{aligned}\ddot{L}^{A,S,i}(\theta^i,\xi^i) &= -\sum_{t=0}^{T-1} G_t^i + \sum_{k\in K}\sum_{j\in J^{i,k}}\sum_{t=0}^{T-1}(S_t^k - C_t^{i,j,k} - H_t^k)\xi_t^{i,j,k} \\ &\quad + \sum_{t=0}^{T-1}\theta_t^i(A_{t+1}-A_t) - \theta_T^i A_T \\ &\quad - \pi(\Gamma^i + \Pi^i(\xi^i) - \theta_T^i)^+.\end{aligned}$$

In this case

- In equilibrium, **production** and **trading** strategies remain the same $(\theta^\dagger,\xi^\dagger) = (\theta^*,\xi^*)$
- **Abatement costs** and **Emissions reductions** are also the same
- New equilibrium prices $(A^\dagger, S^\dagger)$ given by

$$A_t^\dagger = A_t^* \quad \text{for all } t = 0,\dots,T \tag{1}$$

$$S_t^{\dagger k} = S_t^{*k} + H_t^k \quad \text{for all } k\in K, t = 0,\dots,T-1 \tag{2}$$

- Cost of the tax passed along to the end consumer

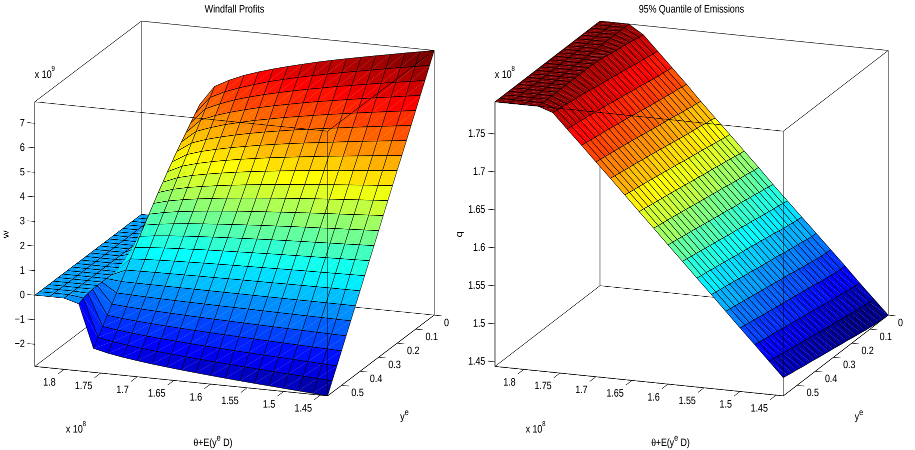
- **Currently Regulator Specifies**
  - Penalty π
  - Overall Certificate Allocation θ_0 $(=\sum_{i\in I}\theta_0^i)$
- **Alternative Scheme (Still) Controlled by Regulator**
  (i) **Sets penalty level** π
  (ii) **Allocates allowances**
    - θ_0' at inception of program $t=0$
    - then **proportionally to production**

$$y\xi_t^{i,j,k} \quad \text{to agent } i \text{ for producing } \xi_t^{i,j,k} \text{ of good } k \text{ with technology } j$$

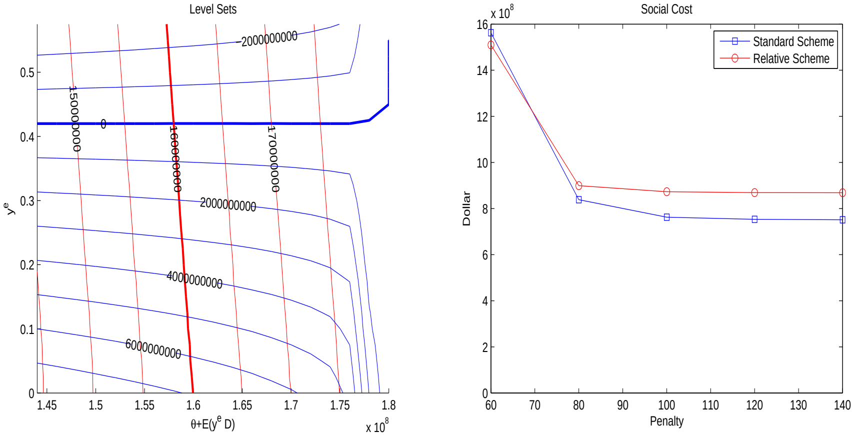
  (iii) **Calibrates** y, e.g. in **expectation**.

$$y=\frac{\theta_0-\theta_0'}{\sum_{t=0}^{T-1}\sum_{k\in K}\mathbb{E}\{D_t^k\}}$$

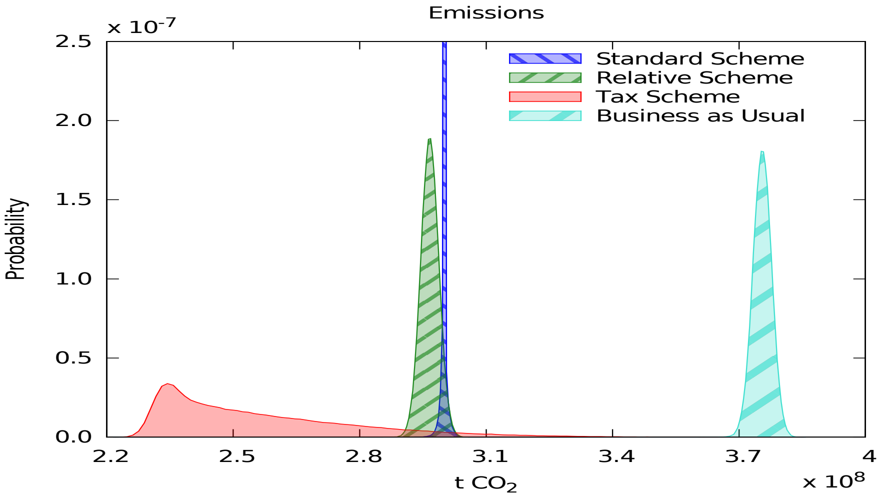
So total number of credit allowance is the same in expectation, i.e.
$\theta_0=\mathbb{E}\{\theta_0'+y\sum_{t=0}^{T-1}\sum_{k\in K}D_t^k\}$



Windfall profits (left) and 95% percentile of total emissions (right) as functions of the relative allocation parameter and the expected allocation

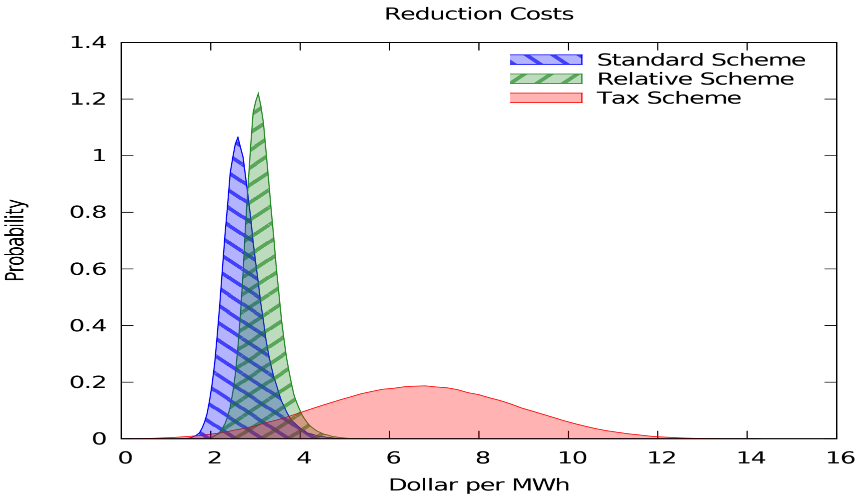


(left) Level sets of previous plots. (right) Production costs for electricity for one year as function of the penalty level for both the absolute and relative schemes.

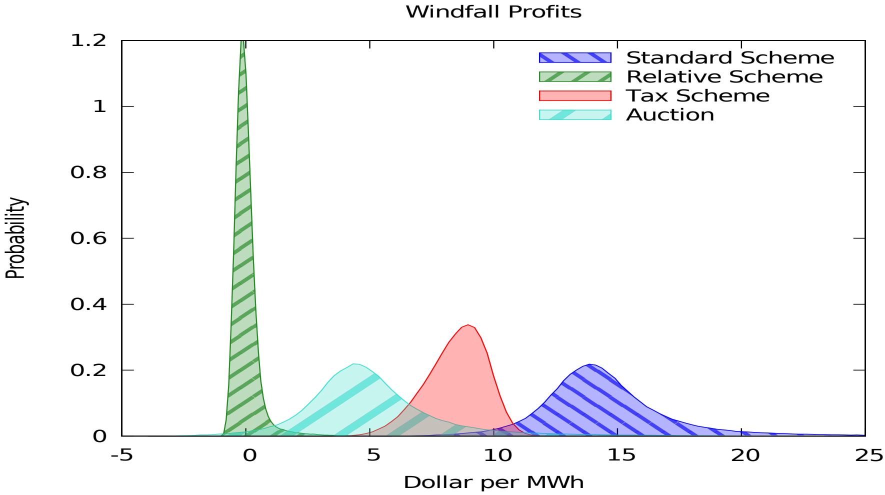


Yearly emissions from electricity production for the Standard Scheme, the Relative Scheme, a Tax Scheme and BAU.

Reduction (Abatement) Costs



Yearly abatement costs for the Standard Scheme, the Relative Scheme and a Tax Scheme.



Histograms of the yearly distribution of windfall profits for the Standard Scheme, a Relative Scheme, a Standard Scheme with 100% Auction and a Tax Scheme

Conclusions

- Market Mechanisms **CANNOT** solve all the pollution problems
- **Cap-and-Trade Schemes CAN Work!**
  - Given the right emission target
  - Using the appropriate tool to allocate emissions credits
  - Significant Windfall Profits for Standard Schemes
- **Taxes**
  - Politically unpopular (**though things are changing**)
  - Cannot reach emissions targets without inefficiencies
- **Auctioning**
  - Fairness is Smoke Screen: Re-distribution of the cost
- **Relative Schemes**
  - Can Reach Emissions Target
  - Can Control Windfall Profits
  - Optimal in Minimizing Social Costs
- **Extensions of the Present Work (Sharpening the Tools)**
  - Including Risk Averse Agents and Inelastic Demands
  - Statistical Analysis of Equilibrium Prices
  - Exogenous Prices and Large Scale Case Studies
  - Other Schemes (e.g. **California Low Emissions Fuel Standards**)

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(**RC-Espinoza-Touzi**)

- Relax the **demand inelasticity** assumption
- Agent based model ("Representative Agent" form already considered in **Seifert-Uhrig-Homburg-Wagner**, **RC-Fehr-Hinz**, **Hinz**, ...)
- Include preferences (relax the **risk neutrality** assumption of **RC-Fehr-Hinz**)
- Major restriction: **COMPLETE MARKET MODEL**

Mathematical Set-Up (continuous time)

- $(\Omega, \mathcal{F}, \mathbb{P})$ **historical** probability structure
- W 1-D Wiener process on $(\Omega, \mathcal{F}, \mathbb{P})$
- $T > 0$ finite horizon (end of the **single** compliance period)
- $\mathbb{F} = \{\mathcal{F}_t;\ 0 \leq t \leq T\}$ filtration of W

Goal of equilibrium analysis is to derive pollution permit price $\{A_t;\ 0 \leq t \leq T\}$ allowing firms to **maximize their expected utilities simultaneously**

Assume allowance price $A = \{A_t;\ 0 \le t \le T\}$ exists.

- A is a $\mathbb{F}$-martingale under $\mathbb{Q}$
- $dA_t = Z_t\,dB_t$ for some adapted process Z s.t. $Z_t \neq 0$ a.s. and B 1-D Wiener process for (unique) spot martingale measure $\mathbb{Q}$
- $A_T = \pi \mathbf{1}_{[\kappa,\infty)}(E_T)$ where
  - π is the penalty
  - $E_t = \sum_{i \in \mathcal{I}} E_t^i$ is the aggregate of the E_t^i representing the **cumulative emission** up to time t of firm i
  - κ $(= \theta_0)$ is the cap imposed by the regulator

Assume the following dynamics **under** $\mathbb{P}$

$$dE_t^i = (b_t^i - \xi_t^i)dt + \sigma_t^i dW_t, \qquad E_0^i = 0.$$

- Think of $\{E_t^i(\xi_t^i \equiv 0)\}_{0 \le t \le T}$ as the cumulative emissions of firm i in BAU
- Think of the control $\{\xi_t^i)\}_{0 \le t \le T}$ as the abatement strategy of firm i
- Assumptions on emission drifts b_t^i and volatilities σ_t^i to be articulated later

NB: All the individual control problems are driven by the **SAME** Brownian motion W

Abatement costs for firm i given by cost function $c_t^i : \mathbb{R} \to \mathbb{R}$

- c_t^i is C^1 and strictly convex
- c_t^i satisfies Inada-like conditions for each $t \in [0, T]$

$$(c_t^i)'(-\infty) = -\infty \qquad \text{and} \qquad (c_t^i)'(+\infty) = +\infty.$$

- $c_t^i(0) = \min c_t^i$ ($\xi^i \equiv 0$ corresponds to BAU)

Typical example for c^i

$$\lambda |x|^{1+\alpha},$$

for some $\lambda > 0$ and $\alpha > 0$.

Each firm chooses its **abatement strategy** ξ^i and its **investment** θ^i in allowances. Its **wealth** is given by

$$X_t^i = X_t^{i,\xi,\theta} = x^i + \int_0^T \theta_t^i dA_t - \int_0^T c_t^i(\xi_t^i) dt - E_T^i A_T.$$

Preferences of firm i given by a C^1, increasing, strictly concave **utility function** $U^i : \mathbb{R} \to \mathbb{R}$ satisfying Inada conditions:

$$(U^i)'(-\infty) = +\infty \qquad \text{and} \qquad (U^i)'(+\infty) = 0.$$

The optimization problem for firm i is:

$$V(x^i) := \sup_{(\xi^i,\theta^i)\in\mathcal{A}^i} \mathbb{E}^{\mathbb{P}}\{U^i(X_T^{i,\xi^i\theta^i})\}$$

$\mathcal{A}^i$ set of admissible strategies for firm i

Use Convex Duality

Proposition

If an equilibrium allowance price $\{A_t\}_{0\le t\le T}$ exists, then there is optimal abatement and investment strategies $(\hat{\xi}^i, \hat{\theta}^i)$ such that

$$\hat{\xi}_t^i = [(c_t^i)']^{-1}(A_t).$$

NB: The optimal abatement strategy $\hat{\xi}^i$ is independent of the utility function U^i!
NB: The existence of the optimal investment strategy is guaranteed by the **completeness** assumption!

Complete Market $\Longrightarrow$ Representative Agent (Informed Central Planner) approach

- Recall
$$dE_t^i = \left[\tilde{b}_t^i - [(c_t^i)']^{-1}(A_t)\right] dt + \sigma_t^i dB_t, \qquad E_0^i = 0, \text{ for each } i$$
- Assume
  - $\forall i, \tilde{b}_t^i = \tilde{b}^i(t)E_t^i$ or $\forall i, \tilde{b}_t^i = \tilde{b}^i(t)$
  - $\forall i, \sigma_t^i = \sigma^i(t)$.
- Set
$$b := \sum_{i\in\mathcal{I}} \tilde{b}^i, \ \sigma := \sum_{i\in\mathcal{I}} \sigma^i, \text{ and } f := \sum_{i\in\mathcal{I}} [(c^i)']^{-1}.$$

Therefore we have the following FBSDE

$$dE_t = \{b(t, E_t) - f_t(A_t)\}dt + \sigma(t)dB_t, \qquad E_0 = 0 \tag{3}$$
$$dA_t = Z_t dB_t, \qquad Y_T = \pi \mathbf{1}_{[\kappa,+\infty)}(E_T), \tag{4}$$

with $b(t, E_t) = b(t)E_t^\beta$ with $\beta \in \{0, 1\}$ and f increasing.

Theorem

For any $\pi > 0$ and $\kappa \in \mathbb{R}$, FBSDE (3)-(4) admits a unique solution $(E, A, Z) \in M^2$. Moreover, A_t is nondecreasing w.r.t π and nonincreasing w.r.t κ.

Proof

- Approximate the singular terminal condition $\pi \mathbf{1}_{[\kappa,+\infty)}(E_T)$ by increasing and decreasing sequences $\{\varphi_n(E_T)\}_n$ and $\{\psi_n(E_T)\}_n$ of smooth monotone functions of E_T
- Use comparison results for BSDEs to control the limits

One possible way to solve a FBSDE is to

- Postulate that the solution is of the form $A_t = \alpha(t, E_t)$
- Inject this form for A_t into the FBSDE
- Derive a (nonlinear) PDE that the function $\alpha(t, x)$ MUST satisfy
- Solve this PDE by a backward inductive scheme

CLAIM

This is exactly what **Hinz** is doing (in discrete time) to compute prices !

A Slightly Different Model

Single good (e.g. **electricity**) regulated economy, with price dynamics given **exogenously**!

$$\frac{dP_t}{P_t} = \mu(t, P_t)dt + \sigma(t, P_t)dW_t$$

Firm i

- Controls its *instataneous rate of production* q_t^i
- **Production** over $[0, t]$
$$Q_t^i := \int_0^t q_t^i dt.$$
- **Costs of production** given by $c_t^i : \mathbb{R}_+ \mapsto \mathbb{R}$ C^1 strictly convex satisfying Inada-like conditions
$$(c_t^i)'(0) = 0, \qquad (c_t^i)'(+\infty) = +\infty$$
- **Cumulative emissions** $E_t^i := e^i Q_t^i$
- **P&L** (wealth)

$$X_t^i = X_t^{i,q^i,\theta^i} = x^i + \int_0^T \theta_t^i dA_t - \int_0^T [P_t q_t^i - c_t^i(q_t^i)]dt - e^i Q_T^i A_T.$$

Individual Firm Optimization Problem

Proposition

If such an equilibrium exits, then there is an optimal control $(\hat{q}^i, \hat{\theta}^i)$, and we have:

$$\hat{q}^i_t = [(c^i)']^{-1}(P_t - e^i Y_t).$$

NB: As before the optimal production schedule $\hat{q}^i$ **DOES NOT DEPEND** upon the utility function!

- Set $E_t := \sum_{i \in \mathcal{I}} E_t^i$ for the total aggregate emissions up to time t
- Define $f(p, y) := \sum_{i \in \mathcal{I}} \varepsilon^i [(c^i)']^{-1}(p - \varepsilon^i y)$

Then the corresponding FBSDE under $\mathbb{Q}$ reads

$$dP_t = \sigma(t, P_t) dB_t, \qquad P_0 = p \tag{5}$$

$$dE_t = f(P_t, A_t) dt, \qquad E_0 = 0 \tag{6}$$

$$dA_t = Z_t dB_t, \qquad Y_T = \lambda 1_{[\kappa, +\infty)}(E_T). \tag{7}$$

NB: The volatility of the forward equation is **degenerate!**

Theorem(Conjectured)

Let $\pi > 0$ and $\kappa \in \mathbb{R}$, then the above FBSDE has a unique solution (P, E, A, Z). Moreover, A_t is nondecreasing in π and nonincreasing in κ.

Cetin-Verschuere (Dec-07 vs Dec-08 futures contracts)

- S_t value at time t of Dec-08 EUA futures contract

$$dS_t = S_t[\mu + \alpha\theta_t]dt + S_t\sigma dW_t$$

- σ, μ, α constants, $S_0 = s$
- θ_t two-state continuous-time Markov chain **independent** of Wiener process W_t
  - $\theta_t = 1$ market is *long allowances* at time t
  - $\theta_t = -1$ market is *short allowances* at time t
- $T = Dec - 07$ end of Phase I
- P_t value at time $t \leq T$ of Dec-07 EUA futures contract

$$P_T = \begin{cases} S_T + \pi & \text{if } \theta_T \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

- View Dec-07 contract as **derivative** written on Dec-08 contract
- Pricing & Hedging in **Incomplete Market** (**two** sources of randomness, **one** underlier)

Filtering Techniques

- Observe $\mathcal{F}^S = \{\mathcal{F}^S_t\}_t$ filtration of S_t
- One time announcement of true value of θ at time T

$$\mathcal{G}_t = \begin{cases} \mathcal{F}^S_t & \text{for } t < T \\ \mathcal{F}^S_t \vee \sigma(\theta_T) & \text{for } t = T \end{cases}$$

- Optional projection $\overline{\theta}_t = \mathbb{E}\{\theta_t | \mathcal{F}^S_t\}$
- $\overline{W}_t = \int_0^t \frac{1}{\sigma S_s}[dS_s - (\mu - \alpha\overline{\theta}_s)S_s ds]$ is a $\mathcal{G}$ Brownian motion
- $d\overline{\theta}_t = -2\lambda\overline{\theta}_t dt + \frac{\alpha}{\sigma}(1 - \overline{\theta}^2_t)d\overline{W}_t$ with $\overline{\theta}_0 = 2p - 1$, and $p = \mathbb{P}\{\theta_0 = 1\}$.
- $Z_t = \mathbf{1}_{\{t=T\}}(\theta_T - \overline{\theta}_T)$ is a $\mathcal{G}$ martingale orthogonal to $\overline{W}$
- $\overline{P}_t$ fair price of P_t

$$\overline{P}_t = \mathbb{E}^*\{\frac{1-\theta_T}{2}(S_T + \pi)|\mathcal{G}_t\}$$

where $\mathbb{E}^*$ is expectation w.r.t. **minimal martingale measure** $\mathbb{P}^*$ (**Foellmer-Schweizer**)

Dynamics under $\mathbb{P}^*$

$$
\begin{aligned}
dS_t &= \sigma S_t dW_t^*, \\
d\overline{\theta}_t &= -\left(2\lambda\overline{\theta}_t + \frac{\alpha}{\sigma^2}(1-\overline{\theta}_t^2)(\mu+\alpha\overline{\theta}_t)\right)dt + \frac{\alpha}{\sigma}(1-\overline{\theta}_t^2)dW_t^*
\end{aligned}
$$

where W^* is a $(\mathcal{G}, \mathbb{P}^*)$ Brownian motion.

What Happened in April 06?

- **TRUE** value θ_{t_0} of θ_t revealed at time t_0
- Replace $\overline{\theta}_t$ by $\tilde{\theta}_t = \mathbb{E}\{\theta_t | \mathcal{F}_t^S, \theta_{t_0}\}$ for $t > t_0$
- Fair price of Dec-07 contract now given by

$$P_t = \begin{cases} Z_t^h + h(t, S_t, \overline{\theta}_t) & \text{for } t < t_0 \\ h(t, S_t, \overline{\theta}_t) - Z_t(S_t + \pi)/2 & \text{for } t > t_0 \end{cases}$$

and

$$\Delta P_{t_0} = h(t_0, S_{t_0}, \theta_{t_0}) - h(t_0, S_{t_0}, \overline{\theta}_{t_0})$$

where h is the solution of a specific PDE (full observation model) and

$$Z_t^h = \mathbb{E}^*\{h(t_0, S_{t_0}, \theta_{t_0}) - h(t_0, S_{t_0} | \mathcal{G}_t\}$$

Explicit formula for the size of the jump in price!