

Risk-Constrained Multi-Stage (Renewable) Power Investment



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THE OHIO STATE UNIVERSITY

President Obama's 2016 State of the Union Address

“Now we’ve got to accelerate the transition away from dirty energy. Rather than subsidize the past, we should invest in the future...”



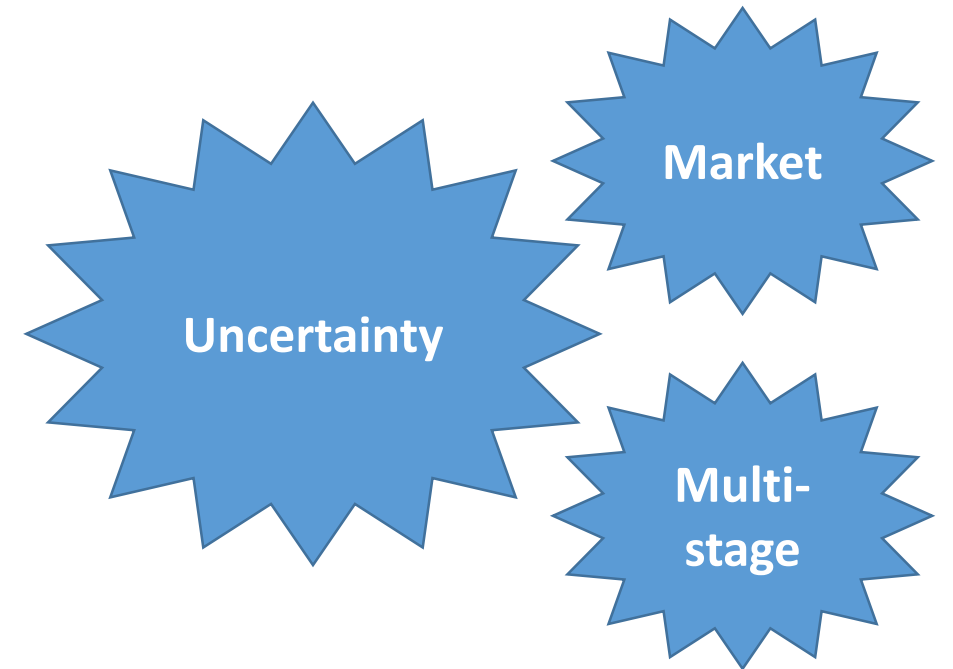
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1. Motivation & approach

Power investor: seeks to determine the power capacity to be built that **maximizes** its **expected profit** while **controlling its risk of profit volatility**.

- ✓ Where to build?
- ✓ **When** to build?
- ✓ Which capacity to build?



1. Motivation & approach

- ✓ Where to build (**the network exist!**)?
 - At nodes where construction is possible
 - At nodes with the best (renewable) investment conditions
 - At nodes “well connected” to the system

1. Motivation & approach

✓ When to build (investment is necessarily multi-stage)?

It depends on:

- Demand growth uncertainty
- Fuel cost uncertainty
- Investment cost uncertainty

1. Motivation & approach

✓ Which capacity to build?

It depends on:

- Renewable production uncertainty
- Equipment failure rates

1. Motivation & approach

✓ How to solve this problem?

- Stochastic model
- Complementarity model (**we have a market!**)
- Multi-stage model
- Risk-constrained model

1. Motivation & approach

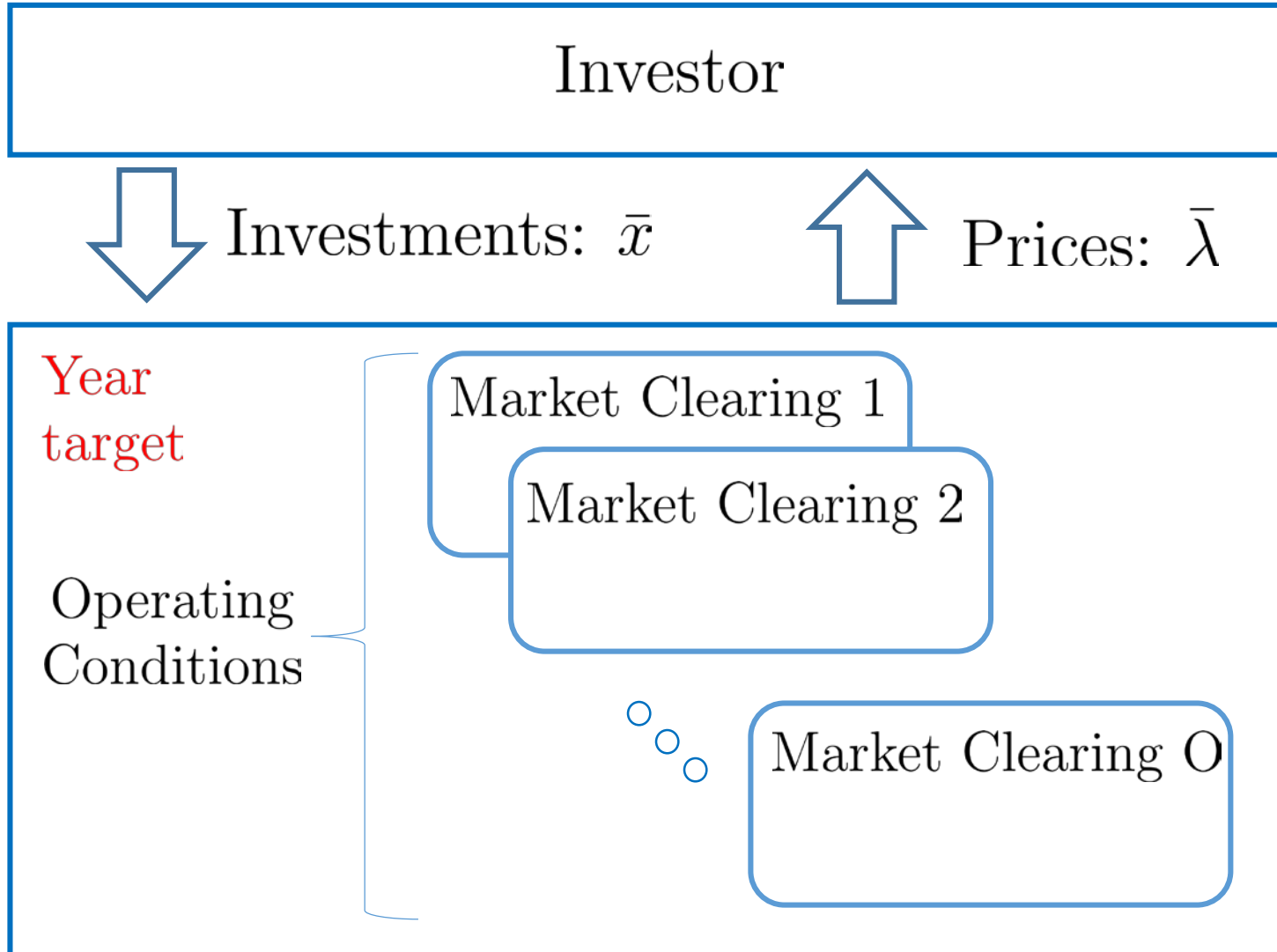
- ✓ Increasing model complexity
 - Static, long-term deterministic, short-term stochastic
 - Static, long-term stochastic, short-term stochastic
 - Dynamic, long-term stochastic, short-term stochastic

1. Motivation & approach

Uncertainty

- Long-term (scenarios): **from year to year**
 - ✓ Demand growth
 - ✓ Investment cost
 - ✓ Fuel cost
- Short-term (operating conditions): **within a year**
 - ✓ Renewable production + Demand level
 - ✓ Equipment failure

Static, ST stochastic, LT deterministic



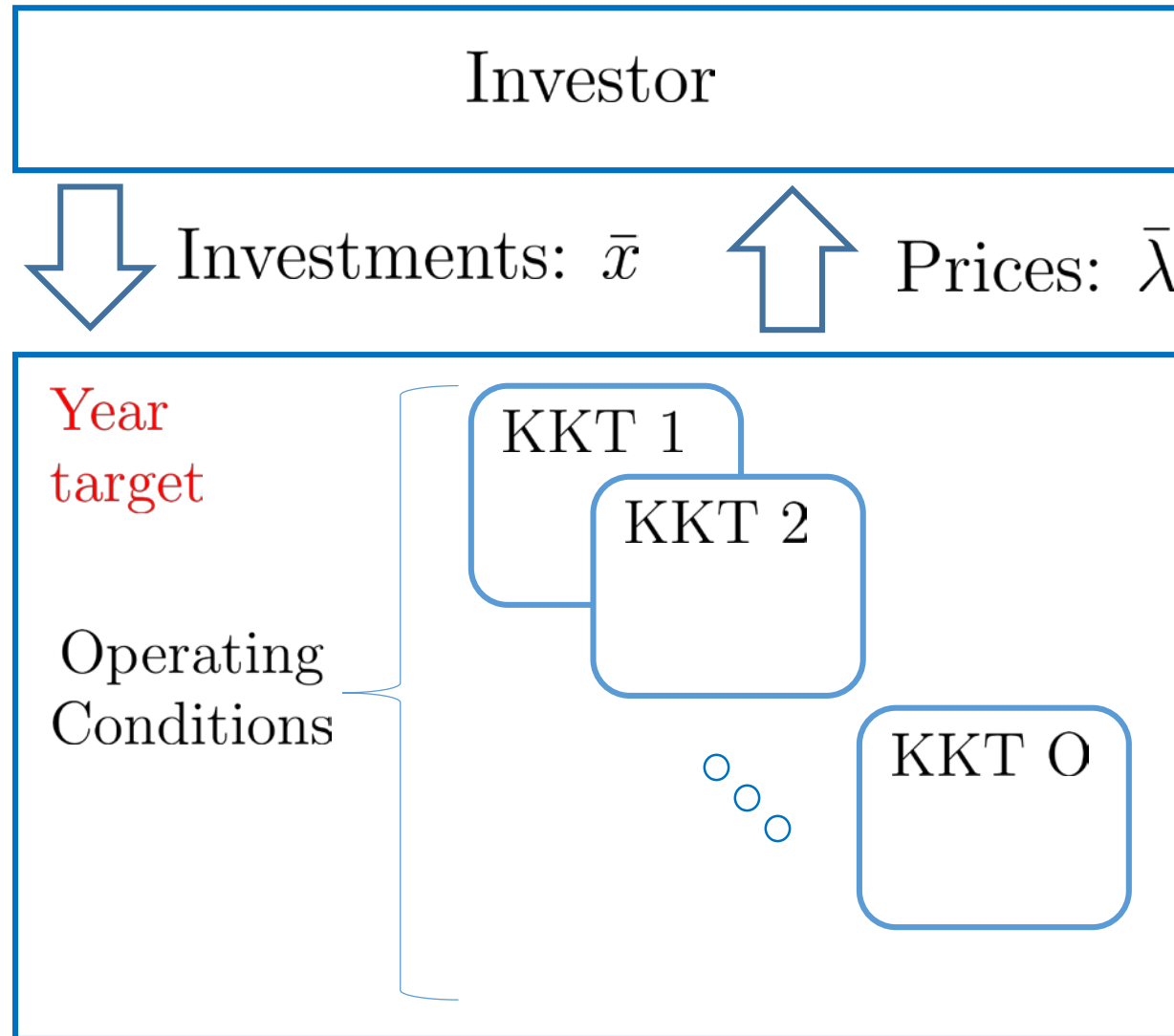
ST: Short-Term

LT: Long-Term

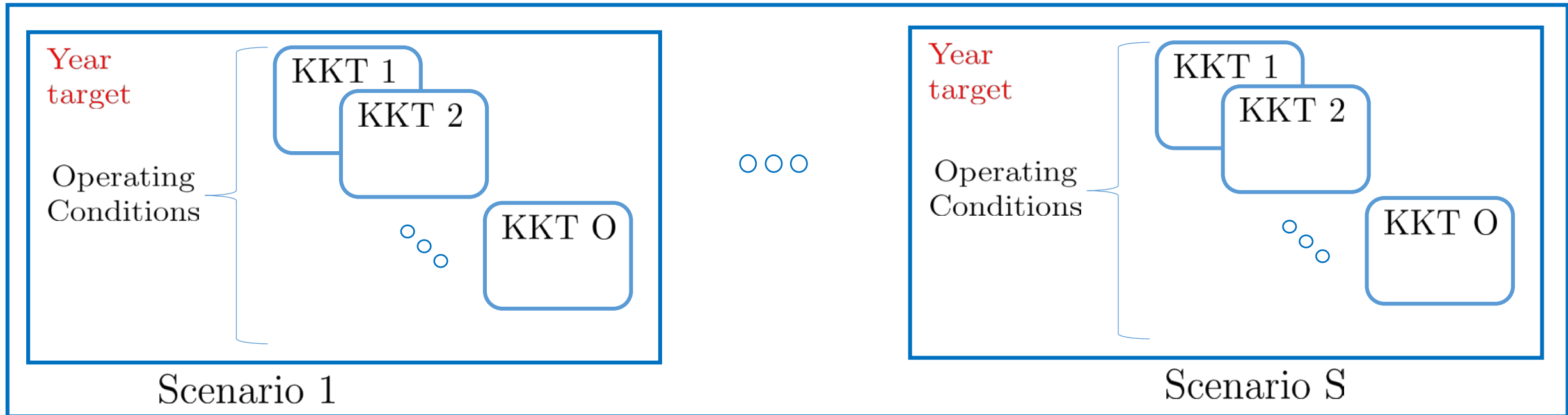
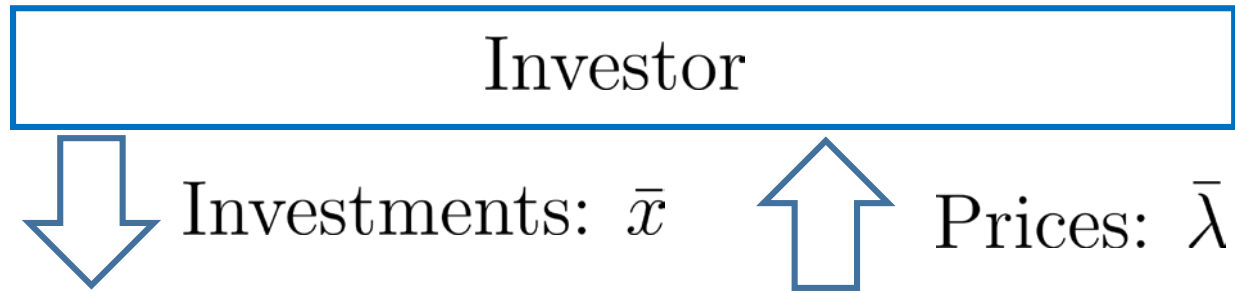
Short-Term Stochastic:
(Operating Conditions)

- Wind/solar production + demand level
- Contingencies

Static, ST stochastic, LT deterministic



Static, ST stochastic, LT stochastic



Scenarios: demand growth, investment cost change, fuel cost change

Dynamic, ST stochastic, LT stochastic

Investor

Investments: \bar{x}_1
↓ Prices: $\bar{\lambda}_1$ ↑

Year 1

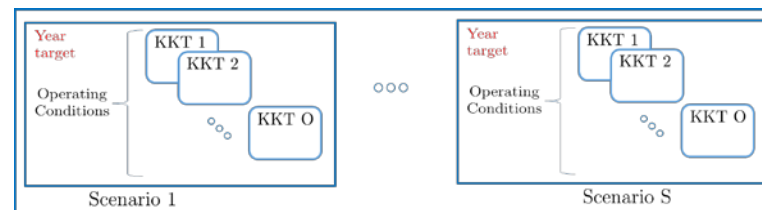
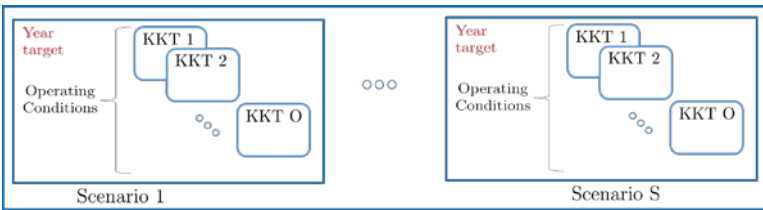
Investments: \bar{x}_2
↓ Prices: $\bar{\lambda}_2$ ↑

Year 2

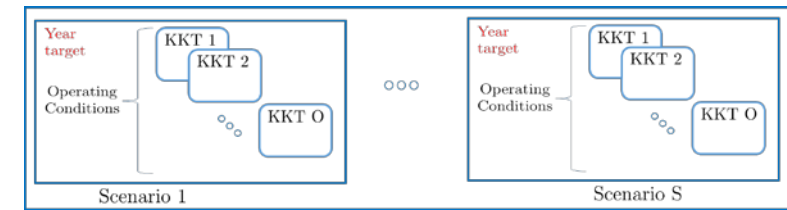
...

Investments: \bar{x}_Y
↓ Prices: $\bar{\lambda}_Y$ ↑

Year Y



...



Risk Control



Nonanticipativity

Risk Control



Nonanticipativity

Risk Control



Risk Control

1. Motivation & approach

- ✓ Maximization of the wind investor profit
- ✓ Control of the risk of profit volatility
- ✓ Pool based electricity market:
 - The wind producer is paid the LMP of its node
- ✓ Given transmission capacity (dc model)

2. Problem description

The risk-constrained multi-stage wind power investment problem above comprises an upper-level problem (1) and a collection of lower-level problems (2).

2. Problem description

The optimization variables of each of the lower-level problems (1) are $\Delta_{\text{LL},\nu}^{(t)}(\gamma) = \left\{ g_{ib,\nu}^{(t)}(\gamma), \forall i, b; f_{k,\nu}^{(t)}(\gamma), \forall k; \delta_{n,\nu}^{(t)}(\gamma), \forall n \right\}, \forall t, \nu, \gamma;$

while the upper-level problem (2) includes these optimization variables and the additional variables $\Delta_{\text{UL}}^{(t)}(\gamma) = \left\{ X_n^{(t)}(\gamma), \forall n; \zeta; \eta(\gamma); P_{n,\nu}^{\text{W},(t)}(\gamma), \forall n, \nu \right\}, \forall t, \gamma.$

2. Problem description

Maximize $\Delta_{UL}^{(t)}(\gamma), \Delta_{LL,\nu}^{(t)}(\gamma)$

$$\begin{aligned}
 & \sum_{\gamma \in \Omega_\gamma} \tau(\gamma) \left\{ \sum_{t \in \Omega^T} \left[\sum_{\nu \in \Omega_\nu} \vartheta_\nu \sum_{n \in \Omega^N} \lambda_{n,\nu}^{(t)}(\gamma) P_{n,\nu}^{W,(t)}(\gamma) \right. \right. \\
 & \left. \left. - a^{(t)} \sum_{n \in \Omega^N} c_n^{(t)}(\gamma) X_n^{(t)}(\gamma) \right] \right\} \\
 & + \beta \left(\zeta - \frac{1}{1-\alpha} \sum_{\gamma \in \Omega_\gamma} \tau(\gamma) \eta(\gamma) \right) \tag{1a}
 \end{aligned}$$

stage (year)

scenario

Operating condition

subject to

2. Problem description

The objective function (1a) of the upper-level problem represents

the maximization of the expected profit plus a coefficient times the CVaR.

2. Problem description

The first line of (1a) is the revenue obtained by selling wind power in the pool, computed as the wind power production for each load demand/wind power production condition (ν) times the LMP of the bus at which the wind plant is located.

LMPs are computed as the dual variable associated with the balance constraints (2b).

2. Problem description

The revenue for each load demand/wind power production condition is multiplied for the corresponding number of hours ϑ_ν , thus obtaining the expected yearly revenue.

2. Problem description

The second line of (1a) is the wind investment cost, which is multiplied in each period by an amortization factor $a^{(t)}$.

The amortization costs represent the equivalent amount of money to be paid in each period.

2. Problem description

Revenues and costs are computed for each scenario and thus are multiplied by the weight of the scenario $\tau(\gamma)$.

2. Problem description

Finally, the third line of the objective function (1a) is the CVaR multiplied by a factor β

to materialize the tradeoff between profit and risk, so that the higher the value of β , the more risk averse the wind power investor is.

2. Problem description

$$P_{n,\nu}^{\text{W},(t)}(\gamma) \leq k_{n,\nu}^{\text{W}} \sum_{m \leq t} X_n^{(m)}(\gamma), \quad \forall t, \forall n, \forall \nu, \forall \gamma \quad (1b)$$

$$\sum_{n \in \Omega^N} c_n^{(t)}(\gamma) X_n^{(t)}(\gamma) \leq c_{\max}^{(t)}, \quad \forall t, \forall \gamma \quad (1c)$$

$$0 \leq \sum_{t \in \Omega^T} X_n^{(t)}(\gamma) \leq X_n^{\max}, \quad \forall n, \forall \gamma \quad (1d)$$

$$X_n^{(1)}(\gamma) = X_n^{(1)}, \quad \forall n, \forall \gamma \quad (1e)$$

$$X_n^{(t)}(\gamma_l) = X_n^{(t)}(\gamma_{\tilde{l}}),$$

$$\forall n, \forall t \neq 1, \forall l, \tilde{l} : \Upsilon^{(m)}(\gamma_l) = \Upsilon^{(m)}(\gamma_{\tilde{l}}), \forall m < t \quad (1f)$$

2. Problem description

$$P_{n,\nu}^{\text{W},(t)}(\gamma_l) = P_{n,\nu}^{\text{W},(t)}(\gamma_{\tilde{l}}),$$

$$\forall n, \forall \nu, \forall t, \forall l, \tilde{l} : \Upsilon^{(m)}(\gamma_l) = \Upsilon^{(m)}(\gamma_{\tilde{l}}), \forall m \leq t \quad (1g)$$

$$\zeta - \sum_{t \in \Omega^T} \left[\sum_{\nu \in \Omega_\nu} \vartheta_\nu \sum_{n \in \Omega^N} \lambda_{n,\nu}^{(t)}(\gamma) P_{n,\nu}^{\text{W},(t)}(\gamma) \right. \\ \left. - a^{(t)} \sum_{n \in \Omega^N} c_n^{(t)}(\gamma) X_n^{(t)}(\gamma) \right] \leq \eta(\gamma), \quad \forall \gamma \quad (1h)$$

$$\eta(\gamma) \geq 0, \quad \forall \gamma \quad (1i)$$

2. Problem description

Equations (1b)-(1d) model the wind power operation and investment constraints for all scenarios γ .

2. Problem description

Constraints (1b) limit the wind power production to the installed wind capacity times a factor $k_{n,\nu}^W$ modeling the wind power capacity factor for each operating condition.

Constraints (1c) imposes a cap on the investment budget for each period.

Constraints (1d) limit the total wind capacity to be installed at each bus of the system throughout the whole planning horizon.

2. Problem description

Constraints (1e)-(1g) are non-anticipativity constraints, i.e., constraints that prevent anticipating information.

2. Problem description

Constraints (1e) impose that the investment decisions at the beginning of the planning horizon do not depend on any scenario realization,

while constraints (1f) impose that, for periods others than the first, the investment decision variables depend on the scenario realization on the previous periods but they are unique for all the possible scenario realizations in the future.

2. Problem description

Constraints (1g) are non-anticipativity constraints for the wind power generation.

Finally, constraints (1h) and (1i) allow incorporating the CVaR risk metric.

2. Problem description

The upper–level problem (1) is constrained by a collection of lower–level problems (2),

which represent the market clearing for each scenario, for each period and for each load demand/wind power production condition within each scenario and period.

2. Problem description

where $\lambda_{n,\nu}^{(t)}(\gamma) \in \arg \left\{ \right.$

Minimize $\Delta_{LL,\nu}^{(t)}(\gamma)$

$$\sum_{i \in \Omega^G} \sum_{b \in \Omega_i} c_{ib} g_{ib,\nu}^{(t)}(\gamma) \quad (2a)$$

subject to

2. Problem description

$$\begin{aligned}
 & \sum_{i \in \Psi_n^G} \sum_{b \in \Omega_i} g_{ib,\nu}^{(t)}(\gamma) - \sum_{k|o(k)=n} f_{k,\nu}^{(t)}(\gamma) + \sum_{k|r(k)=n} f_{k,\nu}^{(t)}(\gamma) \\
 & + P_{n,\nu}^{W,(t)}(\gamma) = \sum_{j \in \Psi_n^D} d_j^{\max,(t)}(\gamma) k_{j,\nu}^D, \quad \forall n
 \end{aligned} \tag{2b}$$

$$f_{k,\nu}^{(t)}(\gamma) = B_k \left(\delta_{o(k),\nu}^{(t)}(\gamma) - \delta_{r(k),\nu}^{(t)}(\gamma) \right), \quad \forall k \tag{2c}$$

$$-f_k^{\max} \leq f_{k,\nu}^{(t)}(\gamma) \leq f_k^{\max}, \quad \forall k \tag{2d}$$

$$0 \leq g_{ib,\nu}^{(t)}(\gamma) \leq g_{ib}^{\max}, \quad \forall i, \forall b \tag{2e}$$

$$-\pi \leq \delta_{n,\nu}^{(t)}(\gamma) \leq \pi, \quad \forall n \setminus n: \text{ref.} \tag{2f}$$

$$\delta_{n,\nu}^{(t)}(\gamma) = 0, \quad n: \text{ref.} \tag{2g}$$

2. Problem description

$$\left. \begin{aligned} &\forall t, \forall \nu, \forall \gamma \\ &\Delta_{\text{LL},\nu}^{(t)}(\gamma_l) = \Delta_{\text{LL}}^{(t)}(\gamma_{\tilde{l}}), \\ &\forall \nu, \forall t, \forall l, \tilde{l} : \Upsilon^{(m)}(\gamma_l) = \Upsilon^{(m)}(\gamma_{\tilde{l}}), \forall m \leq t \end{aligned} \right\} \quad (2h)$$

2. Problem description

The objective function (2a) represents the minimization of the generation cost, equivalent in this case to the maximization of the social welfare since loads are considered constant within each operating condition, period and scenario.

2. Problem description

Equations (2b) represent the power balance at each bus of the system.

Constraints (2c) define the power flows through lines, which are limited to the transmission capacities by constraints (2d).

Constraints (2e) enforce power bounds for blocks of generation units other than wind power units.

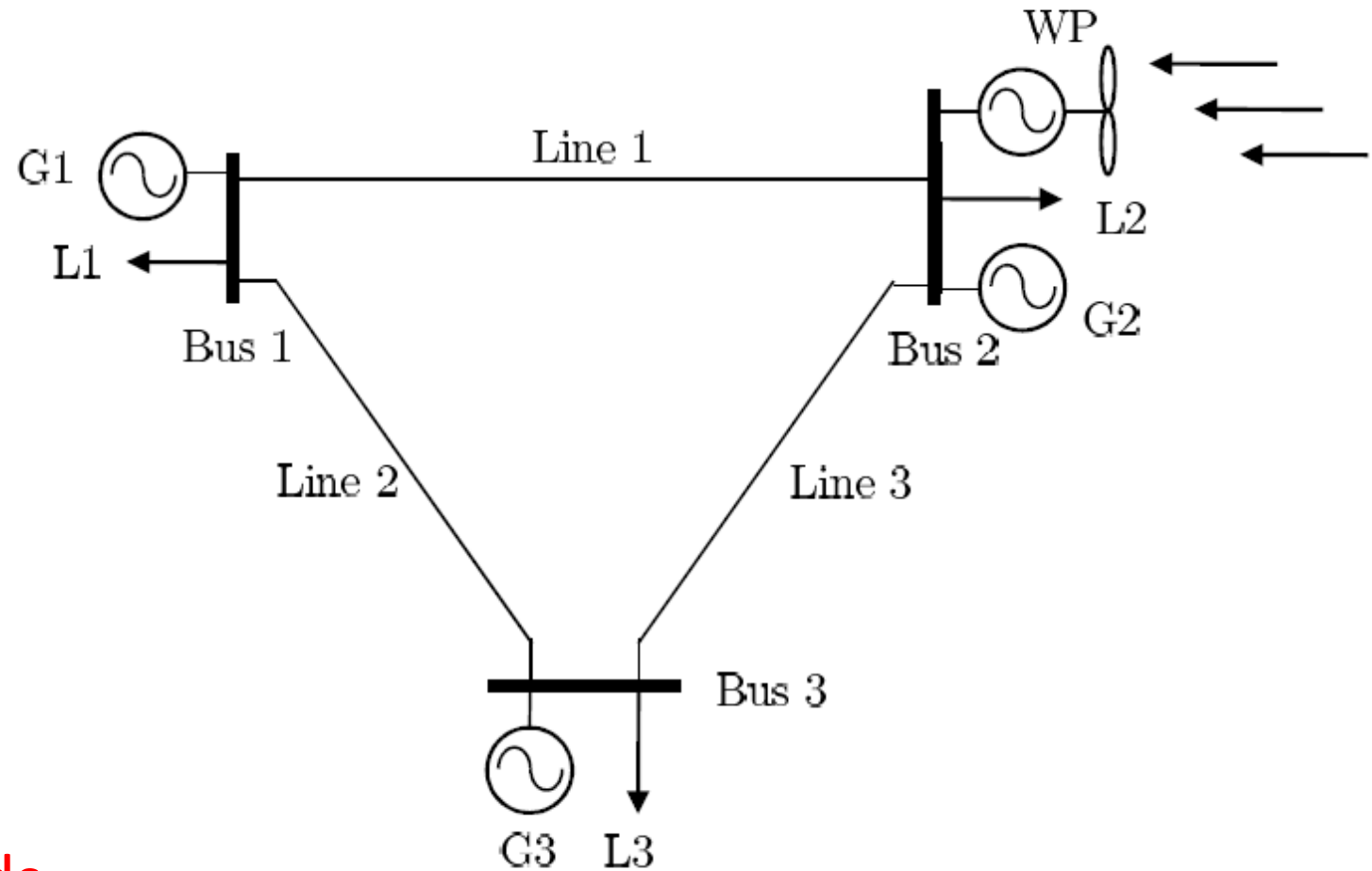
2. Problem description

Constraints (2f) and (2g) limit the voltage angle and fix the voltage angle at the reference bus, respectively.

Finally, constraints (2h) are non-anticipativity constraints that enforce that variables $\Delta_{LL,\nu}^{(t)}$ depend on the scenario realization of previous periods but they do not depend on the possible scenario realizations in future periods.

3. Case Study

3-bus system:



Two five-years periods

3. Case Study

3-bus system: **Just investment cost uncertainty**

- ✓ Investment cost known in period 1
- ✓ 3 investment cost scenario realizations in period 2: high (H), medium (M) and low (L)
- ✓ Risk-neutral ($\beta=0$) and risk-averse ($\beta=1$) solutions

3. Case Study

Results

3-bus system: investment cost uncertainty

Scenario	Risk-neutral		Risk-averse	
	Period 1	Period 2	Period 1	Period 2
H		0		0
M	114.3 MW	39.0 MW	153.3 MW	0
L		185.7 MW		146.7 MW

3. Case Study

3-bus system: **Just wind/demand uncertainty**

- ✓ 3 wind/demand conditions in period 1: H, M and L
- ✓ 3 wind/demand conditions in period 2 for each condition in period 1: H, M and L
- ✓ Risk-neutral ($\beta=0$) and risk-averse ($\beta=1$) solutions

3. Case Study

Results

3-bus system: **wind/demand uncertainty**

Condition	Risk-neutral		Risk-averse	
	Period 1	Period 2	Period 1	Period 2
HH, HM, HL		108.3 MW		108.3 MW
MH, MM, ML	152.3 MW	0.8 MW	60.7 MW	92.3 MW
LH, LM, LL		0		0

3. Case Study

3-bus system: **investment cost and wind/demand uncertainty**

- ✓ Investment cost known for period 1
- ✓ Two scenario realizations of investment cost in period 2: M, L
- ✓ 2 wind/demand conditions in period 1: H, L
- ✓ 2 wind/demand conditions in period 2 for each condition in period 1: H, L
- ✓ Risk-neutral ($\beta=0$) and risk-averse ($\beta=1$) solutions

3. Case Study

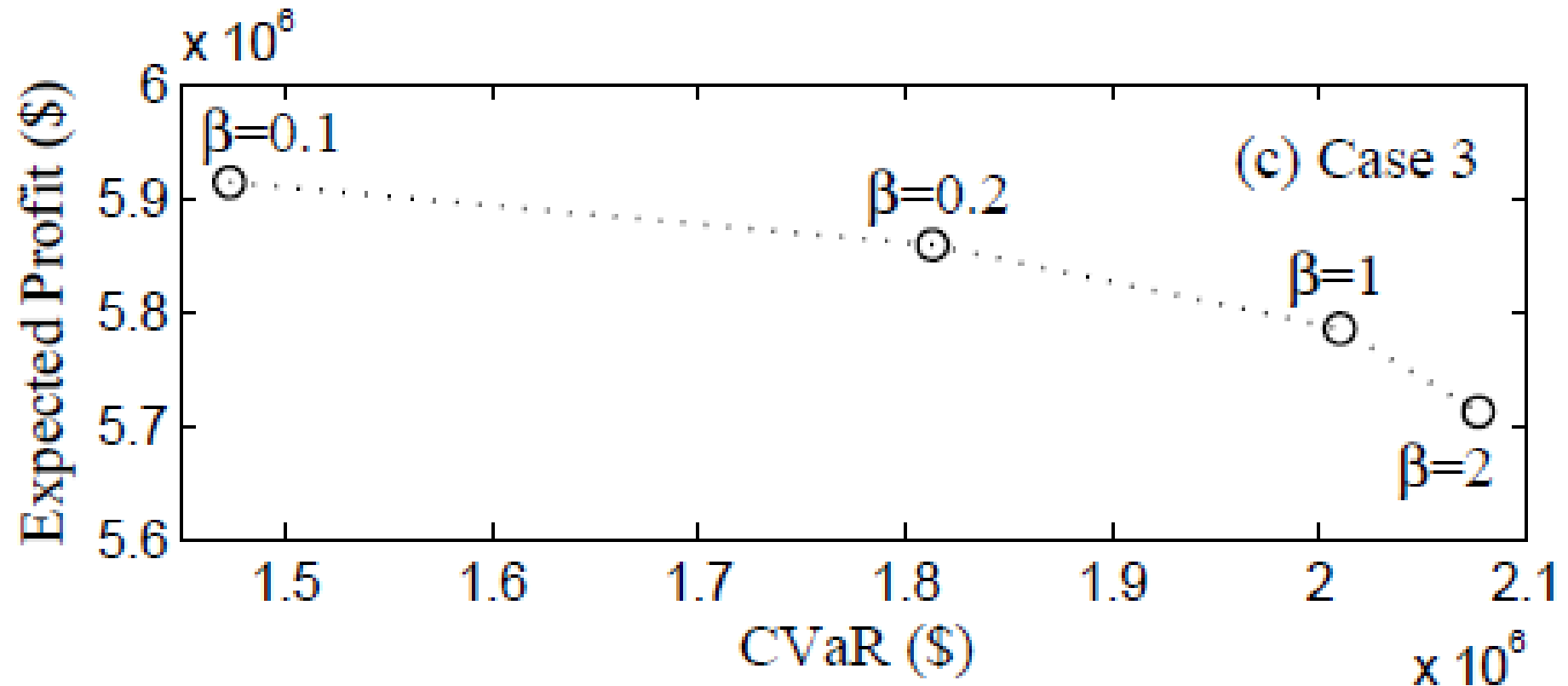
Results

3-bus system: investment cost and wind/demand uncertainty

Investment cost scenario	Demand/wind condition	Risk-neutral		Risk-averse	
		Period 1	Period 2	Period 1	Period 2
M	HH, HL		198.8 MW		136.0 MW
M	LH, LL		92.4 MW		78.7 MW
L	HH, HL	67.2 MW	232.8 MW	130.0 MW	170.0 MW
L	LH, LL		232.8 MW		170.0 MW

3. Case Study

3-bus system: Investment cost and wind/demand uncertainty (efficient frontier)



3. Case Study

IEEE 118-bus system:

- ✓ 3 five-year periods
- ✓ 32 wind/demand conditions and investment costs scenarios
- ✓ 2 potential locations of wind plants
- ✓ Risk-neutral ($\beta=0$) and risk-averse ($\beta=1$) solutions

3. Case Study

IEEE 118-bus system: **results**

- ✓ Different risk-aversion levels result in different investment strategies
- ✓ Computational issues:
 - ✓ Intractable if MILP is solved directly
 - ✓ Using Benders: around 20 h on a Linux-based server with four processors clocking at 2.9 GHz and **250 GB of RAM** (compatible with time requirements in investment studies)

4. Conclusions

1. A risk-constrained multi-stage modeling is a must for deciding renewable power investment
2. “Tractable” model for systems of realistic size via decomposition
3. Different risk-aversion levels: different investment strategies

Reading

- Baringo, L.; Conejo, A. J.; , "Risk-Constrained Multi-Stage Wind Power Investment," Power Systems, IEEE Transactions on, vol. 28, no. 1, pp.401-411, Feb. 2013

Thank
you



Notation: constants

$a^{(t)}$ Amortization factor.

B_k Susceptance of line k .

c_{ib} Price offered by the b th block of the i th generation unit.

$c_{\max}^{(t)}$ Budget for investment in wind power.

$c_n^{(t)}(\gamma)$ Investment cost of wind power at bus n .

Notation: constants

$d_j^{\max, (t)}$ (γ) Peak load of the j th demand.

f_k^{\max} Transmission capacity of line k .

g_{ib}^{\max} Upper limit of the b th block of the i th generation unit.

$k_{j,\nu}^{\mathbf{D}}$ Load level of the j th demand.

$k_{n,\nu}^{\mathbf{W}}$ Wind power capacity factor at bus n .

Notation: constants

$o(k)$ Sending-end bus of line k .

$r(k)$ Receiving-end bus of line k .

X_n^{\max} Maximum wind power capacity that can be installed at bus n .

Notation: constants

α Confidence level used to compute the CVaR.

β Weighting parameter modeling the tradeoff between expected profit and CVaR.

ϑ_ν Number of hours comprising the ν th load demand/wind power production condition.

$\tau(\gamma)$ Weight of scenario γ .

Notation: variables

$f_{k,\nu}^{(t)}(\gamma)$ Power flow through line k .

$g_{ib,\nu}^{(t)}(\gamma)$ Power produced by the b th block of the i th generation unit.

$P_{n,\nu}^{\mathbf{W},(t)}(\gamma)$ Wind power produced at bus n .

$X_n^{(t)}(\gamma)$ Wind power to be installed at bus n .

$\delta_{n,\nu}^{(t)}(\gamma)$ Voltage angle at bus n .

$\eta(\gamma), \zeta$ Auxiliary variables to compute the CVaR.

Notation: Indices and sets

$\Delta_{\mathbf{LL},\nu}^{(t)}(\gamma)$ Set of variables of the lower-level problem in the t th period, γ th scenario and ν th load demand/wind power production condition.

$\Delta_{\mathbf{UL}}^{(t)}(\gamma)$ Set of variables of the upper-level problem in the t th period and γ th scenario.

Notation: Indices and sets

Ψ_n^D Set of indices of the demands located at bus n .

Ψ_n^G Set of indices of the units located at bus n .

Ω_i Set of indices of the blocks of the i th generation unit.

Ω^G Set of indices of generation units.

Ω^K Set of indices of transmission lines.

Ω^N Set of indices of buses.