Mechanism design and allocation algorithms for energy-network markets with piece-wise linear costs and quadratic externalities

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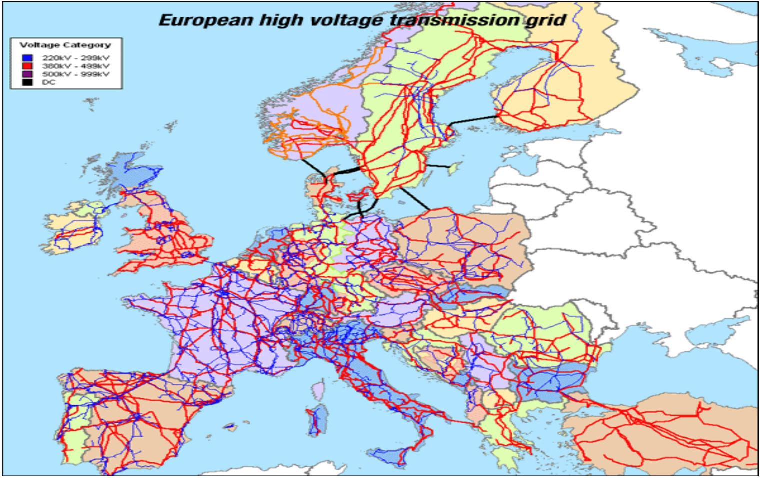
Outline

- Introduction and motivation
- Modeling market and Equilibrium. Discontinuous Games
- Nash and beyond
- Intrinsic market Power
- Efficient regulations and Extended Mechanism Design
- Conclusions

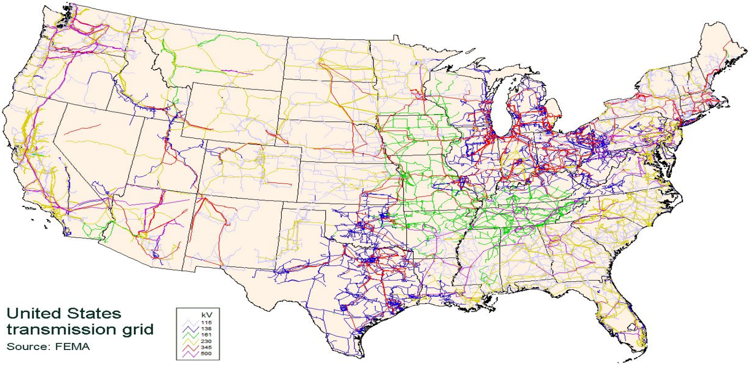
Motivations

- Most of ISOs have few generation companies: oligopoly
- Transmission networks highly congested in some areas
- Intrinsic market power produced by externalities and information asymmetries

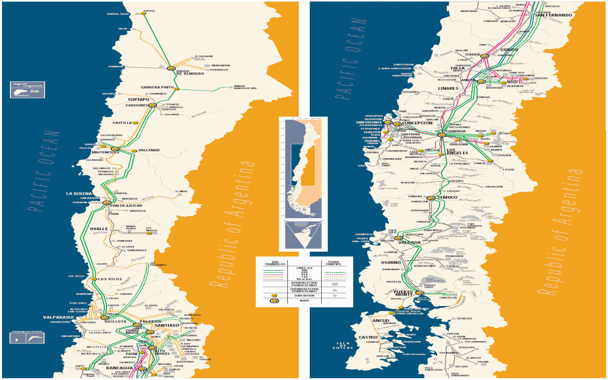
Transmission Europe



Transmission US



Transmission Chile



A generation short term market: day-ahead mandatory pool

- Today: generators taking into account an estimation of the demand bid *increasing piece-wise linear cost functions or equivalently piece-wise constant ”price”*. Even general convex cost functions.
- Tomorrow: the (ISO) using this information and knowing a realization of the demand, minimizes the sum of the costs to satisfy demands at each node considering all the transmission constraints: ”dispatch problem”.
- Tomorrow: the (ISO) sends back to generators the optimal quantities and ”prices” (multipliers associated to supply = demand balance equation at each node)

ISO problem or dispatch $DP(c, d)$

The (ISO) knows a realization of the demand $d \in \mathbb{R}^V$, receives the costs functions bid $(c_i)_{i \in G}$ and compute: $(q_i)_{i \in G}$, $(\lambda_i)_{i \in G}$

$$\min_{(h,q)} \quad \sum_{i \in G} c_i(q_i). \tag{1}$$

$$\sum_{e \in K_i} \frac{r_e}{2} h_e^2 + d_i \leq q_i + \sum_{e \in K_i} h_e sgn(e, i), \quad i \in G \tag{2}$$

$$q_i \in [0, \bar{q}_i], \quad i \in G, \tag{3}$$

$$0 \leq h_e \leq \overline{h}_e \tag{4}$$

We denote $Q(c,d) \subset I\!R^G$ the generation component of the optimal solution set associated to each cost vector submitted $c = (c_i)$ and demand d.
We denote $\Lambda(c,d) \subset I\!R^G$ the set of multipliers associated to the supply=demand in the ISO problem.

Modeling Generators

1. At each node $i \in G$ we have a generator with payoff

$$u_i(\lambda, q) = \lambda q - \bar{c}_i(q)$$

   $\bar{c}_i$ is the real cost.

2. The strategic set for each player i denoted S_i:

   $\{c_i : \mathbb{R} \to \mathbb{R}_+ |$ convex, nondecreasing, bounded subgradients or

   $\partial c_i \subset [0, p^*]$, p^* is a *price cap*.

Equilibrium

An equilibrium is (q, λ, m) such that q is a selection of $Q(\cdot, \cdot)$ and λ is a selection of $\Lambda(\cdot, \cdot)$ and $m = (m_i)_{i \in G}$ is a mixed-strategy equilibrium of the generator game in which each generator submits costs $c_i \in S_i$ with a payoff

$$\mathbb{E}u_i(\lambda_i(c, \cdot), q_i(c, \cdot)) = \int_D [\lambda_i(c, d) q_i(c, d) - \bar{c}_i(q_i(c, d))] d\mathbb{P}(d),$$

Neutral or risk averse

Literature

- In some cases, for example, using a supply function equilibria approach there are previous works by Anderson, Philpott, or using variational inequality approach by Pang, Ralph, Ferris or also using game theory by Hogan, Smeers, Wilson, Joskow, Tirole, Hobbs, Oren, Borestein, Wolak...

Nash equilibrium

Consider a game $G = (X_i, u_i)^N$ that consists of N players where each player $i = 1, \dots, N$ has a strategy set X_i and a payoff function $u_i : X \to \mathbb{R}$, where $X = \Pi_{i \in N} X_i$.

Nash equilibrium (x_i^*)

$$x_i^* \in \text{ argmax } \{u_i(x_i, x_{-i}^*) | \, x_i \in X_i\}$$

Nash equilibrium

For the sake of simplicity, we assume that each X_i is contained in a metric vectorial space:

- If for all i the strategy set X_i is a compact set, and u_i is a bounded function, we say that G is a *compact game*.
- If for all i the set X_i is convex and for each $x_{-i} \in X_{-i}$, $u_i(\cdot, x_i)$ is a (concave) quasiconcave function, then we say that G is a *convex game* (*quasiconvex game*)

Nash equilibrium existence

A convex compact game $G = (X_i, u_i)^N$ satisfying:

- $u_i(\cdot, \cdot)$ is upper semicontinuous
- $u_i(x_i, \cdot)$ is lower semicontinuous for all x_i

has a Nash equilibrium point.

Extensions: generalized games, convergence-stabilty lopsided convergence

Discontinuos games: tie-breaking rules

Consider the following two-player game: Let the payoff for the i player be given by

$$u_i(x_i, x_{-i}) = \begin{cases} l_i(x_i) & \text{if} \quad x_i < x_{-i}, \\ \varphi(x_i) & \text{if} \quad x_i = x_{-i}, \\ m_i(x_{-i}) & \text{if} \quad x_i > x_{-i}, \end{cases} \tag{5}$$

where $x_i \in [0, 1]$. Assume that for all i and $x \in [0, 1]$ (a) l_i and m_i are continuous functions, l_i is nondecreasing $\varphi(x)$ is a convex combination of $l_i(x)$ and $m_i(x)$;
sign $[l_i(x) - \varphi(x)] =$ sign $[\varphi_{-i}(x) - m_{-i}(x)]$.

Existence discontinuos games

Reny (1999) Econometrica

Theorem

A compact quasiconcave game possesses a Nash equilibrium if it is also a better reply secure game.

Bagh and Jofre (2006) Econometrica

Theorem

If $(X_i, u_i)^N$ is weakly reciprocally upper semicontinuous and payoff secure, then it is better reply secure.

Assumptions

S1. For all $d \in D$, there exists $\delta_d > 0$ such that

$$\Omega(d) \neq \emptyset, \quad \|\hat{d} - d\| \leq \delta_d.$$

S2. D is compact
S3. (1) Either $\mathbb{P}$ is non atomic; or (2) given two convex sets $M, N \subset \mathbb{R}^G$, $u(M \times N)$ is convex.
S4 $u_i : \mathbb{R}^2 \to \mathbb{R}$ is continuous.

Equilibrium existence

Theorem

If each S_v is a nonempty closed set for the point-wise convergence, then there exists an equilibrium (q, λ, m) for the bid-based generator pool game.

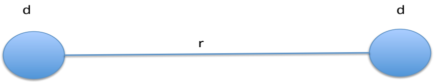
Example: In real system... increasing piece-wise constant cost functions

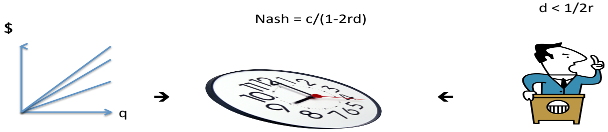
Two-node case

Two nodes case

Symmetric Nash equilibrium

Profit = multiplier × quantity − cost × quantity





the ISO Problem: two-node case

Given that each generator reveals a cost c_i, the (ISO) solves:

$$\begin{aligned}
\min_{q,h} \quad & \sum_{i=1}^{2} c_i q_i \\
s.t. \quad & q_i - h_i + h_{-i} \geq \tfrac{r}{2}[h_1^2 + h_2^2] + d \quad \text{for} \quad i = 1, 2 \\
& q_i, h_i \geq 0 \quad \text{for} \quad i = 1, 2
\end{aligned}$$

Result

- Escobar and J. (ET (2010)) equilibrium exists but producers charge a price above marginal cost:
- $$Nash = \bar{c}/(1 - 2rd)$$

Sensitivity formula

Proposition

Let $c \in \prod_{i\in G} S_i$ *and* $c_i - \hat{c}_i$ *a Lipschitz function with constant* κ*. Then,*

$$|Q_i(c,d) - Q_i(\hat{c}_i, c_{-i}, d)| \leq \kappa\eta,$$

where $\eta = 2\frac{(1+r_i\overline{h}_i)^2}{\min_{i\in G} r_i c_i^+(0)} \in]0, +\infty[$ *and* $c_i^+(0) = \lim_{y\to 0+} \frac{c_i(y)-c_i(0)}{y}$*.*

Why? losses $=>$ the second-order growth

Market Power formula

Proposition

The equilibrium prices p_i satisfy

$$\mathbb{E}|p_i - \gamma| \geq \frac{\mathbb{E}[Q_i(p_i, p_{-i}, d)]}{\bar{\eta}_i}$$

where $\bar{\eta}_i = 2\frac{|K_i|^2\left(1+\max\{r_e\overline{h}_e \,:\, e\in K_i\}\right)^2}{p_* \min_{e\in K} r_e}$

$\gamma(p_{-i}, d)$ *is a measurable selection of* $\partial\bar{c}_i(Q_i(p_i, p_{-i}, d))$.

Market Power formula

Proposition

Linear case: $\bar{c}_i(q) = \bar{c}_i q$, *then*

$$p_i - \bar{c}_i \geq \frac{\mathbb{E}[Q_i(p_i, p_{-i}, d)]}{\bar{\eta}}.$$

The Questions

In an electric network with **transmission costs** and **private information**:

- Does the usual (price equal Lagrange multiplier) regulation mechanism minimize costs for the society?
- If not, what is the mechanism that achieves this objective?
- How does the performance of both systems compare?

Methodology:

- Bayesian Game Theory
- Mechanism Design

Framework

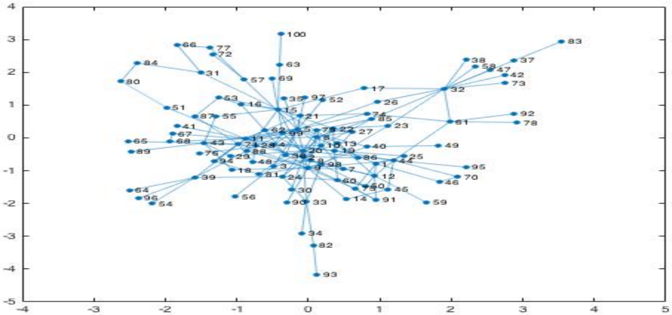
- A network with demand d at each node.
- One producer at each node, with piece-wise linear cost of production $c_i \sim F_i[\underline{c}_i, \overline{c}_i]$. Common knowledge !
- Transmission costs rh^2, with h the amount sent from one node to another.

ISO for piece-wise linear cost functions

Problem

$$
\begin{aligned}
\underset{(q,h)}{\textit{minimize}} \quad & \sum_{i=1}^{n}\sum_{j=1}^{N} q_{i,j}c_{i,j} \\
& \sum_{j=1}^{N} q_{i,j} + \sum_{i'\in V(i)} h_{i',i} - h_{i,i'} - \frac{h_{i,i'}^2 + h_{i',i}^2}{2} r_{i,i'} \geq d_i \quad (\lambda_i) \\
& \forall (i,i') \in E : h_{i,i'} \geq 0 \quad (\gamma_{i,i'}) \\
& \forall i \in I, j \in J : q_{i,j} \geq 0 \quad (\mu_{i,j}) \\
& \forall i \in I, j \in J : q_{i,j} \leq \bar{q} \quad (\nu_{i,j}).
\end{aligned} \tag{6}
$$

100 nodes network



The ISO Problem: two-node case

Given that each generator reveals a cost c_i, the ISO solves:

$$
\begin{aligned}
\min_{q,h} \quad & \sum_{i=1}^{2} c_i q_i \\
s.t. \quad & q_i - h_i + h_{-i} \geq \tfrac{r}{2}[h_1^2 + h_2^2] + d \quad \text{for} \quad i = 1, 2 \\
& q_i, h_i \geq 0 \quad \text{for} \quad i = 1, 2
\end{aligned}
$$

The Solution for ISO problem

If we define

$$H(x,y) = d + \frac{1}{2r}\left(\frac{x-y}{x+y}\right)^2 - \frac{1}{r}\left(\frac{x-y}{x+y}\right)$$

and

$$\overline{q} = 2\left[\frac{1-\sqrt{1-2dr}}{r}\right]$$

then the solution to this problem can be written as

$$q_i(c_i, c_{-i}) = \begin{cases} H(c_i, c_{-i}) & \text{if } \; H(c_i, c_{-i}) \geq 0 \; \text{ and } \; H(c_{-i}, c_i) \geq 0 \\ \overline{q} & \text{if } \; H(c_{-i}, c_i) < 0 \\ 0 & \text{if } \; H(c_i, c_{-i}) < 0 \end{cases}$$

$$\lambda_i(c_i, c_{-i}) \equiv p_i(c_i, c_{-i}) = c_i \quad \text{if } \; H(c_i, c_{-i}) \geq 0$$

The Bayesian Game: benchmark

The game:

- 2 players. Strategies $c_i \in C_i = [\underline{c}_i, \overline{c}_i]$, i=1,2.
- Payoff $u_i(c_i, c_{-i}) = (\lambda_i(c_i, c_{-i}) - \mathbf{c}_i)q_i(c_i, c_{-i})$,

where $\mathbf{c}_i$ is the real cost. The Equilibrium:

- A strategy $b_i : [\underline{c}_i, \overline{c}_i] \longrightarrow I\!R^+$ (convex at equilibrium!)
- In a Nash equilibrium

$$\overline{b}(c) \in arg\max_x \int_{C_{-i}} [\lambda_i(x, \overline{b}(c_{-i})) - c]q_i(x, \overline{b}(c_{-i}))f_{-i}(c_{-i})dc_{-i} \tag{7}$$

Numerical Approximation

- For simplicity $C_i = [1, 2]$.
- Let $k \in \{0, ..., n-1\}$, and $b(c) = b_k$ for $c \in [\frac{k}{n}, \frac{k+1}{n}]$.
- The weight of each interval is given by $w_k = F(\frac{k+1}{n}) - F(\frac{k}{n})$.
- The approximate equilibrium is characterized by:

$$b_k \in arg\max_x \sum_{l=0}^{n-1} [\lambda_i(x, b_l) - r_k] q_i(x, b_l) w_l \quad \text{for all} \quad k \in \{0, ..., n-1\} \tag{8}$$

Optimal Mechanism. Principal Agent Model (Myerson)

- A *direct revelation mechanism* $M = (q, h, x)$ consists of an *assignment rule* $(q_1, q_2, h_1, h_2) : C \longrightarrow R^4$ and a *payment rule* $x : C \longrightarrow R^2$.
- The ex-ante expected profit of a generator of type c_i when participates and declares c_i' is

$$U_i(c_i, c_i'; (q, h, x)) = E_{c_{-i}}[x_i(c_i', c_{-i}) - c_i q_i(c_i', c_{-i})]$$

- A mechanism (q, h, x) is feasible iff:

$$\begin{aligned}
U_i(c_i, c_i; (q, h, x)) &\geq U_i(c_i, c_i'; (q, h, x)) \quad \text{for all} \quad c_i, c_i' \in C_i \\
U_i(c_i, c_i; (q, h, x)) &\geq 0 \quad \text{for all} \quad c_i \in C_i \\
q_i(c) - h_i(c) + h_{-i}(c) &\geq \frac{r}{2}[h_1^2(c) + h_2^2(c)] + d \quad \text{for all} \quad c \in C \\
q_i(c), h_i(c) &\geq 0 \quad \text{for all} \quad c \in C
\end{aligned}$$

The Regulator's Problem

Using the revelation principle, the regulator's problem can be written as:

$$\min \int_C \sum_{i=1}^{2} x_i(c) f(c) dc \tag{9}$$

subject to $\quad (q, h, x) \quad$ being "feasible"

Existence: Knuster-Tarski fixed point theorem (monotone relations)

The Regulator's Problem (II)

It can be rewritten as

$$
\begin{array}{ll}
\min & \int\limits_C \sum\limits_{i=1}^{2} q_i(c)[c_i + \frac{F_i(c_i)}{f_i(c_i)}] f(c) dc \\
\text{s.t} & \int\limits_{C_{-i}} q_i(c_i, c_{-i}) f_{-i}(c_{-i}) dc_{-i} \ \text{ is non-increasing in } \ c_i \\
 & q_i(c) - h_i(c) + h_{-i}(c) \geq \frac{r}{2}[h_1^2(c) + h_2^2(c)] + d \ \text{ for all } \ c \in C \\
 & q_i(c), h_i(c) \geq 0 \ \text{ for all } \ c \in C
\end{array}
$$

We denote by $J_i(c_i) = c_i + \frac{F_i(c_i)}{f_i(c_i)}$ the virtual cost of agent i. We assume it is increasing (Monotone likelihood ratio property: true for any log concave distribution)

Solution

An optimal mechanism is given by

$$\hat{q}_i(c_i, c_{-i}) = \begin{cases} H(J_i(c_i), J_{-i}(c_{-i})) & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) \geq 0 \text{ ar} \\ \overline{q} & \text{if } H(J_{-i}(c_{-i}), J_i(c_i)) < 0 \\ 0 & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) < 0 \end{cases}$$

$$\hat{x}_i(c_i, c_{-i}) = c_i \hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{\overline{c}_i} \hat{q}_i(s, c_{-i}) ds$$

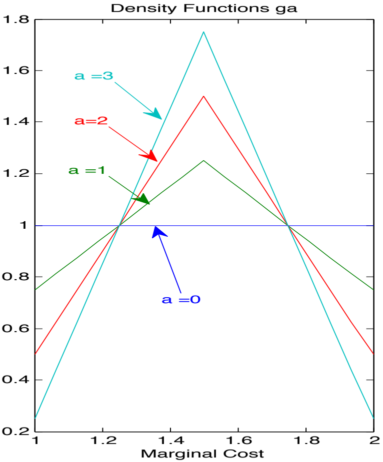
Such mechanism is dominant strategy incentive compatible.

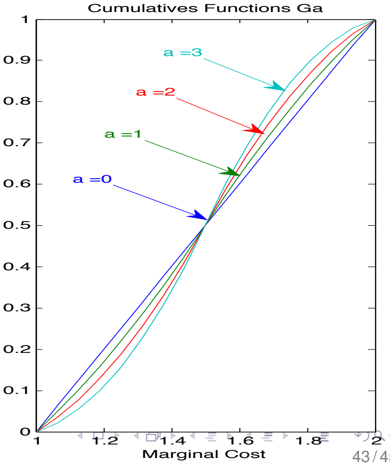
Comparing Benchmark with Optimal Mechanism

We consider the family of distributions with densities

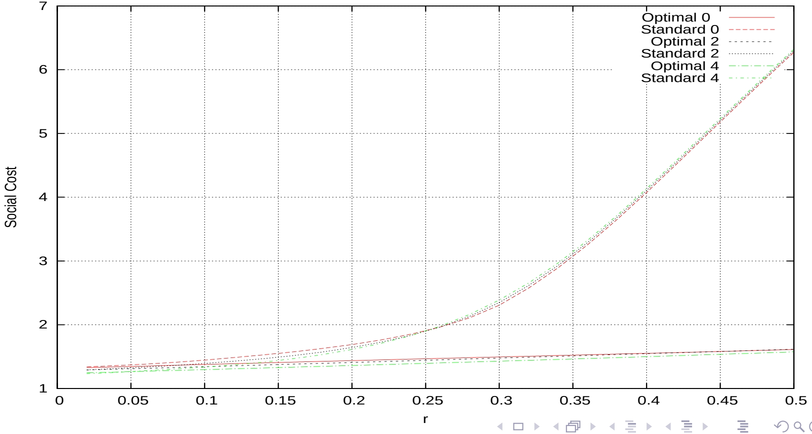
$$f_a(x) = \begin{cases} a(x-1) + (1 - \frac{a}{4}) & \text{if} \quad x \leq 1.5 \\ -a(x-1) + (1 + \frac{3a}{4}) & \text{if} \quad x \geq 1.5 \end{cases}$$

Asymmetric information





Social costs for different mechanisms



Robustness and Practical Implementation

- The optimal mechanism is detail free. If the designer is wrong about common beliefs, then the mechanism is still not bad:

$$||X_f - X_{\tilde{f}}|| \leq ||x||_1 ||f - \tilde{f}||_\infty \leq \bar{c}\bar{q}||f - \tilde{f}||_\infty$$

- The assignment rule is computationally simple to implement. It requires solving **once** the dispatcher problem, with modified costs.
- However, the payments are computationally difficult

$$c_i \hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{\bar{c}_i} \hat{q}_i(s, c_{-i}) ds$$

- Bagh, Adib; Jofre, Alejandro. Weak reciprocally upper-semicontinuity and better reply secure games: A comment. *Econometrica*, Vol. 74 Issue 6 (2006), 1715-1721.
- Figueroa, N. Jofre, A. and Heymann B. Cost-Minimizing regulations for a wholesale electricity market. Submitted (2015)
- Heymann B. and Jofre, A. Mechanism design and allocation algorithms for network markets with piece-wise linear costs and quadratic externalities. (2016)
- Escobar, Juan and Jofre, Alejandro. Monopolistic Competition in Electricity Markets. *Economics Theory*, 44, Number 1, 101-121 (2010)
- Escobar, Juan and Jofre, Alejandro. Equilibrium analysis of electricity auctions. Submitted.

Escobar, Juan F.; Jofre, Alejandro. Equilibrium analysis for a network market model. Robust optimization-directed design, 63–72, Nonconvex Optim. Appl., 81, Springer, New York, (2006).