Two applications of bilevel programming in energy modelling

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in collaboration with

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in Energy Economics

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outline of the presentation

- bilevel optimization
- peak load management through price signals
  - context
  - preemptive model
  - non-premptive model
  - mixed model
  - algorithmic considerations
- V2G–G2V and the smart grid
  - context
  - mathematical model
  - MIP reformulation
  - numerical results
- ongoing research

bilevel optimization

$$I \qquad \max_{x \in X, y \in Y(x)} \quad f(x,y)$$

$$\text{s.t.} \quad y \in \arg\min_{y' \in Y(x)} g(x,y')$$

$$II \qquad \max_{x \in X, y \in Y(x)} \quad f(x,y)$$

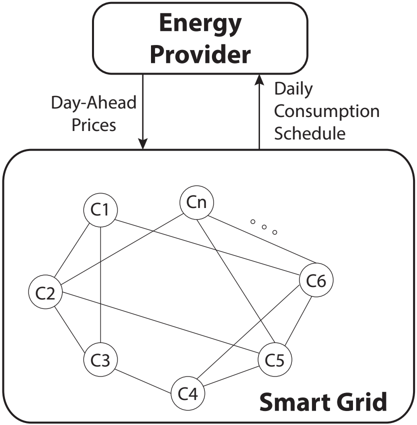
$$\text{s.t.} \ \ y \in Y(x)$$

$$\langle F(y), y - y' \rangle \leq 0 \quad \forall y' \in Y(x)$$

PEAK LOAD MANAGEMENT: context

- energy sources:
  renewable vs. non-renewable, reliable vs. unreliable
- demand response
- role of the smart grid
- dynamic pricing for load shaping (residential):
  peak clipping, valley filling, load shifting
- revenue optimization

preemptive model



Objectives:

Minimize peak
Maximize revenue

Minimize payment
Minimize inconvenience

preemptive model: notation

T : horizon
$h \in H$: hour
$n \in N$: customer
$a \in A_n$: appliance, set of appliances for customer n
$E_{n,a}$: demand
λ_n : inconvenience coefficient
Γ : peak load
p^h : tariff
$p^h_{\max}$: upper bound
$x^h_{n,a}$: consumption
$\beta^{\max}_{n,a}$: upper bound on consumption
$C_{n,a}(h)$: inconvenience $= \lambda_n \times E_{n,a} \times \frac{h - TW^b_{n,a}}{TW^e_{n,a} - TW^b_{n,a}}$
κ : trade-off between revenue and peak

preemptive model: formulation

$$
\begin{aligned}
\max_{p,\Gamma} \quad & \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in T_{n,a}} p^h x_{n,a}^h - \kappa\Gamma \\
\text{s.t.} \quad & \Gamma \geq \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in T_{n,a}} x_{n,a}^h \\
& 0 \leq p^h \leq p_{\max}^h
\end{aligned}
$$

$$
\begin{aligned}
\min_{x} \quad & \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in T_{n,a}} (p^h + C_{n,a}(h)) x_{n,a}^h \\
\text{s.t.} \quad & 0 \leq x_{n,a}^h \leq \beta_{n,a}^{\max} \\
& \sum_{h\in T_{n,a}} x_{n,a}^h \geq E_{n,a}
\end{aligned}
$$

preemptive model: MIP

$$\max_{\substack{p,\Gamma,x\\ w,v,\psi}} \quad -\sum_{\substack{n\in N\\ a\in A_n\\ h\in T_{n,a}}} \beta^{\max}_{n,a} w^h_{n,a} + \sum_{\substack{n\in N\\ a\in A_n}} E_{n,a} v_{n,a} - \sum_{\substack{n\in N\\ a\in A_n\\ h\in T_{n,a}}} C^h_{n,a} x^h_{n,a} - \kappa\Gamma$$

s.t.

$$\Gamma \geq \sum_{n,a} x^h_{n,a} \qquad\qquad w^h_{n,a} - M_1 \xi^h_{n,a} \leq 0$$

$$0 \leq p^h \leq p^h_{\max} \qquad\qquad \sum_h x^h_{n,a} + M_2 \epsilon_{n,a} \leq M_2 + E_{n,a}$$

$$0 \leq x^h_{n,a} \leq \beta^{\max}_{n,a} \qquad\qquad v_{n,a} - M_2 \epsilon_{n,a} \leq 0$$

$$\sum_h x^h_{n,a} \geq E_{n,a} \qquad\qquad x^h_{n,a} - M_3 \psi^h_{n,a} \leq 0$$

$$-w^h_{n,a} + v_{n,a} - p^h \leq C^h_{n,a} \qquad\qquad w^h_{n,a} - v_{n,a} + p^h + M_1 \psi^h_{n,a} \leq M_1 - C^h_{n,a}$$

$$-x^h_{n,a} + M_1 \xi^h_{n,a} \leq M_3 - \beta^{\max}_{n,a}$$

$$x^h_{n,a} - M_3 \psi^h_{n,a} \leq 0$$

$$\xi^h_{n,a}, \psi^h_{n,a} \in \{0,1\};\ w^h_{n,a} \geq 0 \qquad\qquad \epsilon_{n,a} \in \{0,1\};\ v_{n,a} \geq 0$$

preemptive model: competition

$$
\begin{aligned}
\max_{p,\Gamma} \quad & \sum_{n,a,h} p^h x^h_{n,a} - \kappa\Gamma \\
\text{s.t.} \quad & \Gamma \geq \sum_{n,a,h} x^h_{n,a} \\
& 0 \leq p^h \leq p^h_{\max} \\
\min_{x,\bar{x}} \quad & \sum_{n,a,h} (p^h + C_{n,a}(h)) x^h_{n,a} + \sum_{n,a,h} (\bar{p}^h + C_{n,a}(h)) \bar{x}^h_{n,a} \\
\text{s.t.} \quad & x^h_{n,a} + \bar{x}^h_{n,a} \leq \beta^{\max}_{n,a} \\
& \sum_{h} x^h_{n,a} + \bar{x}^h_{n,a} \geq E_{n,a} \\
& x^h_{n,a}, \bar{x}^h_{n,a} \geq 0
\end{aligned}
$$

experimental framework

- sensitivity w.r.t. trade-off parameter κ
- sensitivity w.r.t. to time window width
- random generation of time windows
- comparison of monopolistic and competitive models

20% TWW instances: 10 customers, 3 appliances

	mono.			comp.		
κ	revenue	inconv.	total	revenue	inconv.	total
200	78.02	21.49	99.51	78.31	21.17	99.48
400	77.15	21.59	98.74	78.01	20.13	98.14
600	75.76	21.85	97.61	77.84	18.74	96.58
800	73.52	22.16	95.68	78.07	16.81	94.88
1000	71.50	22.48	93.99	77.63	16.28	93.91
Average	75.19	21.91	97.10	77.97	18.63	96.60

100% TWW instances: 10 customers, 3 appliances

κ	mono. revenue	inconv.	total	comp. revenue	inconv.	total
200	85.12	14.16	99.28	85.25	14.06	99.30
400	82.87	14.55	97.42	84.05	13.90	97.95
600	80.13	15.29	95.42	84.67	12.69	97.35
800	75.67	16.29	91.96	84.33	11.84	96.17
1000	74.95	16.44	91.39	83.70	11.45	95.15
Average	79.75	15.35	95.10	84.40	12.79	97.19

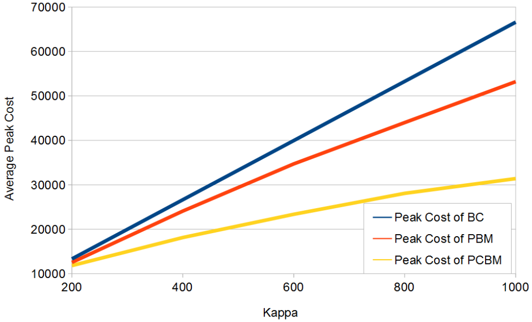
running times 20% instances

	cpu		gap		# unsolved	
κ	mono	comp.	mono	comp	mono	comp
200	1.10	1.20	0.00%	0.00%	0	0
400	3.10	3.50	0.00%	0.00%	0	0
600	6.80	13.10	0.00%	0.00%	0	0
800	8.90	56.60	0.00%	0.00%	0	0
1000	17.90	63.00	0.00%	0.00%	0	0

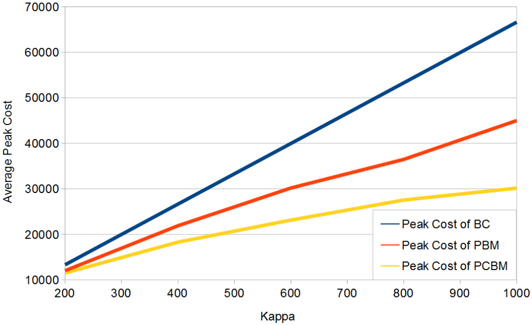
running times 100% instances

	cpu		gap		# unsolved	
κ	mono	comp.	mono	comp	mono	comp
200	28.10	321.10	0.00%	0.00%	0	0
400	339.50	592.67	0.00%	0.55%	0	1
600	2040.20	1232.88	0.00%	3.87%	0	2
800	4666.40	2350.67	0.00%	3.73%	0	4
1000	7707.00	2034.14	0.00%	6.04%	0	3

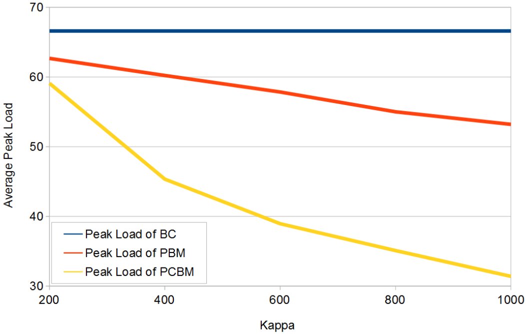
peak cost vs. κ (20% instances)



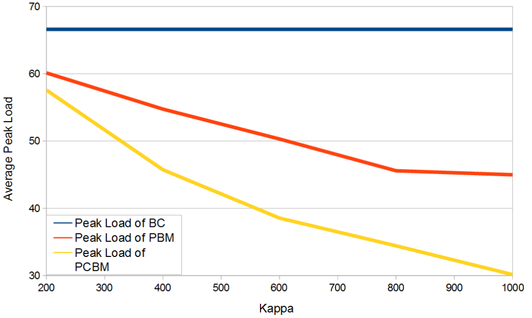
peak cost vs. κ (100% instances)



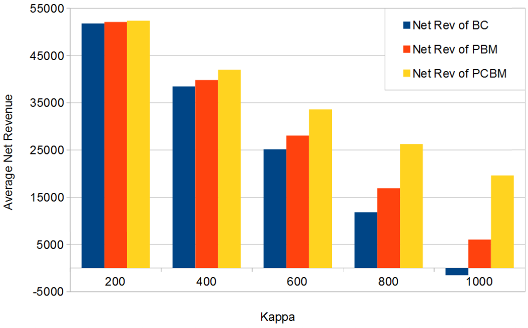
peak load vs. κ (20% instances)



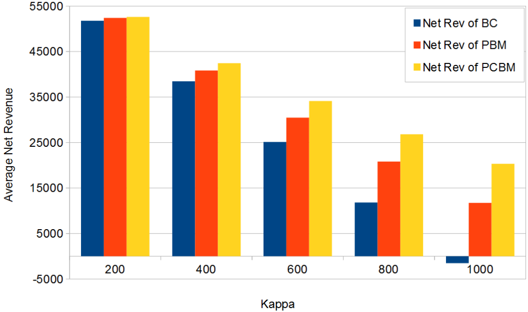
peak load vs. κ (100% instances)



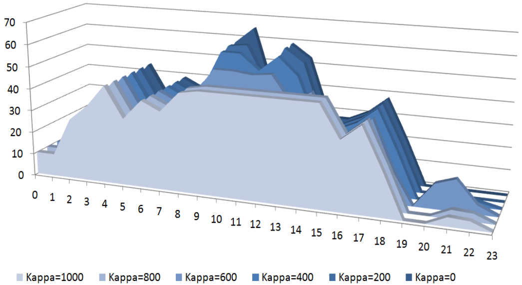
net revenue vs. κ (20% instances)



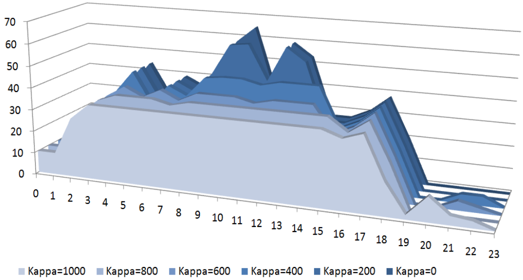
net revenue vs. κ (100% instances)



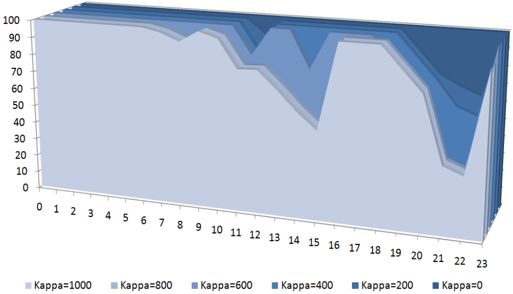
load distribution (monopolistic case)



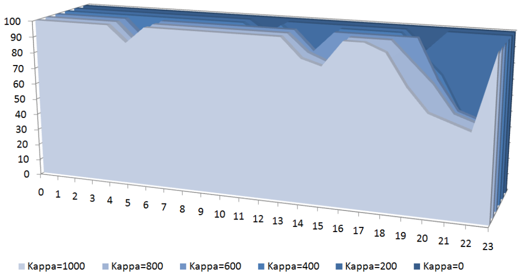
load distribution (competitive case)



tariffs (monopolistic case)



tariffs (competitive case)



a non-preemptive model (with competition)

$$
\begin{aligned}
k_{n,a} :&\quad \text{consumption} \\
l_{n,a} :&\quad \text{time duration} \\
C_{n,a}(h) :&\quad \text{inconvenience} = \lambda_n \times k_{n,a} \times l_{n,a} \times \tfrac{h - TW^b_{n,a}}{TW^e_{n,a} - TW^b_{n,a}} \\
x^h_{n,a} &\quad : \text{starting time (binary variable)}
\end{aligned}
$$

- lower level is a particular instance of assignment (easy), hence lower level integrality constraints may be lifted
- however, integrality has to be enforced at the upper level

bilevel programming formulation

$$\max_{p,\Gamma} \quad \sum_{n,a,h} k_{n,a} x_{n,a}^{h} \sum_{h'=h}^{h+l_{n,a}} p^{h'} - \kappa\Gamma$$

$$\text{s.t.} \quad \Gamma \geq \sum_{n,a} \sum_{h'=h-l_{n,a} \in T_{n,a}} k_{n,a} x_{n,a}^{h}$$

$$0 \leq p^{h} \leq p_{\max}^{h}$$

$$\min_{x,\bar{x}} \quad \sum_{n,a,h} k_{n,a} x_{n,a}^{h} + \sum_{h'=h}^{h+l_{n,a}} p^{h'} + \sum_{n,a,h} C_{n,a}(h)(x_{n,a}^{h} + \bar{x}_{n,a}^{h})$$

$$\text{s.t.} \quad 0 \leq x_{n,a}^{h} + \bar{x}_{n,a}^{h} = 1$$

$$x_{n,a}^{h},\ \bar{x}_{n,a}^{h} \in \{0, 1\}$$

a mixed model (with competition)

Notation becomes more involved, but you get the idea.

algorithmics: first heuristic

1. for a given price, find the maximum load and the corresponding slot
2. decrease tariffs slots following the critical one
3. solve lower level problem
4. find corresponding optimal tariffs (inverse optimization)
5. if best incumbent solution is improved, repeat the procedure, else halt

algorithmics: second heuristic

1. binary search w.r.t. peak load
2. for fixed Γ solve the corresponding MIP
   and implement inverse optimization

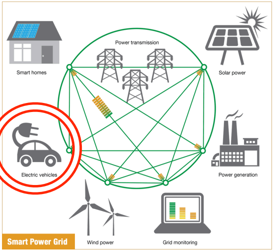
algorithmics: other ideas

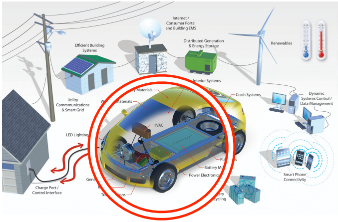
1. schedule shifting
2. divide-and-stitch

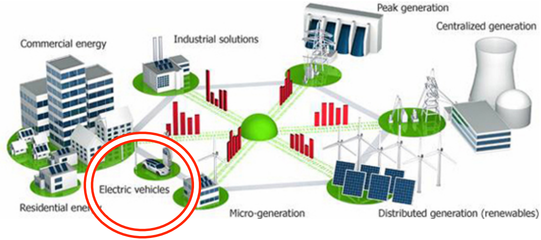
V2G–G2V AND THE SMART GRID: outline

- context and motivation
- bilevel formulation
- MPEC formulation
- MILP formulation
- illustrative case
- the Ontario Power System
- conclusion and perspectives

EV in the smart grid ecosystem

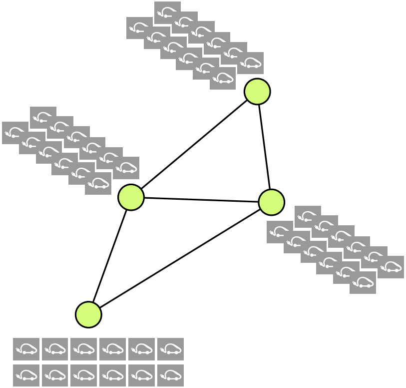


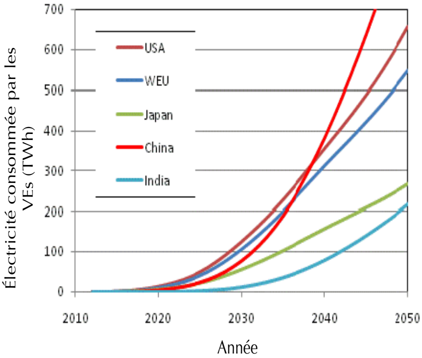




EV in the smart grid ecosystem

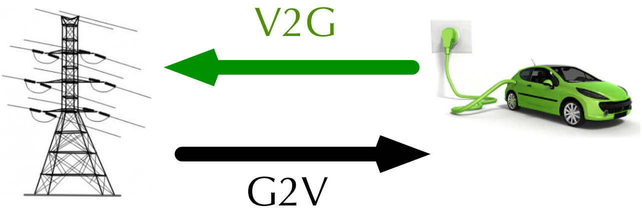
- EV load estimation





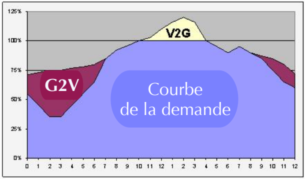
EV in the smart grid ecosystem

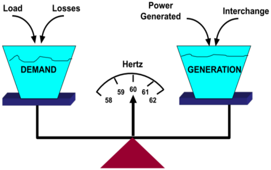
- bi-directional energy transfer

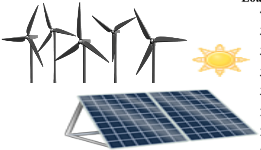


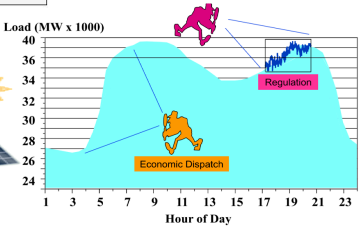
EV in the smart grid ecosystem

- EVs = an opportunity









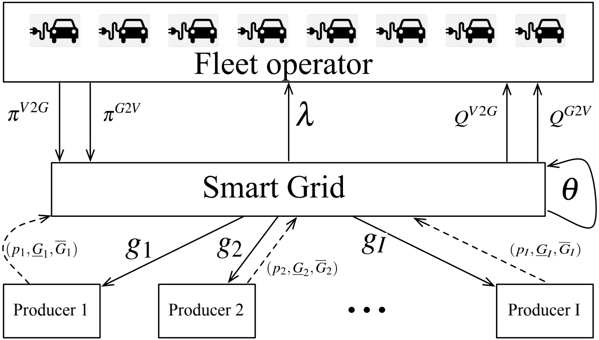
context and motivation

stakeholders

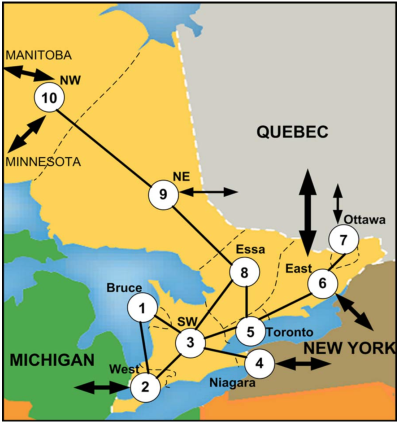
- EV fleet operator (FO)
  - satisfies EV power demand
  - two-way communication with smart grid
    - offer prices: V2G
    - bid prices: G2V
- smart grid (SG)
  - input: offers of the producers, offers and bids of the FO
  - economic constraints (market clearing): supply/demand balance
  - technical constraints: optimal power flow (OPF)
  - output: accepted quantities (production, G2V, V2G), voltage angles

power supplied by competition is exogenous to the model

context and motivation (continued)

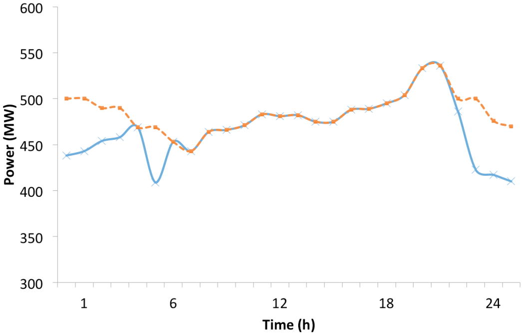


spatial aspect: Ontario power grid



- generation, demand, G2V, V2G : by node
- nodes are connected by transmission lines

temporal aspect: one-day horizon



- optimization horizon: one day
- 24 time slots
- hourly decisions

bilevel formulation

Stackelberg game

- **leader :** fleet operator (FO)
  - max profit (revenue V2G – cost G2V)
  - decisions: bids and offer prices
  - constraints:
    - battery
    - EV demand
- **follower :** smart grid (SG)
  - max social welfare (consumer surplus + producer surplus)
  - decisions: accepted quantities (production, G2V, V2G), voltage angles
  - constraints
    - supply/demand equilibrium
    - optimal power flow

bilevel formulation (continued)

main notation

sets	**description**	**index**
N	grid nodes	n
M_n	grid nodes connected to n	m
T	time slots	t
I	producers	i

constants	
$Q^{\downarrow}_{nt}$	battery level of incoming vehicles
$Q^{\uparrow}_{nt}$	battery level of outgoing vehicles
$\overline{P}_{n,m}$	capacity of power line (n, m)
p_{nit}	offer of producer i on node n in period t
D_{nt}	non-EV electricity demand on node n in period t

bilevel formulation (continued)

constants

$\underline{L}_{n,t}$	Minimum level of the battery
$\overline{L}_{n,t}$	Maximum level of the battery
$B_{n,m}$	susceptance of line (n, m)
$\overline{P}_{n,m}$	transmission capacity of line (n, m)

lower and upper bounds on the variables are under-scored and over-scored, respectively

bilevel formulation (continued)

decision variables

leader (FO)

π_{nt}^{G2V}	G2V price bids
π_{nt}^{V2G}	V2G price offers

follower (SG)

g_{nit}	inputs from producer i
Q_{nt}^{G2V}	charging quantities
Q_{nt}^{V2G}	discharge quantities
θ_{nt}	voltage angles
λ_{nt}	dual variables of supply-demand equilibrium = local marginal price in node n at period t

bilevel formulation: upper level

$$
\begin{aligned}
\max_{\pi^{G2V},\pi^{V2G}} \quad & f(\pi^{G2V},\pi^{V2G}) \\
\text{s.t.} \quad & L_{nt} = L_{n,t-1} + Q_{nt}^{G2V} - Q_{nt}^{V2G} + Q_{nt}^{\downarrow} - Q_{nt}^{\uparrow} \\
& \underline{L}_{nt} \leq L_{nt} \leq \overline{L}_{n} \\
& \underline{\pi}_{nt}^{G2V} \leq \pi_{nt}^{G2V} \leq \overline{\pi}_{nt}^{G2V} \\
& \underline{\pi}_{nt}^{V2G} \leq \pi_{nt}^{V2G} \leq \overline{\pi}_{nt}^{V2G}
\end{aligned}
$$

bilevel formulation: lower level

$$
\begin{aligned}
\min_{Q^{G2V}, Q^{V2G}, g, \theta} \quad & \overbrace{\sum_{nit} p_{nit} g_{nit}}^{\text{producers}} - \overbrace{\sum_{nt} (\pi^{G2V}_{nt} Q^{G2V}_{nt} - \pi^{V2G}_{nt} Q^{V2G}_{nt})}^{\text{fleet operator}} \\
\text{s.t.} \quad & \sum_{i} g_{nit} + Q^{V2G}_{nt} - Q^{G2V}_{nt} - D_{nt} = \sum_{m \in M_n} B_{nm}(\theta_{nt} - \theta_{mt}) \\
& \underline{G}_{nit} \leq g_{nit} \leq \overline{G}_{nit} \\
& \underline{Q}^{G2V}_{nt} \leq Q^{G2V}_{nt} \leq \overline{Q}^{G2V}_{nt} \\
& \underline{Q}^{V2G}_{nt} \leq Q^{V2G}_{nt} \leq \overline{Q}^{V2G}_{nt} \\
& \underline{\theta}_{n} \leq \theta_{nt} \leq \overline{\theta}_{n} \\
& -\overline{P}_{nm} \leq B_{nm}(\theta_{nt} - \theta_{mt}) \leq \overline{P}_{nm} \\
& \theta_{0,t} = 0
\end{aligned}
$$

bilevel formulation: objective function

two proposals

$$f_1(\pi^{G2V}, \pi^{V2G}) = \sum_{nt} (\pi_{nt}^{V2G} Q_{nt}^{V2G} - \lambda_{nt} Q_{nt}^{G2V})$$

or

$$f_2(\pi^{G2V}, \pi^{V2G}) = \sum_{nt} \lambda_{nt} (Q_{nt}^{V2G} - Q_{nt}^{G2V}).$$

bilevel formulation

$$\max_{\pi^{G2V},\pi^{V2G}} \quad \sum_{nt}(\pi^{V2G}_{nt}Q^{V2G}_{nt} - \lambda_{nt}Q^{G2V}_{nt})$$

s.to

State of Charge →
$$L_{nt} = L_{n,t-1} + Q^{G2V}_{nt} - Q^{V2G}_{nt} + Q^{\downarrow}_{nt} - Q^{\uparrow}_{nt}$$

EV demand →
$$\underline{L}_{nt} \leq L_{nt} \leq \overline{L}_{n}$$

$$\underline{\pi}^{G2V}_{nt} \leq \pi^{G2V}_{nt} \leq \overline{\pi}^{G2V}_{nt}$$

$$\underline{\pi}^{V2G}_{nt} \leq \pi^{V2G}_{nt} \leq \overline{\pi}^{V2G}_{nt}$$

$$\min_{Q^{G2V},Q^{V2G},g,\theta} \quad \overbrace{\sum_{nit} p_{nit}g_{nit}}^{\text{Producers}} - \overbrace{\sum_{nt}(\pi^{G2V}_{nt}Q^{G2V}_{nt} - \pi^{V2G}_{nt}Q^{V2G}_{nt})}^{\text{Leader: FO}}$$

Offer/Demand → s.to
$$\sum_i g_{nit} + Q^{V2G}_{nt} - Q^{G2V}_{nt} - D_{nt} = \sum_{m\in M_n} B_{nm}(\theta_{nt} - \theta_{mt})$$

$$\underline{G}_{nit} \leq g_{nit} \leq \overline{G}_{nit}$$

$$\underline{Q}^{G2V}_{nt} \leq Q^{G2V}_{nt} \leq \overline{Q}^{G2V}_{nt}$$

$$\underline{Q}^{V2G}_{nt} \leq Q^{V2G}_{nt} \leq \overline{Q}^{V2G}_{nt}$$

$$\underline{\theta}_{n} \leq \theta_{nt} \leq \overline{\theta}_{n}$$

Transmission capacity →
$$-\overline{P}_{nm} \leq B_{nm}(\theta_{nt} - \theta_{mt}) \leq \overline{P}_{nm}$$

Reference Node →
$$\theta_{0,t} = 0$$

bilevel formulation: dual information

lower level constraints

$$
\begin{aligned}
\sum_i g_{nit} + Q^{V2G}_{nt} - Q^{G2V}_{nt} - D_{nt} & \\
= \sum_{m \in M_n} B_{nm}(\theta_{nt} - \theta_{mt}) & \qquad \lambda_{n,t} \\
\underline{G}_{nit} \le g_{nit} \le \overline{G}_{nit} & \qquad y^{11}_{n,i,t}, y^{12}_{n,i,t} \\
\underline{Q}^{G2V}_{nt} \le Q^{G2V}_{nt} \le \overline{Q}^{G2V}_{nt} & \qquad y^{21}_{n,t}, y^{22}_{n,t} \\
\underline{Q}^{V2G}_{nt} \le Q^{V2G}_{nt} \le \overline{Q}^{V2G}_{nt} & \qquad y^{31}_{n,t}, y^{32}_{n,t} \\
\underline{\theta}_n \le \theta_{nt} \le \overline{\theta}_n & \qquad y^{41}_{n,t}, y^{42} n, t \\
-\overline{P}_{nm} \le B_{nm}(\theta_{nt} - \theta_{mt}) \le \overline{P}_{nm} & \qquad y^{51}_{n,m,t}, y^{52}_{n,m,t} \\
\theta_{0,t} = 0 & \qquad y^{6}_{0,t}
\end{aligned}
$$

MPEC formulation

KKT conditions

$$
\begin{aligned}
p_{nit} + \lambda_{nt} - y^{11}_{nit} + y^{12}_{nit} &= 0 \\
-\pi^{G2V}_{nt} - \lambda_{nt} - y^{21}_{nt} + y^{22}_{nt} &= 0 \\
\pi^{V2G}_{nt} + \lambda_{nt} - y^{31}_{nt} + y^{32}_{nt} &= 0 \\
-\sum_{m \in M_n} B_{nm}((\lambda_{nt} - \lambda_{mt}) & \\
+(y^{51}_{nmt} - y^{51}_{mnt}) - (y^{52}_{nmt} - y^{52}_{mnt})) - y^{41}_{nt} + y^{42}_{nt} &= 0 \qquad n \neq 0 \\
-\sum_{m \in M_0} B_{0m}((\lambda_{0t} - \lambda_{mt}) & \\
+(y^{51}_{0mt} - y^{51}_{m0t}) - (y^{52}_{0mt} - y^{52}_{m0t})) - y^{41}_{0t} + y^{42}_{0t} + y^{6}_{0t} &= 0
\end{aligned}
$$

MPEC formulation (continued)

KKT conditions (continued)

$$
\begin{aligned}
y^{11}_{nit}(\underline{G}_{nit} - g_{nit}) &= 0 \\
y^{12}_{nit}(g_{nit} - \overline{G}_{nit}) &= 0 \\
y^{21}_{nt}(\underline{Q}^{G2V}_{nt} - Q^{G2V}_{nt}) &= 0 \\
y^{22}_{nt}(Q^{G2V}_{nt} - \overline{Q}^{G2V}_{nt}) &= 0 \\
y^{31}_{nt}(\underline{Q}^{V2G}_{nt} - Q^{V2G}_{nt}) &= 0 \\
y^{32}_{nt}(Q^{V2G}_{nt} - \overline{Q}^{V2G}_{nt}) &= 0 \\
y^{41}_{nt}(\underline{\theta} - \theta_{nt}) &= 0 \\
y^{42}_{nt}(\theta_{nt} - \overline{\theta}) &= 0 \\
y^{51}_{nmt}(-B_{nm}(\theta_{nt} - \theta_{mt}) - \overline{P}_{nm}) &= 0 \\
y^{52}_{nmt}(B_{nm}(\theta_{nt} - \theta_{mt}) - \overline{P}_{nm}) &= 0
\end{aligned}
$$

MPEC formulation (continued)

KKT conditions (cont.)

$$\sum_i g_{nit} + Q^{V2G}_{nt} - Q^{G2V}_{nt} - D_{nt}$$

$$= \sum_{m \in M_n} B_{nm}(\theta_{nt} - \theta_{mt}) \qquad \lambda_{n,t}$$

$$\underline{G}_{nit} \le g_{nit} \le \overline{G}_{nit} \qquad y^{11}_{n,i,t}, y^{12}_{n,i,t}$$

$$\underline{Q}^{G2V}_{nt} \le Q^{G2V}_{nt} \le \overline{Q}^{G2V}_{nt} \qquad y^{21}_{n,t}, y^{22}_{n,t}$$

$$\underline{Q}^{V2G}_{nt} \le Q^{V2G}_{nt} \le \overline{Q}^{V2G}_{nt} \qquad y^{31}_{n,t}, y^{32}_{n,t}$$

$$\underline{\theta}_n \le \theta_{nt} \le \overline{\theta}_n \qquad y^{41}_{n,t}, y^{42}_{n,t}$$

$$-\overline{P}_{nm} \le B_{nm}(\theta_{nt} - \theta_{mt}) \le \overline{P}_{nm} \qquad y^{51}_{n,m,t}, y^{52}_{n,m,t}$$

$$\theta_{0,t} = 0 \qquad y^{6}_{0,t}$$

$$y^{11}_{nit}, y^{12}_{nit}, y^{21}_{nt}, y^{22}_{nt}, y^{31}_{nt}, y^{32}_{nt}, y^{41}_{nt}, y^{42}_{nt}, y^{51}_{nmt}, y^{52}_{nmt} \ge 0$$

$$\lambda_{nt}, y^{6}_{0t} \quad \text{free}$$

MPEC formulation (continued)

two non-linearities

- complementarity slackness constraints

$$\begin{aligned} y_{nit}^{11}(\underline{G}_{nit} - g_{nit}) &= 0 \quad \forall n, \forall i, \forall t \\ y_{nit}^{12}(g_{nit} - \overline{G}_{nit}) &= 0 \quad \forall n, \forall i, \forall t \end{aligned}$$

$$\dots$$

- upper level objective

$$f_1(\pi^{G2V}, \pi^{V2G}) = \sum_{nt} (\pi_{nt}^{V2G} Q_{nt}^{V2G} - \lambda_{nt} Q_{nt}^{G2V})$$

or

$$f_2(\pi^{G2V}, \pi^{V2G}) = \sum_{nt} \lambda_{nt} (Q_{nt}^{V2G} - Q_{nt}^{G2V}).$$

MILP formulation

linearization

- complementarity slackness constraints

$$y_{nit}^{11}(\underline{G}_{nit} - g_{nit}) \quad = 0 \quad \forall n, \forall i, \forall t$$

$$\ldots$$

big M constants and binary variables ω

$$\begin{aligned} (g_{nit} - \underline{G}_{nit}) &\leq M^{11}(1 - \omega_{nit}^{11}) \\ y_{nit}^{11} &\leq M^{11}\omega_{nit}^{11} \end{aligned}$$

MILP formulation (continued)

linearization

- leader's objective f_1

$$f_1(\pi^{G2V}, \pi^{V2G}) = \sum_{nt} (\pi_{nt}^{V2G} Q_{nt}^{V2G} - \lambda_{nt} Q_{nt}^{G2V})$$

includes two nonlinear terms

MILP formulation (continued)

Linearization:

- leader's objective f_1

$$f_1(\pi^{G2V}, \pi^{V2G}) = \sum_{nt} (\pi_{nt}^{V2G} Q_{nt}^{V2G} - \lambda_{nt} Q_{nt}^{G2V})$$

strong duality yields

$$\begin{aligned} PG - \pi^{G2V} Q^{G2V} + \pi^{V2G} Q^{V2G} = \\ D\lambda + \underline{G} y^{11} \\ -\overline{G} y^{12} + \underline{Q}^{G2V} y^{21} - \overline{Q}^{G2V} y^{22} + \underline{Q}^{V2G} y^{31} - \overline{Q}^{V2G} y^{32} \\ +\underline{\theta} y^{41} - \overline{\theta} y^{42} - \overline{P} y^{51} - \overline{P} y^{52} \end{aligned}$$

MILP formulation (continued)

linearization

- leader's objective f_1

$$f_1(\pi^{G2V}, \pi^{V2G}) = \sum_{nt} (\pi^{V2G}_{nt} Q^{V2G}_{nt} - \lambda_{nt} Q^{G2V}_{nt})$$

complementarity slackness yields

$$\pi^{G2V} Q^{G2V} \;\; = \;\; -\lambda Q^{G2V} + y^{22} \overline{Q}^{G2V} - y^{21} \underline{Q}^{G2V}$$

and then

$$\pi^{V2G} Q^{V2G} + \lambda Q^{G2V} = -PG + D\lambda + \underline{G} y^{21} - \overline{G} y^{12} + \underline{Q}^{V2G} y^{31} \\ -\overline{Q}^{V2G} y^{32} + \underline{\theta} y^{41} - \overline{\theta} y^{42} - \overline{P}(y^{51} + y^{52})$$

MILP formulation (continued)

linearization:

- leader's objective f_1

$$f_1(\pi^{G2V}, \pi^{V2G}) = \sum_{nt} (\pi^{V2G}_{nt} Q^{V2G}_{nt} - \lambda_{nt} Q^{G2V}_{nt})$$

$$\begin{aligned} f_1(\pi^{G2V}, \pi^{V2G}) &= \pi^{V2G} Q^{V2G} + \lambda Q^{G2V} - 2\lambda Q^{G2V} \\ &= \Gamma - 2\lambda Q^{G2V} \end{aligned}$$

linearization of λQ^{G2V} through binary expansion

$$Q^{G2V}_{nt} = \sum_{r=0}^{r=\lfloor \log_2(\overline{Q}^{G2V}_{nt}) \rfloor} w_{n,t,r} 2^r$$

MILP formulation (continued)

Linearization:

- leader's objective f_1

$$
\begin{aligned}
f_1(\pi^{G2V}, \pi^{V2G}) &= \pi^{V2G} Q^{V2G} + \lambda Q^{G2V} - 2\lambda Q^{G2V} \\
&= \Gamma - 2\lambda Q^{G2V}
\end{aligned}
$$

$$
\max_{X} \quad f_1(X_1) = \Gamma - \sum_{n \in N} \sum_{t \in T} \sum_{r=0}^{r=\lfloor \log_2(\overline{Q}_{nt}^{G2V}) \rfloor} 2^{r+1} z_{n,t,r}
$$

decision variables regrouped into the vector vector

$$
\begin{aligned}
X_1 = [&\pi^{G2V}, \pi^{V2G}, g, Q^{G2V}, Q^{V2G}, \theta, \lambda, y^{11}, y^{12}, \\
&y^{21}, y^{22}, y^{31}, y^{32}, y^{41}, y^{42}, y^{51}, y^{52}, y^{6}, \omega^{11}, \\
&\omega^{12}, \omega^{21}, \omega^{22}, \omega^{31}, \omega^{32}, \omega^{41}, \omega^{42}, \omega^{51}, \omega^{52}, \\
&w, z, \alpha]
\end{aligned}
$$

MILP formulation (continued)

linearization

- add constraints

$$
\begin{aligned}
z_{n,t,r} - \lambda_{n,t} &\leq M_1'(1 - \alpha_{n,t,r}) \\
-z_{n,t,r} + \lambda_{nt} - M(1 - w_{n,t,r}) &\leq M_2'(1 - \alpha_{n,t,r}) \\
z_{n,t,r} - M_0 w_{n,t,r} &\leq M_3(1 - \alpha_{n,t,r}) \\
-z_{n,t,r} &\leq M_4(1 - \alpha_{n,t,r}) \\
-z_{n,t,r} + \lambda n, t &\leq M_1' \alpha_{n,t,r} \\
z_{n,t,r} - \lambda_{nt} - M(1 - w_{n,t,r}) &\leq M_2' \alpha_{n,t,r} \\
-z_{n,t,r} - M_0 w_{n,t,r} &\leq M_3' \alpha_{n,t,r} \\
z_{n,t,r} &\leq M_4' \alpha_{n,t,r}
\end{aligned}
$$

$$\alpha_{n,t,r} \in \{0, 1\}$$

MILP formulation (continued)

- leader's objective f_2

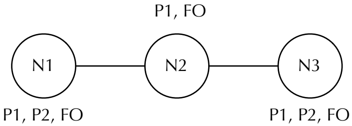
$$f_2(\pi^{G2V}, \pi^{V2G}) = \sum_{nt} \lambda_{nt}(Q_{nt}^{V2G} - Q_{nt}^{G2V}).$$

from duality:

$$\begin{aligned}
\lambda Q^{V2G} - \lambda Q^{G2V} &= PG - D\lambda \\
&\quad -\underline{G} y^{21} + \overline{G} y^{12} \\
&\quad -\underline{\theta} y^{41} + \overline{\theta} y^{42} \\
&\quad +\overline{P}(y^{51} + y^{52}).
\end{aligned}$$

3-node network

3-node power system:

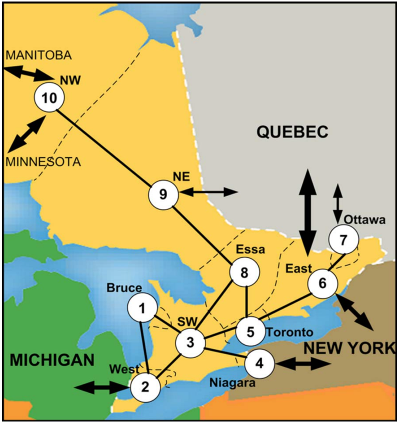


	f_1	f_2
objective	8446$	1777$
social welfare	15710$	24821$
cpu	1.02 sec.	0.04 sec.

3-node network: numerical results

		Node 1				Node 2				Node 3			
		t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4
	Demand[a]	30	40	80	135	0	50	0	0	30	40	80	50
	Generation[b]	130	130	130	130	20	20	20	20	130	130	130	130
P1	$\underline{G}$	0	0	0	0	0	0	0	0	0	0	0	0
	G[c]$:f_1$	**0**	**0**	**0**	**60**	**0**	**13**	**0**	**1.77**	**0**	**0**	**20**	**10.22**
	$G:f_2$	**0**	**0**	**0**	**59.44**	**0**	**0**	**0**	**13.33**	**0**	**0**	**0**	**0**
	$\overline{G}$	60	60	60	60	20	20	20	20	40	40	40	40
	price	50	50	50	50	50	50	50	50	50	50	50	50
P2	$\underline{G}$	0	0	0	0	0	0	0	0	0	0	0	0
	$G:f_1$	**55.11**	**70**	**70**	**70**	-	-	-	-	**31.88**	**60**	**60**	**60**
	$G:f_2$	**70**	**62.22**	**70**	**70**	-	-	-	-	**50**	**60**	**60**	**60**
	$\overline{G}$	70	70	70	70	-	-	-	-	60	60	60	60
	price	40	40	40	40	-	-	-	-	40	40	40	40
FO	$\underline{L}$	0	5	5	10	0	0	0	0	0	5	5	10
	$Q^{G2V}:f_1$	**9**	**11**	**0**	**17**	**10**	**0**	**20**	**0**	**8**	**12**	**0**	**20**
	$Q^{G2V}:f_2$	**0**	**0**	**12.22**	**0**	**0**	**2.22**	**17.77**	**0**	**0**	**5.55**	**0**	**0**
	$Q^{V2G}:f_1$	**0**	**0**	**15**	**0**	**0**	**10**	**0**	**20**	**0**	**0**	**15**	**0**
	$Q^{V2G}:f_2$	**20**	**0**	**0**	**12.22**	**20**	**0**	**0**	**0**	**20**	**0**	**0**	**5.55**
	$\overline{L}$	20	20	20	20	20	20	20	20	20	20	20	20
	$\pi^{G2V}:f_1$	**40**	**48**	**.**	**66**	**40**	**.**	**50**	**.**	**.**	**40**	**.**	**50**
	$\pi^{G2V}:f_2$	**.**	**.**	**40**	**.**	**.**	**40**	**40**	**.**	**.**	**40**	**.**	**.**
	$\pi^{V2G}:f_1$	**.**	**.**	**50**	**.**	**.**	**48**	**.**	**58**	**.**	**.**	**50**	**.**
	$\pi^{V2G}:f_2$	**24**	**32**	**.**	**32**	**40**	**.**	**.**	**32**	**40**	**.**	**32**	**24**
	N1 - N2[d]$:f_1$	**+14.5**	**+17.1**	**+4.5**	**−19.8**	**−14.5**	**−17.1**	**−4.5**	**+19.8**	-	-	-	-
	N1 - N2$:f_2$	**+18**	**+20**	**+2**	**−16**	**−18**	**−20**	**−2**	**+16**	-	-	-	-
	N2 - N3$:f_1$	-	-	-	-	**+5.5**	**−7.2**	**−13.5**	**−0.2**	**−5.5**	**+7.2**	**+13.5**	**+0.2**
	N2 - N3$:f_2$	-	-	-	-	**0**	**−23**	**+18**	**−4**	**0**	**+23**	**−18**	**+4**

Ontario power grid



- 10 internal zones
- 5 neighbouring jurisdictions (provinces and US states)
- nuclear, hydro, gas, wind, solar, biofuel, no coal

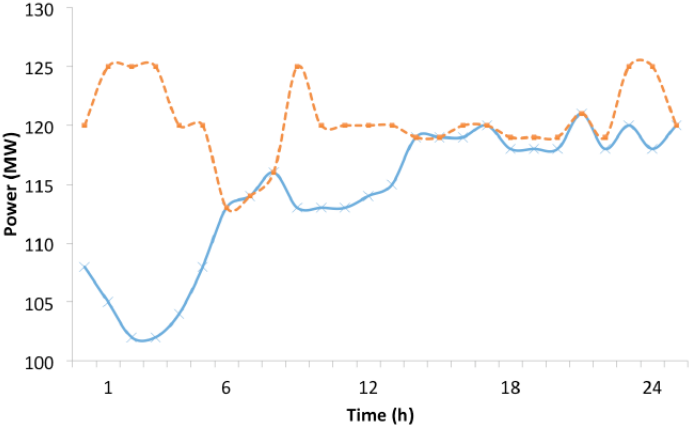
demand: annual patterns

	total (TW)	mean	S.D.
peak	2	0.4	0.3
medium	0.8	0.26	0.2
low	0.8	0.2	0.1

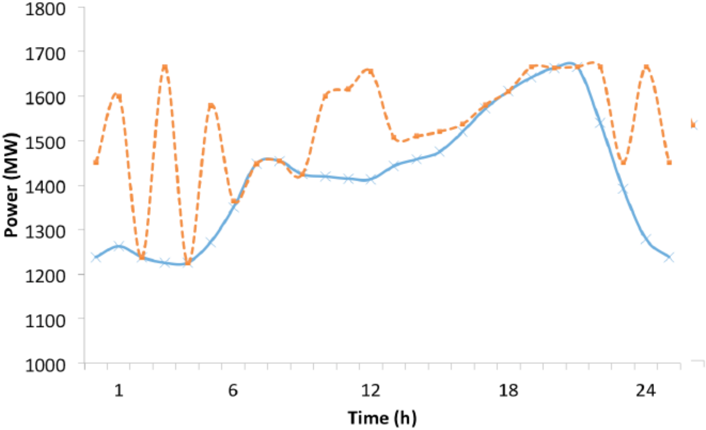
demand: daily patterns (continued)

		night		day-time		evening	
node	MW	mean	S.D.	mean	S.D.	mean	S.D.
Toronto	5806	165	60	97	60	696	500
SW	2322	66	50	39	30	278	250
West	1161	33	20	20	10	139	100
Ottawa	929	26	10	16	5	111	100
Essa	464	13	5	8	5	55	50
NE	243	6	4	4	2	29	20
East	232	6	4	4	2	27	20
Niagara	220	6	3	4	2	26	20
NW	162	4	3	3	2	19	15
Bruce	69	2	1	1	1	8	5

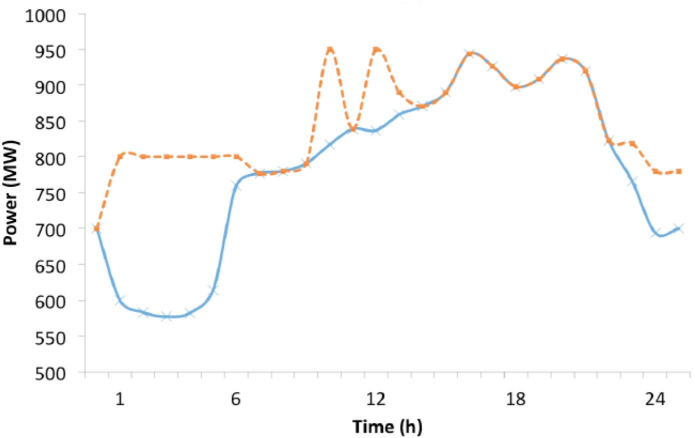
results: node 1 (Bruce: mainly nuclear)



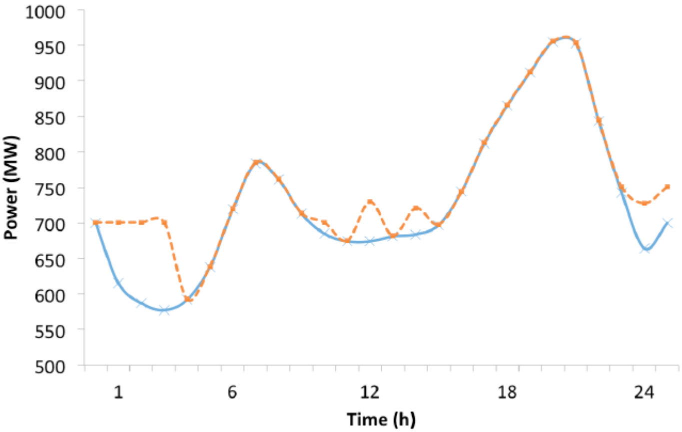
results: node 2 (West: mainly natural gas)



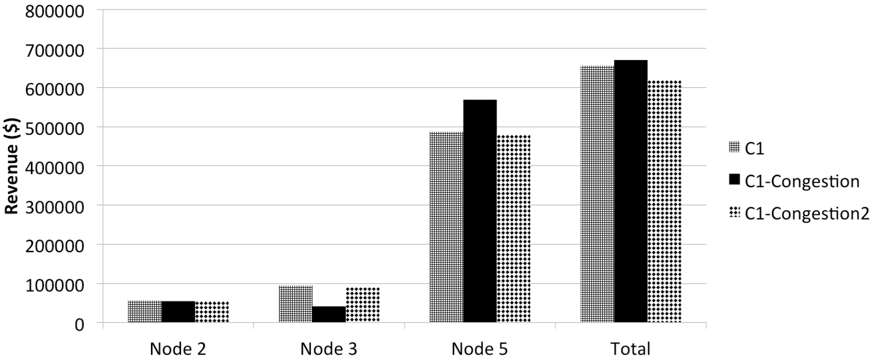
results: node 7 (Ottawa: co-generation)



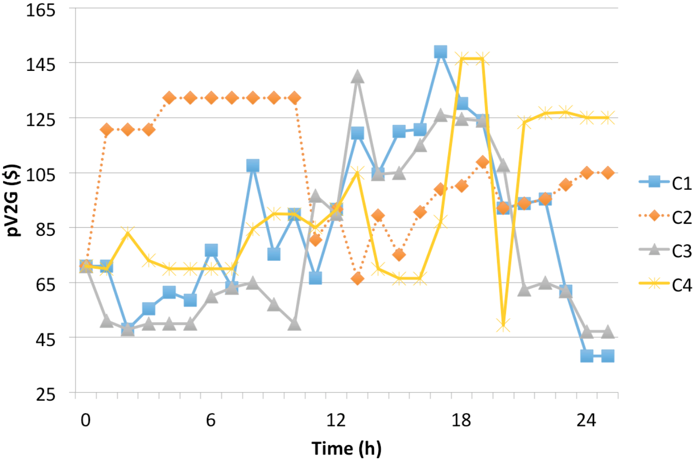
results: node 8 (Essa: hydro-electric)



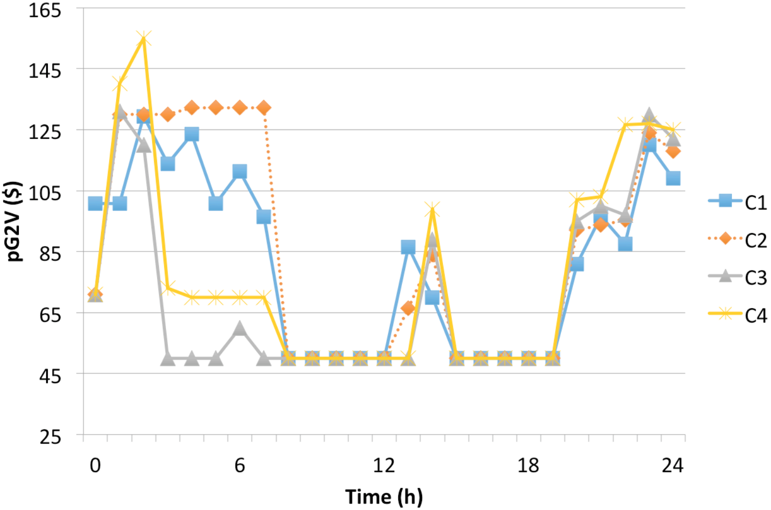
effect of congestion on revenue



price offers: Toronto



price bids: Toronto



nodal transfers

	EV Demand	G2V	V2G	exchange rate
node 1	70	150	80	3.28
node 2	1161	2500	1339	3.30
node 3	2322	5000	2678	3.30
node 4	220	335	115	2.04
node 5	5806	18000	12194	5.20
node 6	232	300	68	1.58
node 7	929	1500	571	2.22
node 8	464	520	56	1.24
node 9	243	420	177	2.45
node 10	162	315	153	2.88

ongoing research

- first model: embed production costs
- algorithmic issues (scalable algorithms)
- stochastic models