

# Learning and Correcting Static and Dynamic Properties of Materials in Atomistic Simulations

Michele Ceriotti  
<http://cosmo.epfl.ch>

IPAM Workshop, November 2017, L.A.





**CCMX**  
Competence Centre for  
Materials Science and Technology



**MARVEL**  
NATIONAL GREEK OF COMBUSTION RESEARCH



**Sandip De, Felix Musil,**  
Andrea Grisafi, David Wilkins  
**Venkat Kapil, M. Rossi**

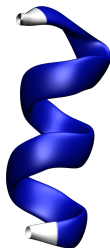
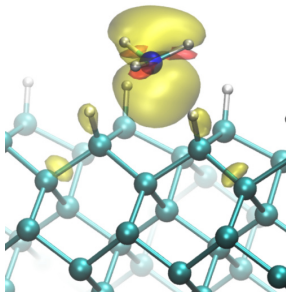
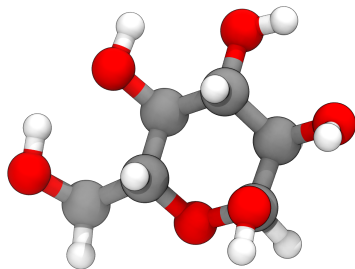
G. Csányi, A. Bartók, C. Poelking,  
J. Kermode, N. Bernstein

**Hiring post-docs with machine-learning background/interests**

# Machine-Learning Static Properties of Materials and Molecules

# A Universal Predictor of Atomic-Scale Properties

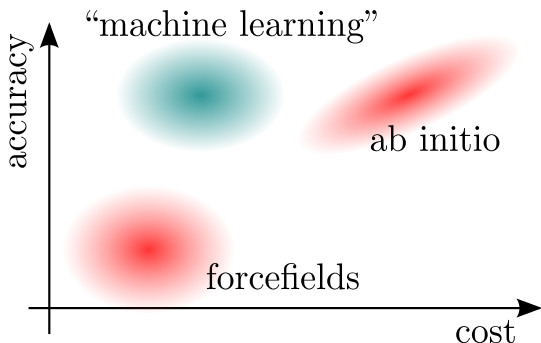
- The Schrödinger Equation allows – in principle! – to predict the properties of any kind of molecule or material
- Prohibitive computational cost
- A proliferation of ad-hoc electronic-structure methods and empirical potentials tuned to specific problems



$$\hat{H}\Psi = E\Psi$$

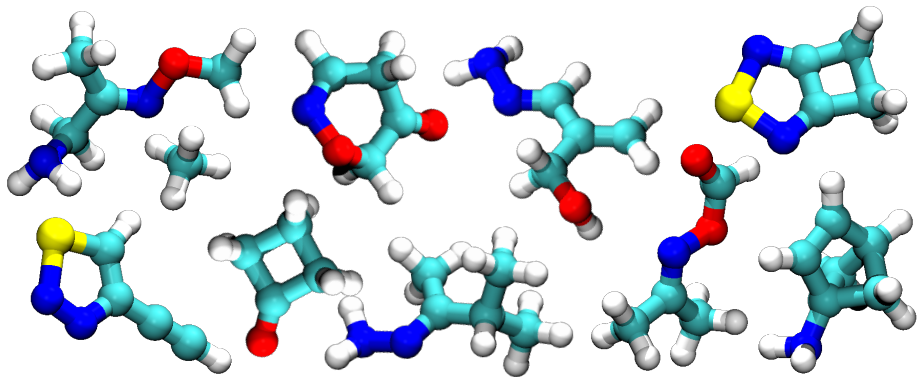
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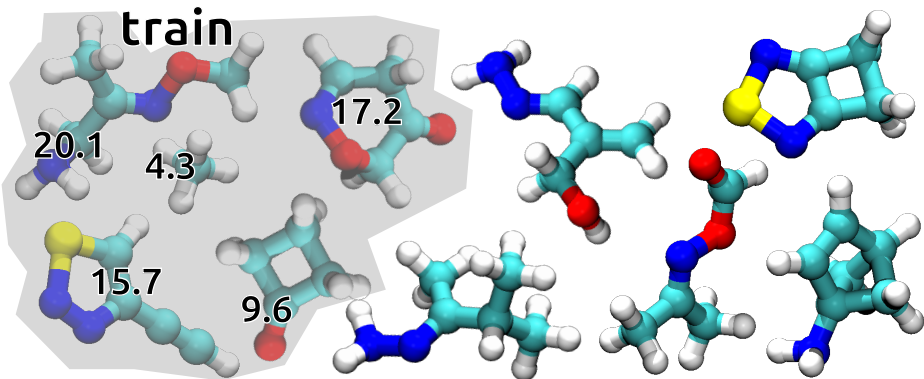
# Machine-Learning as a Universal Interpolator

- Machine-learning can be regarded as a sophisticated interpolation between a few known values of the properties
- Can it be made as accurate and general as the Schrödinger equation?



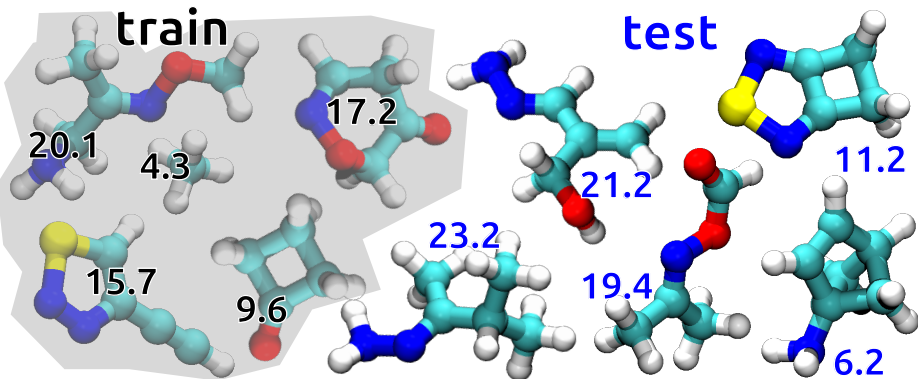
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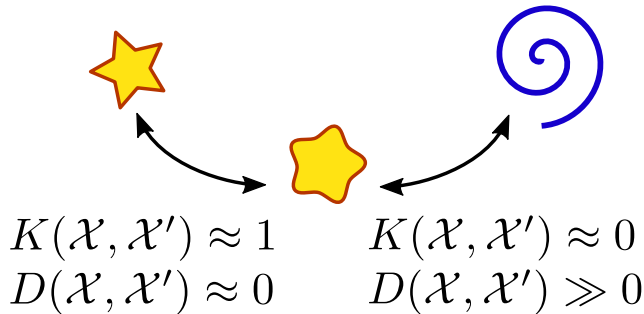
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$$E(\mathcal{A}) = \sum_i w_i K(\mathcal{A}, \mathcal{A}_i)$$

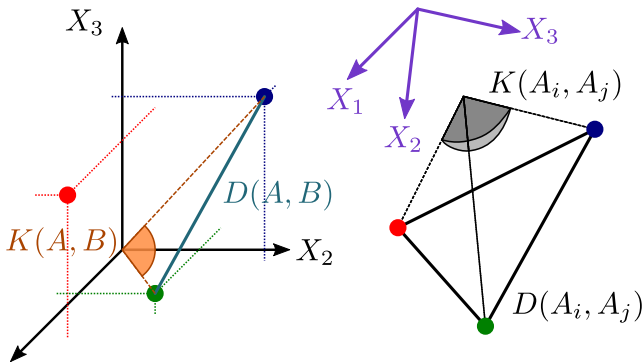
# Measuring distances between materials

- The crucial ingredient in machine-learning is a method to compare the items whose properties should be predicted
- A distance  $D(A, B)$  or a kernel function  $K(A, B)$  can be used to assess the (dis)-similarity between items in a set
- Under reasonable assumptions one can always convert a distance  $D(A, B)$  to a kernel and vice versa



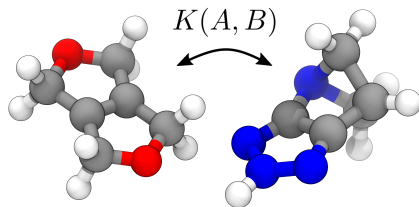
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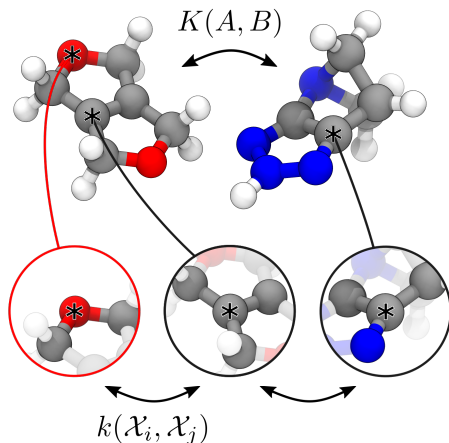
# A General-Purpose Similarity Kernel

- How to compare two atomic *structures*?
- Start from a comparison of *local environments*!
- We use SOAP kernels that are smooth, and invariant to translations, rotations and permutations of identical atoms



# A General-Purpose Similarity Kernel

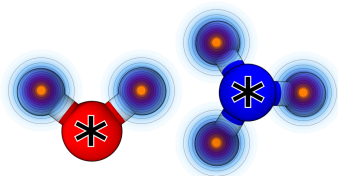
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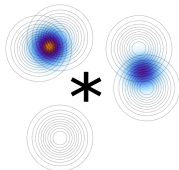
Prodan, Kohn, PNAS (2005)

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$$\rho(\mathbf{x}) = \sum_i g(\mathbf{x} - \mathbf{x}_i)$$



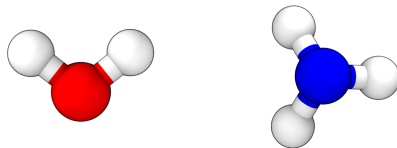
$$k(\mathcal{X}, \mathcal{X}') \sim \int \rho(\mathbf{x}) \rho'(\mathbf{x})$$

Bartók, Kondor, Csányi, PRB (2013)

# Structures as Combinations of Local Environments

- One can write *structural* kernels as a combination of *local* kernels
- Entropy-regularized Wasserstein distance interpolates between “best-match” and “average” constructions

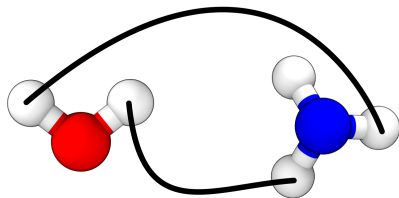
$$K(A, B) = \sum_{i \in A, j \in B} P_{ij} k(\mathcal{X}_i, \mathcal{X}_j)$$



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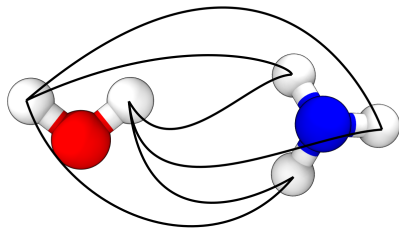
$$\hat{K}(A, B) \propto \max_{\mathbf{P} \in \mathcal{U}} \sum_{ij} P_{ji} C_{ij}^{A,B}$$

De, Bartók, Csányi, Ceriotti, PCCP (2016); M. Cuturi, NIPS (2013);

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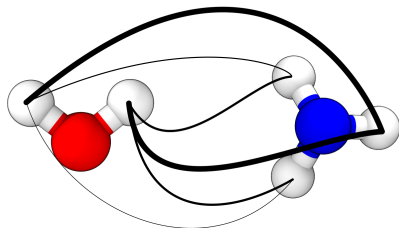
$$P_{ij} = \frac{1}{N_A N_B}$$

De, Bartók, Csányi, Ceriotti, PCCP (2016); M. Cuturi, NIPS (2013);

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$$\hat{K}^\gamma(A, B) \propto \max_{\mathbf{P} \in \mathcal{U}} \sum_{ij} P_{ji} (C_{ij}^{A,B} - \gamma \ln P_{ji})$$

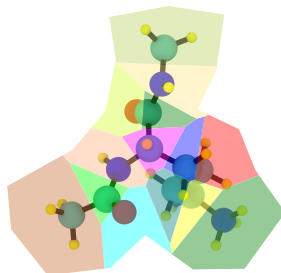
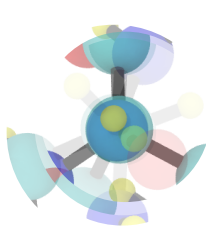
De, Bartók, Csányi, Ceriotti, PCCP (2016); M. Cuturi, NIPS (2013);

# Additive Property Models & Beyond

- Crucial observation: learning with an average kernel is equivalent to learning an atom-centered *additive* energy model

$$\begin{aligned} E(A) &= \sum_i W_i K(A, A_i) \\ K(A, B) &= \sum_{i \in A, j \in B} k(\mathcal{X}_i, \mathcal{X}_j) \end{aligned} \iff \begin{aligned} \epsilon(\mathcal{X}) &= \sum_i w_i k(\mathcal{X}, \mathcal{X}_i) \\ E(A) &= \sum_{i \in A} \epsilon(\mathcal{X}_i) \end{aligned}$$

- Entropy-regularized match provides a natural strategy to go beyond additive models



$$K(A, B) = \sum_{i,j} k(\mathcal{X}_i^A, \mathcal{X}_j^B)$$

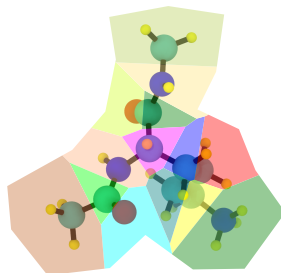
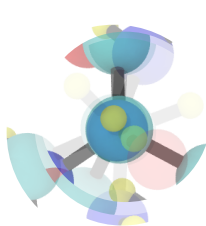
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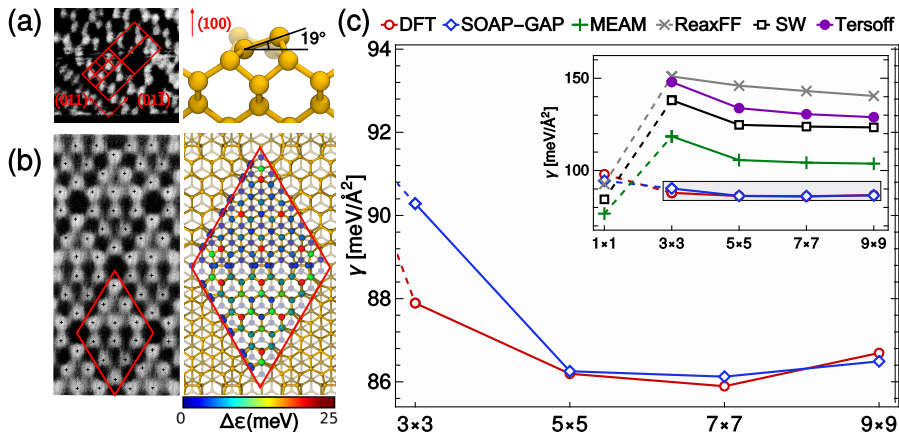


$$K(A, B) = \sum_{i,j} k(\mathcal{X}_i^A, \mathcal{X}_j^B)$$

$$E(A) = \sum_i \epsilon(\mathcal{X}_i^A)$$

# Silicon Surfaces - Complexity in a Simple Material

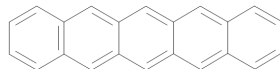
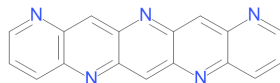
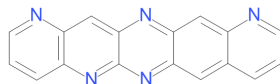
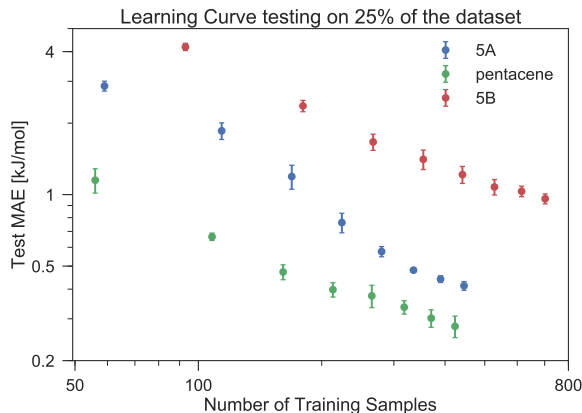
- A SOAP-GAP model for Si can capture the dimer tilt in Si(100)-2x1, and the delicate energy balance that determines the stability of the Si(111) 7x7 DAS reconstruction



Bartok, De, Kermode, Bernstein, Csanyi, Ceriotti, arxiv:1706.00179

# Accurate Predictions for Molecular Crystals

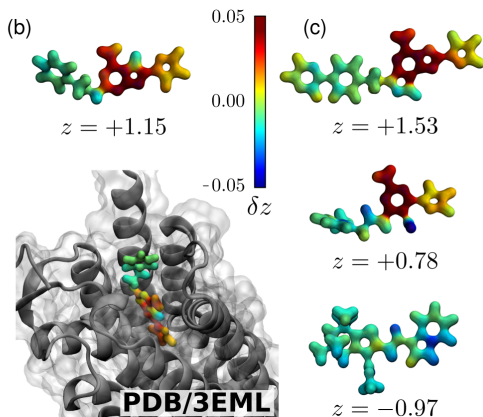
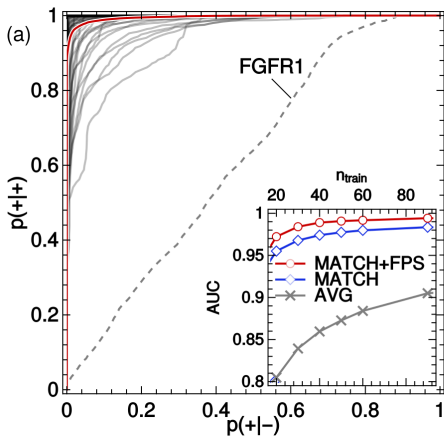
- Substituted pentacenes - model systems for molecular electronics
- Easily achieve sub-kcal/mol prediction accuracy, with REMatch-SOAP kernels



Data from G. Day and J. Yang

# Recognizing Active Ligands for Receptor Proteins

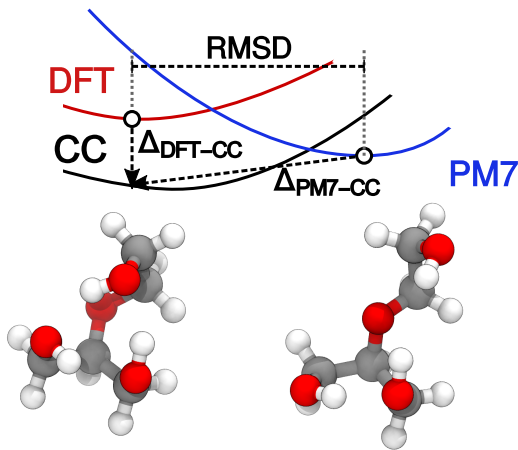
- A SOAP-REMatch-based KSVM classifies active and inactive ligands with 99% accuracy; non-additive model is crucial!
- Sensitivity analysis help identify the active “warhead” and could guide drug design and optimization



Bartok, De, Kermode, Bernstein, Csanyi, Ceriotti, arxiv:1706.00179

# 100k Molecules with Coupled-Clusters

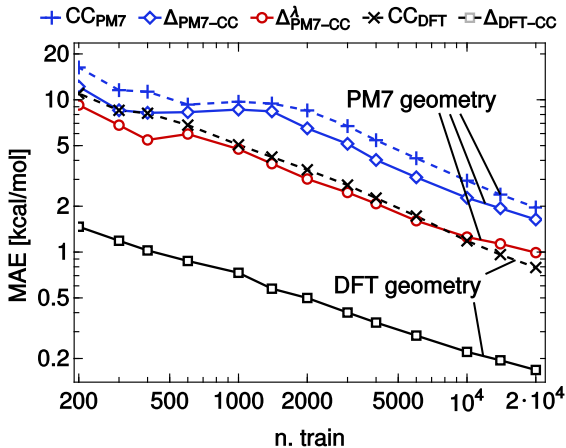
- CCSD(T) Energetics on the GDB9 - 114k *useful* predictions based on 20k training calculations
- 1kcal/mol error for predicting CCSD(T) based on PM7 geometries;  
0.18kcal/mol error for predicting CCSD(T) based on DFT geometries!



Ramakrishnan et al. Scientific Data (2014)

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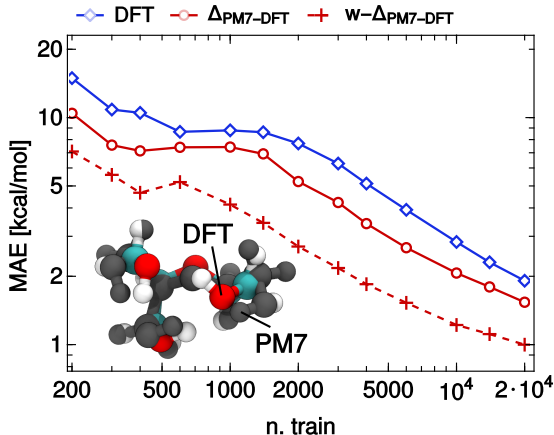
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De, Bartók, Csányi, Ceriotti, PCCP (2016);  
Bartok, De, Kermode, Bernstein, Csanyi, Ceriotti, arxiv:1706.00179

# Useful Semi-empirical Predictions

- In some cases PM7 optimization fails catastrophically and gives a different compound than DFT. These structures *hurt* predictions
- Solution: weight the structures in the train phase by using a diagonal regularization  $\propto e^{-|x_{\text{DFT}} - x_{\text{PM7}}|^2 / \lambda^2}$



Bartok, De, Kermode, Bernstein, Csanyi, Ceriotti, arxiv:1706.00179

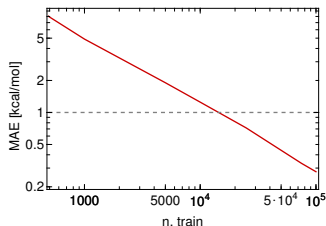
# Let's Talk About Benchmarks

- What happens if we increase the train set fraction?
- Can we improve the accuracy by tuning the kernel?

## GDB9, MAE on atomization energies (eV)

KRR	CM	0.128
	BOB	0.0667
	BAML	0.0519
	ECFP4	4.25
	HDAD	<b>0.0251</b>
	HD	0.0644
	MARAD	0.0529

90% TRAIN, arxiv:1702.0553



**KRR/SOAP: 0.012 eV  
(1kJ/mol)**

80% TRAIN, arxiv:1706.00179

# Let's Talk About Benchmarks

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## QM7B, MAE on atomization energies (kcal/mol)

Repr.	Kernel	E / kcal mol <sup>-1</sup>	
		RMSE	MAE
CM	Laplacian	5.48	3.54
BoB	Laplacian	3.32	1.95
BAML [17]	Laplacian	2.54	1.15
<del>SOAP [35]</del>	<del>REMatch</del>	<del>1.61</del>	<del>0.92</del>
MBTR	Linear	1.81	0.82
MBTR	Gaussian	<b>0.94</b>	<b>0.60</b>

75% TRAIN, arxiv:1704.06439

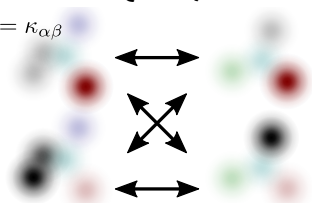
**KRR/SOAP: 0.40 kcal/mol**  
75% TRAIN, arxiv:1706.00179

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QM7B, MAE on atomization energies (kcal/mol)

$$\langle \alpha | \beta \rangle = \kappa_{\alpha\beta}$$



$$\int d\hat{R} \left| \sum_{\alpha\beta} \kappa_{\alpha\beta} \int \rho_{\alpha}(\mathbf{x}) \rho'_{\beta}(\hat{R}\mathbf{x}) \right|^2$$

$$\kappa_{\alpha\beta} = e^{-(E_{\alpha} - E_{\beta})^2 / 2\lambda^2}$$

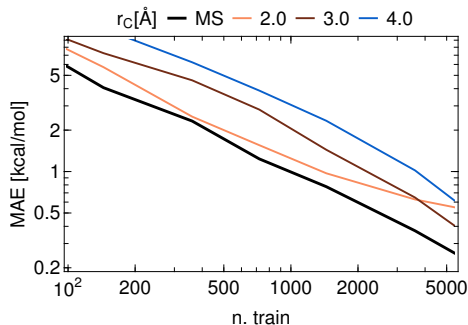
KRR/SOAP: 0.40 kcal/mol  
**ALCHEMY: 0.33 kcal/mol**  
75% TRAIN, arxiv:1706.00179

Bartok, De, Kermode, Bernstein, Csanyi, Ceriotti, arxiv:1706.00179

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## QM7B, MAE on atomization energies (kcal/mol)



KRR/SOAP: 0.40 kcal/mol  
**MULTISCALE: 0.26 kcal/mol**  
75% TRAIN, arxiv:1706.00179

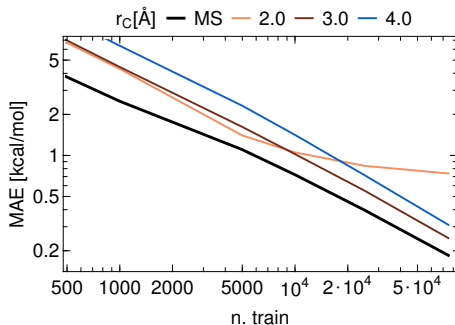
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Bartok, De, Kermode, Bernstein, Csanyi, Ceriotti, arxiv:1706.00179

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## Multiscale for GDB9 atomization energies (kcal/mol)



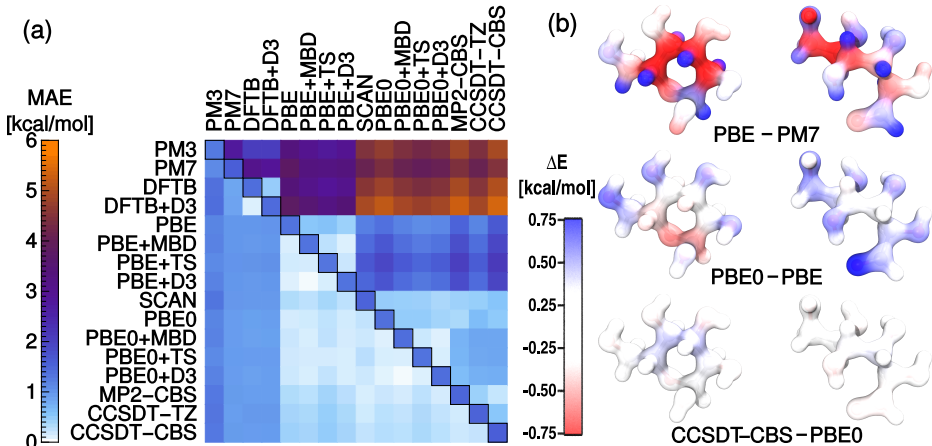
KRR/SOAP: 0.27 kcal/mol  
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80% TRAIN, arxiv:1706.00179

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Bartok, De, Kermode, Bernstein, Csanyi, Ceriotti, arxiv:1706.00179

# Understanding Errors in Quantum Chemistry

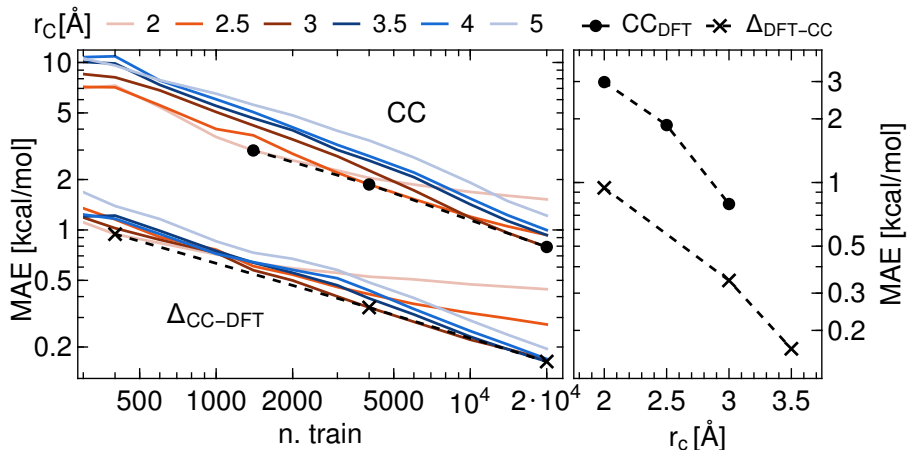
- Learning differences between electronic structure methods is simpler
- Atom-centered energetics give insight into the impact of approximations



Von Lilienfeld & C, JCTC 2015; M. Marianski et al., J. Chem. Theory Comput. 12, 6157 (2016);  
Bartok, De, Kermode, Bernstein, Csanyi, Ceriotti, arxiv:1706.00179

# Assessing the Range of Chemical Interactions

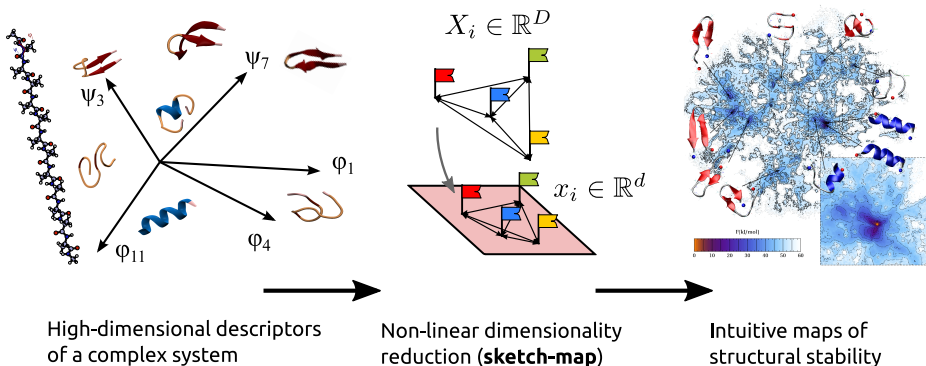
- Different SOAP ranges saturate at different levels of accuracy
- We can use the information to understand “how much” energy information is available within a given cutoff



Bartok, De, Kermode, Bernstein, Csanyi, Ceriotti, arxiv:1706.00179

# A Map to Navigate Materials & Molecules

- Kernel-induced distances can be also used as the basis of clustering and dimensionality-reduction techniques
- Generate insightful representations of the (free)-energy landscape of complex systems

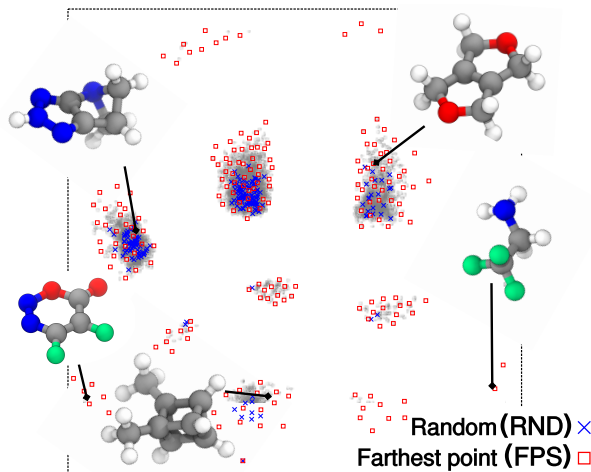


Ceriotti, Tribello, Parrinello, PNAS (2011); [interactive.sketchmap.org](http://interactive.sketchmap.org)



# Train Set Selection & Active Learning

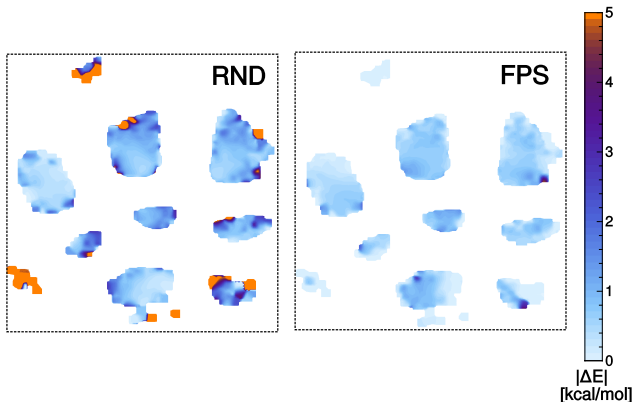
- The train set should cover uniformly the relevant space
- FPS is a simple, constructive strategy to optimize the training set, opening doors to active learning



Bartok, De, Kermode, Bernstein, Csanyi, Ceriotti, arxiv:1706.00179  
Browning et al., JPCL 2017; Ceriotti, Tribello, Parrinello, PNAS (2011); <http://sketchmap.org>

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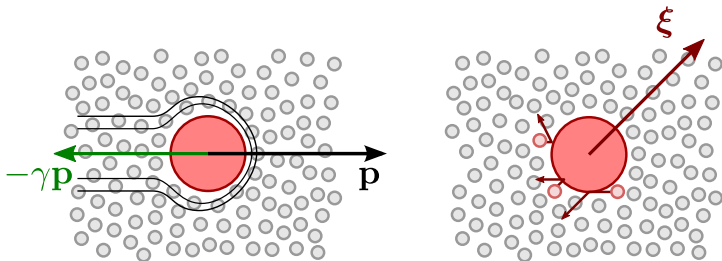
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How About Dynamics?

# Colored-noise Langevin dynamics

- The Langevin equation is Markovian:  $\dot{\rho}(t)$  only depends on  $\rho(t)$
- Can we formulate a non-Markovian generalized Langevin equation?

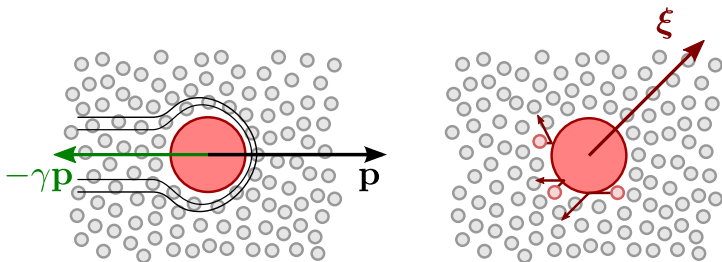
$$\dot{\rho}(t) = -\gamma\rho(t) + \sqrt{2m\gamma/\beta}\xi(t), \quad \langle \xi(t)\xi(0) \rangle = \delta(t)$$



# Colored-noise Langevin dynamics

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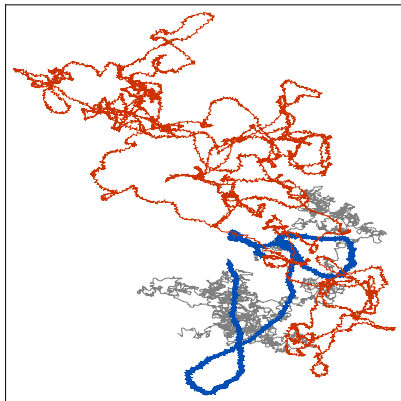
$$\dot{p}(t) = - \int_0^\infty K(s) p(t-s) ds + \sqrt{2m/\beta} \zeta(t), \quad \langle \zeta(t) \zeta(0) \rangle = K(t)$$



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$$\dot{p}(t) = - \int_0^\infty K(s) p(t-s) ds + \sqrt{2m/\beta} \zeta(t), \quad \langle \zeta(t) \zeta(0) \rangle = K(t)$$



Details of the noise **dramatically change dynamics!**

The trajectories of a particle subject to white and colored-noise LE with the same diffusion coefficient yield very different short-times behavior.

# Colored-noise Langevin dynamics

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- Non-Markovian generalized Langevin equations are used in many contexts as a physical model
  - Can we exploit the flexibility to manipulate the sampling properties of a molecular dynamics trajectory?
  - Can we make it as general and “user-friendly” as possible?

# A GLE framework for molecular dynamics

## Non-Markovian GLE

$$\begin{aligned}\dot{q}(t) &= p(t)/m \\ \dot{p}(t) &= -V'(q) - \int_0^\infty K(t') p(t-t') dt' + \sqrt{2mT}\zeta(t) \\ \langle \zeta(t)\zeta(0) \rangle &= TK(t)\end{aligned}$$

- We want to use a non-Markovian GLE, but it is inconvenient
- A large class of non-Markovian dynamics can be **mapped onto a Markovian dynamics** in an extended phase space
- The Markovian GLE corresponds to a (possibly complex) exponential memory kernel  $K(t) = a_{pp}\delta(t) - \mathbf{a}_p^T e^{-\mathbf{A}t} \bar{\mathbf{a}}_p$
- Except for the non-linear potential, this stochastic differential equation is an **Ornstein-Uhlenbeck process**  $\dot{\mathbf{u}} = -\mathbf{A}\mathbf{u} + \mathbf{B}\xi$  which can be solved analytically.

<http://gle4md.org>; Ceriotti, Bussi, Parrinello, Phys. Rev. Lett. 102&103 (2009)

# A GLE framework for molecular dynamics

## Markovian GLE

$$\begin{aligned} \dot{q}(t) &= p(t)/m \\ \begin{pmatrix} \dot{p} \\ \dot{\mathbf{s}} \end{pmatrix} &= \begin{pmatrix} -V'(q) \\ \mathbf{0} \end{pmatrix} - \begin{pmatrix} a_{pp} & \mathbf{a}_p^T \\ \bar{\mathbf{a}}_p & \mathbf{A} \end{pmatrix} \begin{pmatrix} p \\ \mathbf{s} \end{pmatrix} + \mathbf{B}\xi \\ \mathbf{B}\mathbf{B}^T &= T(\mathbf{A} + \mathbf{A}^T) \end{aligned}$$

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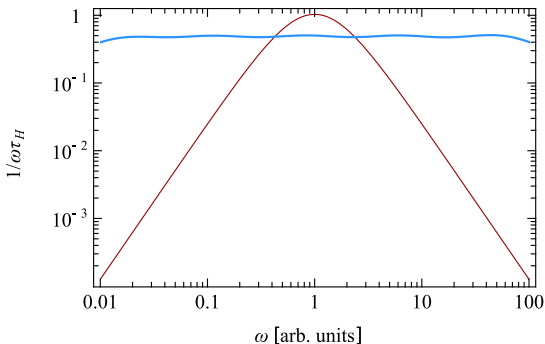
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# A Thermostat in All Colors

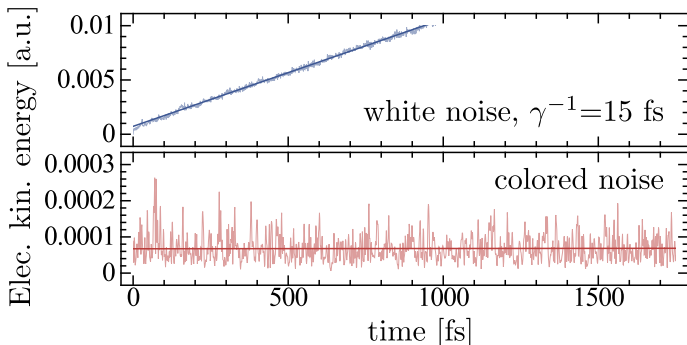
- GLEs give exquisite control over canonical sampling
- Efficient sampling for forcefield and ab initio molecular dynamics
- Controlling resonances in multiple-time step methods
- Frequency-dependent thermalization
- Accelerate modelling of nuclear quantum effects



Ceriotti, Bussi, Parrinello, PRL 020601 (2009)

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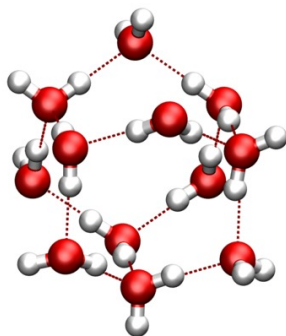
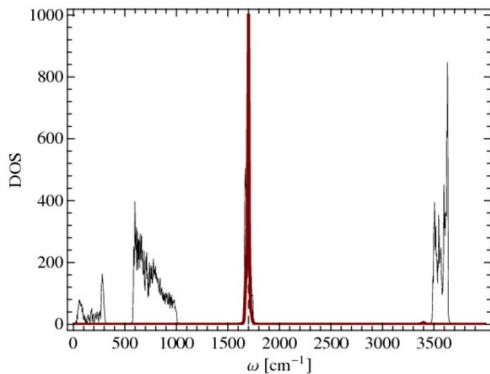
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	$\langle T \rangle / K$	$\langle V_b \rangle / \text{kcal/mol}$	$D / \text{\AA}^2 / \text{ps}$
REF	300.0	0.5227	0.25
White noise, 12 fs	301.7	0.5278	0.095
White noise, 20 fs	302.3	0.5246	0.019
GLE, 12 fs	301.5	0.5245	0.24
GLE, 20 fs	302.0	0.5236	0.20

Morrone, Markland, Ceriotti & Berne, JCP (2011).

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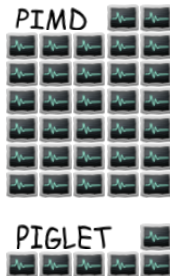
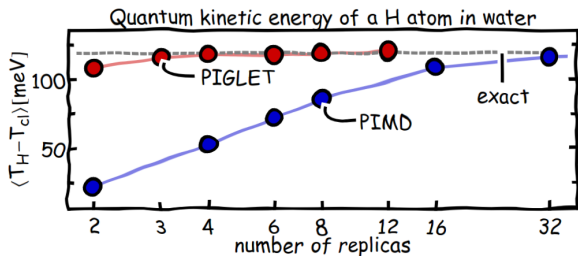
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Dettoni, Ceriotti, Melis, Hunger, Colombo, Donadio, JCTC (2017).

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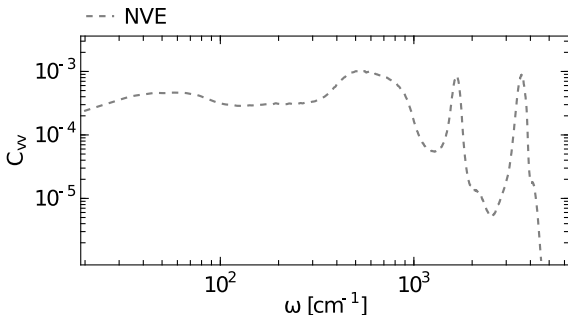
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Ceriotti, Manolopoulos, PRL (2012)

# Controlling and Correcting Dynamics

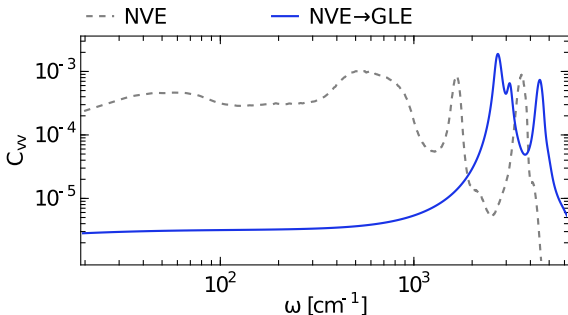
- Can we control (& correct) the disturbance introduced by (Generalized) Langevin dynamics on top of microcanonical MD?
- We can predict the spectrum of a GLE-thermostatted HO of frequency  $\omega_0$ , and write the perturbed power spectrum as a convolution
$$c_w^{\text{GLE}}(\omega) = \int d\omega_0 C(\omega, \omega_0) c_w^{\text{NVE}}(\omega_0)$$
- It is also possible to deconvolute a (G)LE spectrum: great potential to extract dynamical properties from thermostatted MD



M. Rossi, V. Kapil, Ceriotti, JCP (2018)

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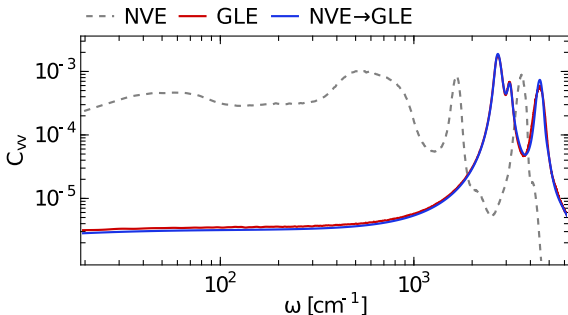
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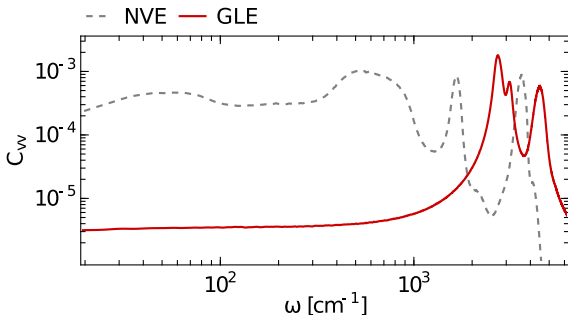
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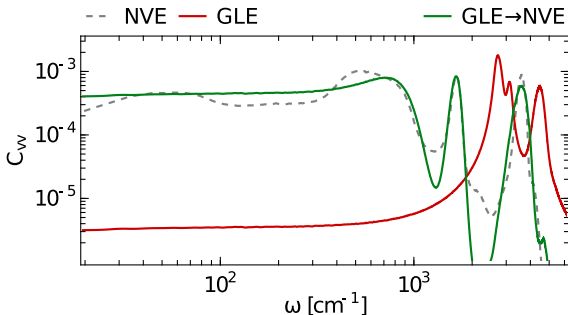
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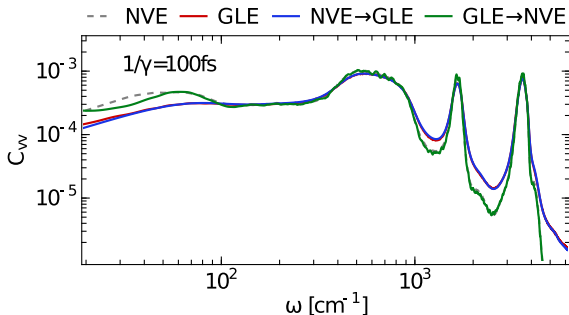
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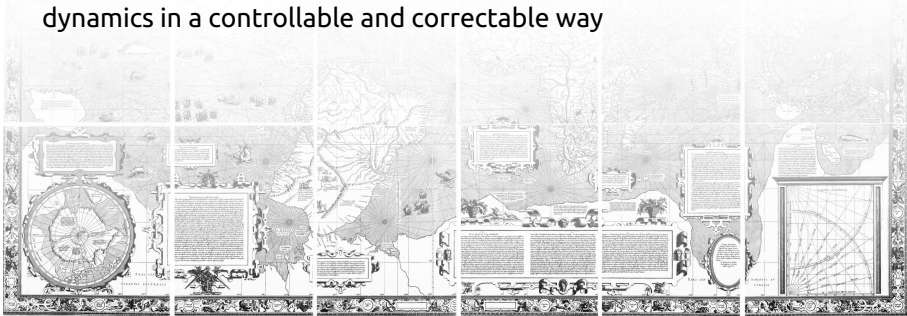
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M. Rossi, V. Kapil, Ceriotti, JCP (2018)

# Outlook

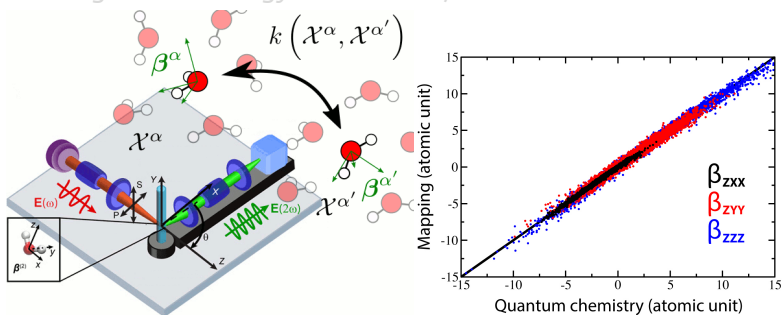
- Building structural kernels from local-environments SOAP fingerprints
  - “Nearsightedness” of electronic matter: excellent accuracy on benchmark DBs - **~1kJ/mol** for 80%GDB9 and 75%QM7b(multi-scale)
  - Beyond additive models using entropy-regularized kernels
  - Potentials for solids, 99% accuracy in predicting drug activity, silicon & molecular crystals, “useful” predictions: PM7→CC
- Ingredients for effective learning: sound mathematical foundation, train set sparsification, cross-species learning & multi-scale kernels.
- Colored noise Langevin equations as a mean to manipulate sampling and dynamics in a controllable and correctable way



(Development) code available on <http://cosmo-epfl.github.io> & <http://sketchmap.org/>

# Symmetry-Adapted Gaussian Process Regression

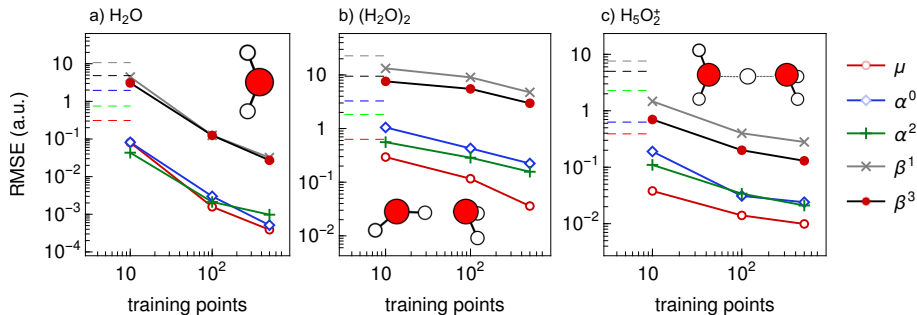
- How about machine-learning tensorial properties  $\mathbf{T}$ ? The kernel should be covariant to rigid rotations - need a symmetry-adapted framework
$$k_{\mu\nu}(\mathcal{X}, \mathcal{X}') = \langle T_{\mu}(\mathcal{X}); T_{\nu}(\mathcal{X}') \rangle \rightarrow k_{\mu\nu}(S\mathcal{X}, S'\mathcal{X}') = S_{\mu\mu'} k_{\mu'\nu'}(\mathcal{X}, \mathcal{X}') S_{\nu\nu'}$$
- For rigid molecules, one can convert in the local frame and learn individual components
- A more general strategy for fluxional, flexible environments:  $\lambda$ -SOAPs



Liang, Tocci, Wilkins, Grisafi, Roke, & **MC**, PRB (2017);  
Glielmo, Sollich, & De Vita, PRB (2017); Grisafi, Wilkins, Csányi, & **MC**, arXiv:1709.06757.

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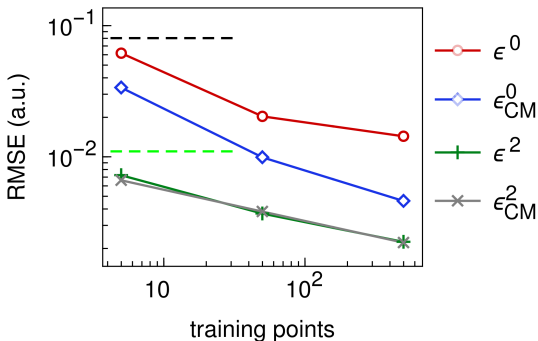
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