Stochastic modelling of urban structure

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Stochastic Modelling of Urban Structure, L. Ellam et al, Submitted (2017)

Overview

Motivation

Problem statement

What are the workings of cities and regions, and how will they evolve over time?

Problem statement

- Over half of the world's population live in a city.
- We should be interested in:
  - What is happening in the city;
  - How the city is evolving; and
  - How can we enable a better quality of life.
- Applications: planning, policy and decision making (e.g. retail, health, crime, education, transport etc.).
- Requires an understanding of the underlying mechanisms and behaviours.
- On going task of matching socio-economic theories with empirical evidence.

This talk: Improving insights into urban and regional systems with the development of data-driven mathematical models.

What can mathematics and statistics offer?

- Mathematical models can represent socio-economic theories.
- Can help explain the behaviour of complex systems.
- Simulations may provide insights:
  - 'Flight simulators' for urban and regional planners.
  - 'What if' forecasting capabilities.
  - How does the system respond to change/initiatives?
  - Which parameters/mechanisms are most important?
  - What is the long term behaviour?
- Mathematical modelling long history (> 50 years) in urban and regional analysis
  - *Equilibrium values and dynamics of attractiveness terms in production-constrained spatial-interaction models* (Harris and A. Wilson, 1978).

Data-driven methodologies

- Data sets are becoming routinely available:
  - Social media,
  - Location tracking,
  - Travel ticketing,
  - Census data,
  - Ad-hoc reports etc.
- The range of statistical models available is arguably somewhat limited.
- Ignoring either data or mathematical models seems unwise.
- A structured approach that's consistent with the available data is desirable.
- Build upon well-established mathematical formalisms with this new found data (rather than throwing the baby out with the bath water).

Mathematical and statistical challenges

"All models are wrong; some models are useful."

- Urban and regional systems are complex in nature:
  - Phase transitions; and
  - Multimodality and rare events.
- An emergent behaviour arises from the actions of many interacting individuals.
- Seamless integration of mathematical models with data.
- Quality of model vs. quality of data?
- Uncertainty should be addressed in the modelling process:
  - Model uncertainty and system fluctuations;
  - Parameter uncertainty; and
  - Observation error and bias.

Example: London retail system

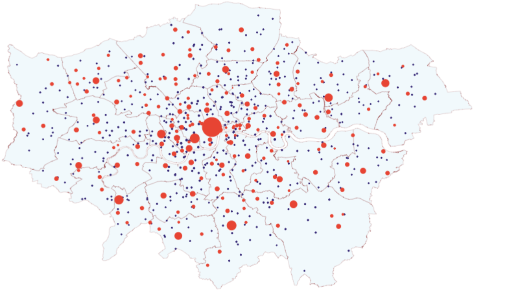


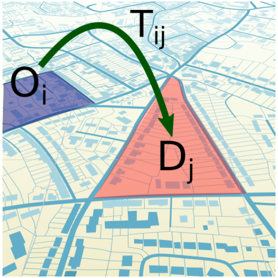
Figure: London retail structure for 2008 (left) and 2012 (right). The locations of retail zones and residential zones are red and blue, respectively. Sizes are in proportion to floorspace and spending power, respectively. $N = 625$ and $M = 201$.

Components of an urban and regional system

- Three key features:
  - Activities at location;
  - Flows between locations (spatial interaction); and
  - Structures that facilitate flows.
- Broad range of applications:
  - Retail: town centres, shopping centers, distribution centres;
  - Health: surgeries, hospitals;
  - Education: schools, colleges, universities;
  - Transport: airports, car parks;
  - Population: residence.
- Spatial interaction of activities can be modelled with statistical averaging procedures (Boltzmann).
- Evolution of structures relating to the activities can be modelled using differential equations (Lotka-Volterra).
- New approach: Expand on BLV models with Overdamped Langevin dynamics.

Modelling urban structure (forward problem)

Urban and regional systems



- Urban structure:
  - N origin zones and M destination zones.
  - Origin quantities $\{O_i\}_{i=1}^{N}$.
  - Destination quantities $\{D_j\}_{j=1}^{M}$.
- Spatial interaction:
  - Flows of activities at location are denoted $\{T_{ij}\}_{i,j}^{N,M}$, where T_{ij} is the flow from zone i to j.

Modelling flows

Flows from origin zones:

$$O_i = \sum_{j=1}^{M} T_{ij}, \quad i = 1, \ldots, N.$$

Flows to destination zones (demand function):

$$D_j = \sum_{i=1}^{N} T_{ij}, \quad j = 1, \ldots, M.$$

Nomenclature: A singly-constrained system since $\{O_i\}_{i=1}^{N}$ is fixed and $\{D_j\}_{j=1}^{M}$ is undetermined.

Modelling flows

Fixed total benefit (or capacity):

$$\sum_{i=1}^{N}\sum_{j=1}^{M} T_{ij}x_j = X,$$

where x_j is the attractiveness, of zone j.

Fixed total transport cost:

$$\sum_{i=1}^{N}\sum_{j=1}^{M} T_{ij}c_{ij} = C,$$

where c_{ij} is the cost of transporting a unit from zone i to zone j.

A production-constrained spatial interaction model

The destination flows are obtained by maximizing an entropy function.

$$D_j = \sum_{i=1}^{N} O_i \frac{\exp\left(\alpha x_j - \beta c_{ij}\right)}{\sum_{k=1}^{M} \exp\left(\alpha x_k - \beta c_{ik}\right)}.$$

- x_j is the attractiveness of j.
- c_{ij} is the inconvenience of transporting from zone i to j.
- α is the attractiveness scaling parameter.
- β is the cost scaling parameter.
- $\alpha x_j - \beta c_{ij}$ is the net utility from transporting from zone i to j.

A statistical theory of spatial distribution models, A. Wilson, Transportation research (1967)

Equilibrium structures

Urban structure is a vector of sizes: $w = \{w_1, \dots, w_N\} \in \mathbb{R}^M_{>0}$.

The evolution of urban structure can be modelled by the Lotka-Volterra equations

$$\frac{dw_j}{dt} = \epsilon\Big(D_j - \kappa w_j\Big) w_j, \quad j = 1, \dots, M,$$

for some initial condition $w(0) = w$.

Equilibrium condition

$$\sum_{i=1}^{N} O_i \frac{\exp\left(\alpha \ln w_j - \beta c_{ij}\right)}{\sum_{k=1}^{M} \exp\left(\alpha \ln w_k - \beta c_{ik}\right)} = \kappa w_j, \quad j = 1, \dots, M.$$

Equilibrium values and dynamics of attractiveness terms in production-constrained spatial-interaction models. B. Harris A. Wilson, Environment and Planning (1978)

Stochastic modelling of urban structure

Work in terms of attractiveness $x = \{x_1, \ldots, x_M\} \in \mathbb{R}^M$ with $w_j(x_j) = \exp(x_j)$.

The evolution of urban structure can be modelled by overdamped Langevin dynamics:

$$dx = -\nabla V(x)dt + \sqrt{2\gamma^{-1}}dB,$$

for some initial condition $x(0) = x$ and standard M-dimensional Brownian motion B.

Equilibrium distribution

Urban structure is realization of the Boltzmann distribution

$$\pi(x) = \frac{1}{Z}\exp\Big(-\gamma V(x)\Big), \quad Z := \int_{\mathbb{R}^M} \exp\big(-\gamma V(x)\big)\,dx,$$

specified by the potential function $V : \mathbb{R}^M \to \mathbb{R}$ and 'inverse-temperature' $\gamma > 0$.

Stochastic Modelling of Urban Structure, L. Ellam et al, Submitted (2017)

Assumptions for the potential function

Interpretation of gradient structure (in terms of net supply/demand Π_j):

$$-\partial_j V(x) = \epsilon \Pi_j, \quad j = 1, \dots, M.$$

$$V(x) = \underbrace{V_{\text{Utility}}(x)}_{\text{Demand}} + \underbrace{V_{\text{Cost}}(x) + V_{\text{Additional}}(x)}_{\text{Supply}}.$$

1. The potential function $V \in C^2(\mathbb{R}^M, \mathbb{R})$ is confining in that $\lim_{\|x\| \to +\infty} V(x) = +\infty$, and

$$e^{-\gamma V(x)} \in L^1(\mathbb{R}^M), \quad \forall \gamma > 0.$$

2. There exists $0 < d < 1$ such that

$$\liminf_{\|x\| \to \infty} (1-d) \|V(x)\|^2 - \nabla^2 V(x) > 0.$$

Utility potential

- For a singly-constrained system we have that:

$$-\partial_j V_{\text{Utility}}(x) = \epsilon D_j = \epsilon \sum_{i=1}^{N} v_{ij}(x) O_i, \qquad \sum_{j=1}^{M} v_{ij}(x) \equiv 1.$$

- Can express weights v_{ij} in terms of utility functions u_{ij}:

$$-\partial_j V_{\text{Utility}}(x) = \epsilon \sum_{i=1}^{N} \frac{\varphi(u_{ij})}{\sum_{k=1}^{M} \varphi(u_{ik})} O_i. \tag{1}$$

- By inspection, we look for a potential function of the form

$$V_{\text{Utility}}(x) = -\epsilon \sum_{i=1}^{N} O_i \left\{ f_i(x) \ln \sum_{j=1}^{M} \varphi(u_{ij}(x)) \right\}. \tag{2}$$

Utility potential

- Inserting Eq. (2) into Eq. (1), we obtain the requirement:

$$\frac{\varphi(u_{ij}(x))}{\sum_{k=1}^{M}\varphi(u_{ik}(x))} = \frac{df_i(x)}{dx_j}\ln\sum_{j=1}^{M}\varphi(u_{ij}(x)) + f_i(x)\frac{d\varphi(u_{ik}(x))}{dx_j}\left(\sum_{k=1}^{M}\varphi(u_{ik}(x))\right)^{-1}. \tag{3}$$

- Eq. (3) is satisfied for $\varphi(x) = \exp(x)$,

$$u_{ij}(x) = \alpha_i x_j + \beta_{ij},$$

provided that each $\alpha_i \neq 0$ and that $f_i = \alpha_i^{-1}$.

We obtain:

$$V_{\text{Inflow}}(x) = -\epsilon\sum_{i=1}^{N}\alpha_i^{-1}O_i\ln\sum_{j=1}^{M}\exp\left(u_{ij}(x)\right).$$

Cost and additional potentials

Cost potential (linear cost):

$$-\partial_j V_{\text{Cost}}(x) = -\epsilon\kappa w_j(x_j), \quad j = 1, \ldots, M,$$

$$V_{\text{Cost}}(x) = \epsilon\kappa \sum_{j=1}^{M} w_j(x_j).$$

Additional potential (constant support):

$$-\partial_j V_{\text{Additional}}(x) = -\epsilon\delta_j, \quad j = 1, \ldots, M,$$

$$V_{\text{Additional}}(x) = \epsilon \sum_{j=1}^{M} \delta_j x_j.$$

Resulting Boltzmann distribution

$$\pi(x) = \frac{1}{Z}\exp\Big(-\gamma V(x)\Big), \quad Z := \int_{\mathbb{R}^M} \exp\big(-\gamma V(x)\big)\,dx,$$

$$\epsilon^{-1}V(x) = \underbrace{-\sum_{i=1}^{N}\alpha_i^{-1}O_i \ln \sum_{j=1}^{M}\exp\big(u_{ij}(x_j)\big)}_{\text{Utility (Spatial Interaction)}} \underbrace{+\kappa\sum_{j=1}^{M} w_j(x_j)}_{\text{Cost}} \underbrace{-\sum_{j=1}^{M}\delta_j x_j}_{\text{Additional}}.$$

- $u_{ij}(x_j) = \alpha x_j - \beta c_{ij}$ is the net utility for a flow from zone i to j.
- Potential assumptions satisfied.

Alternative derivation: random utility maximization

- Individuals make choices to maximize their (random) utility:

$$U_{ij} = u_{ij} + \epsilon_{ij}, \qquad \epsilon_{ij} \sim \text{Gumbel}(0, 1).$$

- The probability that individual from zone i makes choice j is:

$$Pr\left[Y_{ij} = 1\right] = Pr\left[\cap_{j \neq k} \{U_{ij} > U_{ik}\}\right] = \frac{\exp(u_{ij})}{\sum_{k=1}^{M} \exp(u_{ij})}.$$

- The expectation for all individuals gives the same demand flows as before:

$$D_j = \sum_{i=1}^{N} O_i Pr\left[Y_{ij} = 1\right] = \sum_{i=1}^{N} O_i \frac{\exp\left(\alpha x_j - \beta c_{ij}\right)\big)}{\sum_{k=1}^{M} \exp\left(\alpha x_j - \beta c_{ik}\right)\big)}.$$

- The utility potential is the (unscaled) expected welfare:

$$\epsilon^{-1} V_{\text{Utility}}(x) = \alpha^{-1} \sum_{i=1}^{N} O_i \mathbb{E}[\max_j U_{ij}] + \text{const} = -\alpha^{-1} \sum_{i=1}^{N} O_i \ln \sum_{j=1}^{M} \exp\left(u_{ij}(x_j)\right) + \text{const}.$$

Alternative derivation: maximum entropy method

- Total welfare (surplus):

$$\mathbb{E}_\pi\left[\alpha^{-1}\sum_{i=1}^{N} O_i \ln \sum_{j=1}^{M} \exp\left(u_{ij}(x_j)\right)\right] = S,$$

- Total size (capacity):

$$\mathbb{E}_\pi\left[\sum_{j=1}^{M} w_j(x_j)\right] = W,$$

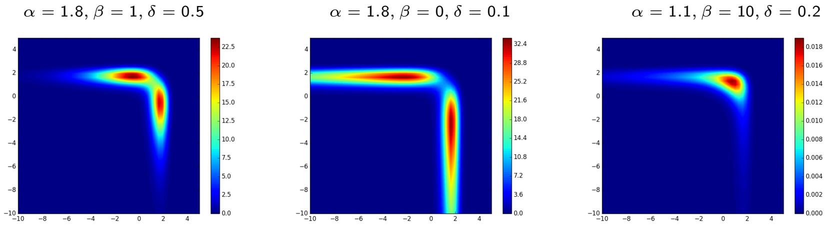
- Expected attractiveness (benefit)

$$\mathbb{E}_\pi\left[\sum_{j=1}^{M} x_j\right] = X.$$

- Then the Boltzmann distribution $\pi(x) = Z^{-1}\exp\left(-\gamma V(x)\right)$ is the maximum entropy distribution (maximal uncertainty) subject to these constraints.

Sampling challenges

- High-dimensionality (use gradient information)
- Multi-modality (use a tempering approach)
- Anisotropy (precondition)



Joint probability distribution:

$$\pi_{\text{pt}}(x_1, \ldots, x_T) = \prod_{t=1}^{T} \pi_t(x_t), \quad \pi_t(x_t) = Z_t^{-1} \exp(-\gamma_t V(x)), \quad \gamma_1 = \gamma.$$

- **HMC move steps** Draw momentum $p_t \sim N(0,1)$, define $H_t(x_t, p_t) = -\ln \pi_t(x_t) + \frac{1}{2} p_t^2$, obtain x_t' and p_t' by simulating Hamiltonian dynamics with a volume preserving time-integrator (e.g. leapfrog) and accept with probability

$$\min\left\{1, H(x_t, p_t) - H(x_t', p_t')\right\}.$$

- **Exchange steps** Propose neighbouring swaps $x_t \mapsto x_{t+1}, x_{t+1} \mapsto x_t$ and accept with probability

$$\min\left\{1, \frac{\pi_t(x_{t+1})\pi_{t+1}(x_t)}{\pi_t(x_t)\pi_{t+1}(x_{t+1})}\right\}.$$

Airports in England

$\alpha = 0.5$

$\alpha = 1.0$

$\alpha = 2.0$

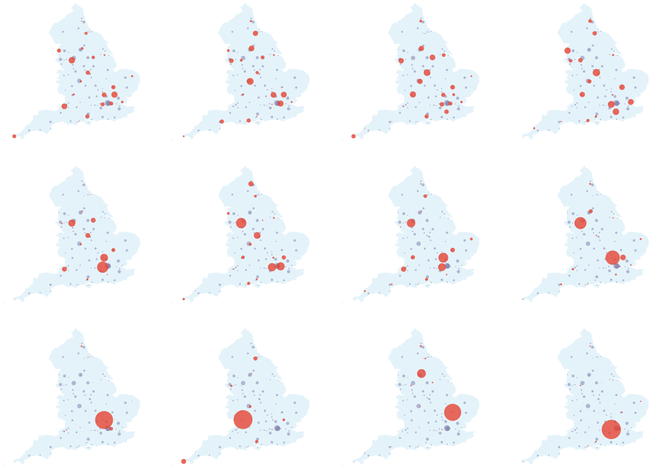


Figure: Approximate draws from $p(x|\theta)$ using HMC combined with parallel tempering.

London Retail system

$\alpha = 0.5$

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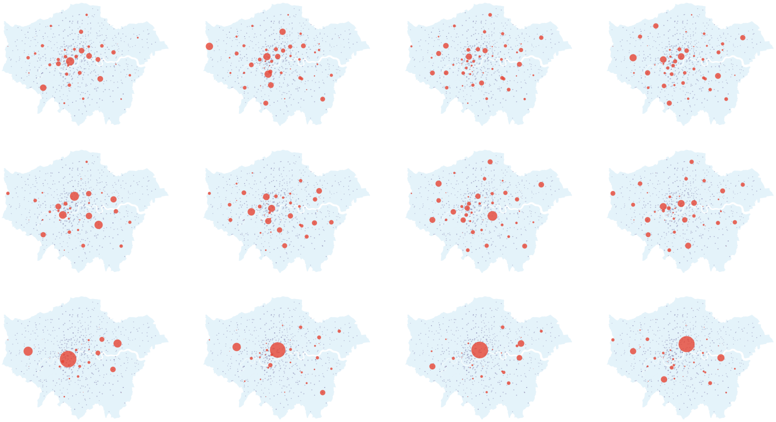

Figure: Approximate draws from $p(x|\theta)$ using HMC combined with parallel tempering.

Overdamped Langevin dynamics

- With the specification of $V(X)$, overdamped Langevin dynamics give the Harris and Wilson model, plus multiplicative noise.

SDE Urban Retail Model

Floorspace dynamics is a stochastic process that satisfies the following Stratonovich SDE.

$$\frac{dW_j}{dt} = \epsilon W_j \left(D_j - \kappa W_j + \delta_j \right) + \sigma W_j \circ \frac{dB_j}{dt},$$

where $\left(B_1, \ldots, B_M\right)^T$ is standard M-dimensional Brownian motion and $\sigma = \sqrt{2/\epsilon\gamma} > 0$ is the volatility parameter.

- Fluctuations (missing dynamics) are modelled as Stratonovich noise.
- The extra parameter δ to represents local economic stimulus to prevent zones from collapsing (needed for stability).
- The Markov process converges exponentially fast to $\pi(x) = Z^{-1} \exp(-\gamma V(x))$.

Inferring the utility function (inverse problem)

A Bayesian model of urban structure

Inverse problem

Given observation data $Y = (Y_1, \ldots, Y_M)$, of log-sizes, infer the parameter values $\theta = (\alpha, \beta)^T \in \mathbb{R}^2_+$ and latent variables $X \in \mathbb{R}^M$ with low-order summary statistics:

$$\mathbb{E}_{X,\theta|Y}[h(X,\theta)] = \int h(X,\theta)\pi(X,\theta|Y)dXd\theta.$$

Assumption (Data generating process)

Assume that the observations $Y_1, \ldots, Y_M$ are i.i.d. realizations of the following hierarchical model:

$$Y_1, \ldots, Y_M | X, \sigma \sim \mathcal{N}(X, \sigma^2 I),$$

$$X|\theta \sim \pi(X|\theta) \propto \exp(-U(X;\theta)),$$

$$\theta \sim \pi(\theta).$$

Posterior distribution

Bayes' rule

The joint posterior is given by

$$\pi(X,\theta|Y) = \frac{\pi(Y|X,\theta)\pi(X,\theta)}{\pi(Y)}, \quad \pi(Y) = \int \pi(Y|X,\theta)\pi(X,\theta)dXd\theta.$$

We have a hierarchical prior given by

$$\pi(X,\theta) = \frac{\pi(\theta)\exp(-U(X;\theta))}{Z(\theta)}, \quad Z(\theta) = \int \exp(-U(X;\theta))dX.$$

The joint posterior is doubly-intractable

$$\pi(X,\theta|Y) = \frac{\pi(Y|X,\theta)\exp(-U(X;\theta))\pi(\theta)}{\pi(Y)Z(\theta)}.$$

Russian roulette

On Russian Roulette Estimates for Bayesian Inference with Doubly-Intractable Likelihoods. AM Lyne, M Girolami, Y Atchade, H Strathmann, D Simpson. Statistical Science (2013)

Metropolis-within-Gibbs with block updates

- We are interested in low-order summary statistics of the form

$$\mathbb{E}_{X,\theta|Y}[h(X,\theta)] = \int h(X,\theta)\pi(X,\theta|Y)dXd\theta.$$

- X and θ are highly coupled, so we use Metropolis-within-Gibbs with block updates.
- X-update. Propose $X' \sim Q_X$ and accept with probability

$$\min\left\{1, \frac{\pi(Y|X',\theta)\,\exp(-U(X';\theta))\,q(X|X')}{\pi(Y|X,\theta)\,\exp(-U(X;\theta))\,q(X'|X)}\right\}.$$

- θ-update. Propose $\theta' \sim Q_\theta$ and accept with probability

$$\min\left\{1, \frac{\pi(Y|X,\theta')\,Z(\theta)\,\exp(-U(X;\theta'))\,q(\theta|\theta')}{\pi(Y|X,\theta)\,Z(\theta')\,\exp(-U(X;\theta))\,q(\theta'|\theta)}\right\}.$$

- Unfortunately the ratio $Z(\theta)/Z(\theta')$ ratio is intractable!

Unbiased estimates of the normalizing constant

- We can use the Pseudo-Marginal MCMC framework if we have an unbiased, positive estimate of $\pi(X|\theta)$, denoted $\hat{\pi}(X|\theta, u)$, satisfying

$$\pi(X|\theta) = \int \hat{\pi}(X|\theta, u)\pi(u|\theta)du.$$

- The Russian Roulette methodology gives an unbiased estimate of $1/Z$:

$$S = \mathcal{V}^{(0)} + \sum_{i=1}^{N} \frac{\mathcal{V}^{(i)} - \mathcal{V}^{(i-1)}}{Pr(N \geq i)},$$

  provided that $\lim_{i \to \infty} \mathbb{E}[\mathcal{V}^{(i)}] = 1/Z$.
- Requires N estimates of $1/Z$ using path sampling e.g. annealed importance sampling or thermodynamic integration, for a random stopping time N.

Unbiased estimates of the normalizing constant

- The Forward Coupling estimator (FCE) gives an unbiased estimate of $1/Z$ with lower variance
- The idea is to find two sequences of consistent estimators $\{\mathcal{V}^{(i)}\}$, $\{\tilde{\mathcal{V}}^{(i)}\}$, each with the same distribution, such that $\mathcal{V}^{(i)}$ and $\tilde{\mathcal{V}}^{(i-1)}$ are "coupled".
- Coupling between $\mathcal{V}^{(i)}$ and $\tilde{\mathcal{V}}^{(i-1)}$ is introduced with a Markov chain that shares random numbers.
- Then the unbiased estimate is given by

$$S := \mathcal{V}^{(0)} + \sum_{i=1}^{N} \frac{\mathcal{V}^{(i)} - \tilde{\mathcal{V}}^{(i-1)}}{Pr(N \geq i)}.$$

Markov Chain Truncation for Doubly-Intractable Inference, C. Wei, and I. Murray, AISTATS (2016)

Annealed importance sampling

- Define the (log-gamma) density $\pi_0(X) \propto exp(-\gamma U_0(X))$ by

$$U_0(X) = \kappa \sum_{j=1}^{M} \exp(X_j) - \delta \sum_{j=1}^{M} X_j.$$

  We can sample from π_0 exactly and easily.
- Define a temperature schedule $0 = t_0 < t_1 < \cdots < t_T = 1$ with bridging distributions

$$\pi_t = Z_t^{-1} \pi_0^{1-t_t} \pi^{t_t}, \quad \text{for} \quad j = 1, \dots, T.$$

Set $w_0^{(n)} = 1/N$ and $X_0 \sim \pi_0$, for $n = 1, \dots, N$. Then for $t = 1, \dots, T$:

- Draw $X_t^{(n)} \sim K_t(\cdot | X_{t-1})$, for $n = 1, \dots, N$.
- Set $w_t^{(n)} = w_{t-1}^{(n)} \frac{\pi_t\left(X_{t-1}^{(n)}\right)}{\pi_{t-1}\left(X_{t-1}^{(n)}\right)}$, for $n = 1, \dots, N$.

Then $Z = \lim_{N \to \infty} \sum_{n=1}^{N} w_T^{(n)}$.

Bridging distributions

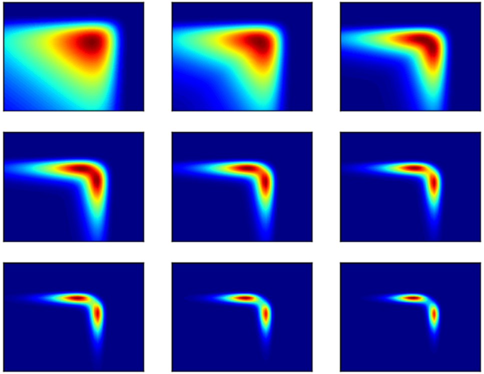

Figure: $\{\pi_j\}_{j=1}^{T=9}$ from left to right, then top to bottom.

The signed measure problem

- S can be negative when $\mathcal{V}^{(i)} < \tilde{\mathcal{V}}^{(i-1)}$ for many i. This is known as the 'sign problem'.
- Rejecting when S is negative would introduce a bias.
- Instead, we note that

$$\mathbb{E}[h(X,\theta)] = \frac{1}{\pi(Y)} \int h(X,\theta)\pi(Y|X,\theta)\hat{\pi}(X|\theta,u)\pi(\theta)\pi(u)dud\theta dX,$$

$$= \frac{\int h(X,\theta)\sigma(X|\theta,u)\check{\pi}(X,\theta,u|Y)dud\theta dX}{\int \sigma(X|\theta,u)\check{\pi}(X,\theta,u|Y)dud\theta dX},$$

where σ is the sign function and we have defined

$$\check{\pi}(X,\theta,u|Y) = \frac{\pi(Y|X,\theta)|\hat{\pi}(X|\theta,u)|\pi(\theta)\pi(u)}{\int \pi(Y|X,\theta)|\hat{\pi}(X|\theta,u)|\pi(\theta)\pi(u)dud\theta dX}.$$

Pseudo-marginal Markov chain Monte Carlo

- We can sample from $\check{\pi}(X, \theta, u|Y)$ using Metropolis-within-Gibbs with block updates.
- X-update. Propose $X' \sim Q_X$ and accept with probability

$$\min\left\{1, \frac{\pi(Y|X', \theta)\, \exp(-U(X'; \theta))\, q(X|X')}{\pi(Y|X, \theta)\, \exp(-U(X; \theta))\, q(X'|X)}\right\}.$$

- (θ, u)-update. Propose $(\theta', u') \sim Q_{\theta,u}$ and accept with probability

$$\min\left\{1, \frac{\pi(Y|X, \theta')|S(\theta, u)|\exp(-U(X; \theta'))\pi(\theta')\pi(u')\, q(\theta|\theta')}{\pi(Y|X, \theta)|S(\theta', u')|\exp(-U(X; \theta))\pi(\theta)\pi(u) q(\theta'|\theta)}\right\}.$$

- Posterior expectations are estimated using

$$\mathbb{E}_{X,\theta|Y}[h(X, \theta)] = \lim_{N\to\infty} \frac{\sum_{i=1}^{N} h(X_i, \theta_i)\sigma(X_i|\theta_i, u_i)}{\sum_{i=1}^{N} \sigma(X_i|\theta_i, u_i)}.$$

Airports in England

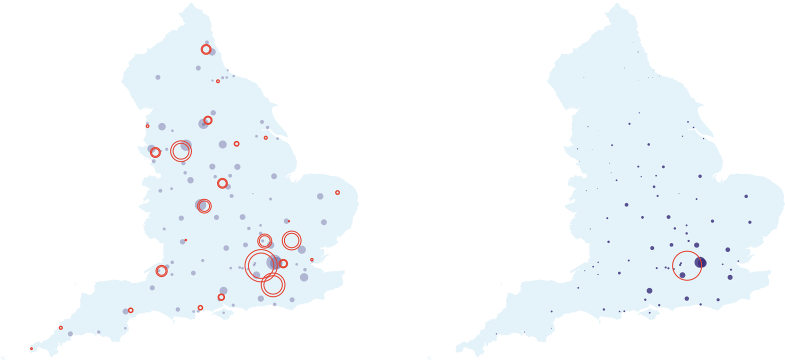

Figure: Left: Visualization of the posterior-marginals for the latent variables $\{x_j\}_{j=1}^M$ over a map of England. Right: Visualization of the posterior demands.

The London Retail system

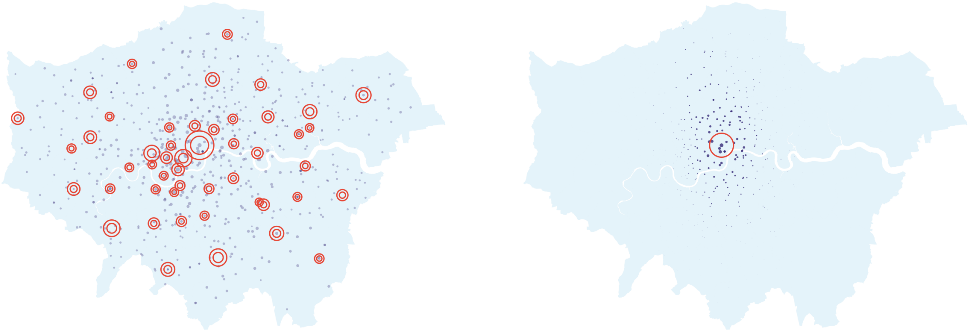

Figure: Left: Visualization of the posterior-marginals for the latent variables $\{x_j\}_{j=1}^M$ over a map of London Right: Visualization of the posterior demands. Observation data obtained

Bayesian model comparison

Open question: Do consumers perceive distance linearly or logarithmically?

$$p(y|H_j) = \int p(y|\theta_j, H_j)p(\theta_j|H_j)d\theta_j,$$

$$p(y|\theta_j, H_j) = \frac{1}{Z(\theta_j; H_j)} \int p(y|x_j, \theta_j, H_j)q(x_j|\theta_j, H_j)dx_j,$$

$$Z(\theta_j; H_j) = \int q(x_j|\theta_j, H_j)dx_j.$$

Bayes' factors

$$BF_{1,2} = \frac{p(y|H_1)}{p(y|H_2)}.$$

Conclusions and outlook

Further work and extensions

- Investigation of new data assimilation methodologies to calibrate models to data available at different scales. For example:
  - Population data;
  - Cost matrix; or
  - Time dependent parameters.
- Deployment of new methodology to a global-scale problem (non-Bayesian approaches?).
- Extension of discrete-choice approach to other socio-economic phenomena e.g. crime.
- Melding of data and models takes us beyond data analytics.

Summary

- We have developed a novel stochastic model to simulate realistic configurations of urban and regional structure.
- Our model is an improvement on existing deterministic models in the literature, as we account for uncertainties arising in the modelling process.
- We presented a Bayesian hierarchical model for urban and regional systems.
- Our model can be used to infer the components of a utility function from observed structure, rather than hidden flow data.
- We have demonstrated our approach using an example of airports in England and retail in London.

Acknowledgment

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- https://iconicmath.org/