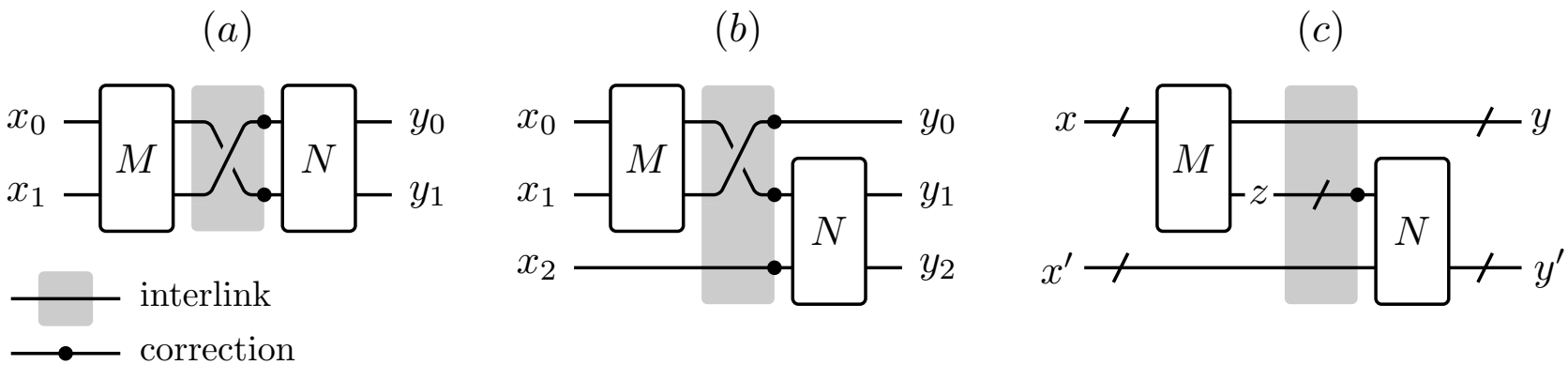
Modular quantum signal processing with gadgets

[2309.16665]

Zane Rossi

joint with Jack Ceroni and Isaac Chuang



October 3, 2023

Quantum algorithms: tough to design and interpret

Despite beautiful work in QPLs [1, 2], we still reason at gate level, most of the time

What can quantum programming *techniques* & *abstractions* learn from their classical cousins?

Quantum algorithm design

Overview

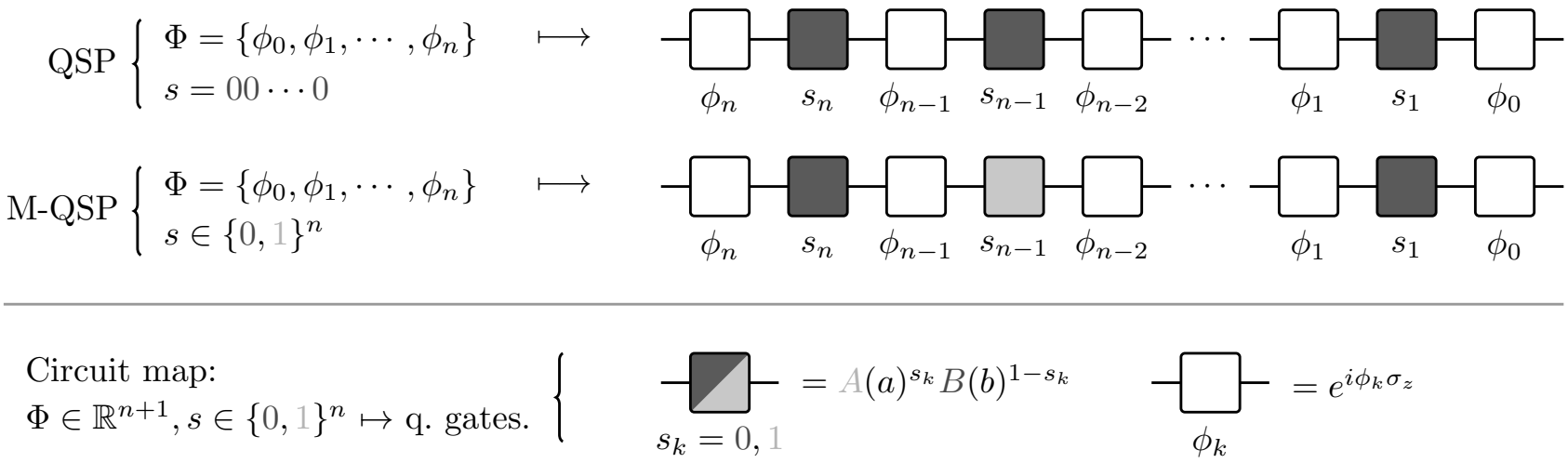
Pinpoint strengths of algorithmic class: QSP

Argue *function-first* algorithmic design with above is (1) desirable but (2) not immediately possible

Turn a desire into a formal problem

Show solution utility, efficiency, functional programming flavor, and introduce package

QSP: functional interpretability



Space- and query-efficient algorithms for spectral mapping; unifies most quantum algorithms [3, 4]

The temptation

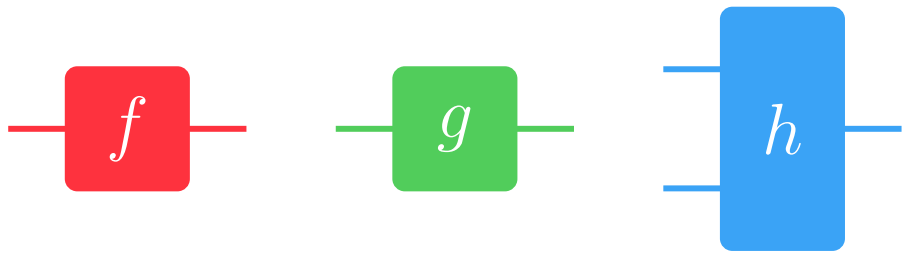
QSP and QSVT achieve* polynomial transforms

$$\langle 0|U(\Phi, x)|0\rangle = P(x)$$

Given protocols achieving $f(x)$, $g(y)$, and $h(z, w)$, it *feels* like we should be able to achieve

$$h \circ (f, g) \equiv h(f(x), g(y)).$$

Moreover, the composite protocol ought to be able to use $f(x)$ and $g(y)$ as contiguous subroutines


f
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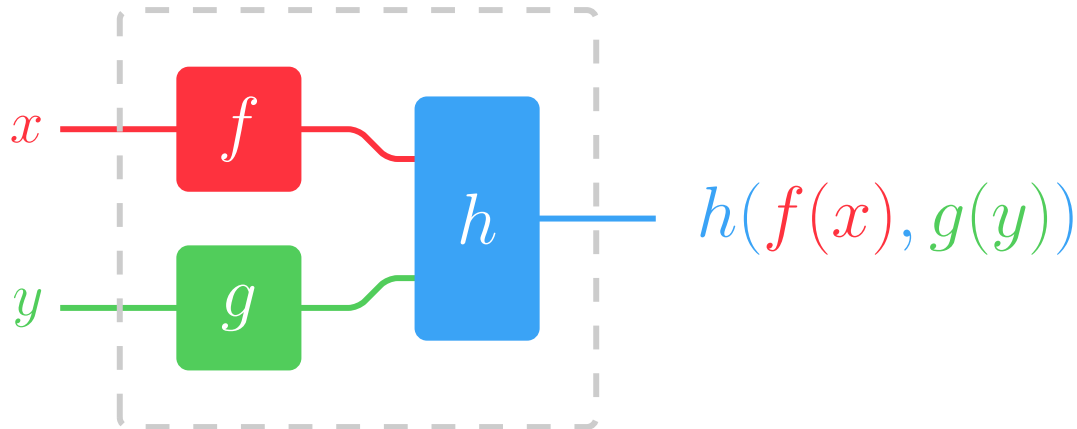
x
y
f
g
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h(f(x), g(y))

The problem

QSP encodes privileged basis: expects X rotations as input but does not produce them as output

Even worse lifting to QSVT or with many oracles

QSP protocols aren't first class functions

Why bother

Modularity and reusability in coherent setting

QSP is great: space efficient, infinity norm dependent, strong numerical footing ...

Multivariable analogues of QSP are *hard*, while multivariable functions (comparators, control-flow, loss) are ubiquitous [5]

A note on QSVT

Commuting operators and spectral mapping

Anything we do in SU(2) is done in preserved QSVT subspaces. Let $\Pi, \tilde{\Pi}$ proj. and $A = \tilde{\Pi} U \Pi$, then

$$A = \sum_{i=1}^{d_{\min}} \xi_i |\tilde{\psi}_i\rangle\langle\psi_i| \mapsto P^{SV}(A) \equiv \sum_{i=1}^{d_{\min}} P(\xi_i) |\tilde{\psi}_i\rangle\langle\psi_i|,$$

$$U = \cdots \oplus \bigoplus_{\xi_i \neq 0,1} \begin{bmatrix} \xi_i & \sqrt{1-\xi_i^2} \\ \sqrt{1-\xi_i^2} & -\xi_i \end{bmatrix}_{\tilde{\mathcal{H}}_i}^{\mathcal{H}_i} \oplus \cdots,$$

Gadgets

An (a, b) gadget is a unitary* superoperator:

$$\bigotimes_{k\in[a]} e^{i\theta_k\sigma_x} \mapsto \bigotimes_{j\in[b]} e^{i\phi_j\sigma_z} e^{i\psi_j\sigma_x} e^{-i\phi_j\sigma_z},$$

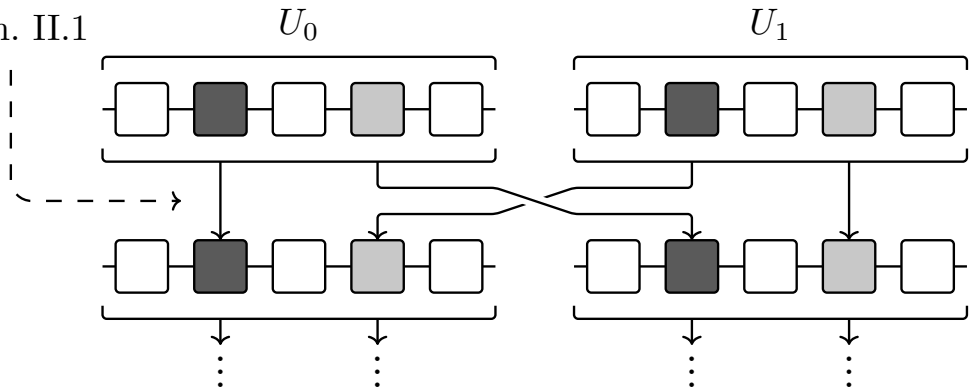
over θ in known range. Each of b output legs achieves $\cos\psi_j$, often polynomial in $\cos\theta_k$, $k \in [a]$.

Functionally interpretable boxes: a inputs, b outputs

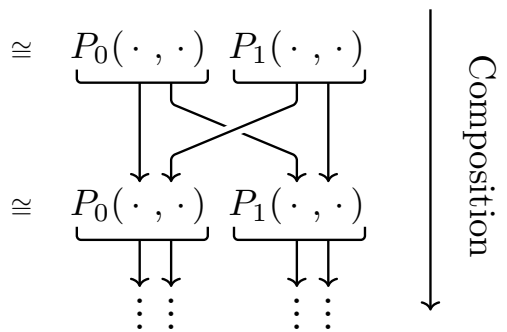
Gadget Rosetta Stone

(a) Self-composition of circuits

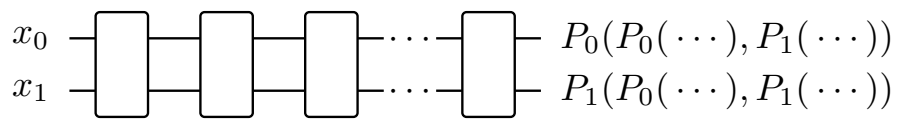
Thm. II.1

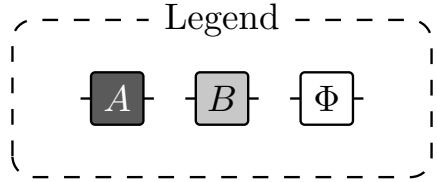


(b) Self-composition of polynomials



(c) Self-composition of $(2, 2)$ gadgets (n.b. superoperators)





Thm 1: linking gadgets 🔗

Let $\varepsilon, \delta > 0$, $\mathfrak{G}$ an (a, b) gadget and (Ξ, S) an atomic (c, d) gadget, where $\Xi \equiv \{\Phi_0, \ldots, \Phi_{d-1}\}$ and $S \equiv \{s_0, \ldots, s_{d-1})$. Suppose $\mathfrak{G}$ achieves

$$F(x) \equiv \{f_0(x_0, \cdots, x_{a-1}), f_1(x_0, \cdots, x_{a-1}), \ldots, f_{b-1}(x_0, \cdots, x_{a-1})\}$$

over $x \in [-1, 1]^{\times a}$, and (Ξ, S) achieves

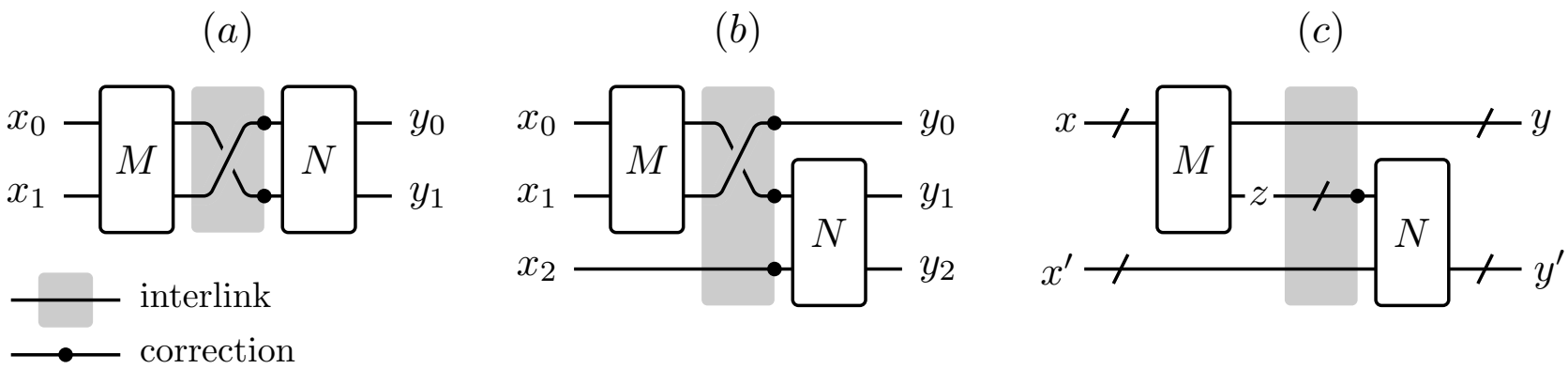
$$G(y) \equiv \{g_0(y_0, \cdots, y_{c-1}), g_1(y_0, \cdots, y_{c-1}), \ldots, g_{d-1}(y_0, \cdots, y_{c-1})\}$$

over $y \in [-1, 1]^{\times c}$. Let $\mathfrak{I} = (B, C, W)$ an interlink. Then, there exists a gadget $\mathfrak{G}'$ which ε-approximately achieves

$$H(x, y') \equiv \bigcup_{k \in [d]} g_k \left(\bigcup_{j \in B} f_{W(j)}(x_0, \ldots, x_{a-1}) \cup \bigcup_{k \notin C} y_k \right) \cup \bigcup_{k \notin B} f_k(x_0, \ldots, x_{a-1})$$

over $(F(x), y') \in \mathcal{D}$, where y' is the subset of y_k such that $k \notin C$ and $\mathcal{D}$ is a domain determined by the correction protocol used.

$\mathfrak{G}'$ uses description of (Ξ, S) and $\widetilde{\mathcal{O}}(d|\Xi|_\infty \zeta)$ black-box calls to result of running $\mathfrak{G}$.



Generally, $\zeta = \widetilde{O}(\text{polylog}(\varepsilon^{-1})\text{poly}(\delta^{-1}))$ for $\delta = \mathcal{O}(1)$. Note $|\Xi|_\infty$ is max length of elem in Ξ.

Thm 2: gadget-poly equivalence ⚖️

Let $\mathcal{L}(x_1, \ldots, x_n)$ polynomials achievable by atomic gadgets over $x_1, \ldots, x_n$. Suppose $P(x_1, \ldots, x_n)$ has degree D and can be split into a tower of $m = \mathcal{O}(\log(D))$ interlinked polynomials,

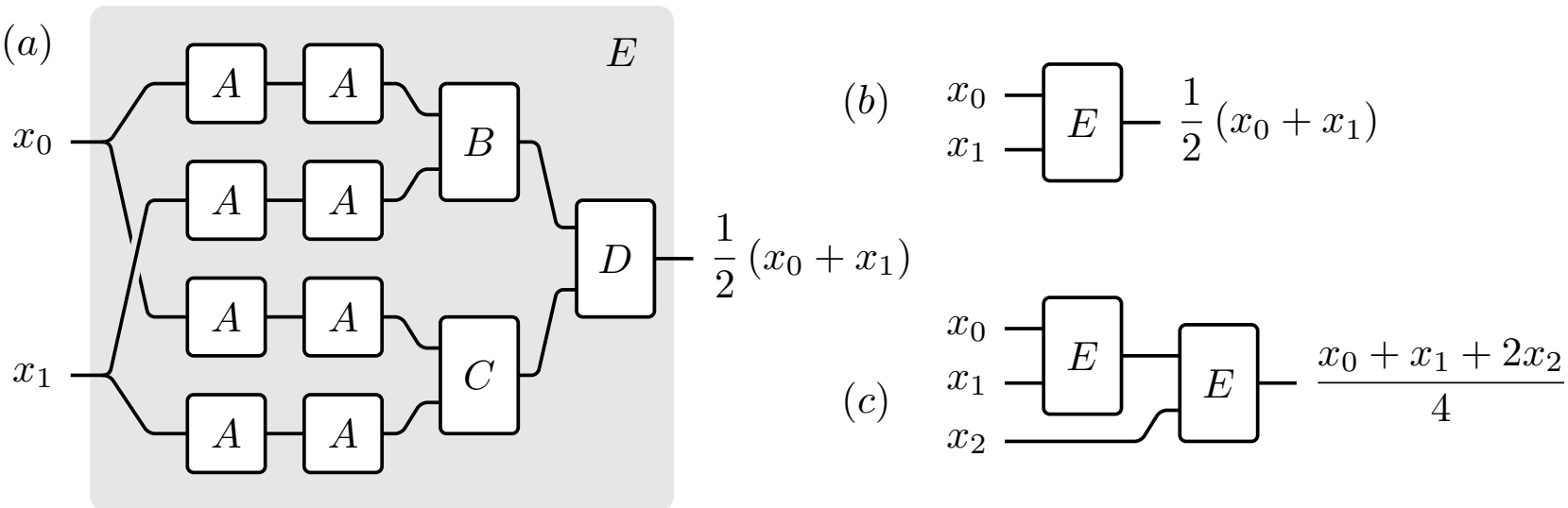
$$P = P^{(m-1)} \circ_{\mathfrak{I}_{m-2}} \left(P^{(m-2)} \circ_{\mathfrak{I}_{m-3}} \circ \left(\cdots \left(P^{(1)} \circ_{\mathfrak{I}_0} P^{(0)} \right) \cdots \right) \right)$$

such that $P_i^{(j)} \in \mathcal{L}(x_1, \ldots, x_n)$ for all i, j. If the P_k are separated from $\{0, \pm 1\}$ by $\delta \in \mathcal{O}(1)$ for all k over $\mathcal{D}_k$. Then, there exists an assemblage of m atomic, snappable gadgets which ε-approximately achieves P on $\mathcal{D}_k$ with aggregate query cost $\widetilde{O}(\text{poly}(D)\,\text{polylog}(\varepsilon))$.

Algebraic structure	Manipulation	Algebraic form	Reference
QSP/M-QSP	Polynomial application	$x_0 \mapsto P(x_0)$,	Thm. D.1
	Multivariable variant†	$x_0, \ldots, x_n \mapsto P(x_0, \ldots, x_n)$,	Thm. D.2
Polynomial ring‡	Addition	$x_0, x_1 \mapsto (x_0 + x_1)/2$,	Thm. IV.3
	Multiplication	$x_0, x_1 \mapsto x_0 x_1$,	Thm. IV.2
	Scalar multiplication	$x_0 \mapsto \min(\alpha x_0, 1), \alpha \in \mathbb{R}$,	Ex. IV.4/5
Composition monoid	Polynomial composition	$f(x_0), g(x_0) \mapsto (f \circ g)(x_0)$,	Thm. III.1
General gadgets	Gadget linking	$\mathfrak{G}, \mathfrak{G}' \mapsto \mathfrak{G}\mathfrak{I}\mathfrak{G}'$,	Thm. III.1

Provided examples	Negation/inversion (Ex. IV.1)	Bandpass function (Ex. IV.11)
	Angle sum/difference (Ex. IV.2)	Majority vote (Ex. IV.12)
	Affine shift (Ex. IV.7)	Functional interpolation (Ex. IV.13)
	Step function (Ex. IV.8)	Chebyshev inverse (Thm. IV.1)
	General mean (Ex. IV.10)	

Sum gadget



A: square root gadget, B, C: angle sum/difference gadgets, D: product gadgets.

Prior work

Cascaded classical filters for sharpening. [6, 7]

Recursively defined quantum (computational) subroutines. [8, 9, 10, 11, 12, 13, 14]

Single-variable self-embedded QSP/QSVT. [15, 16]

Multivariable QSP/QET and LCU-based methods.

Functional programming techniques: quantum and classical. [1, 2, 17]

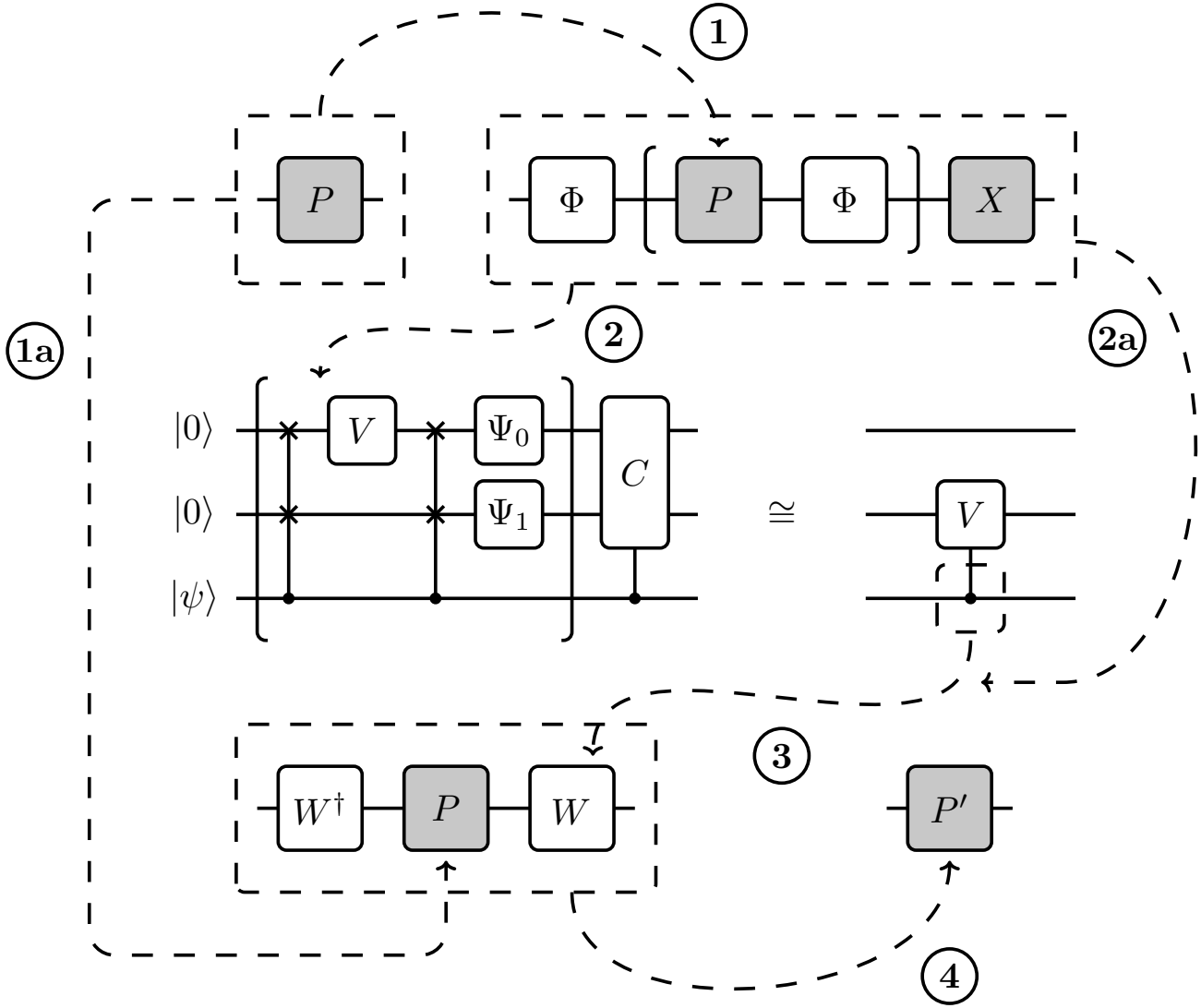
Correction protocol

Main idea: repeated use of twisted signal to obliviously, approximately cancel its own twist

$$e^{i\phi\sigma_z} e^{i\psi\sigma_x} e^{-i\phi\sigma_z} \mapsto \left\{ e^{i\phi\sigma_z} e^{i\psi\sigma_x} e^{-i\phi\sigma_z}, e^{2i\phi\sigma_z} \right\} \mapsto e^{i\psi\sigma_x}.$$

Off diagonal element's phase sent to $\approx \pi/2$

$$\begin{bmatrix} P & iQ\sqrt{1-x^2} \\ * & * \end{bmatrix} \mapsto \begin{bmatrix} P & i\sqrt{1-P^2} \\ * & * \end{bmatrix}$$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Computing costs 💴

For $P(x)$ close to ± 1, QSP suffers 'gimbal lock'

Take $\delta = \mathcal{O}(1)$, and $P(x) \in [-1+\delta, 1-\delta]^b$ over $x \in [-1+\delta, 1-\delta]^a$: simple bounding box

Cost per correction is $\mathcal{O}(\text{poly}(\delta^{-1})\,\text{polylog}(\varepsilon^{-1}))$

On query/space complexity 💾

An (a, b) gadget has a cost for each output leg, up to desired precision, in queries to each input leg:
$a \times b$ cost matrix C

Up to padding rows & cols, C of gadget computed by multiplying C_k of sub-gadgets.

Space/query complexity breaks into cases:
single-variable, psd Q, controlled access.

Is this efficient? Or, why not LCU?

Poly-many, poly-size gadgets, log depth.
Maintains QSP's infinity norm scaling.
Poly-logarithmic in inverse precision.
No post-selection required.
Hierarchical, distributed, modular.
Linking high-degree gadgets is where method shines.
Potentially huge space saving (psd Q); coupled to questions in poly decomposition theorems.
Not truly in competition with LCU.

Method	Queries	Space	Norm
[18]	$\mathrm{poly}(r)d^{\mathrm{poly}(r)}$	$\mathrm{poly}(r)[\mathrm{poly}(d) + \mathrm{poly}(s)]$	$\|\cdot\|_1$
Ours	d	$s + c$	$\|\cdot\|_\infty$

Table: Asymptotic query and space complexity, as well as relevant norm. Here d is generalized degree, r is number of variables, and s is qubits needed for block encodings. Note the case that $c = 0$ is well-understood.

Functional programming 🔧

Semantics and syntax

Monad: monoid in the category of endofunctors

$$\texttt{unit} : T \to M\ T,$$
$$\texttt{bind} : (M\ T, T \to M\ U) \to M\ U,$$

Unwrapped type is scalar value (or X rotation), wrapped is QSP output; this work constructs `bind`

Bonus: attribute grammar & language of gadgets

Python package

gadget_init.py UNREGISTERED

```python
xi_0 = [[0, 0, 0, 0], [0, 0, 0, 0]]
xi_1 = [[0, 0, 0, 0], [0, 0, 0, 0]]

s_0 = [[0, 1, 0], [1, 0, 1]]
s_1 = [[0, 1, 0], [1, 0, 1]]

# Create two (2, 2) gadgets named "g0" and "g1".
g0 = AtomicGadget(2, 2, "g0", xi_0, s_0)
g1 = AtomicGadget(2, 2, "g1", xi_0, s_1)

# Link first output of g0 to first input of g1.
linking_guide = [(("g0", 0), ("g1", 0))]
a0 = g0.wrap_gadget()
a1 = g1.wrap_gadget()
a2 = a0.link_assemblage(a1, linking guide)
```

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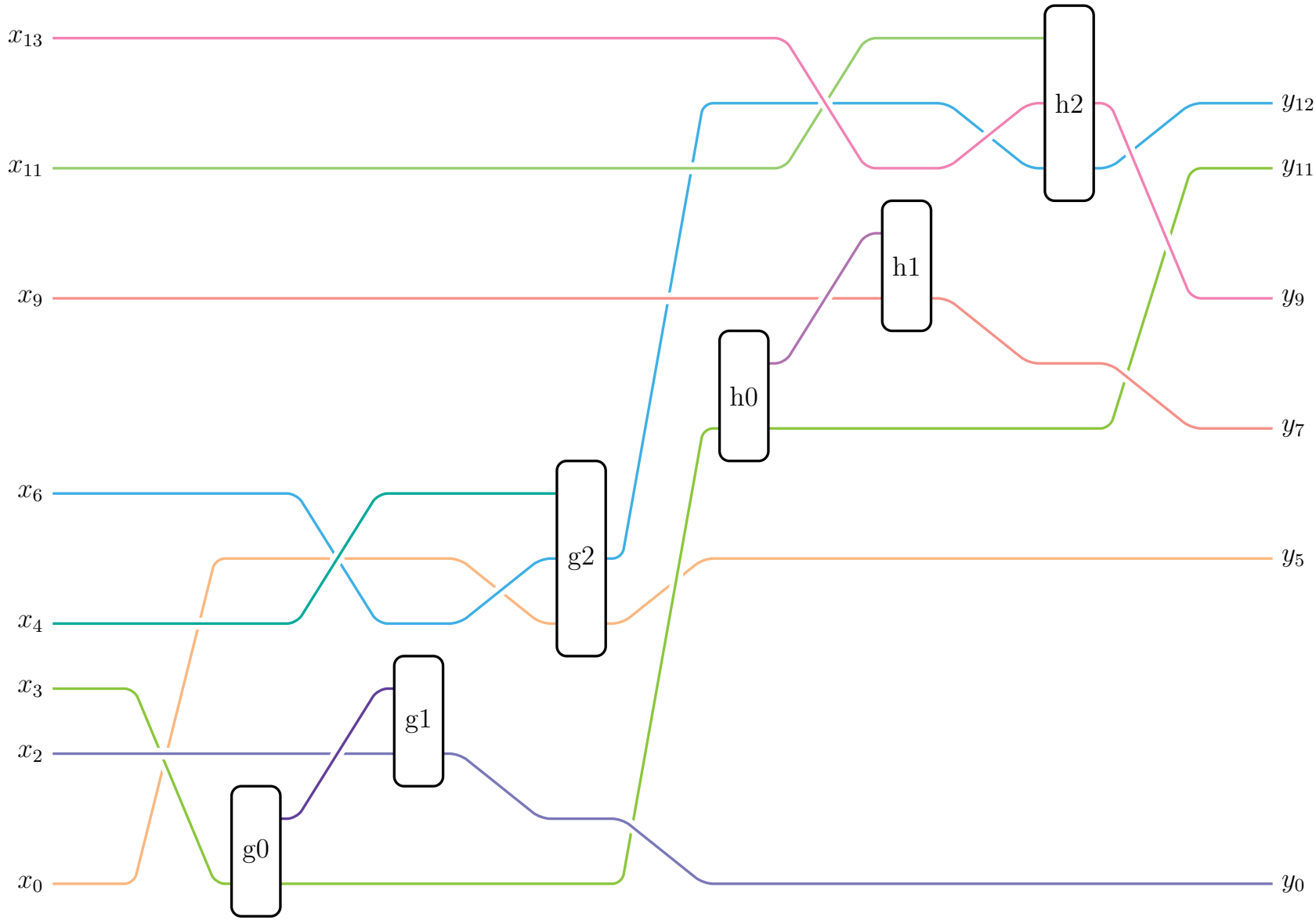
github.com/ichuang/pyqsp/tree/beta


x_{13}
x_{11}
x_9
x_6
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x_3
x_2
x_0
g0
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h0
h1
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y_{12}
y_{11}
y_9
y_7
y_5
y_0

To take home

Efficient, function-first quantum algorithm design

Coherent use of QSP protocols as QSP oracles with functional interpretability

Readymade repo at `ichuang/pyqsp/tree/beta`

Formal syntax and semantics, coupled to deep lit in (Q)PLs: compilation, verification [1, 19, 20]

Ancilla-free gadgets: minimal functional description?

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