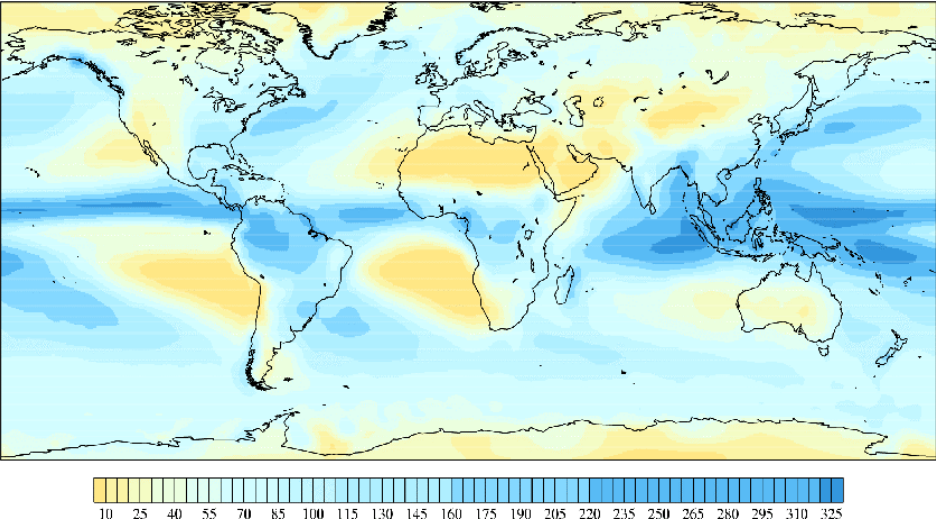


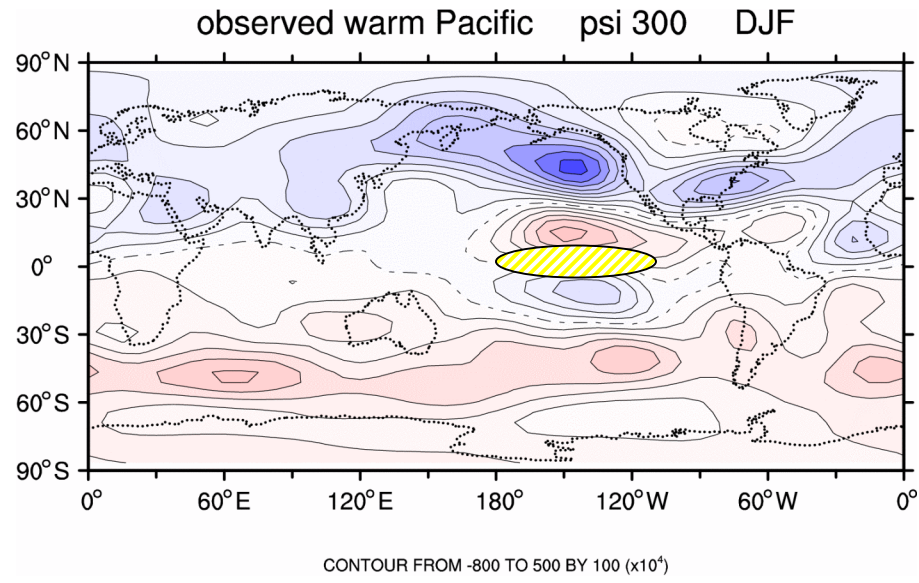
# Annual mean precipitation

Annual precipitation (cm)

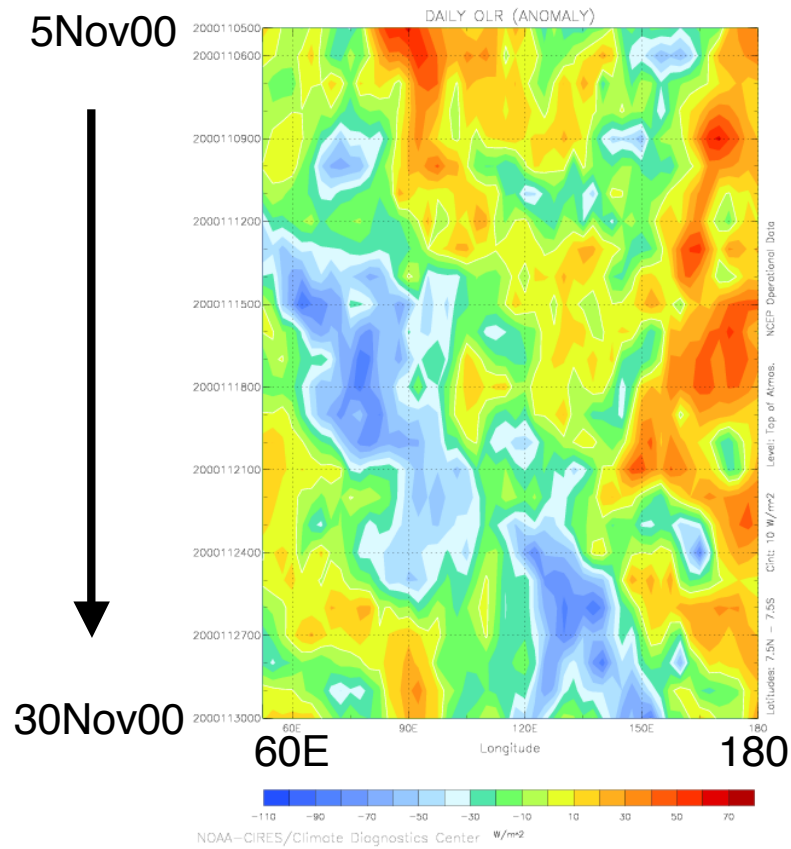


*Wallace & Hobbs*

# Nature's response to El Nino heating



# OLR anomalies



## July - September 1987 OLR

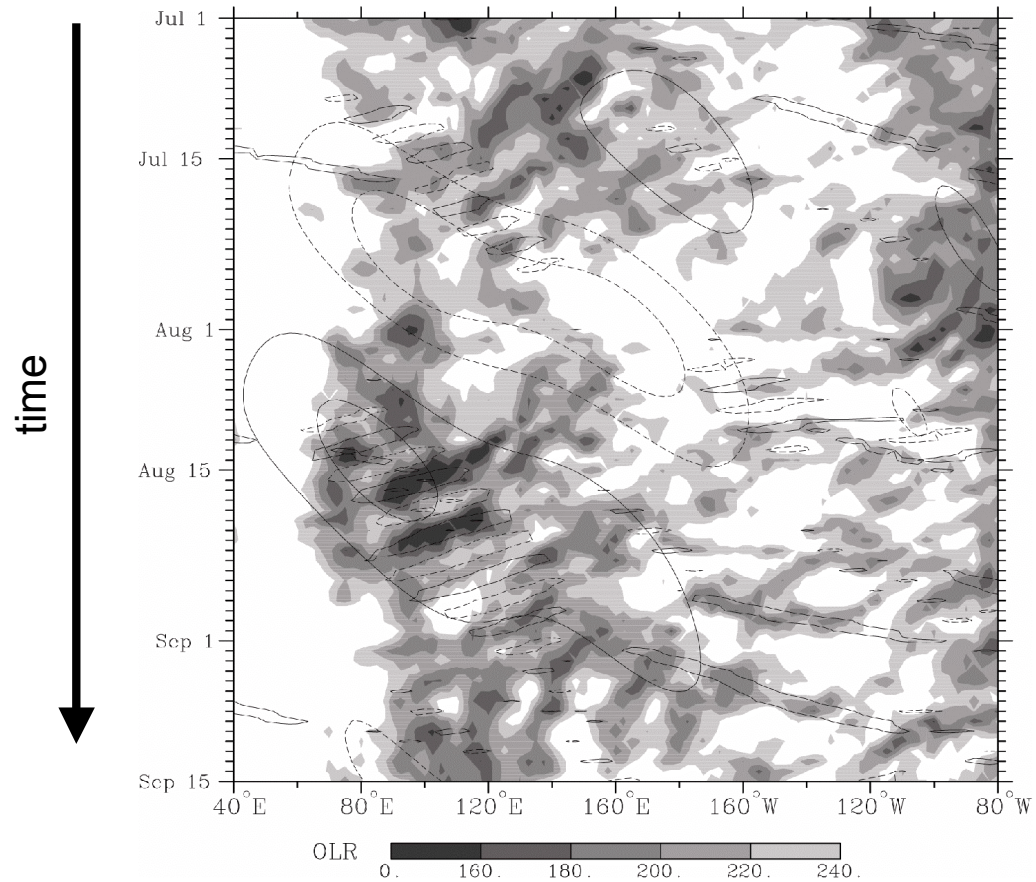
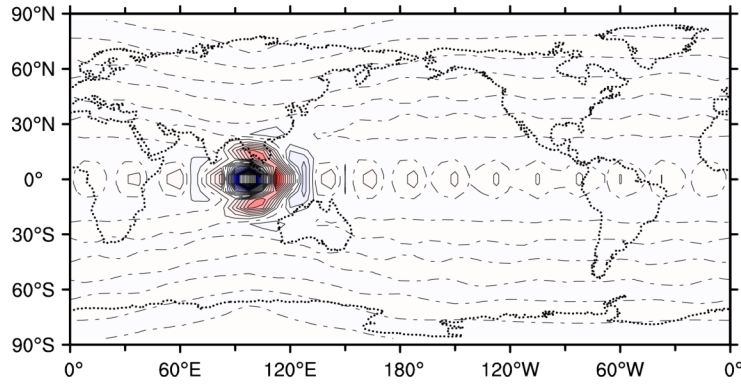


FIG. 6. Time-longitude plot of total OLR (shading, as indicated), filtered ISO, and MRG-TD OLR (contours, solid negative, contour interval  $10 \text{ W m}^{-2}$ , zero contour omitted), and filtered Kelvin wave OLR (contoured at  $-12 \text{ W m}^{-2}$  only), averaged from  $2.5^\circ$  to  $15^\circ\text{N}$ , from 1 Jul to 15 Sep 1987.

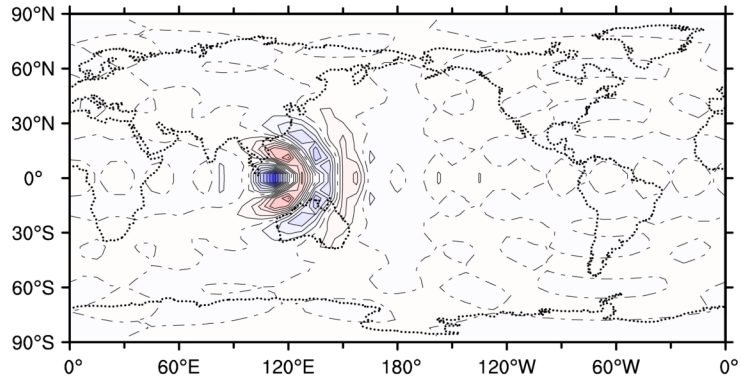
*Straub & Kiladis (2003)*

# Initial pulse in linear model → dispersion

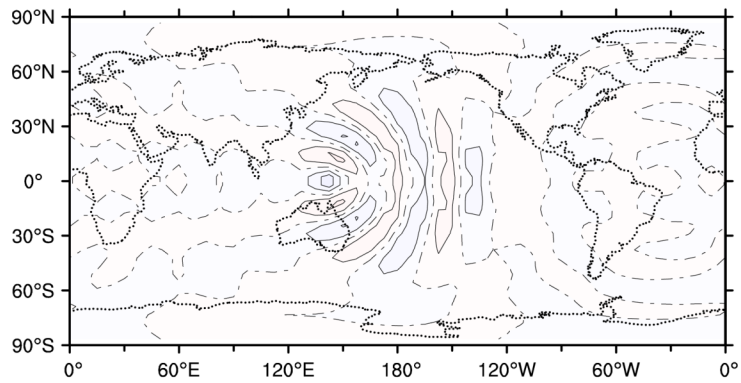
day 1



day 3

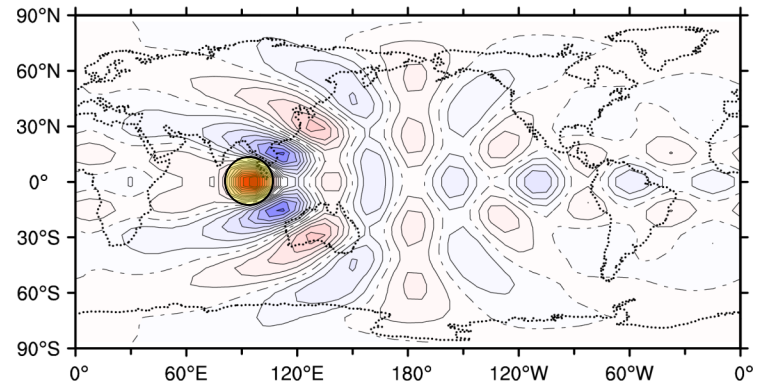


day 7



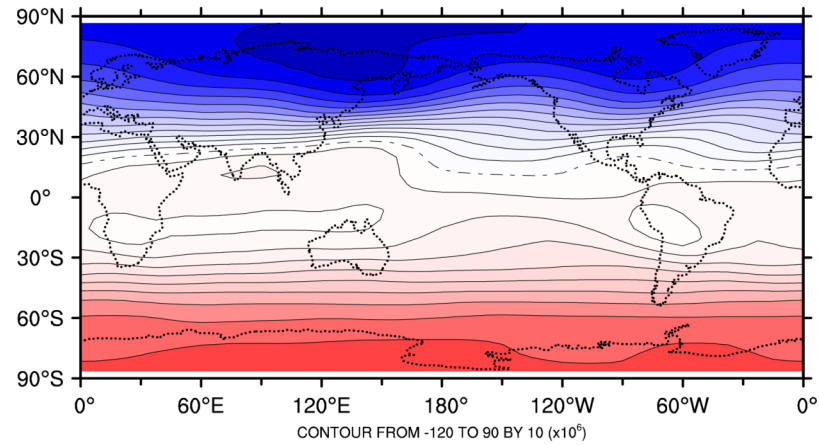
*(vorticity)*

# Linear response to steady source

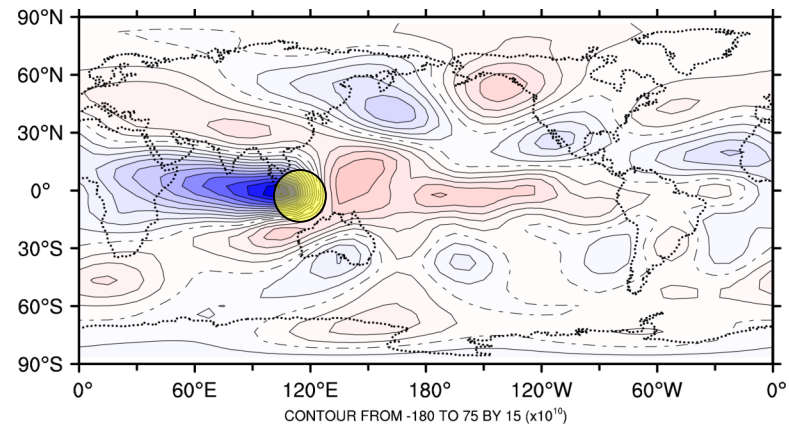
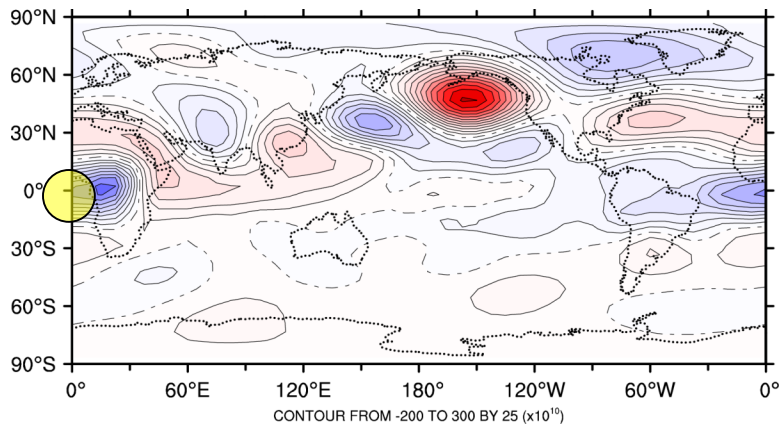


# January mean 300hPa streamfunction

basic state



## Linear response to steady vorticity source

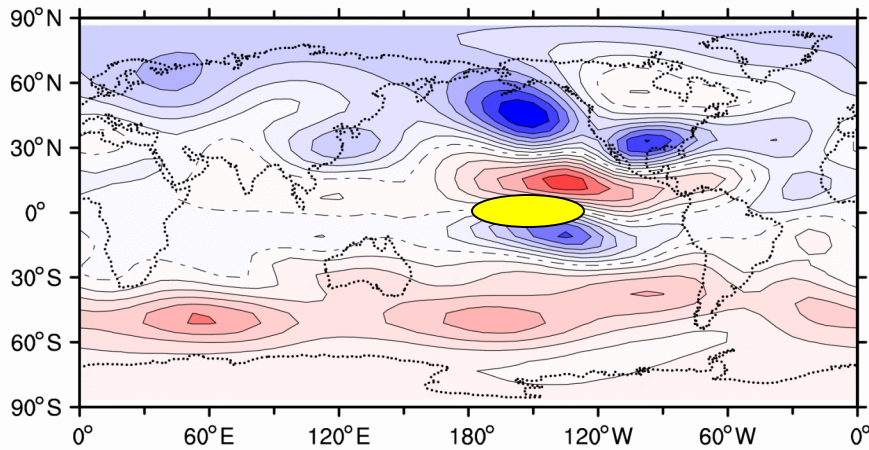


*(streamfunction)*

# GCM response to El Nino SST

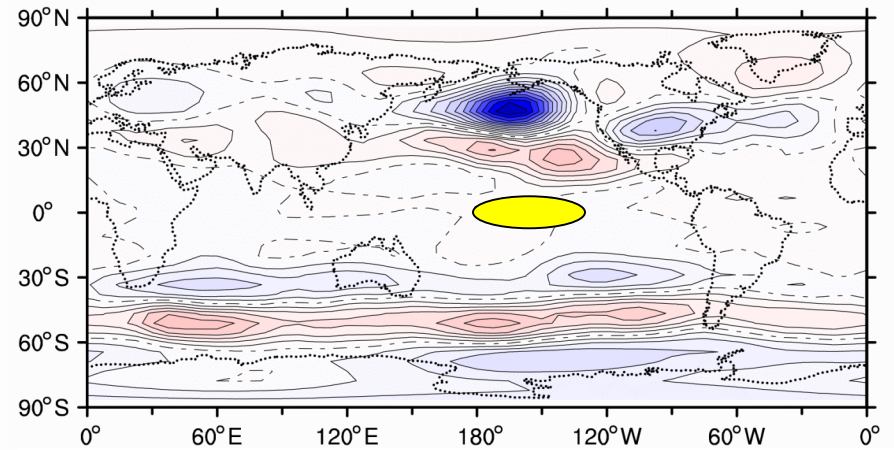
$$\hat{\psi}_{300}$$

psi300 pos nino3.4  
DJF AMIP.22.45

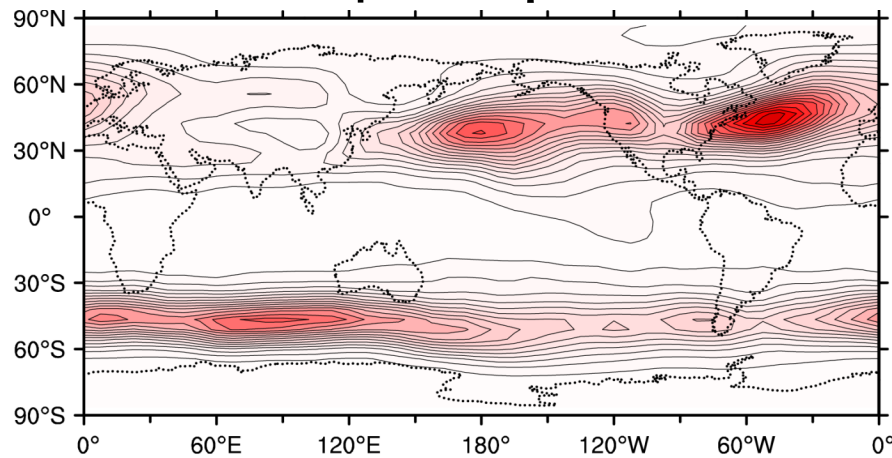


$$-\nabla^{-2} (\vec{v}_{\psi}^{bp} \cdot \nabla \zeta^{bp})$$

psi300tend pos nino3.4  
DJF AMIP.20.45



## 2-7d bandpass $\psi_{300}$ variance



The Atmospheric Response to Time-dependent Tropical Heating  
as Estimated from the Fluctuation Dissipation Theorem  
*Data compression & analysis through data driven modeling*

Grant Branstator  
National Center for Atmospheric Research  
Andrey Gritsun, RAS; Andrew Majda, CIMS

# Fluctuation Dissipation Theorem (Leith, 1975)

Suppose have discretized dynamical system

$$\frac{d\phi}{dt} = F(\phi, \Lambda, f); \phi|_{t=0} = \phi_0; \phi, \phi_0 \in \Phi \quad (\#)$$

Interested in

$$\langle A(\phi) \rangle = \int A(\phi) \rho(\phi) d\phi, \text{ for stationary PDF } \rho$$

Suppose  $f$  is changed by a small  $\delta f$ . Want

$$\text{linear } U \quad \delta \langle A(\phi) \rangle = U \delta f$$

If (*Deker & Haake, 1975; Risken, 1984*)

\* (#) is assumed to include a small noise term so its PDF is not fractal

\* (#) has a F - P equation with unique solution,

then

$$U_{ij}(t) = \int_0^t \left\langle A_i(\phi(t' + \tau)) \frac{\partial \ln \rho(\phi(t'))}{\partial \phi_j(t')} \right\rangle d\tau$$

# Generalized FDT -- Simplifications

For a system like the earth's atmosphere

$$\rho \equiv \rho_0 \exp \left\{ (C^{-1}(0)\phi', \phi') / 2 \right\}$$

So

$$U(t) \equiv \int_0^t \left\langle A(\phi'(t' + \tau))(\phi'(t))^T \right\rangle C^{-1}(0) d\tau$$

Note if  $A = I$ ,

$$U(t) = \int_0^t C(\tau) C^{-1}(0) d\tau$$

Majda, Abramov & Grote (2005) found third order accuracy for  $A = I$  and second order accuracy for quadratic  $A$ .

## Simplified FDT -- Incremental form

For quasiGaussian case with  $A = I$ ,  
the response at  $t$  to a steady forcing switched on at  $t_0$  is

$$U(t, t_0) f = \int_{t_0}^t C(\tau) C^{-1}(0) d\tau f$$

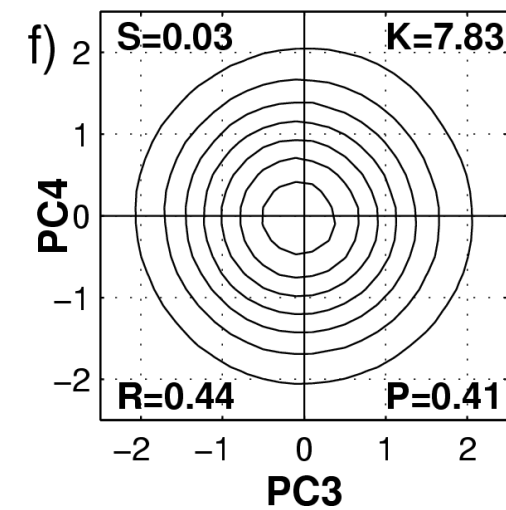
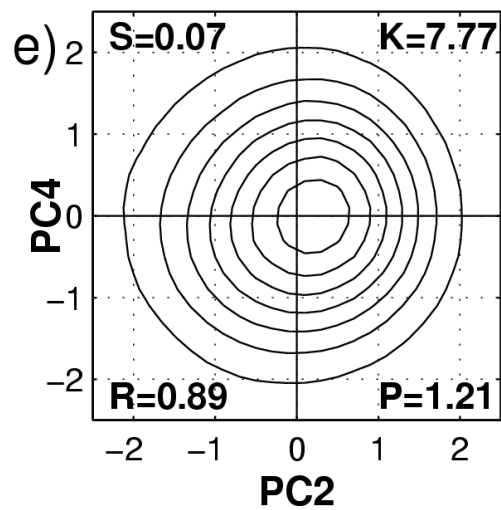
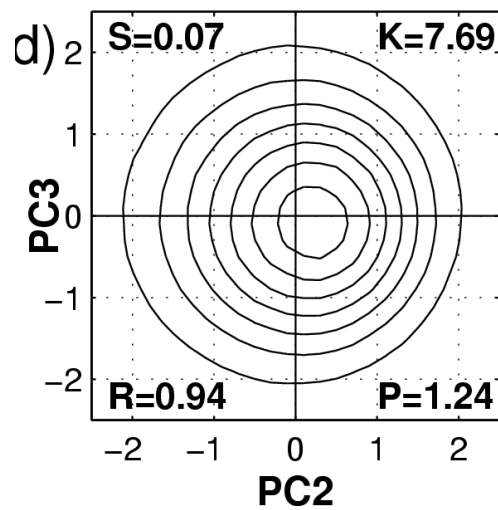
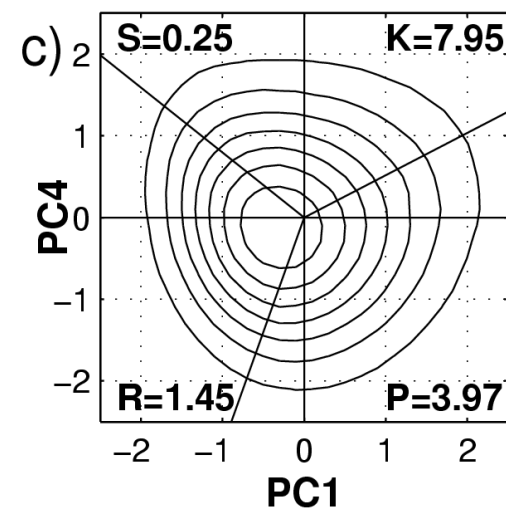
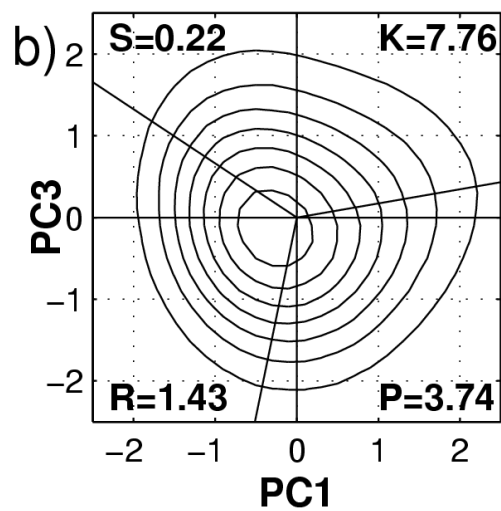
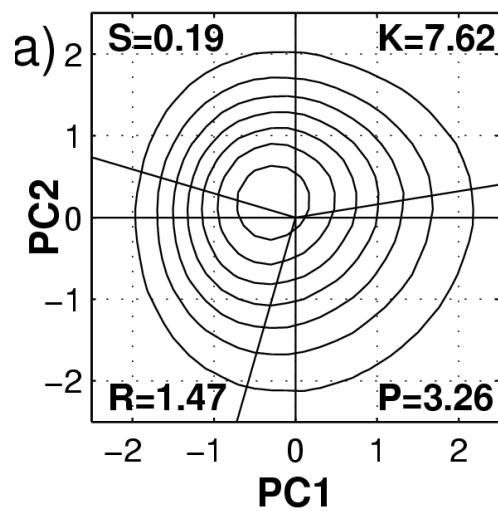
$\Rightarrow$

the operator that gives the response at  $t$  to a pulse forcing at  $t_0$  is

$$\delta U(t, t_0) = C(t - t_0) C^{-1}(0)$$

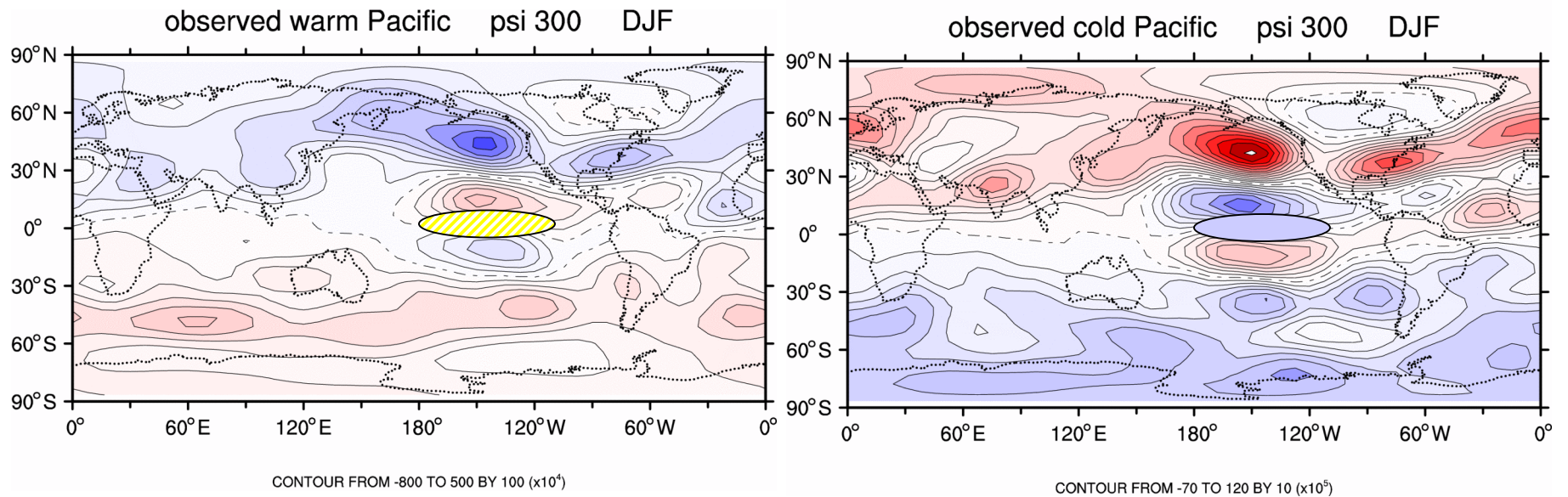
$\Rightarrow$

$$R(t) = \int_{t_0}^t C(t - \tau) C^{-1}(0) f(\tau) d\tau$$



Berner & Branstator (2006)

# Nature's response to El Nino & La Nina



## Test bed & application

Atmospheric general circulation model (NCAR's CCM0)

- \* Avoid sampling limitations when calculating lag covariances
- \* Can sample strictly intrinsic variability
- \* Enables rigorous testing of the resulting operator

Primitive equations, circa 1980 physical parameterizations

Perpetual January, fixed boundary conditions

R15  
9 level } 18352 degrees of freedom

8 million 12hrly simulated states

# Reduce Dimensionality

## 1. Pick fields from

\* ps

→ \* psi x 9

\* chi x 9

→ \* T x 9

\* water vapor mixing ratio x 9

## 2. Truncate each field using EOFs

\* psi 100x9 (>90%)

\* T 496x9 (100%)

## 3. Form multivariate (truncated) fields, normalize by std dev & overweight T, calculate EOFs

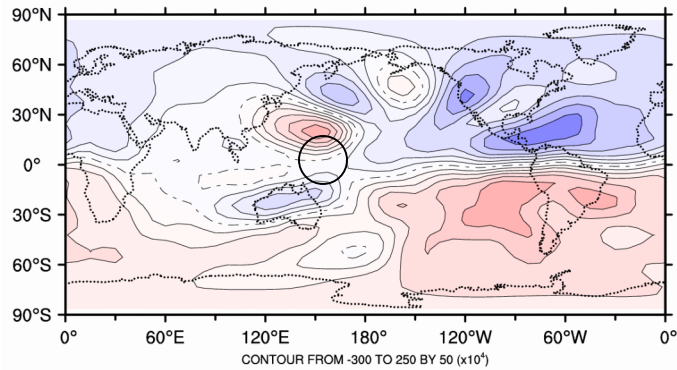
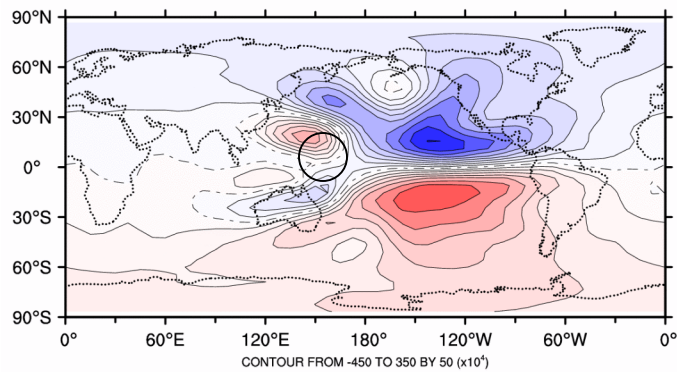
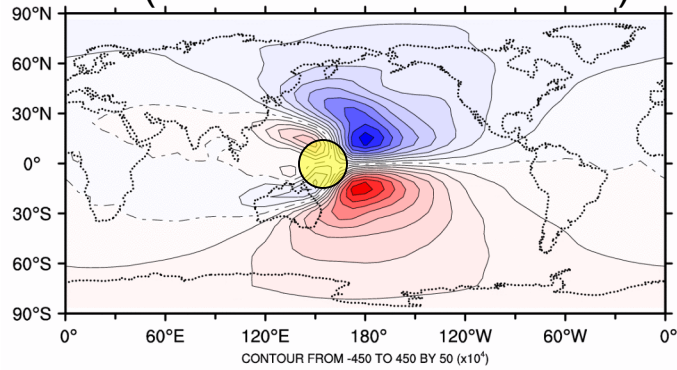
truncate (1800 EOFs, >95%)

**Assume lag covariances vanish for  $\tau > 30d$**

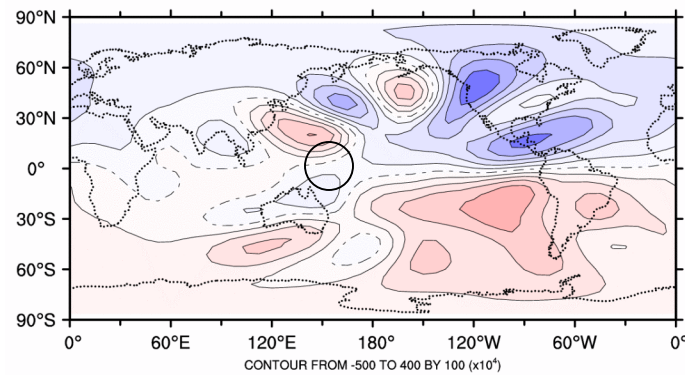
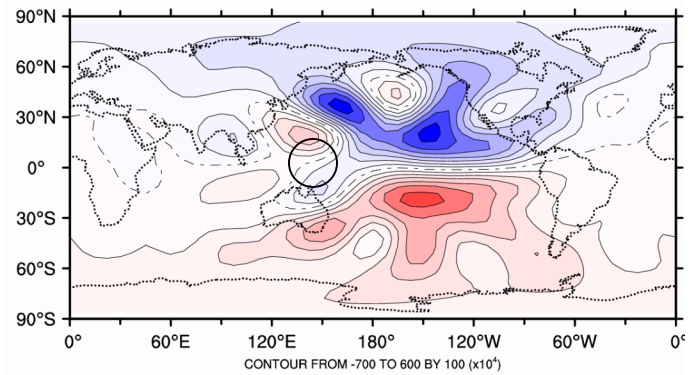
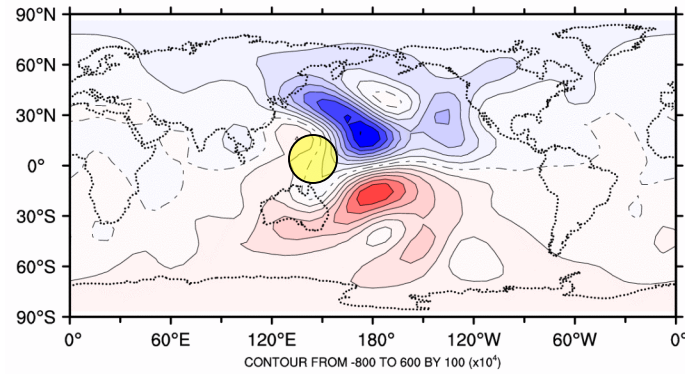
# 3day pulse forcing

## CCM0

(5000 member ensemble)



## FD



day3+0

day3+5

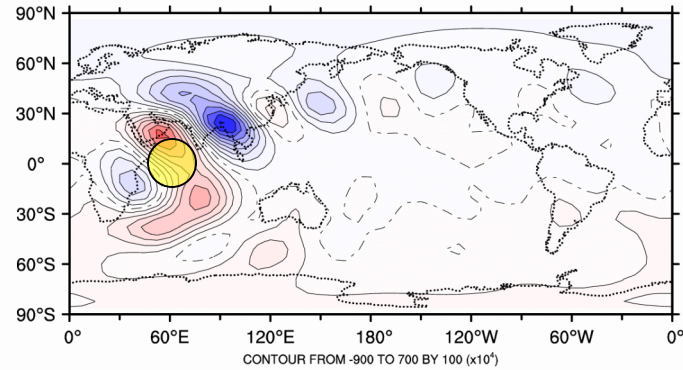
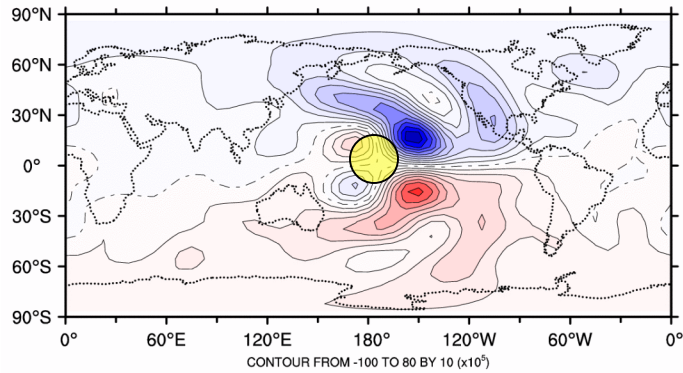
day3+10

# 3day pulse forcing

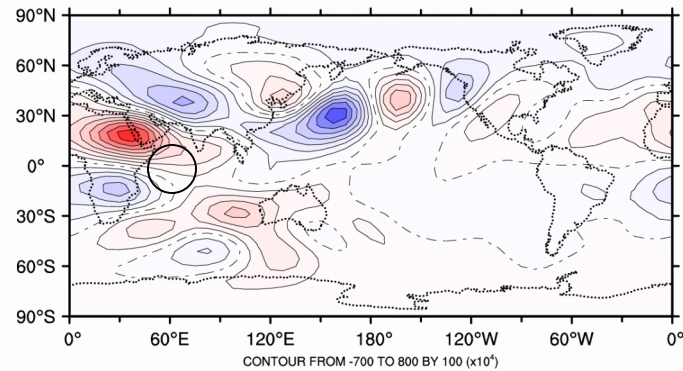
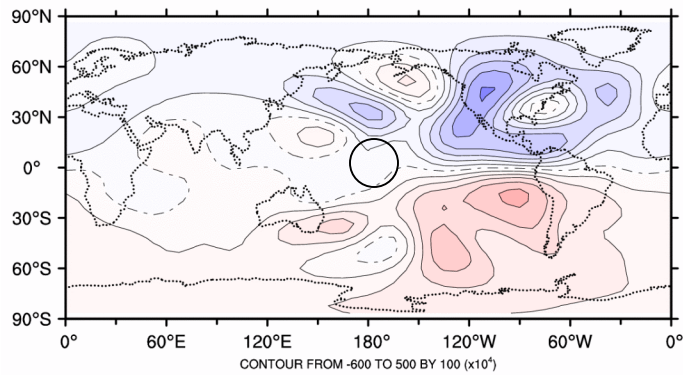
Exploration

FD

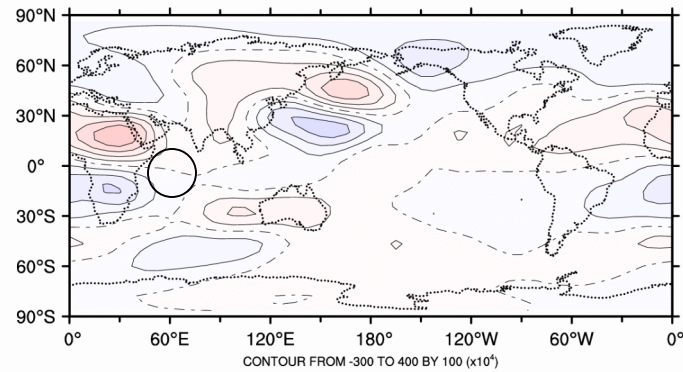
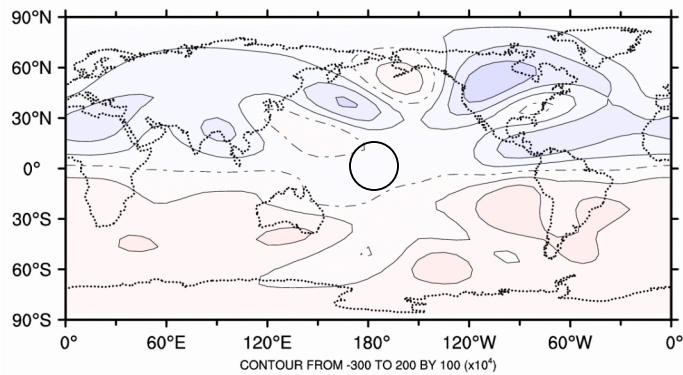
FD



day3+0



day3+5

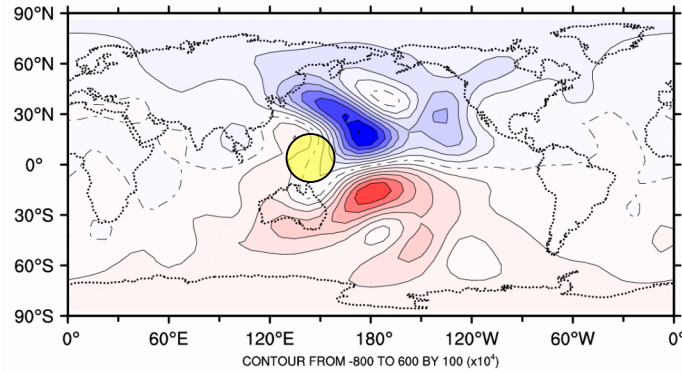


day3+10

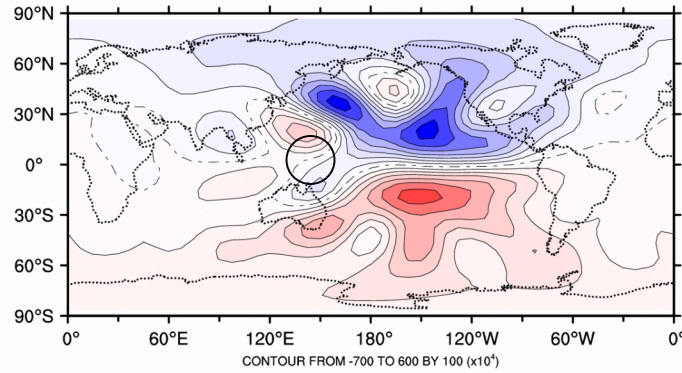
FD

### 3day pulse forcing

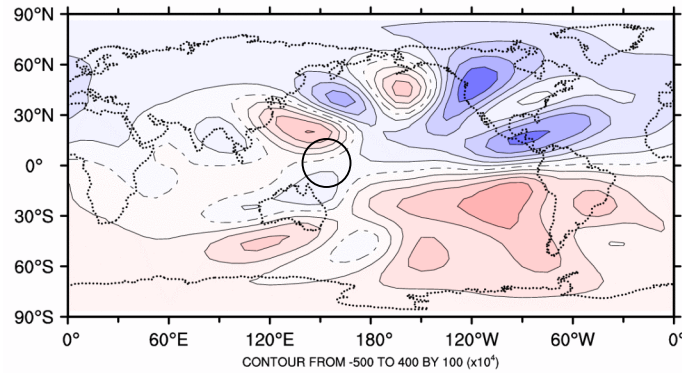
day3+0



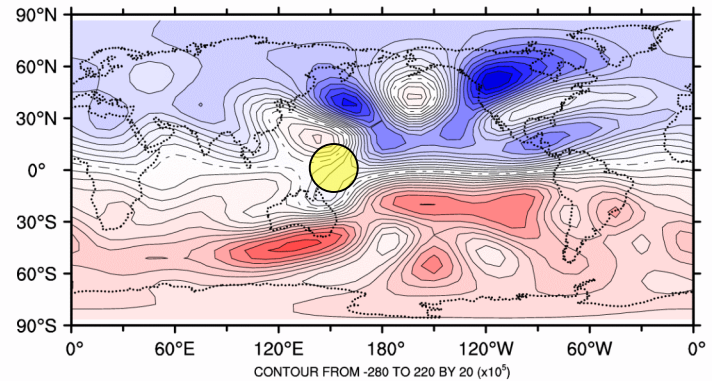
day3+5



day3+10

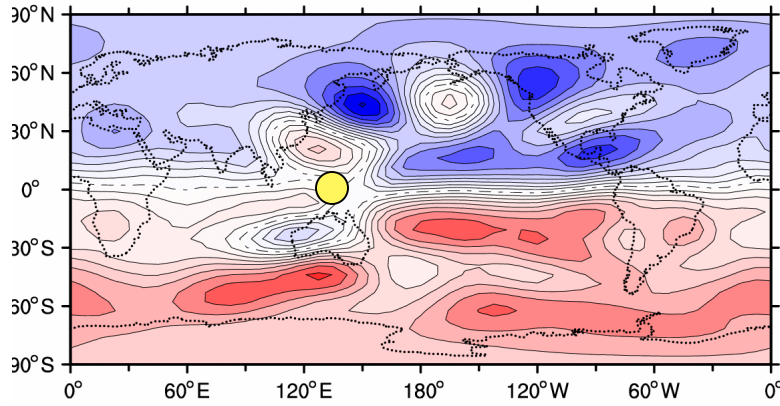


constant forcing  
day 30



# CCM0

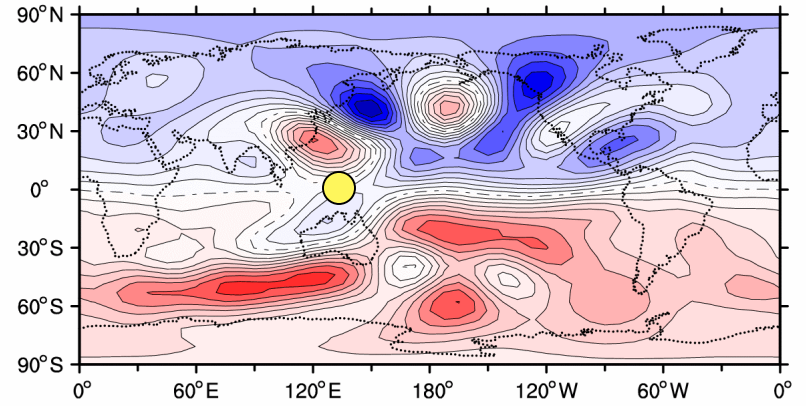
CCM0 strong forcing  
psi336  
( 135.00, 0.00) 2.5C/day



CONTOUR FROM -500 TO 400 BY 50 ( $\times 10^4$ )  
FD vs CCM0 multifield cor= 0.81

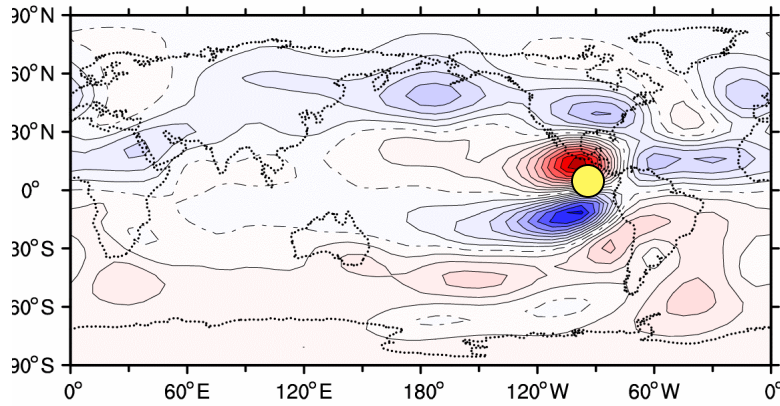
# FDT

FD  
psi336  
( 135.00, 0.00) 2.5C/day



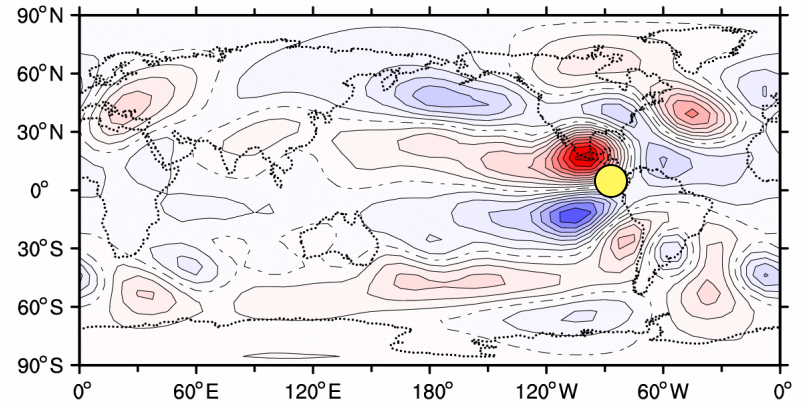
CONTOUR FROM -500 TO 400 BY 50 ( $\times 10^4$ )  
FD vs CCM0 multifield cor= 0.81

CCM0 strong forcing  
psi336  
( -90.00, 0.00) 2.5C/day



CONTOUR FROM -450 TO 500 BY 50 ( $\times 10^4$ )  
FD vs CCM0 multifield cor= 0.77

FD  
psi336  
( -90.00, 0.00) 2.5C/day

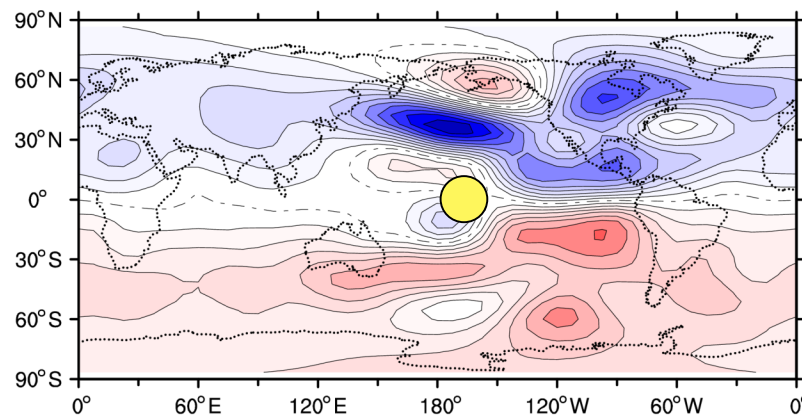
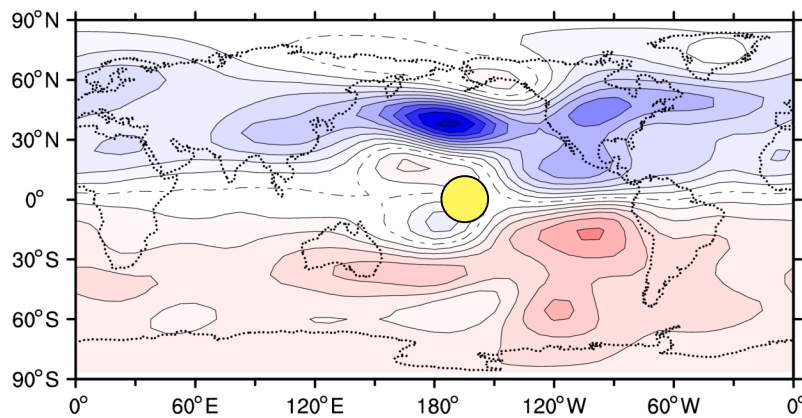


CONTOUR FROM -350 TO 500 BY 50 ( $\times 10^4$ )  
FD vs CCM0 multifield cor= 0.77

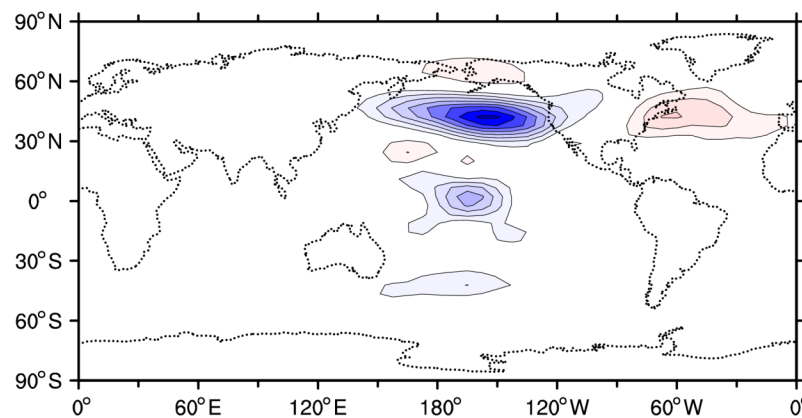
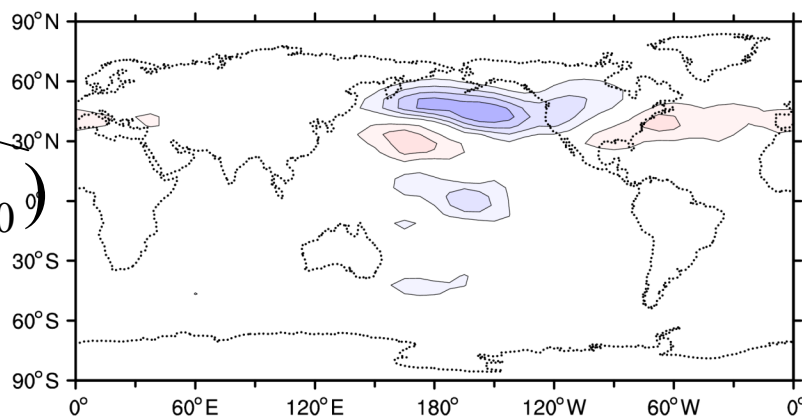
# CCM0 Response

# FD Response

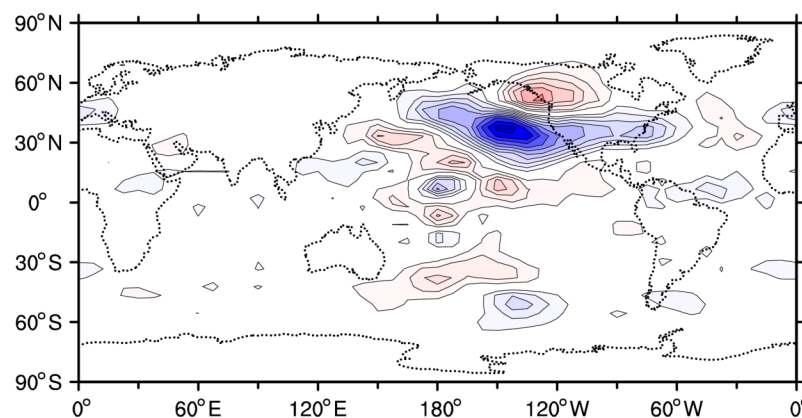
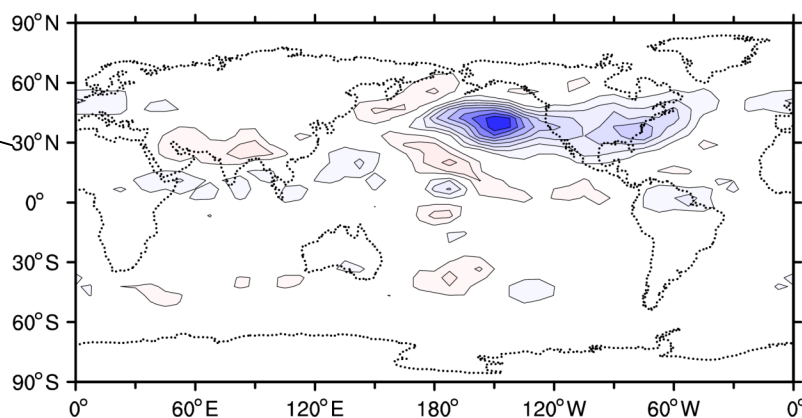
$\hat{\psi}_{300}$



$\text{var}(\psi_{300}^{bp})$

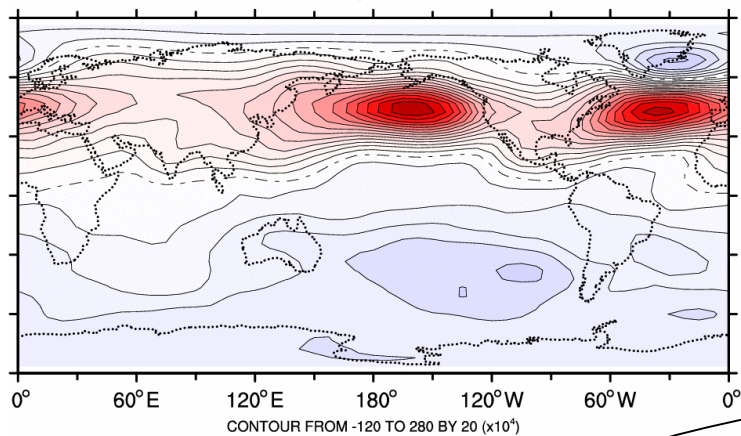


$u_{300}^{bp} v_{300}^{bp}$

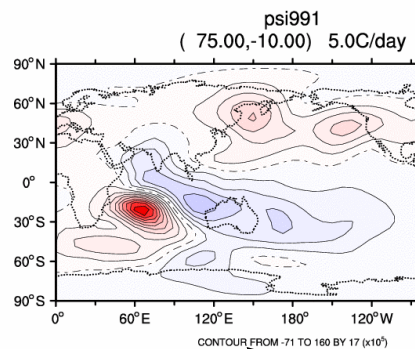


# Control

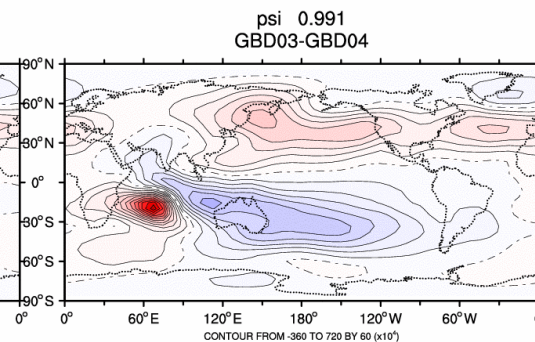
global psi991 EOF1  
regress psi level 1



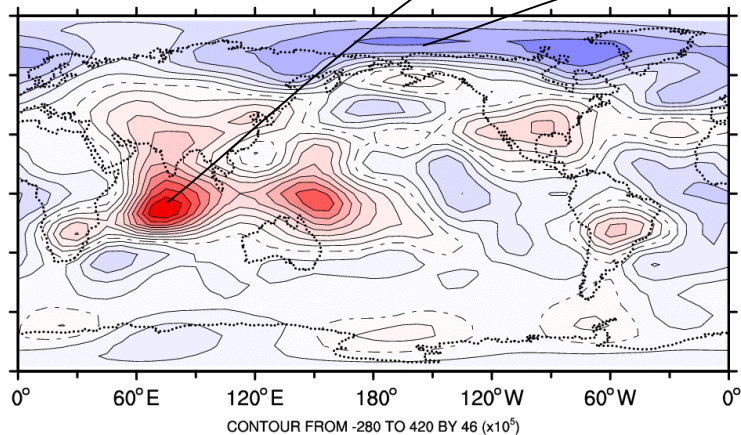
FD



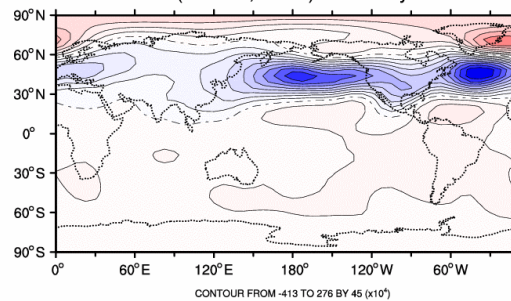
CCM0



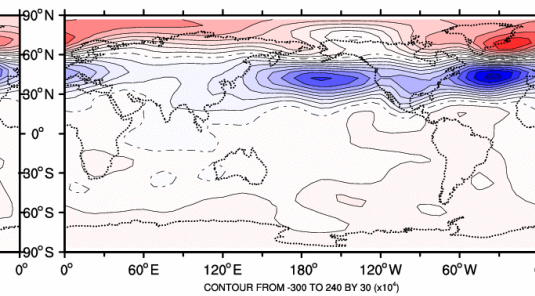
pattern response  
T500 forcing



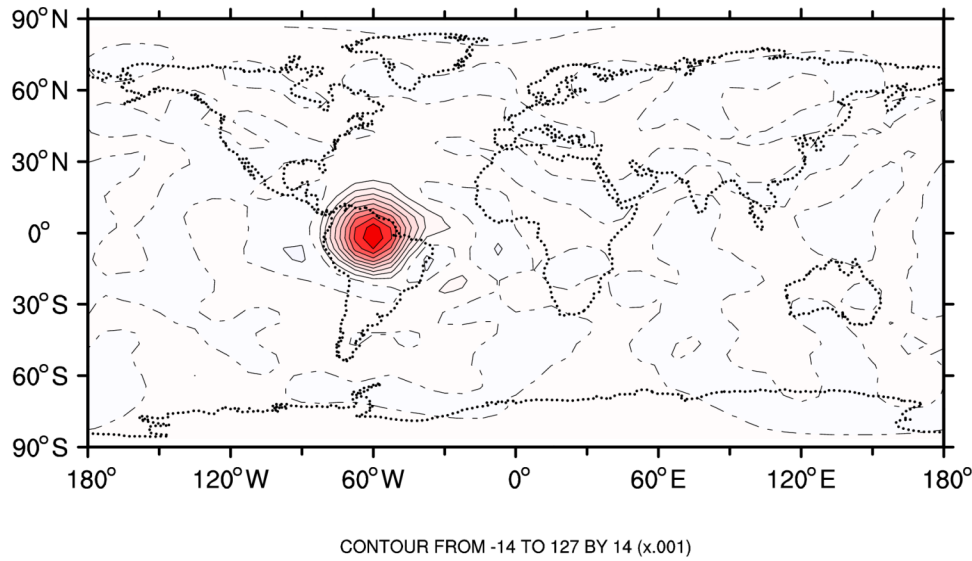
psi991  
(-150.00, 75.00) 5.0C/day



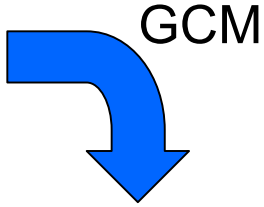
psi 0.991  
GBD14-GBD50



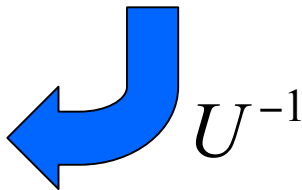
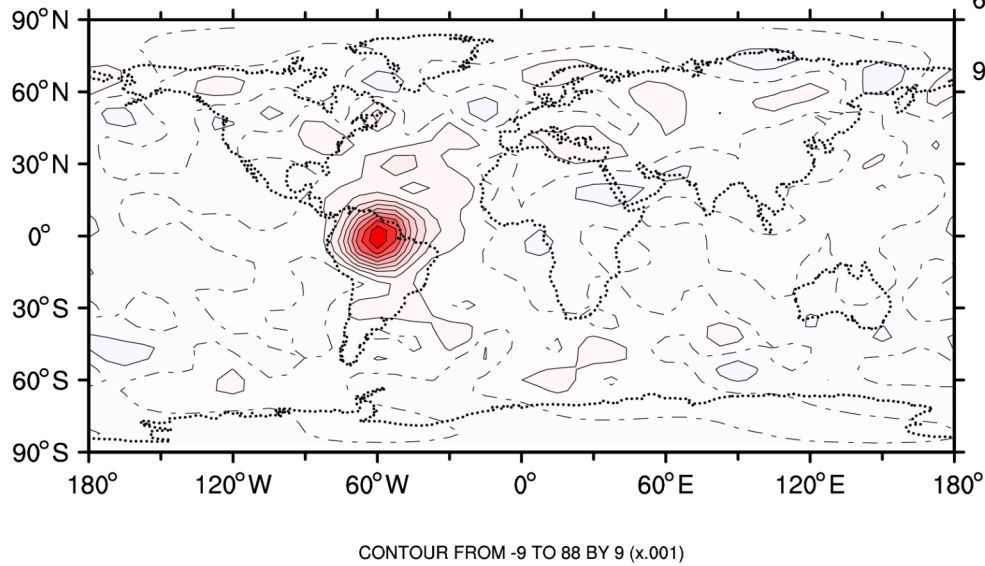
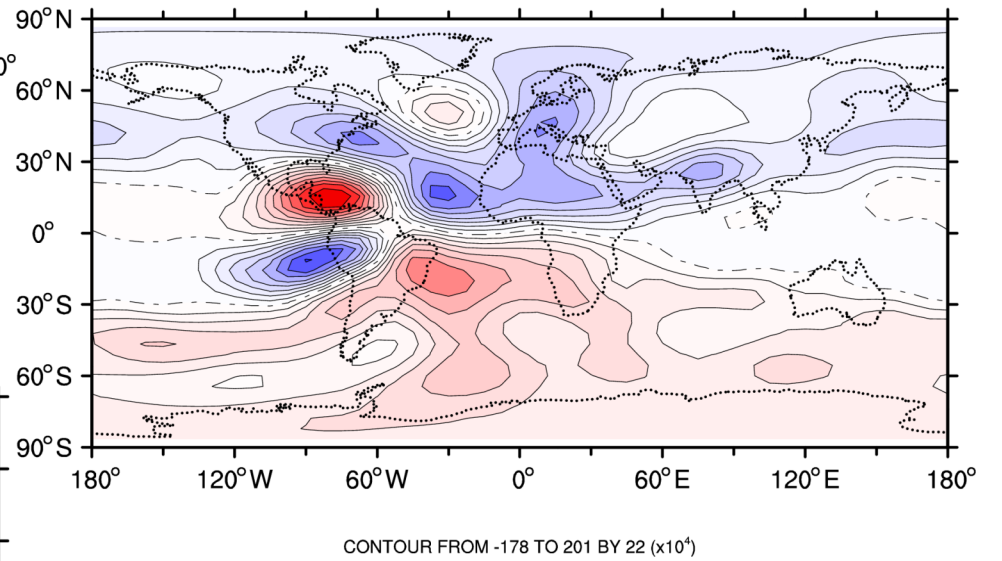
### 500mb heating



Attribution

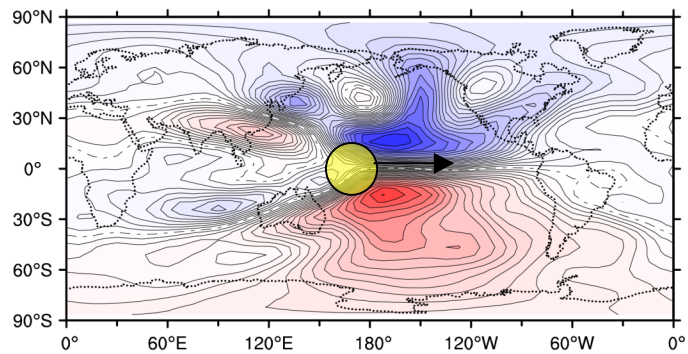
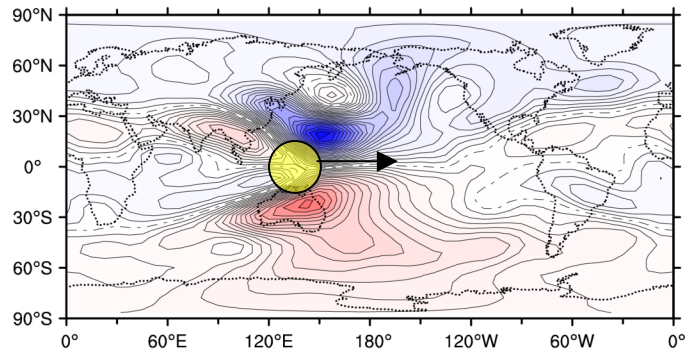
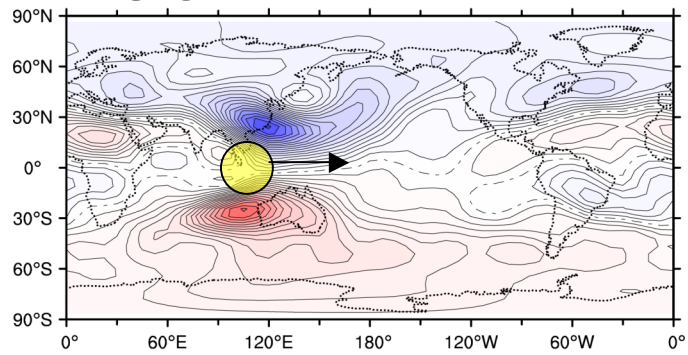


### 336mb streamfunction

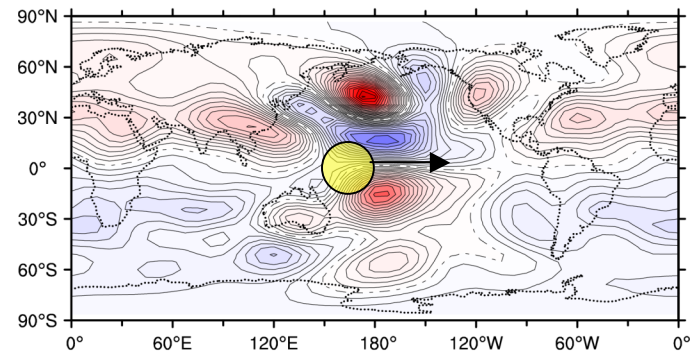
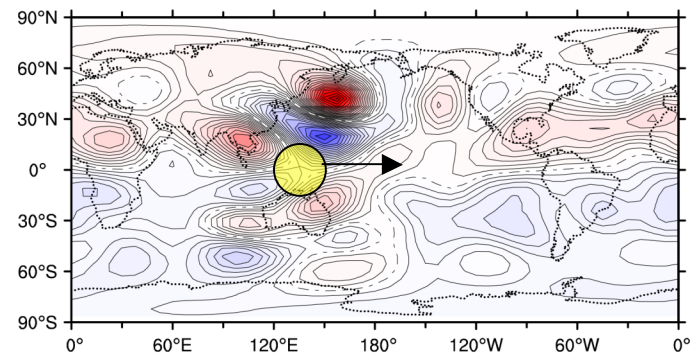
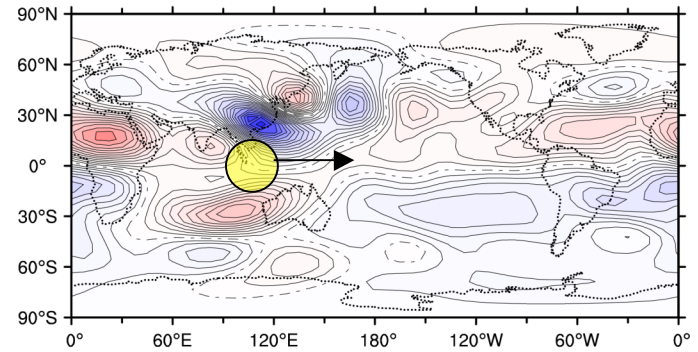


# $\psi_{300}$ response to 8deg/d heat source

## GCM (15000 member ensemble)

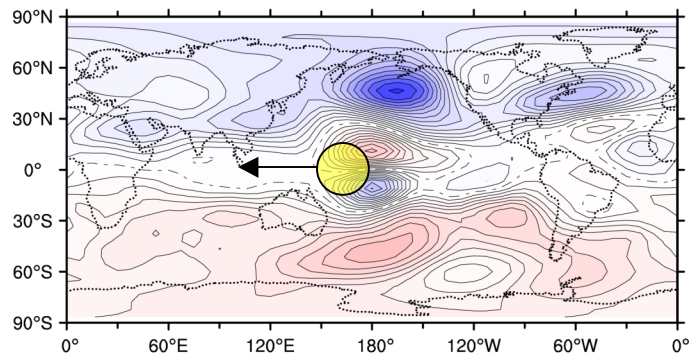
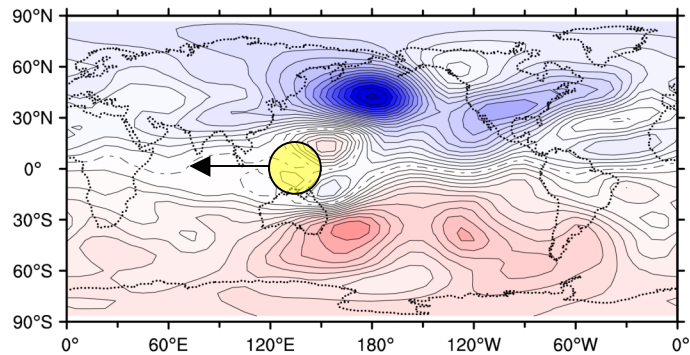
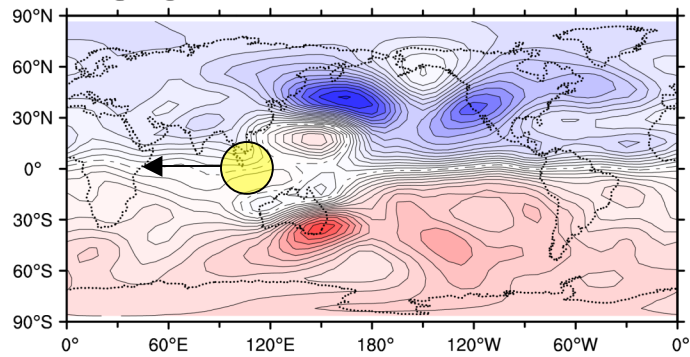


## FDT

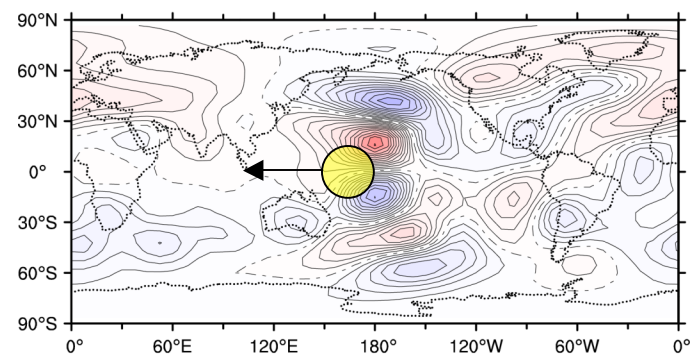
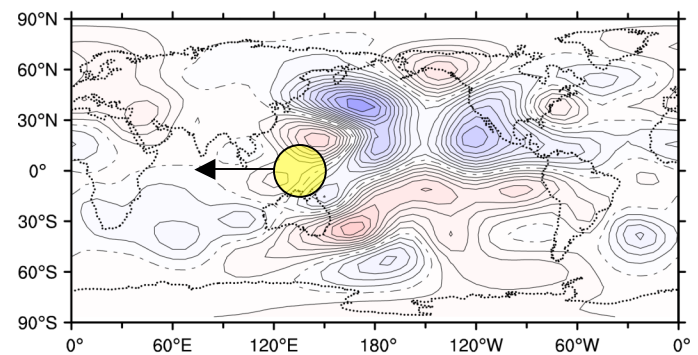
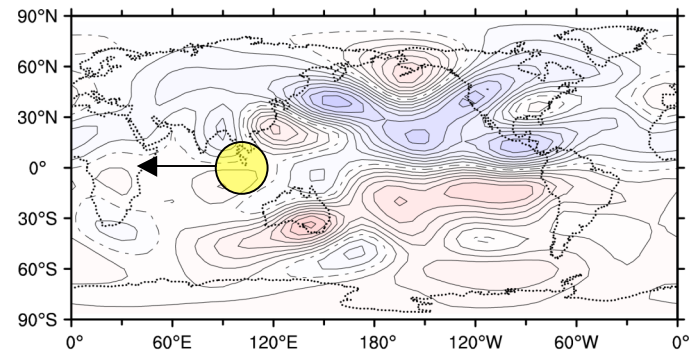


# $\psi_{300}$ response to -8deg/d heat source

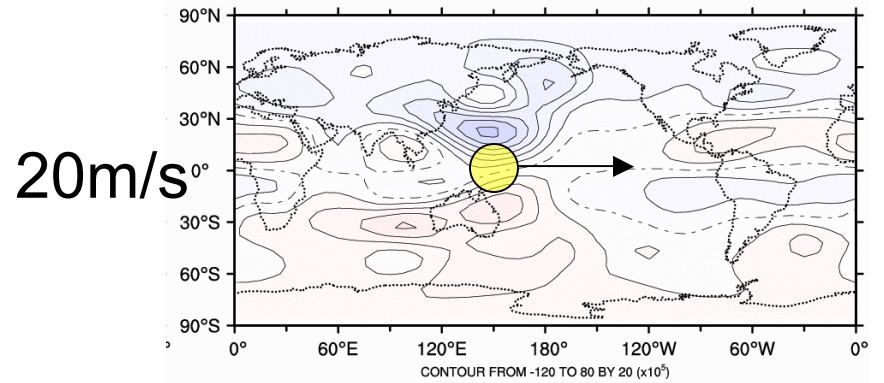
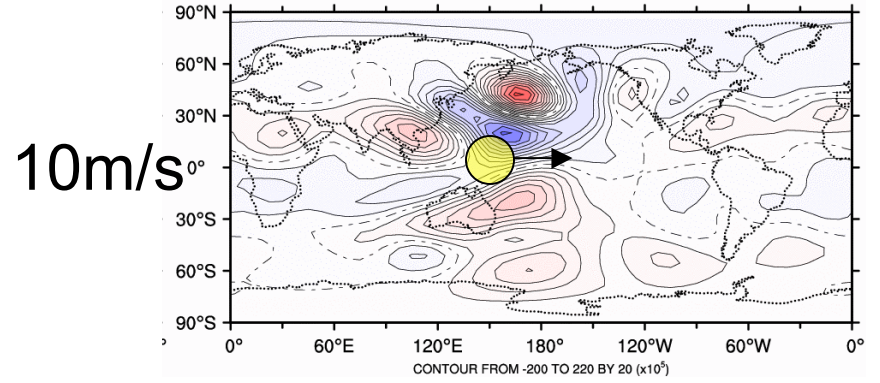
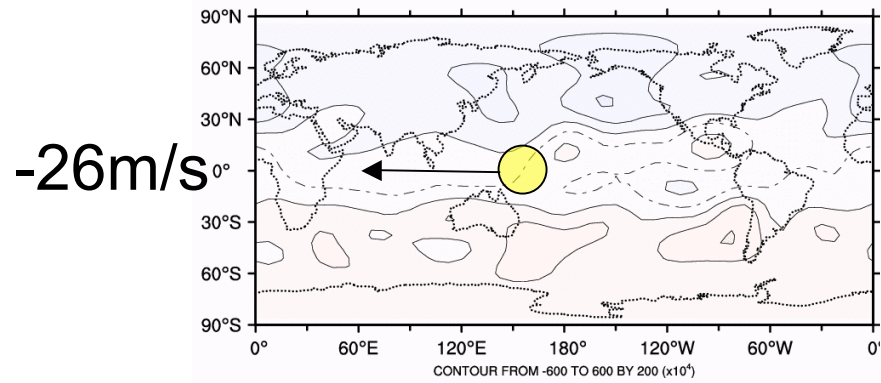
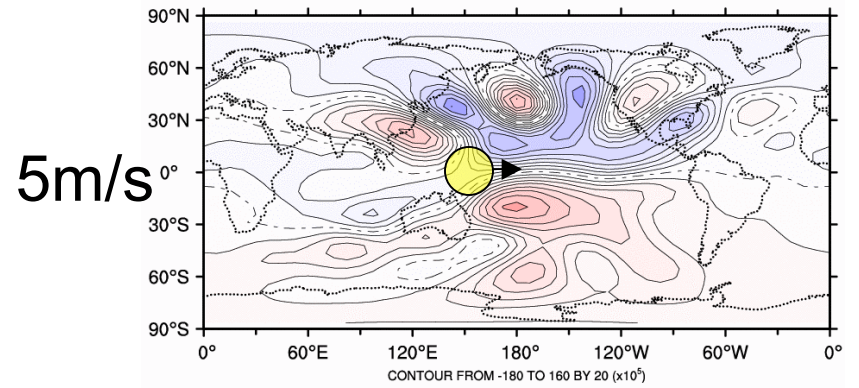
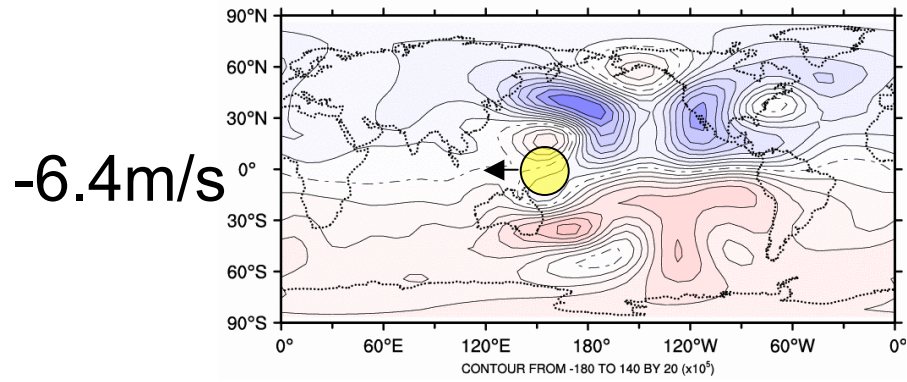
GCM (15000 member ensemble)



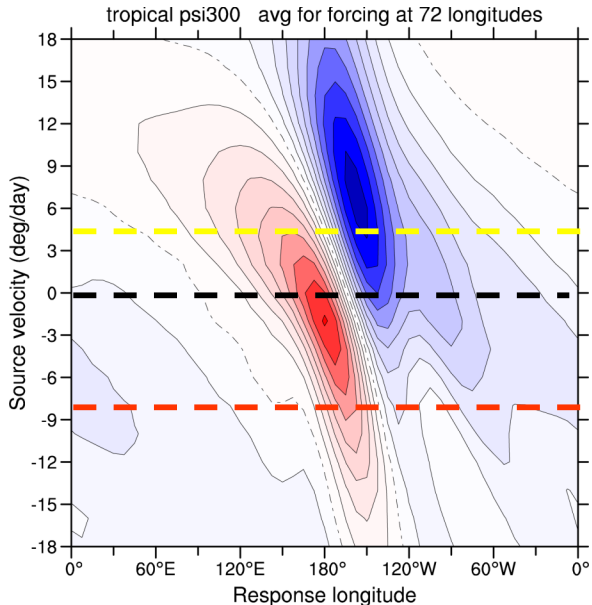
FDT



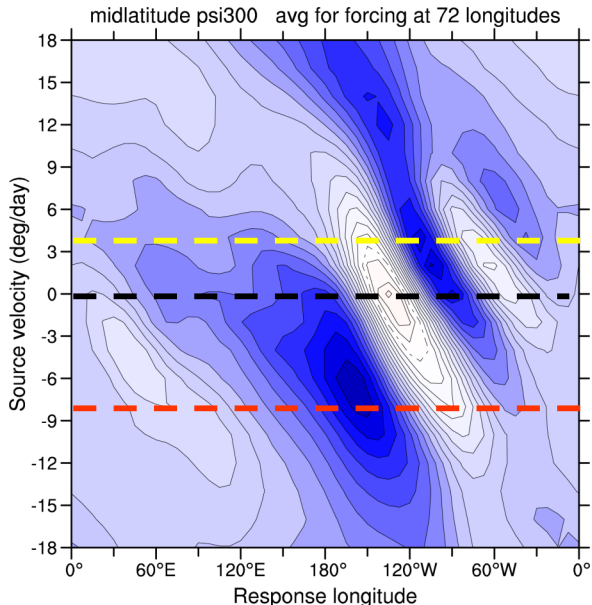
# FD moving source



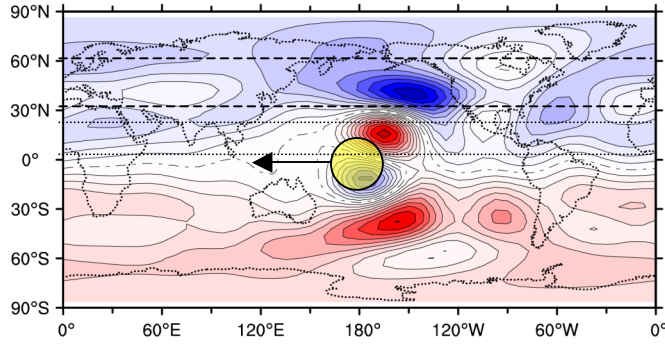
# Response to moving eq heat source



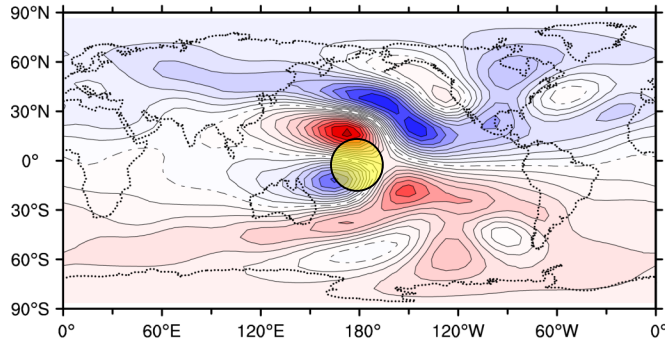
tropical  $\psi_{300}$



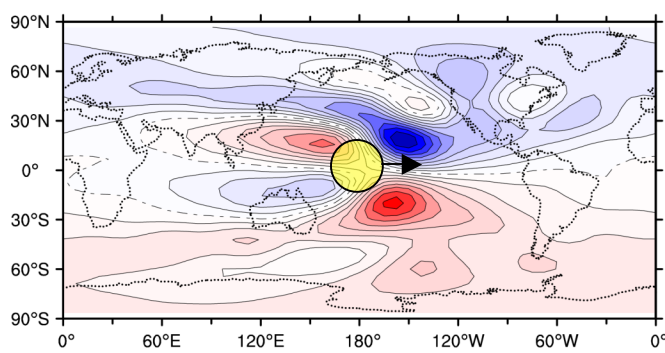
midlatitude  $\psi_{300}$



-8deg/d

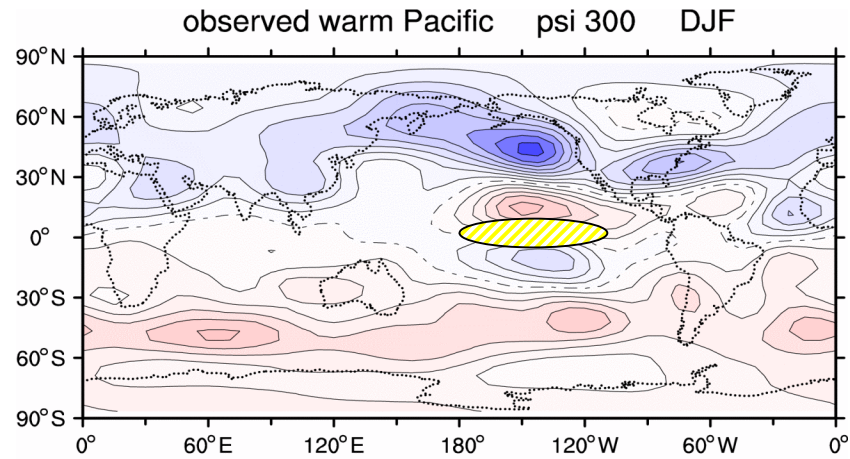


0deg/d

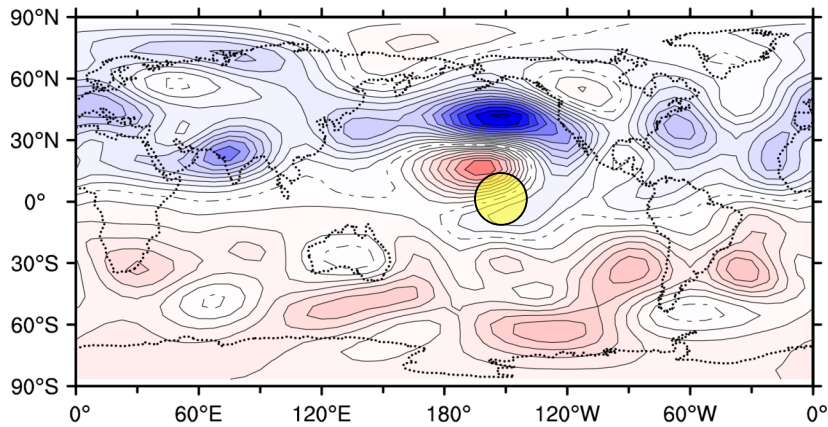


4deg/d

# Response to El Nino in Nature



## FDT response using operator from Nature



*FDT from unfiltered data*