Rain formation in warm clouds: from micro-scale to cloud-scale

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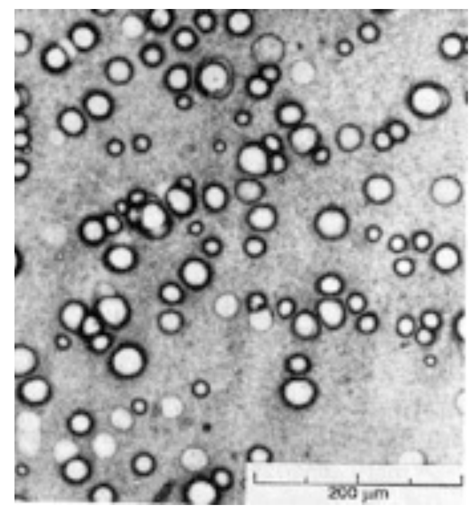
Rain formation in warm clouds:
A highly nonlinear multi-scale problem

cloud-scale

(Picture by Bjorn Stevens)

micro-scale



How do droplets grow from a few micrometers to several millimeters in size?

Why do some clouds rain, but some don‘t?

Overview

- Fundamentals: DSD and SCE, SCBE, etc.
- Two-moment parameterization
- Turbulence effects on rain formation
- A simple kinematic 1D model
- Dynamics and microphysics on the cloud scale
- Similarity theory for orographic warm-rain
- Summary and Outlook

Optional: Stochastic PBL scheme for convective-scale NWP

The size distribution function for droplets

f(x) gives the number density of drops
in the mass interval *[x,x+dx]*

g(x) = x f(x) is the mass distribution function

Stochastic collection equation:

$$\frac{\partial f(x,t)}{\partial t} = \frac{1}{2}\int_0^x f(x-x',t)\,f(x',t)\,K(x-x',x')\,dx' - \int_0^\infty f(x,t)\,f(x',t)\,K(x,x')\,dx'$$

Collision-coalescence kernel:

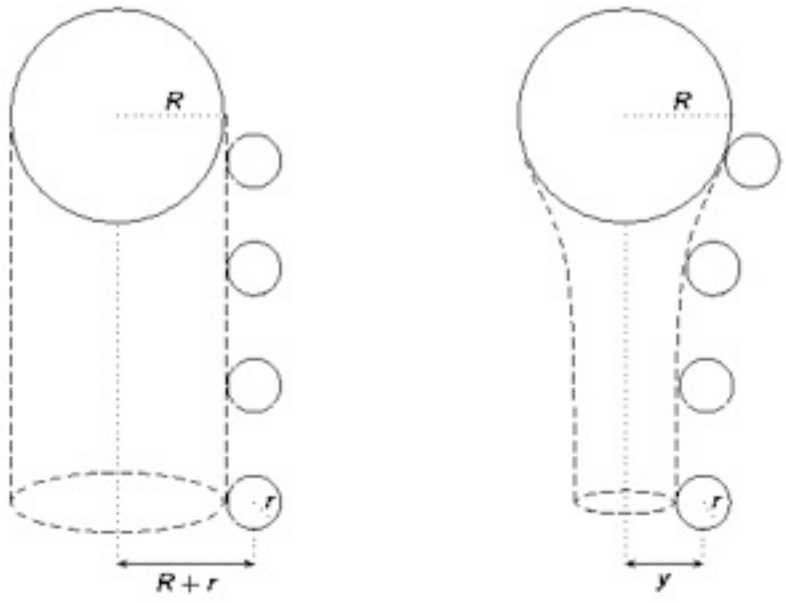
$$K(x,y) = \pi\,[\,r(x)+r(y)\,]^2\,|v_T(x)-v_T(y)|\,E_{coag}(x,y)$$

Gravitational collision-coalescence kernel

Based on collision frequency due to geometry and differential fall speed:

$$K(x,y) = \pi\,[r(x)+r(y)]^2\,|v(x)-v(y)|\,E_{coll}(x,y)\,E_{coal}(x,y)$$

Correction for flow field: collision efficiency



$$E_{coll} = \frac{y^2}{(R+r)^2}$$

Complete budget equation for the DSD

$$\frac{\partial f(x,\vec{r},t)}{\partial t} + \nabla \cdot [\vec{v}(\vec{r},t) f(x,\vec{r},t)] + \frac{\partial}{\partial z}[v_s(x) f(x,\vec{r},t)] + \frac{\partial}{\partial x}[\dot{x} f(x,\vec{r},t)] = \sigma_{coal} + \sigma_{break}$$

with

$$\sigma_{coal} = \frac{1}{2}\int_0^x f(x-x',\vec{r},t)\, f(x',\vec{r},t)\, K(x-x',x')\, dx' - \int_0^\infty f(x,\vec{r},t)\, f(x',\vec{r},t)\, K(x,x')\, dx'$$

and

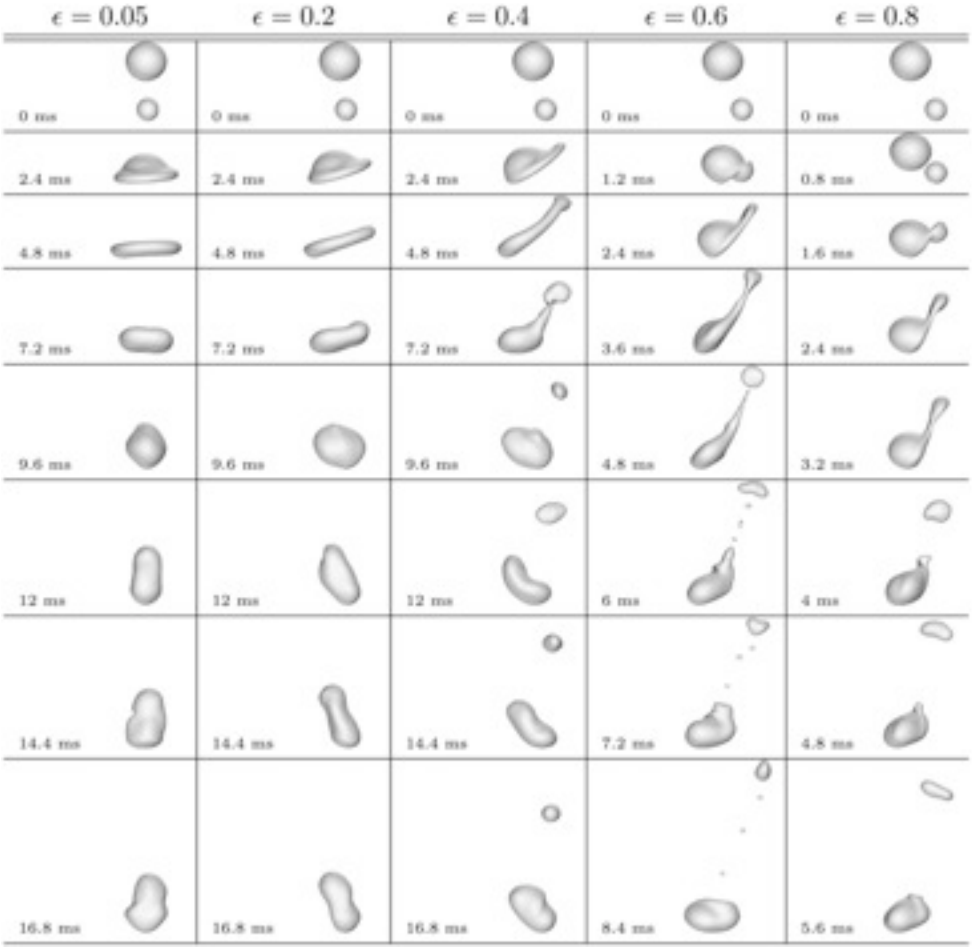
$$\sigma_{break} = \frac{1}{2}\int_0^\infty \int_0^x f(x',\vec{r},t)\, f(x'',\vec{r},t)\, B(x',x'')\, P(x;x',x'')\, dx'\, dx'' - \int_0^\infty f(x,\vec{r},t)\, f(x',\vec{r},t)\, B(x,x')\, dx'.$$

Collisional breakup of drops

- Collisonal breakup is very difficult to measure in the lab. There is only one extensive dataset from the early 80s (Low and List, 1982).
- No complete theory for large Weber numbers (high energy collisions).
- Direct numerical simulations is now possible, and a first dataset has recently been published (Schlottke et al. 2010, Straub et al. 2010).
- From lab data or DNS we can then derive the breakup efficiency B and the fragment distribution P.

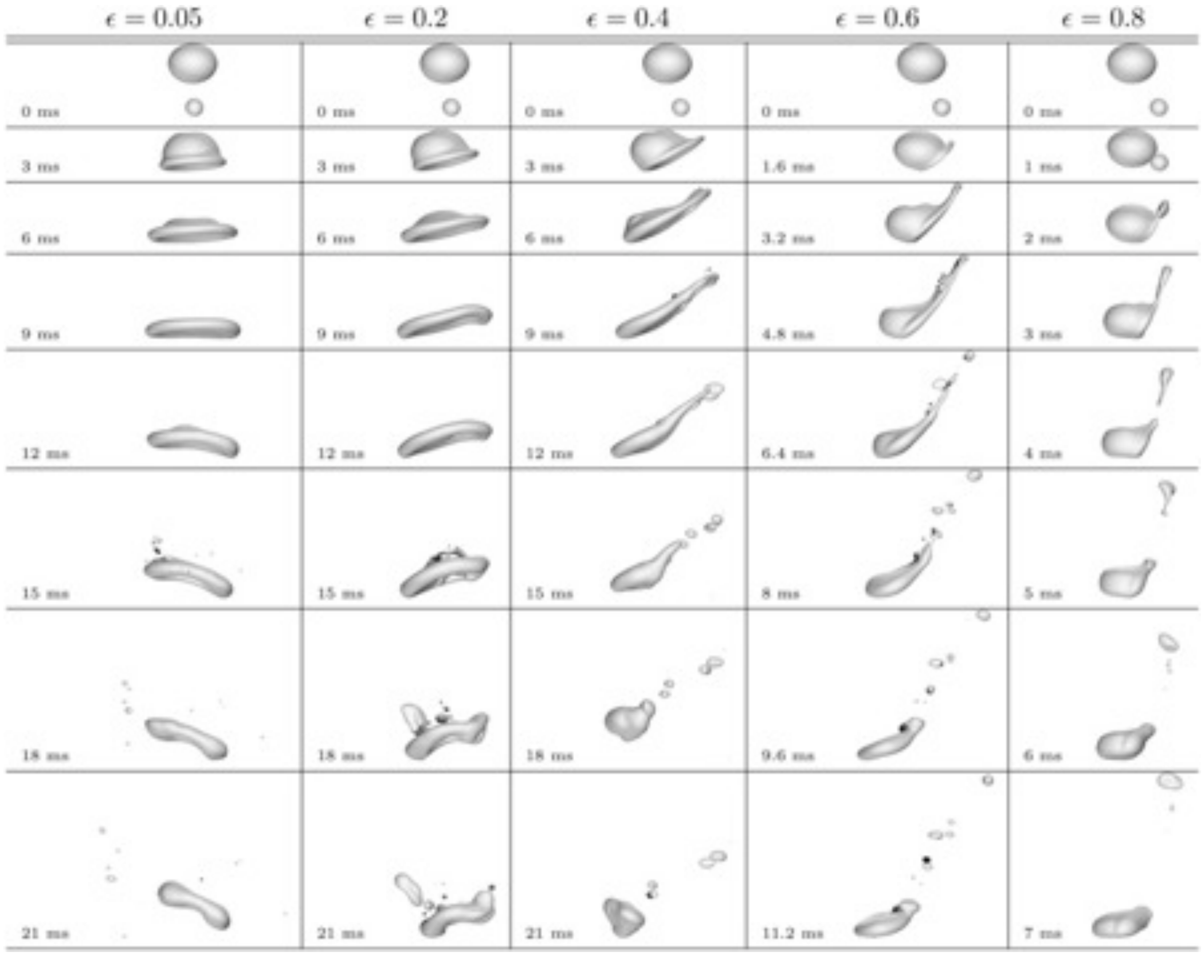
DNS results from Schlottke et al. (2010)

- DNS using ‚volume of fluid‘ methods
- 2.7 mm and 1.6 mm drops
- Low collision energy case CKE = 3.93 μJ
- Depending on eccentricity ϵ we find either coalescence or breakup



DNS results from Schlottke et al. (2010)

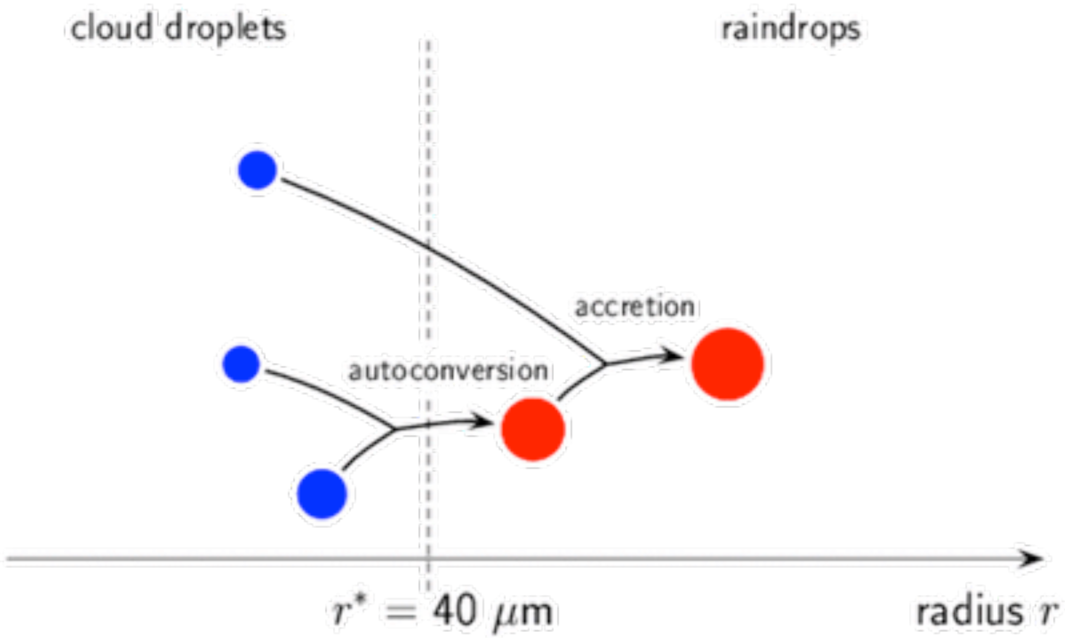
- DNS using ‚volume of fluid‘ methods
- 4.6 mm and 1.8 mm drops
- Higher collision energy case CKE = 12.5 μJ
- Breakup for all eccentricities ϵ



Kessler´s (1969) parameterization

Size domain decomposition:

- cloud droplets $r < 40 \mu m$
- raindrops $r \geq 40 \mu m$



Two variables

- CWC $L_c = \int_0^{x_*} x f(x) dx$
- RWC $L_r = \int_{x_*}^{\infty} x f(x) dx$

autoconversion rate:

$$\left.\frac{\partial L_r}{\partial t}\right|_{au} = \begin{cases} k\,(L_c - L_0), & \text{if } L_c > L_0 = 0.5\ \mathrm{g\ m^{-3}} \\ 0, & \text{else} \end{cases}$$

By using only one (partial) moment for each size domain no distinction between different cloud spectra is possible (e.g. maritime and continental clouds)

Seifert & Beheng (2001) parameterization

Stochastic collection equation:

$$\frac{\partial f(x,t)}{\partial t} = \frac{1}{2}\int_0^x f(x-x',t)\, f(x',t)\, K(x-x',x')\, dx'$$

$$- \int_0^\infty f(x,t)\, f(x',t)\, K(x,x')\, dx'$$

Multiplication with x^k and integration leads to

$$\frac{\partial M^k(t)}{\partial t} = \frac{1}{2}\int_0^\infty \int_0^\infty f(x')\, f(x'')\, K(x',x'')\, \left[(x'+x'')^k - x'^k - x''^k \right] dx'' dx'$$

Can we derive a parameterization directly from this equation?

Integral collection equation

$$\frac{\partial M^k(t)}{\partial t} = \frac{1}{2}\int_0^\infty \int_0^\infty f(x')\, f(x'')\, K(x', x'')\,[\,(x'+x'')^k - x'^k - x''^k\,]\, dx''\, dx'$$

with the piecewise approximation of the kernel (similar to Long's kernel)

$$K(x,y) = \begin{cases} k_{cc}(x^2+y^2), & x, y < x^* \\ k_{cr}(x+y), & x < x^* \vee y < x^* \\ k_{rr}(x+y), & x, y > x^* \end{cases}$$

leads to

$$\begin{aligned} \frac{\partial M^k}{\partial t} = & \frac{k_{cc}}{2}\int_0^\infty \int_0^\infty f_c(x')\, f_c(x'')\, [(x'+x'')^k - x'^k - x''^k]\, (x'^2 + x''^2)\, dx''\, dx' \\ & + \frac{k_{cr}}{2}\int_0^\infty \int_0^\infty [f_c(x')\, f_r(x'') + f_r(x')\, f_c(x'')]\, [(x'+x'')^k - x'^k - x''^k]\, (x'+x'')\, dx''\, dx' \\ & + \frac{k_{rr}}{2}\int_0^\infty \int_0^\infty f_r(x')\, f_r(x'')\, [\,(x'+x'')^k - x'^k - x''^k\,]\, (x'+x'')\, dx''\, dx' \end{aligned}$$

with the partial size distribution functions

$$f_c(x) = \begin{cases} f(x), & x < x^* \\ 0, & x \geq x^* \end{cases}, \qquad f_r(x) = \begin{cases} 0, & x < x^* \\ f(x), & x \geq x^* \end{cases}$$

ODE System after formal integration

$$\frac{\partial N}{\partial t} = -k_{cc} N_c Z_c - k_{cr}(N_c L_r + N_r L_c) - k_{rr} N_r L_r$$

$$\frac{\partial L}{\partial t} = 0$$

$$\frac{\partial Z}{\partial t} = 2k_{cc} L_c M_c^3 + k_{cr}(L_c Z_r + L_r Z_c) + k_{rr} L_r Z_r$$

and for the higher moments:

$$\frac{\partial M^k}{\partial t} = k_{cc} \sum_{n=1}^{k-1} \binom{k}{n} M_c{}^n M_c{}^{k-n+2} + k_{rr} \sum_{n=1}^{k-1} \binom{k}{n} M_r{}^n M_r{}^{k-n+1}$$
$$+ k_{cr} \sum_{n=1}^{k-1} \binom{k}{n} \left(M_c{}^n M_r{}^{k-n+1} + M_r{}^n M_c{}^{k-n+1} \right)$$

Two severe closure problems:

- ➪ nonlinear coupling of the power moments
- ➪ partial moments are unknown

Closure by master function approach

The (partial) size distributions are describable by specific mathematical functions e.g. generalized gamma distributions!

cloud droplets: gamma distribution

$$f_c(x) = A x^{\nu} e^{-B x} \qquad \text{with} \quad \nu = \text{const.}$$

raindrops: exponential distribution

$$f_r(x) = n_0 \, e^{-\lambda D(x)}$$

Higher moments can be formulated as functions of the two moments N and L:

$$Z_c = \frac{\nu + 2}{\nu + 1} \frac{L_c^{\,2}}{N_c}, \quad M_c^3 = \frac{(\nu+2)(\nu+3)}{(\nu+1)^2} \frac{L_c^{\,3}}{N_c^{\,2}}, \quad Z_r = 20 \, \frac{L_r^{\,2}}{N_r}$$

Approximations for all conversion rates can be derived!

(But the necessary condition $f(x^*) = 0$ is never fulfilled)

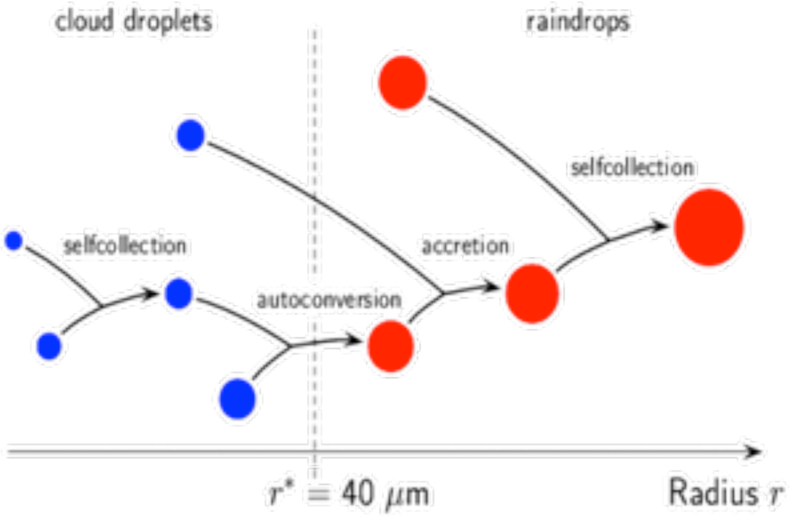
Explizit parameterizations for autoconversion and accretion

autoconversion:

$$\left.\frac{\partial L_r}{\partial t}\right|_{au} = \frac{k_{cc}}{20\,x^*}\frac{(\nu+2)(\nu+4)}{(\nu+1)^2}\,{L_c}^2\,{\bar{x}_c}^2$$

accretion:

$$\left.\frac{\partial L_r}{\partial t}\right|_{ac} = k_{cr}\,L_c\,L_r$$



But the assumptions are too strong, thus a correction is necessary

Invariance and dynamic similarity

The stochastic collection equation

$$\frac{\partial f(x,t)}{\partial t} = \frac{1}{2}\int_0^x f(x-x',t)\,f(x',t)\,K(x-x',x')\,dx' - \int_0^\infty f(x,t)\,f(x',t)\,K(x,x')\,dx'$$

is invariant under the transformation

$$f \to cf, \qquad t \to \frac{t}{c}, \qquad c \in \mathbb{R}.$$

From this mathematical property of the SCE follows:

For each solution $f(x,t)$ of the SCE we find similarity solutions $\tilde{f}(x,t) = c\,f(x,ct)$.

Integration leads to $\tilde{M}^k(t) = cM^k(ct)$ for the power moments and to $c = \tilde{L}/L$.

This gives us the important general result:

The speed of the coagulation processes is proportional to the liquid water content L.

Correction by a similarity function

We define a dimensionless timescale:

$$\tau = 1 - \frac{L_c(t)}{L}$$

- $\tau = 0$ at the beginning of coagulation growth (pure cloud spectrum),

  $\tau = 1$ at the end of the coagulation process (pure raindrop spectrum).

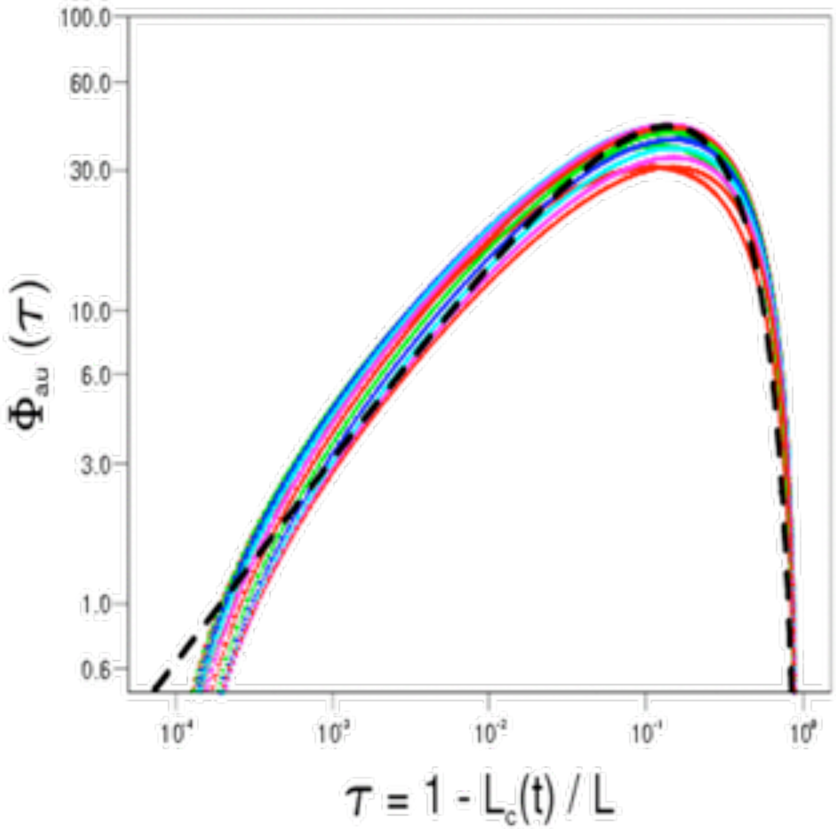
- The time axis $t \in [0, \infty]$ is mapped to $\tau \in [0, 1]$.

Autoconversion:

$$\left.\frac{\partial L_c}{\partial t}\right|_{au} = \frac{k_c}{20\,x^*} \frac{(\nu+2)(\nu+4)}{(\nu+1)^2} L_c{}^2 \bar{x}_c{}^2 \left[1 + \frac{\Phi_{au}(\tau)}{(1-\tau)^2}\right] \approx C_{au}\,\Phi_{au}(\tau)$$

$\Phi_{au}(\tau)$ can be estimated from numerical solutions of the SCE.

Universal autoconversion function



The increase with τ describes the aging of the size distribution which makes the conversion more efficient.

Turbulence effects on warm-rain formation

Three possible ways how turbulence enhances rain formation

- Turbulence modifies the *relative velocity* between the droplets, e.g. by small-scale shears.
- Turbulence modifies the *local droplet concentration* (preferential concentration effect)
- Turbulence modifies the *hydrodynamic interaction* (effect on collision efficiency)

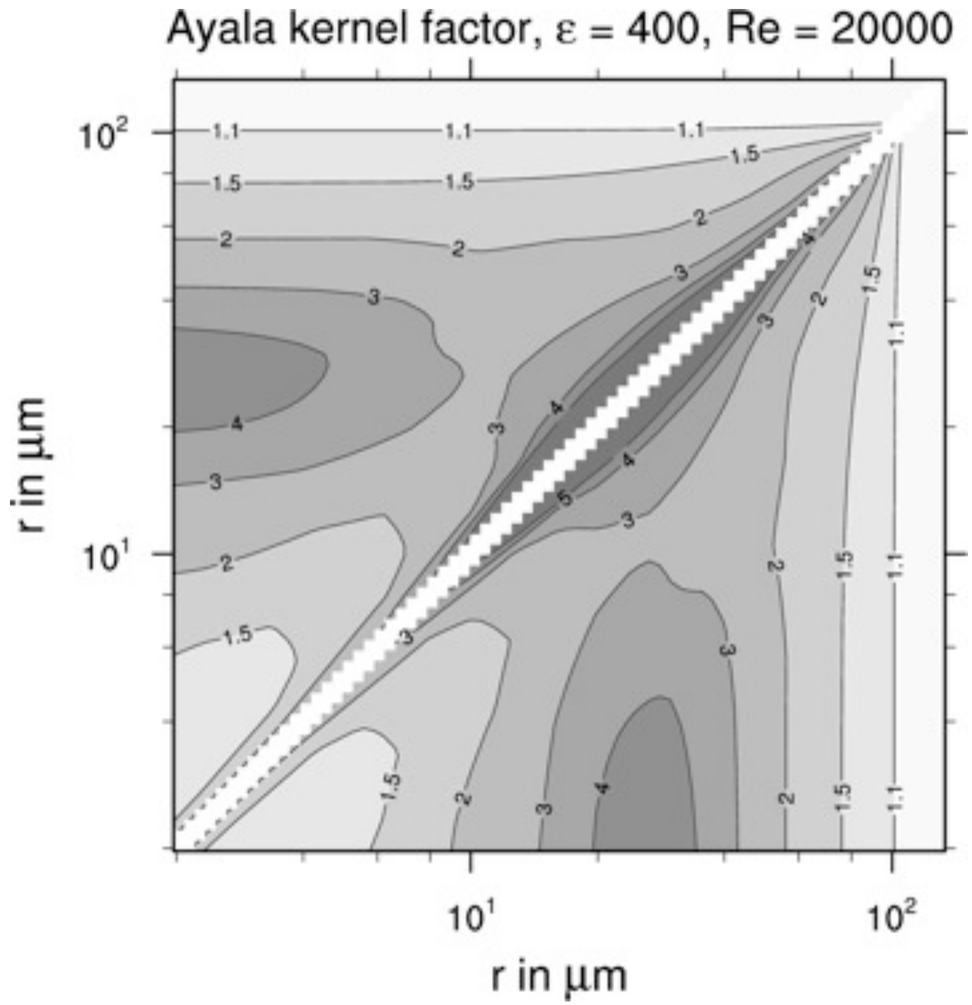
All three mechanisms have been investigated using statistical turbulence theory (e.g. Pinsky et al. 2007, 2008) as well as LES (e.g. Franklin et al. 2005), and DNS (Ayala et al. 2008, Wang et al. 2008).

Turbulent enhancement: Ayala kernel

Based on Hybrid-DNS results, but at relatively low Reynolds number (Ayala et al 2008, Wang et al. 2008).

For drop sizes between 10 and 20 micron the enhancement by turbulence is roughly a factor 4.

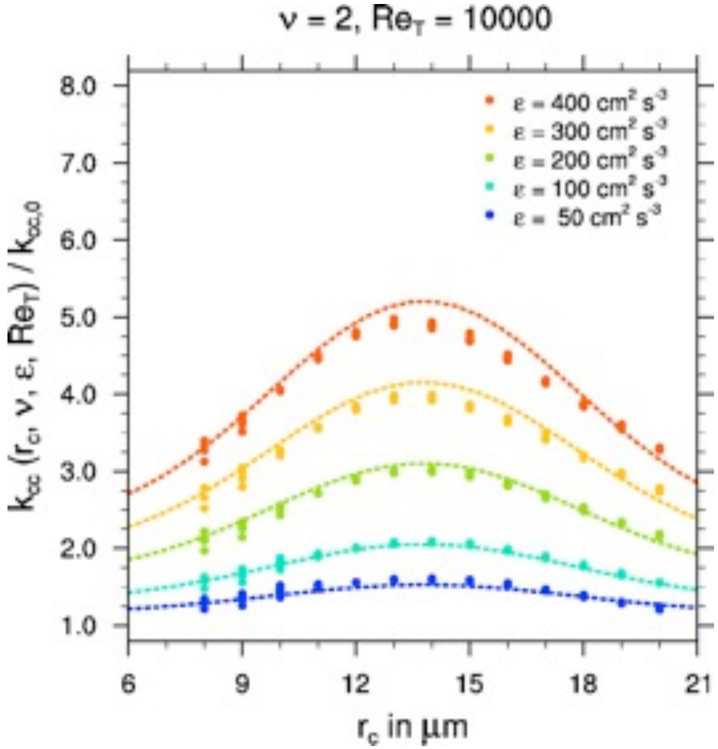
This needs to be included in the autoconversion scheme!



Turbulence effects on autoconversion

We cannot easily repeat the derivation based on a polynomial kernel, because the Ayala kernel is too complicated.

Therefore the enhancement factor is determined empirically by box model simulations and non-linear regression. The effect can be included as a modification of the autoconversion coefficient $\boldsymbol{k_{cc}}$.



$$k_{cc}(\bar{r}_c, \nu, \epsilon, Re_\lambda) = k_{cc,0} \left\{ 1 + \epsilon \, Re_\lambda^{1/4} \left[\alpha_{cc}(\nu) \exp \left\{ - \left[\frac{\bar{r}_c - r_{cc}(\nu)}{\sigma_{cc}(\nu)} \right]^2 \right\} + \beta_{cc} \right] \right\}$$

A simple kinematic 1D cloud model

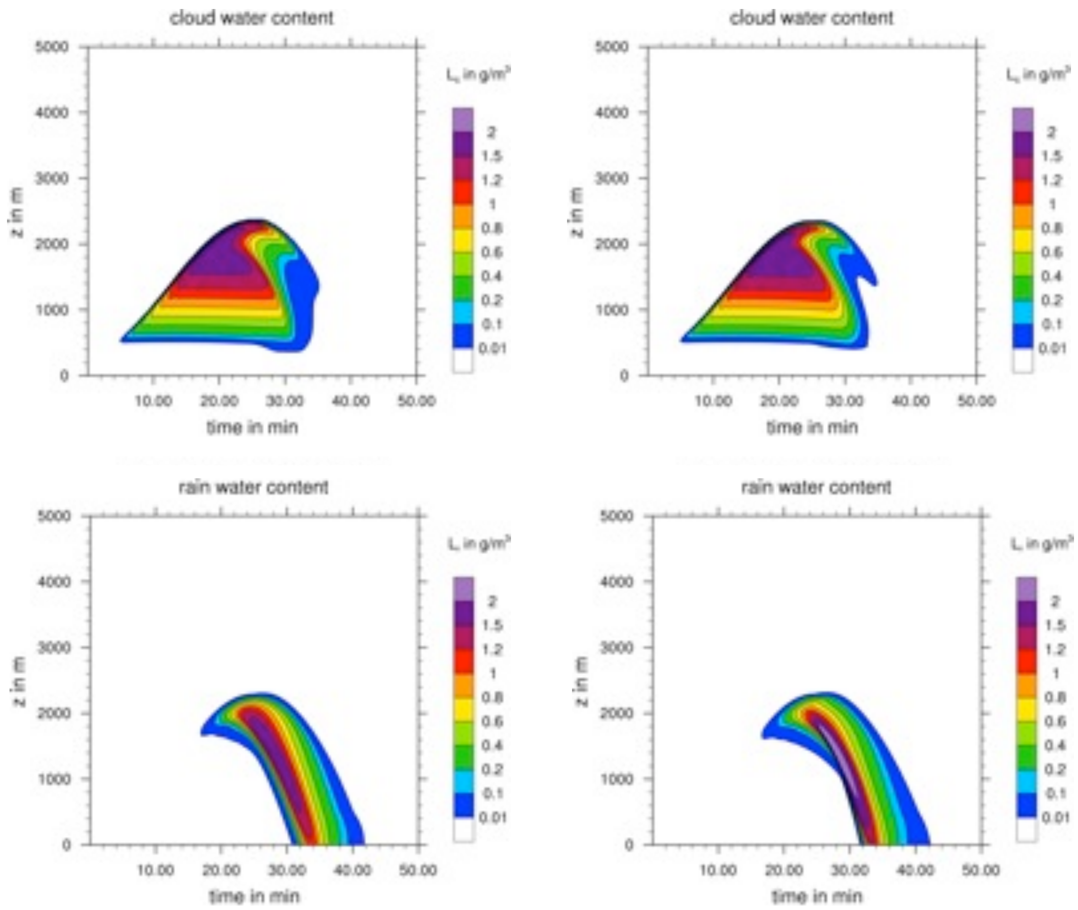
Forced updraft out of a well-mixed boundary layer

Explicit condensational growth and a simplified activation of condensation nuclei.

Two-moment bulk model with Seifert&Beheng parameterization.

Parameterization of evaporation of raindrops following Seifert (2008).

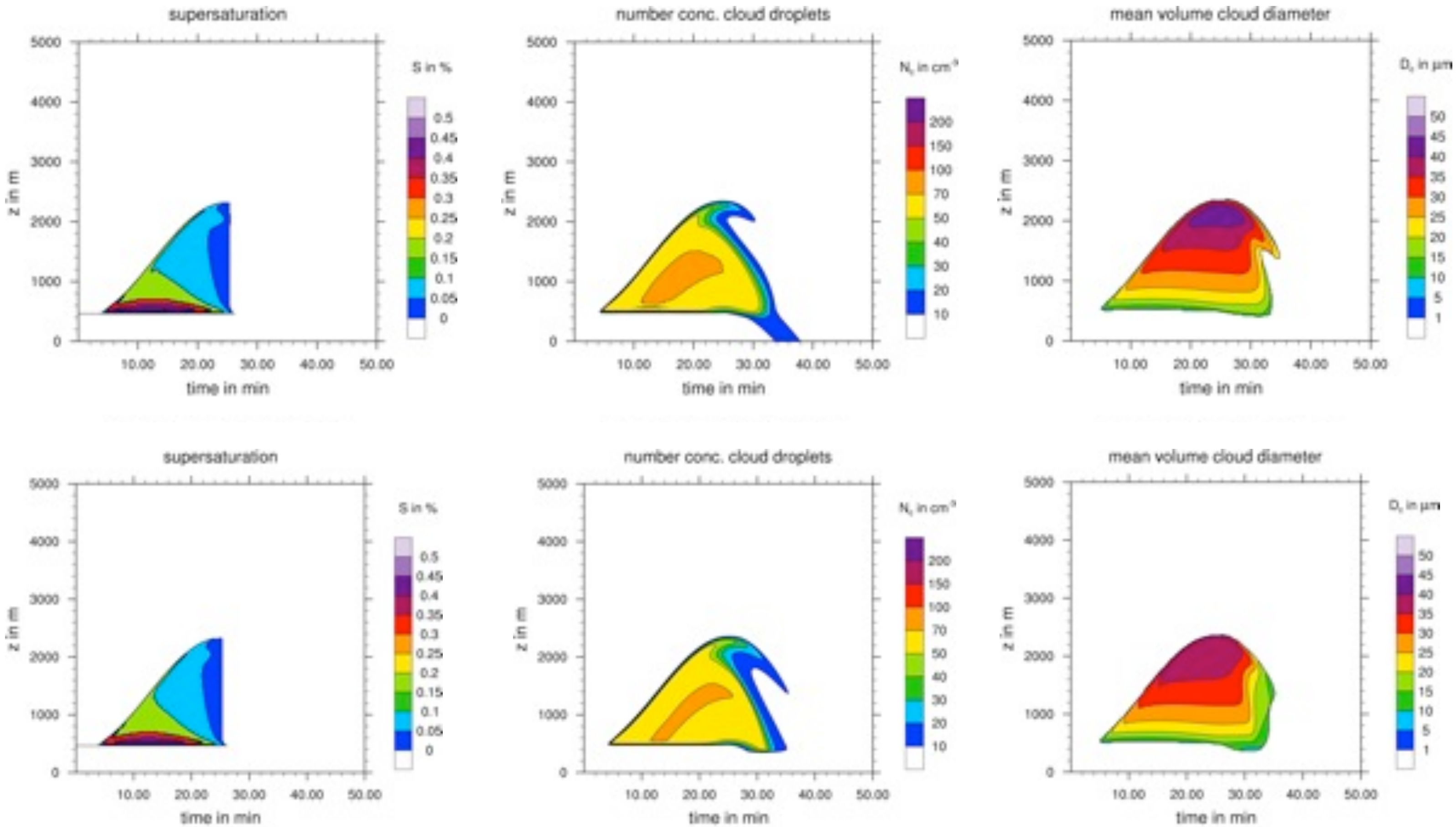
Bin model **Bulk model**



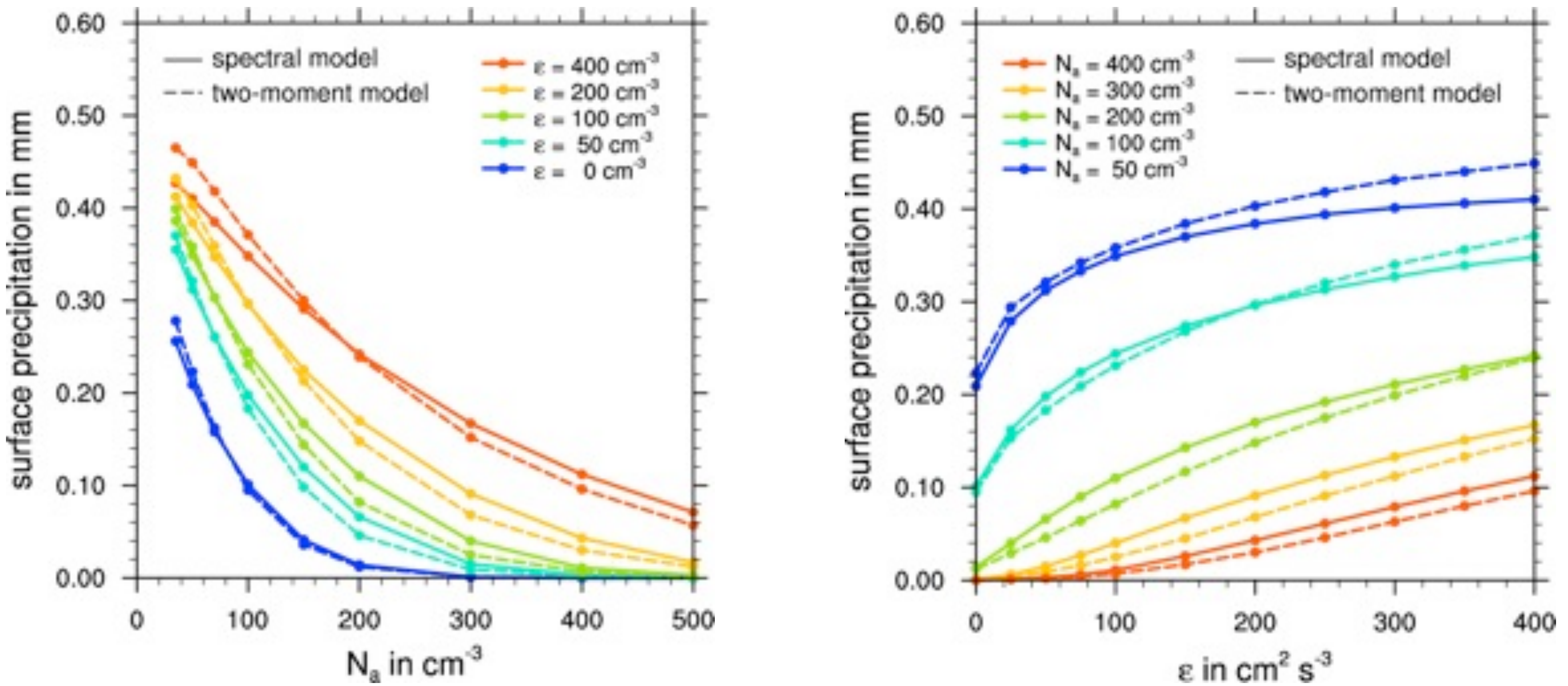
A simple kinematic 1D cloud model: Some more model variables

Bin model

Bulk model



A simple kinematic 1D cloud model:
CCN number and turbulence effects on precipitation



- ➔ **Higher CCN number leads to a reduction of precipitation**
- ➔ **Stronger turbulence increases surface precipitation**
- ➔ **Overall good agreement between bin and bulk model**

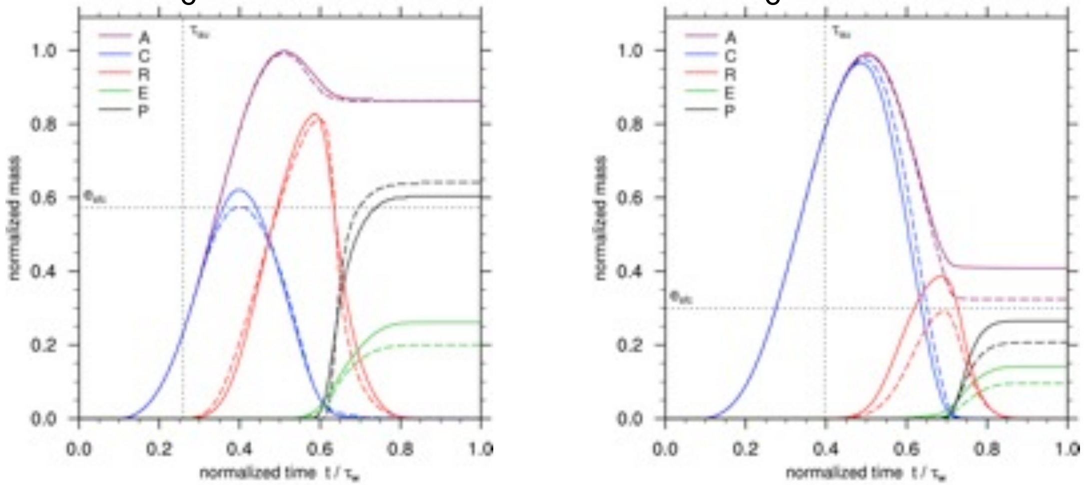
Towards a cloud-scale theory of the precipitation efficiency

The two-moment parameterization works well on the microscale, but can we develop some understanding on the cloud scale?

Mass budget
of a single cloud

A: total condensate
C: cloud water
R: rain water
E: evaporated rain
P: surface precip

$N_c = 35\ cm^{-3}$ $N_c = 300\ cm^{-3}$



➔ Is there a relation between the precipitation efficiency and the time that is needed to form rain?

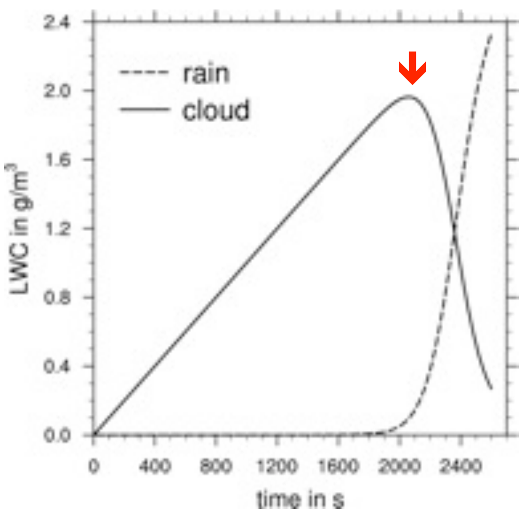
➔ Can we link the timescale of rain formation to the dynamical properties on the cloud scale?

Towards a cloud-scale theory …

Definition of the microphysical timescale

Define a rain formation timescale based on a simple parcel model with constant condensation rate (Stevens and Seifert 2008):

$$\frac{\mathrm{d}q_c}{\mathrm{d}t} = \frac{1}{\tau_{\mathrm{cond}}} - k_{\mathrm{au}}\, q_c^4\, N_c^{-2}\, \phi_{cc}(\epsilon) - k_{cr}\, q_c\, q_r\, \phi_{cr}(\epsilon)$$



$$\tau_* = \left[\beta_* + \sqrt{\beta_*^2 + \frac{\tau_0^4}{(1-\epsilon_*)^4 \phi_{cc}(\epsilon_*)}}\,\right]^{1/2}$$

with

$$\beta_* = \frac{k_{cr}\, N_c^2}{2\, k_{\mathrm{au}}}\, \frac{\epsilon_*\, \phi_{cr}(\epsilon_*)}{(1-\epsilon_*)^3 \phi_{cc}(\epsilon_*)}$$

$$\tau_0 = N_c^{1/2}\, \tau_{\mathrm{cond}}^{3/4}\, k_{\mathrm{au}}^{-1/4}.$$

Time needed for rain formation as a function of condensation time scale τ_{cond} (vertical velocity and lapse rate) as well as droplet number conc. N_c and kernel parameter $k_{au.}$

But the equation is not closed, yet. We need a parameterization for ε_*

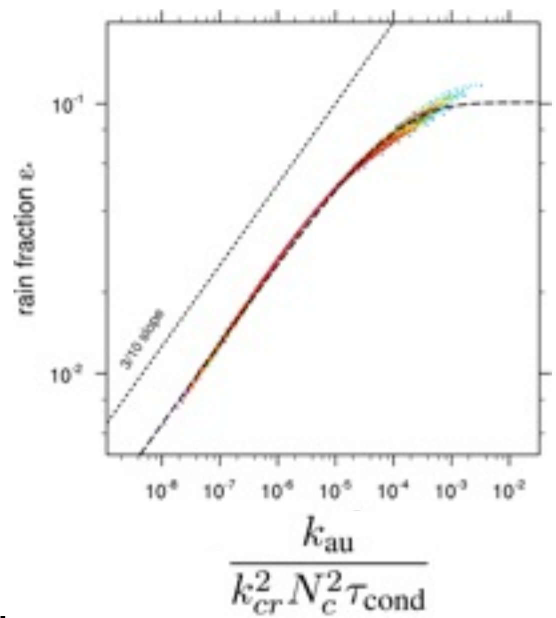
Towards a cloud-scale theory …

The aging of the size distribution

Close equation for rain formation time scale with a parameterization for ε_* based on numerical solutions of the parcel model.

$$\tau_* = \left[\beta_* + \sqrt{\beta_*^2 + \frac{\tau_0^4}{(1-\epsilon_*)^4 \phi_{cc}(\epsilon_*)}} \right]^{1/2}$$

$$\epsilon_* = 0.10 \tanh \left[16.135 \left(\frac{k_{\mathrm{au}}}{k_{cr}^2 N_c^2 \tau_{\mathrm{cond}}} \right)^{3/10} \right]$$



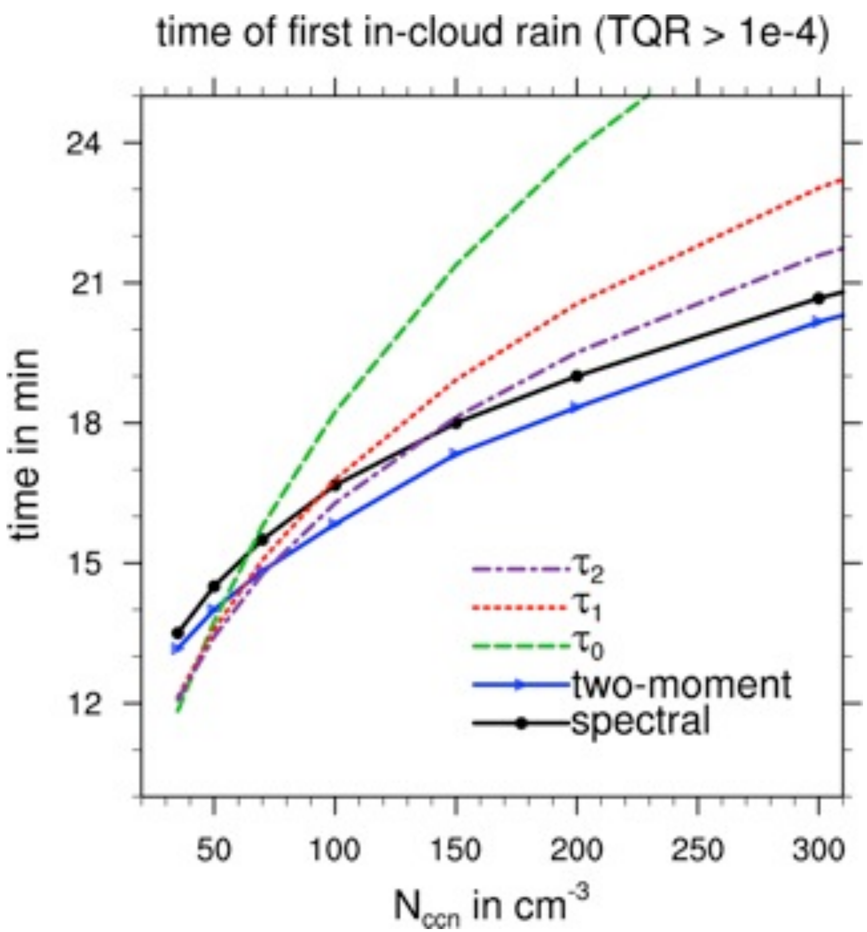
The ε_* and Φ_{cc} describe the aging of the cloud droplet distribution, which is crucial especially for high cloud droplet numbers.

Towards a cloud-scale theory …

Test of the timescale in the 1D model

Comparison of the rain formation timescale in the 1D model.

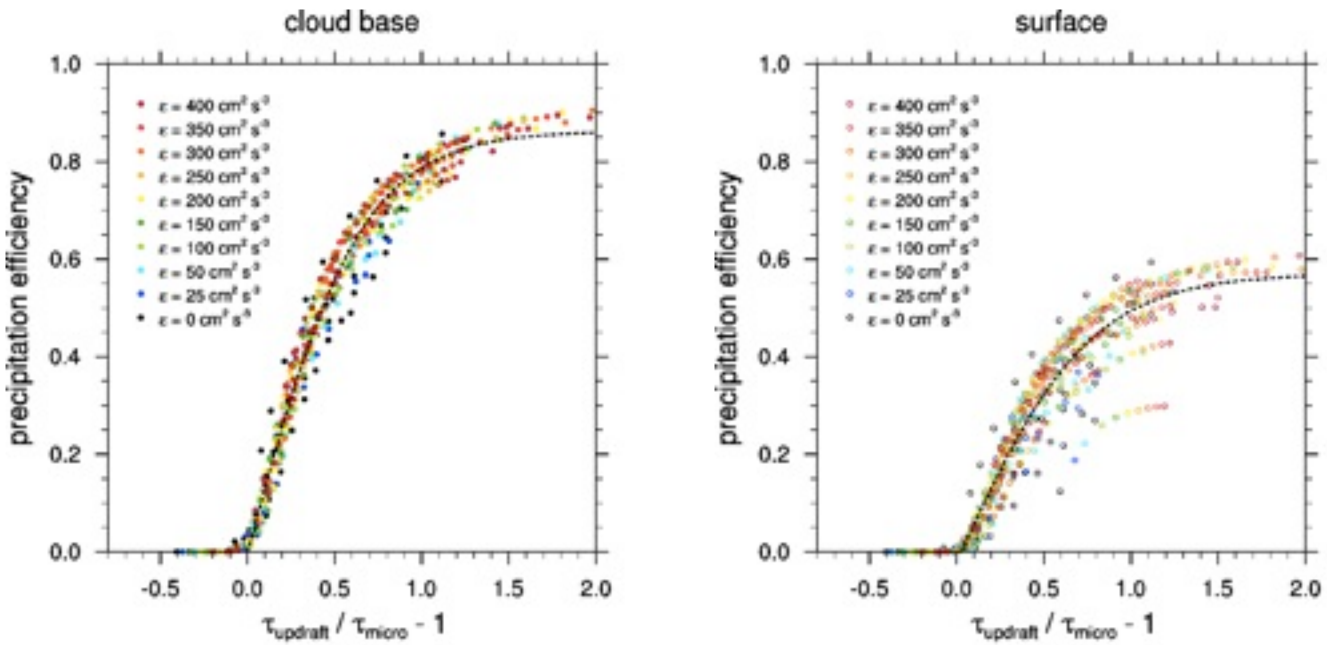
- Good agreement between spectral model and two-moment parameterization.
- The simple timescale τ_0 which neglects the aging of the DSD gives a much too strong sensitivity
- Including the parameterization for ε_* reduces the CCN-sensitivity.



Towards a cloud-scale theory …

The resulting dependency for the precip efficiency

At least for the simple kinematic 1D cloud, the precipitation efficiency is only a function of the **dynamical updraft timescale** and the **microphysical rain formation timescale**.



This includes turbulence effects on rain formation (colors: different dissipation rates from 0 to 400 cm^2/s^3)

Lessons learned
from the timescale approach

- All clouds would rain sooner or later, but the limited lifetime of the cloud leads to a threshold behavior.

- Two effects make the precipitation robust to changes in aerosol/CCN number

  1. Aging of the size distribution
  2. Growth of droplets by accretion

Towards a cloud-scale theory: Orographic precipitation

Can this timescale approach be applied to clouds which have more complicated dynamics?

Let look at the example of idealized warm-rain orographic precipitation:

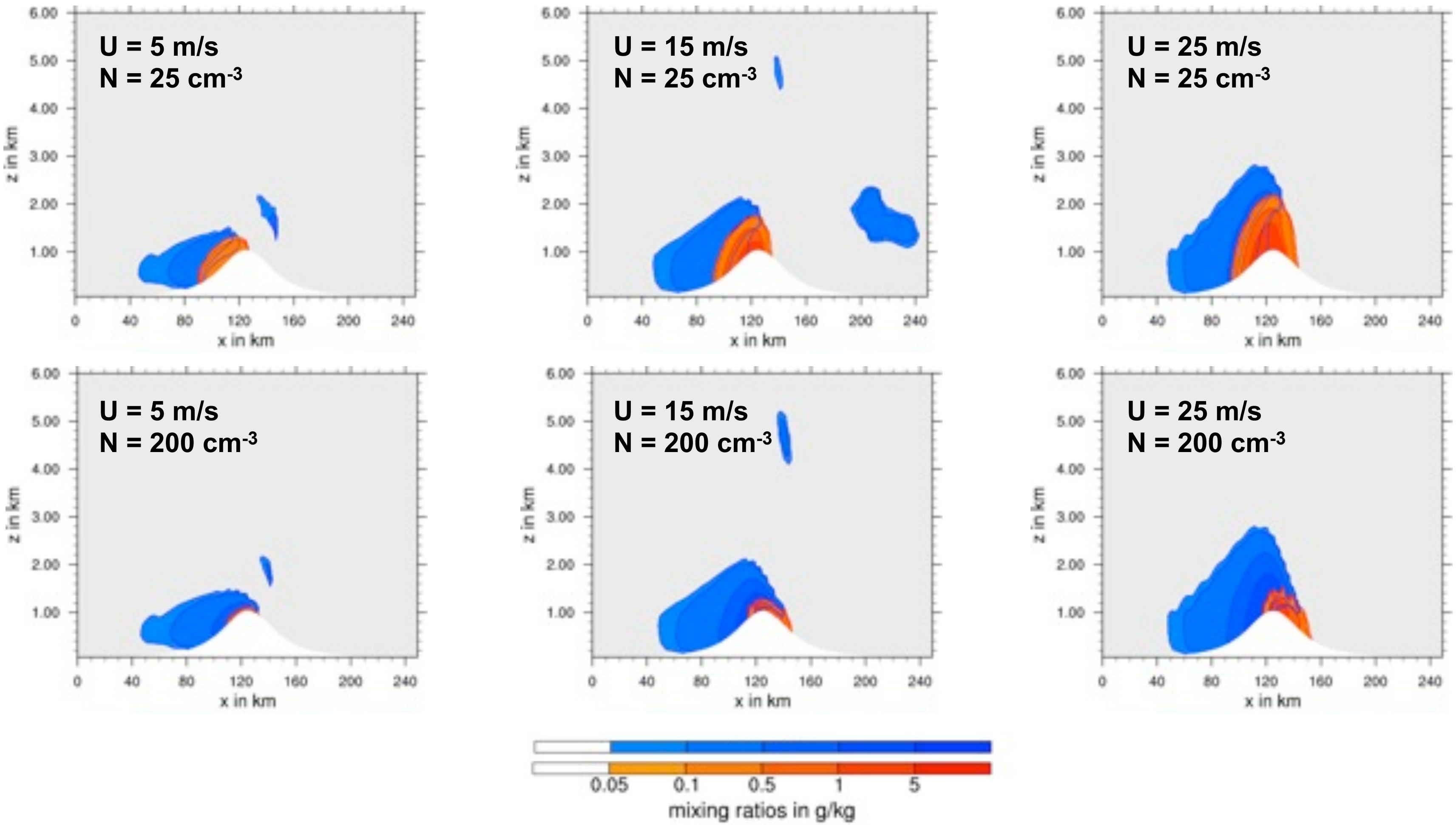
Can we formulate the precipitation efficiency of a warm orographic cloud as a function of the advective and microphysical timescale only?

$\tau_{adv} = a / U$, with a = mountain width, U = horizontal wind speed

Warm-rain orographic precipitation:

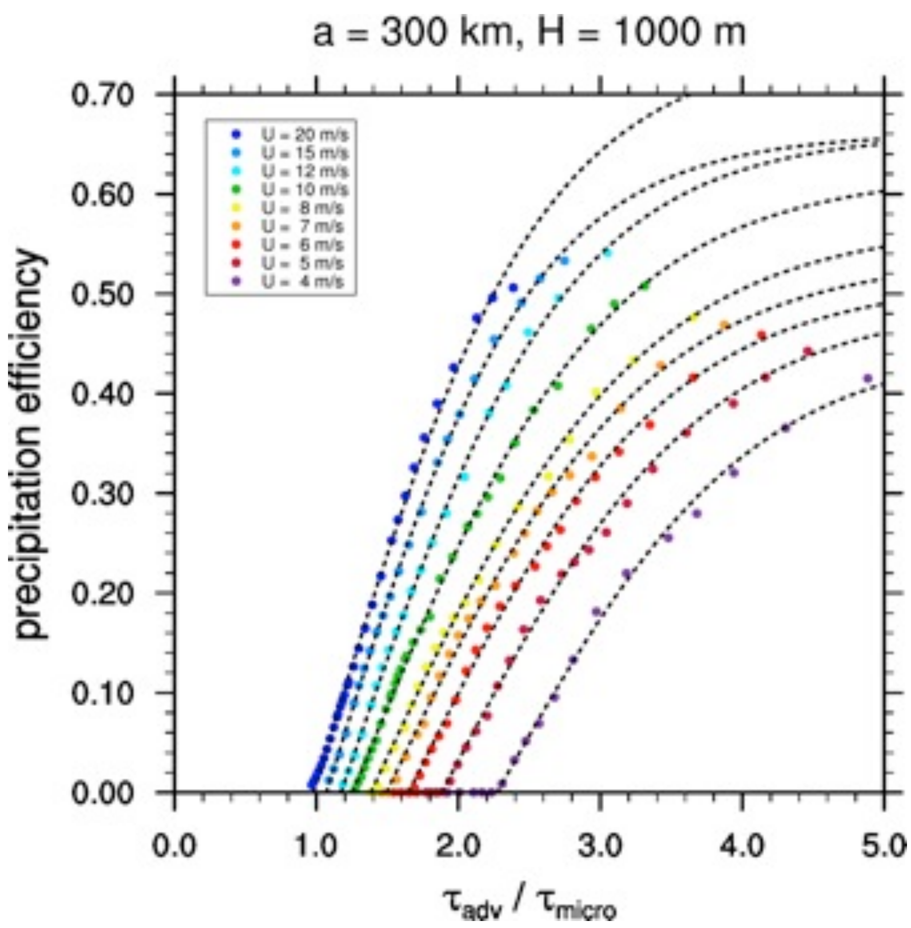
Sensitivity to horizontal velocity and cloud droplet number

xz-cross sections showing cloud and rain water after 6 hours



Warm-rain orographic precipitation: Precipitation efficiency

In principle the system behaves similar to the 1D model, but the ratio of the two timescales alone is not sufficient for similarity.

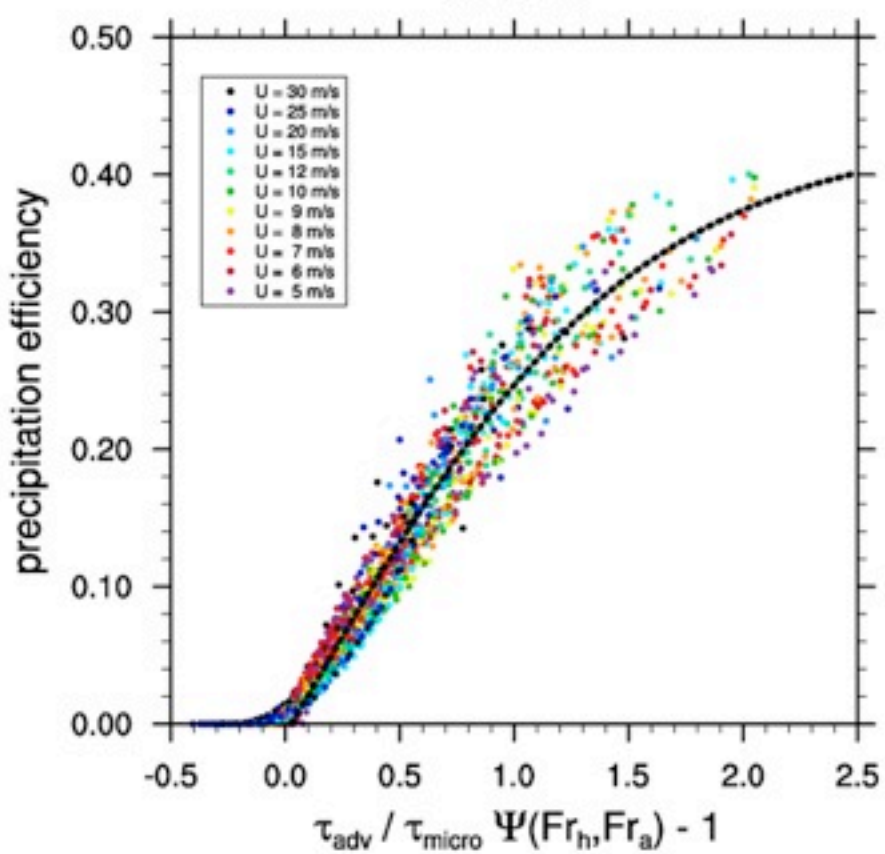


Warm-rain orographic precipitation: Precipitation efficiency

Proper scaling can be achieved by introducing a correction as a function of the Froude numbers.

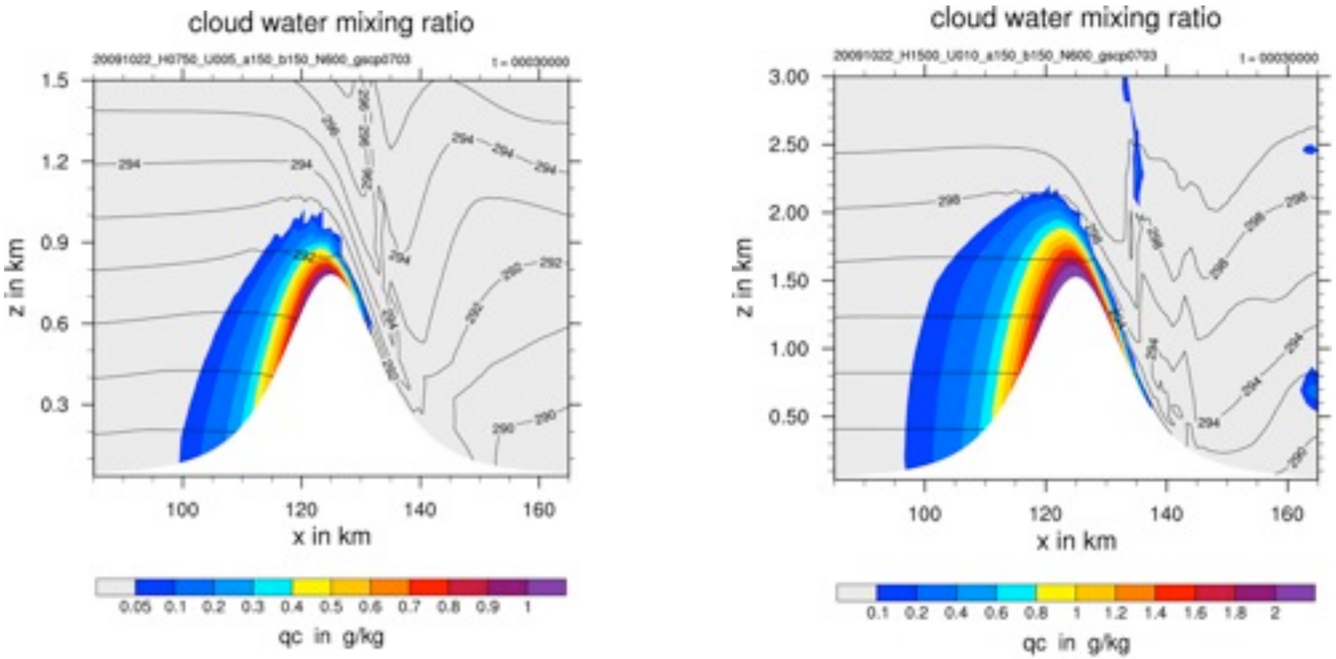
$$\Psi(Fr_h, Fr_a) = \gamma \left[1 + \frac{\alpha}{Fr_h} + \frac{\beta}{Fr_a}\right]^{-1}$$

Why is this correction necessary?



Warm-rain orographic precipitation: Precipitation efficiency

For different Froude numbers U/(Na), at the same U/(NH), the shape of the cloud changes due to the different wave length of the mountain wave



This leads to a modification of precipitation formation, e.g. due to accretion, which is not included in the microphysical timescale, hence the additional Froude number dependency.

Summary and Outlook

- **Warm rain formation is reasonably well understood (much less so for ice clouds).**
- **Remaining uncertainties like turbulence effects or collisional breakup can be investigated by DNS (and lab experiments).**
- **On the scale of individual cloud parcels, e.g. LES models, the two-moment bulk schemes provide an efficient description. Not yet clear whether two or three moments are optimal.**
- **Timescale approach looks promising for the cloud-scale, but application to more complex systems might lead to additional questions, because the timescale approach needs detailed information about the dynamics and geometrie of the clouds.**

Acknowledgements and References

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