Dynamics of Complex Systems with an emphasis on 1/f Noise and Record Dynamics.

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Colaborators:

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and P Anderson, K Christensen, M Hall, D Jones, M Nicodemi, LP Oliveira,

Resistance fluctuations

Example of 1/f spectra

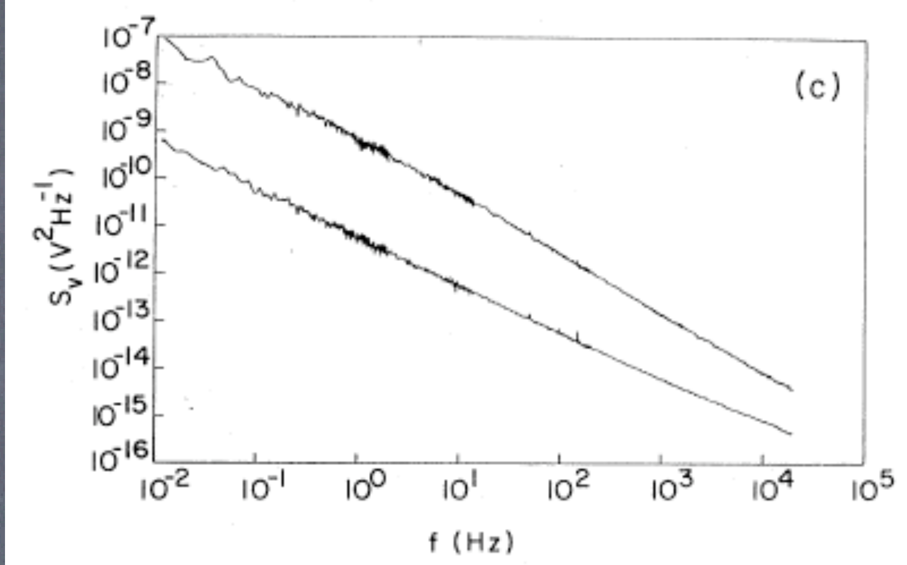
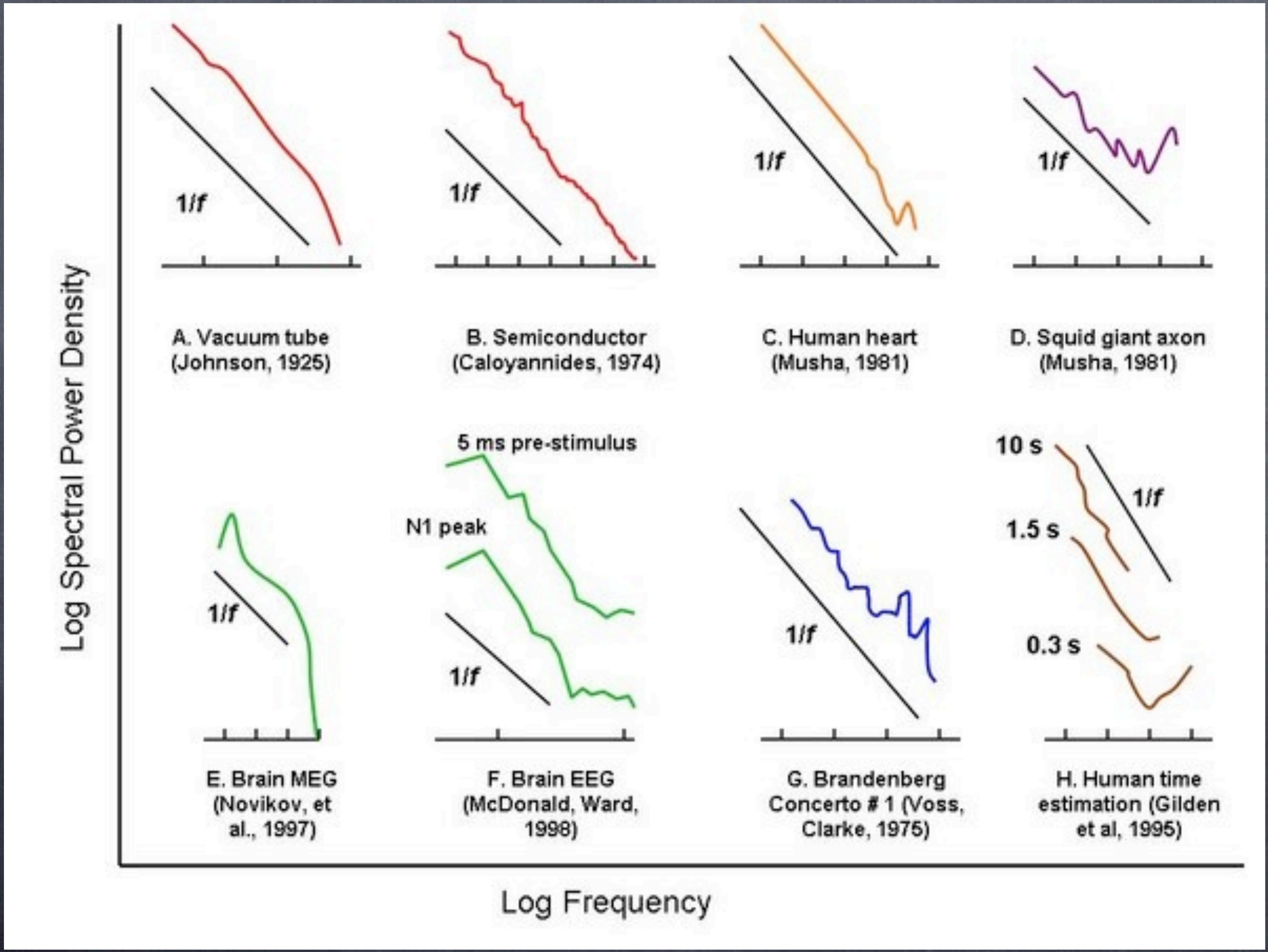


FIG. 1. (a) The basic experimental configuration and typical observations of $1/f$ noise. Schematic diagram of the simplest measuring apparatus for $1/f$ noise. R_s is a large, constant resistor. The unlabeled resistor is the sample. Various modifications, such as the use of ac currents with phase-sensitive detection, bridge circuits, and multiprobe samples, are common. (b) An actual fluctuating voltage from a silicon resistor with about 100 μA of current (1 V average bias), measured in a setup like that shown in part (a). (c) Noise spectra from two thick-film resistors, shown over a very broad range of frequencies. The upper plot is taken from an IrO_2-based film at $T=556$ K, the lower from a ruthenate-based film at $T=300$ K. Each point in each spectrum represents the average square of the Fourier transforms of 1200 1024 point traces, such as that in part (b). Several such spectra, taken at different sampling rates, are stitched together for each broad-band spectrum shown (from Pellegrini, Saletti, Terrini, and Prudenziati, 1983).

Weissman RMP 60 537 (1988)

Example of 1/f spectra - Scholarpedia

http://www.scholarpedia.org/article/1/f_noise



Example of 1/f spectra –

W. Press Commts Astrophys Space Phys 7 103 (1978)

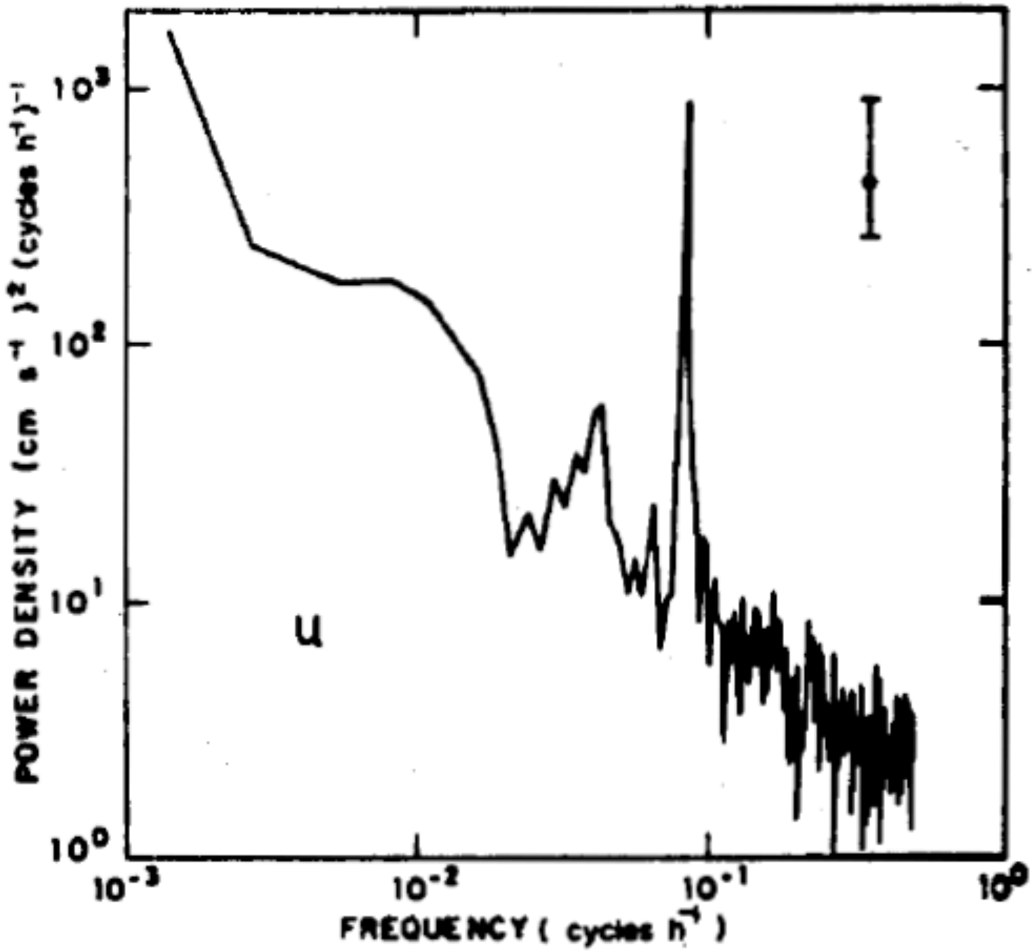


FIGURE 7 Power spectrum of the east-west component of ocean current velocities at a depth of 3100 m at 1°02.3′S, 149°50.7′W. The length of the bar at upper right shows 95% confidence limits. (Reproduced from Taft *et al.*)[20]

Example of 1/f spectra -

W. Press Commts Astrophys Space Phys 7 103 (1978)

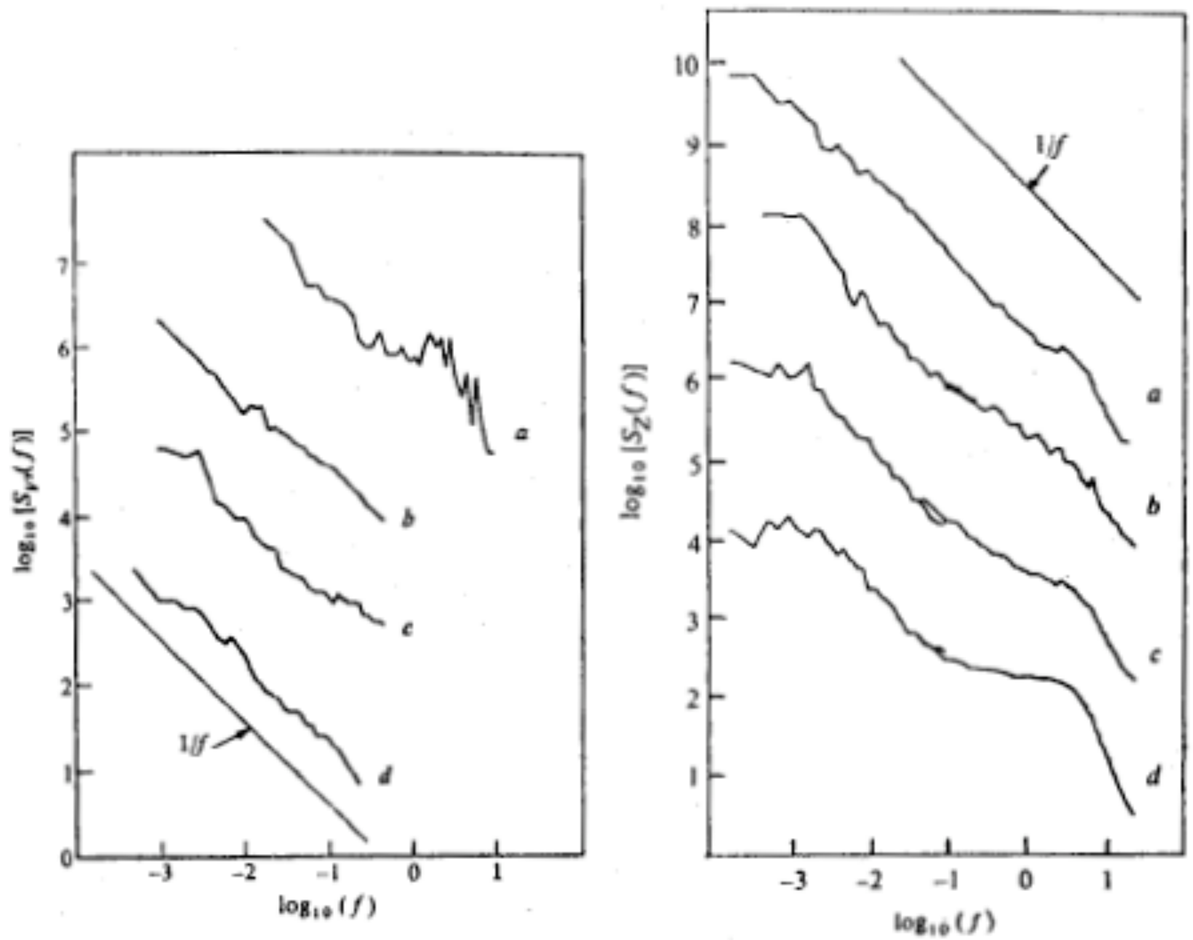


FIGURE 9 (*left*) Loudness fluctuation spectra as a function of frequency *f* (in Hz) for: *a*, Scott Joplin Piano Rags; *b*, classical radio station; *c*, rock station; *d*, news-and-talk station. (Reproduced from Voss and Clarke.[23]) (*right*) Power spectra of pitch fluctuations for four radio stations: *a*, classical; *b*, jazz and blues; *c*, rock; *d*, news-and-talk. (Reproduced from Voss and Clarke (1975).)

Proper power spectrum definition

$$S_X(\omega) \equiv \lim_{T\to\infty} \frac{1}{2T}\mathbb{E}[|\hat{X}_T(\omega)|^2] \quad \text{where} \quad \hat{X}_T(\omega) \equiv \int_{-T}^{T} dt X(t) e^{-i\omega t}$$

Auto-correlation function

$$C_X(t_1, t_2) \equiv \mathbb{E}[x(t_1)x(t_2)]$$

Wiener-Khinchin theorem

$$S_X(\omega) = \int_{-\infty}^{\infty} dt \bar{C}_X(t) e^{-i\omega t}$$

where

$$\bar{C}_X(t) \equiv \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} d\tau C_X(\tau, \tau - t)$$

Even for non-wide sense stationary random process

Lu and Vaswani arXiv:0904.0602

Stationary or not?

$$|\hat{f}(\omega)|^2 \propto \omega^-\beta$$

β<1 => stationary

β=1 => boarder line

β>1 => non-stationary

Power spectrum <-> correlations

$$S(\omega) \sim 1/\omega^{\beta} \ \text{then} \ C(t) \sim 1/t^{1-\beta}$$

Ways to generate 1/f

van der Ziegel - superposition of exponentially correlated signals

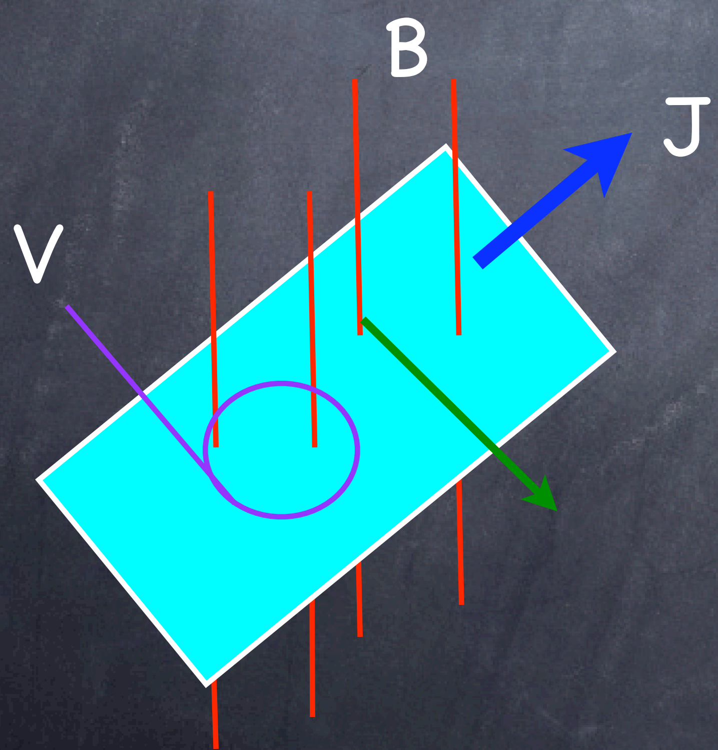
Set of individual signals $$C(t) = A\exp(-t/t_0) \Rightarrow S_{t_0}(\omega) = \frac{4At_0}{1+(\omega t_0)^2}$$

Superimpose according to distribution $P(t_0)$ $$S(\omega) = \int_0^\infty dt_0 S_{t_0}(\omega) P(t_0)$$

Assume $$P(t_0) = \begin{cases} \kappa/t_0 & \text{if } t_0 \in [\tau_1, \tau_2] \\ 0 & \text{otherwise} \end{cases}$$

Leads to $$S(\omega) \simeq \frac{4\kappa A}{\omega} \quad \text{for} \quad \frac{1}{\tau_2} \ll \omega \ll \frac{1}{\tau_1}$$

Experiment on fluctuations in vortex density in thin film
Yeh & Kao, PRL, 53, 1590 (1984)



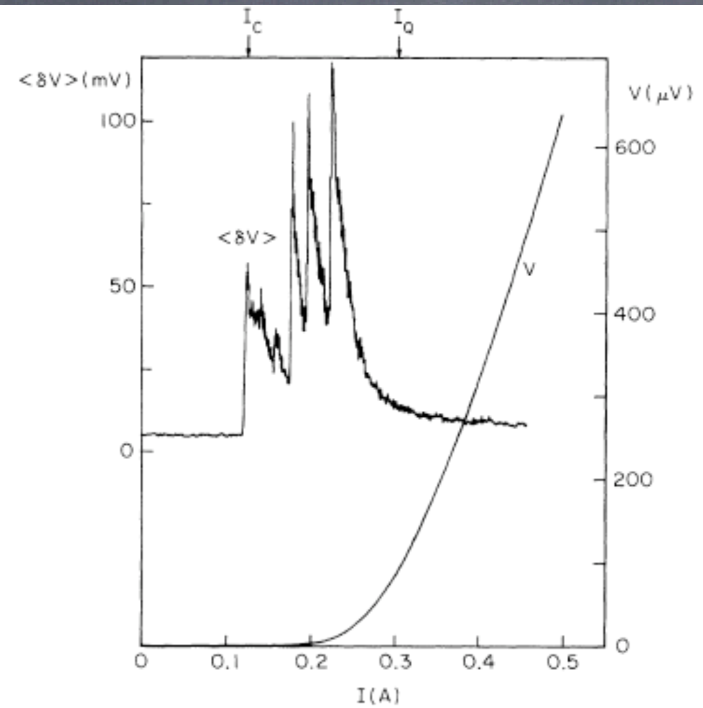


FIG. 1. A representative plot of $\langle \delta V \rangle$ vs transport current I, obtained with $H = 227.5$ Oe and $T = 3.672$ K. I_c marks the critical current at the onset of flux flow. I_Q separates the near-onset region from the quasilinear region. A V-I curve is included for comparison.

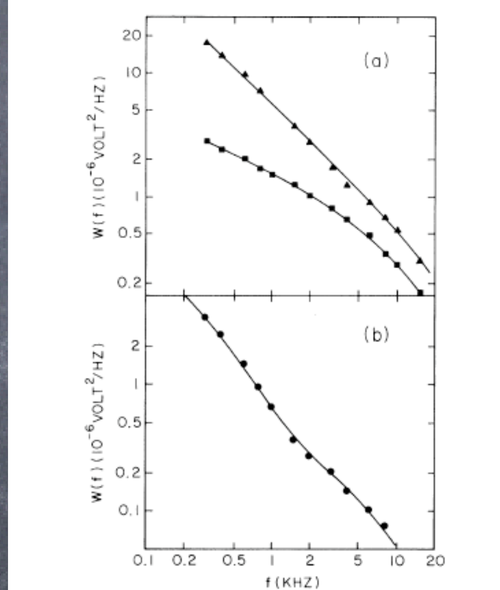
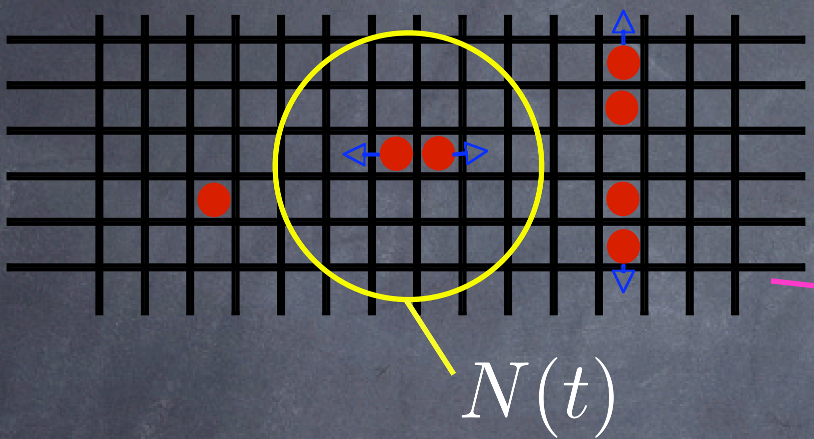
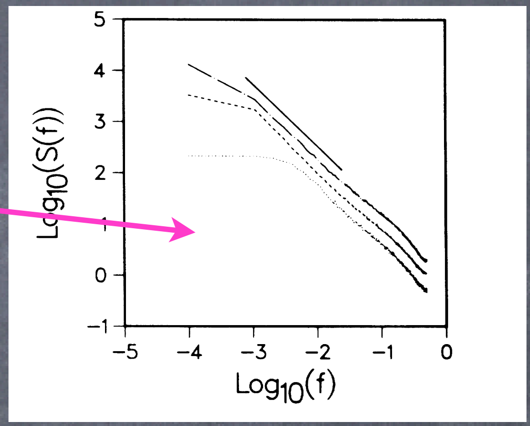


FIG. 3. (a) Typical noise power spectral density in the near-onset region; $H = 52$ Oe, $T = 4.12$ K. The upper curve shows $1/f$ noise, obtained with I set at a maximum of $\langle \delta V \rangle$. The lower curve was obtained with I at a minimum of $\langle \delta V \rangle$. Dots are data points, and lines are drawn to guide eyes. (b) $1/f$ noise and small deviations obtained with I set at a sharp $\langle \delta V \rangle$ maximum in the near-onset region. $H = 182$ Oe, $T = 4.03$ K. Dots are data points, and the curve is a fit obtained by use of Eqs. (1) and (2) with $\tau_1 = 5 \times 10^{-5}$ sec, $\tau_2 = 8 \times 10^{-5}$ sec, $\tau_3 = 6 \times 10^{-4}$ sec, and $\tau_4 = 4 \times 10^{-3}$ sec.

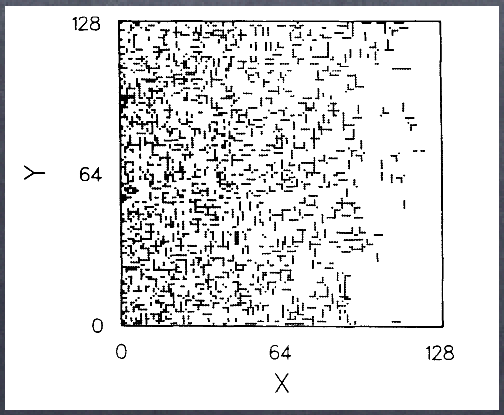
Deterministic lattice gas



1/f power spec



Dissipation occurs on a fractal



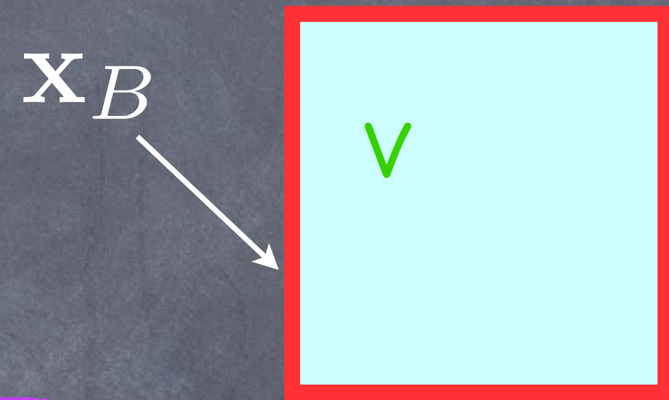
HJ Jensen PRL 64,1 (1990)

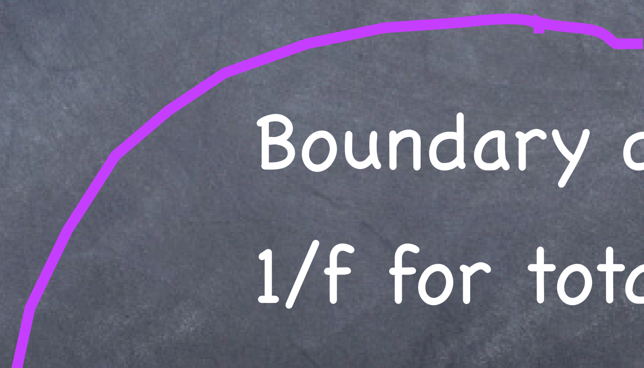
1/f from diffusion

$$\frac{\partial n(\mathbf{x},t)}{\partial t} = \gamma \nabla^2 n(\mathbf{x},t)$$ and $$n(\mathbf{x}_B,t) = \eta(\mathbf{x}_0,t)$$

then $$N(t) = \int_V d\mathbf{x} n(\mathbf{x},t)$$

is 1/f in any dimension.





Boundary driven BTW:

1/f for total $z = \sum_i z_i$

for $f < 1/T_{max}$

See e.g.

- H.J. Jensen, *Self-Organized Criticality*, Cambridge University Press 1998.

$$\frac{\partial n(\mathbf{x},t)}{\partial t} = -\nabla J[n(\mathbf{x},t)] + \delta(x_{\|})J_0(\mathbf{x},t)$$

Effect of non-linearities

J bounded at the boundary -> 1/f

$$\frac{\partial n(\mathbf{x},t)}{\partial t} = \nabla^2 n(\mathbf{x},t) + \text{nonlinearities} + \nabla\eta(\mathbf{x},t)$$

Bulk conservative -> $1/f^{3/2}$

$$\frac{\partial n(\mathbf{x},t)}{\partial t} = \nabla^2 n(\mathbf{x},t) + \text{nonlinearities} + \eta(\mathbf{x},t)$$

Bulk non-conservative -> $1/f^2$

Grinstein, Hwa & Jensen, Phys. Rev. A **45** R559 (1992)

Self-organised Criticality:

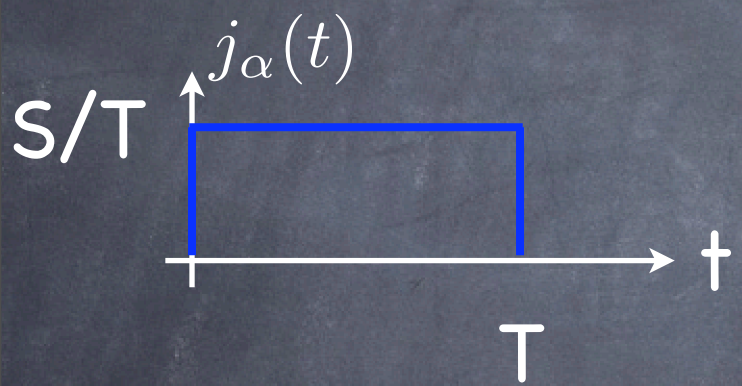
suggested as an explanation of
1/f and fractals

However BTW sandpile: No 1/f as $S(f) \propto 1/f^2$

Self-Organised Criticality:

The BTW sandpile approach

Superposition of (independent) events $J(t) = \sum_\alpha j_\alpha(t - t_\alpha)$



$$P(T) = \frac{1}{T^2}\int_0^\infty dS P(S,T) S^2$$

Assume

$$P(T) \propto T^a \exp(-T/T_0)$$

$$\Downarrow$$

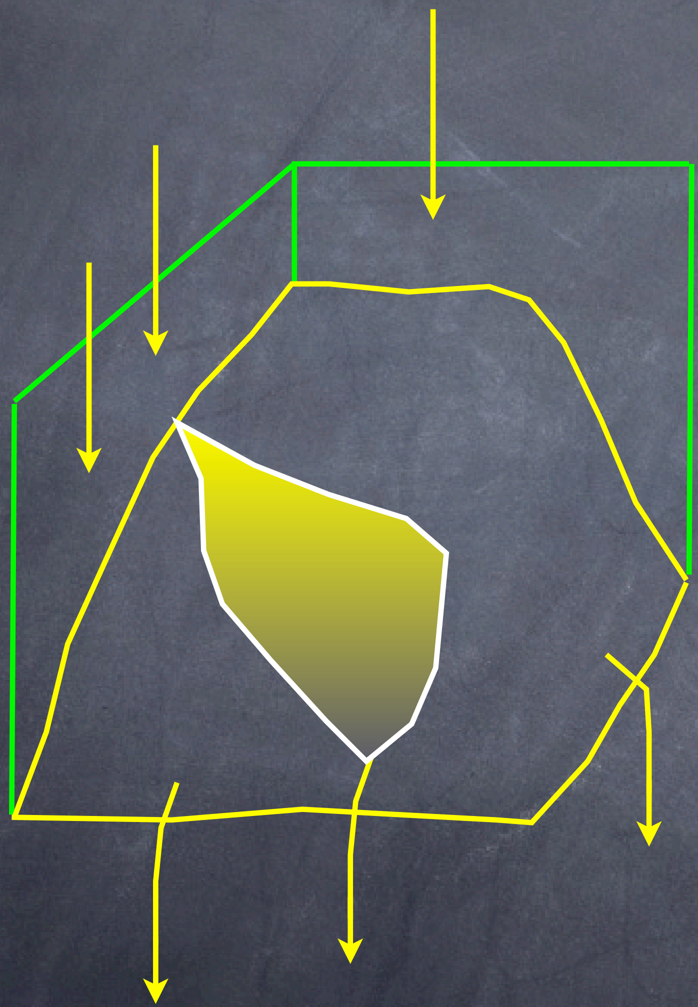
$$S(\omega) \propto \begin{cases} \omega^{-3+a} & \text{when } a < -1 \\ \omega^{-2} & \text{when } a > -1 \end{cases}$$

H. J. Jensen, K. Christensen, and H. C. Fogedby, *1/f Noise, Distribution of Lifetimes and a Pile of Sand*, Phys. Rev. B**40** 7425 (1989).

Edge driven sandpile

$$N(t) = \sum_i z_i(t)$$

$$S_N(\omega) \propto 1/\omega$$



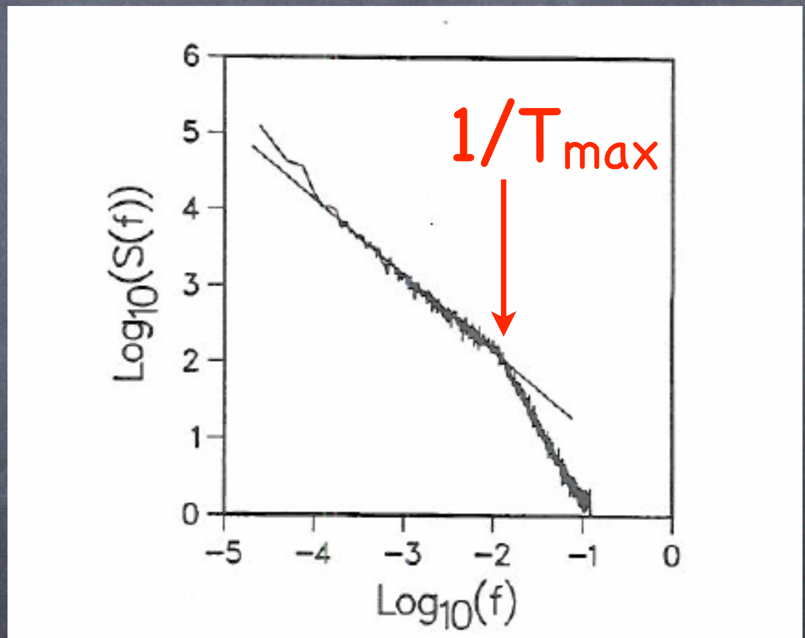


Fig. 1. Power spectrum of the total z-value of a sand pile cellular automaton driven by white noise at the boundary. The straight line has slope equal to -1.

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Record dynamics

Record statistics

Source: *Journal of the Royal Statistical Society. Series B (Methodological)*, Vol. 14, No. 2, (1952), pp. 220-228

THE DISTRIBUTION AND FREQUENCY OF RECORD VALUES

By K. N. CHANDLER

[Received June, 1952]

The Amer Math Month **85**, 2 (1978)

BREAKING RECORDS AND BREAKING BOARDS

NED GLICK

THEORY PROBAB. APPL.
Vol. 32, No. 2
1987

Translated from Russian Journal

RECORDS*

V. B. NEVZOROV

Record dynamics

VOLUME 71, NUMBER 10 PHYSICAL REVIEW LETTERS 6 SEPTEMBER 1993

Slow Dynamics from Noise Adaptation

Paolo Sibani* and Peter B. Littlewood
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 21 August 1992)

We discuss a new mechanism generating long range temporal correlations in dynamical systems coupled to a source of white noise. The external noise induces dynamical events uncorrelated on a *logarithmic* time scale and produces a fluctuating output with "$1/f$" power spectrum. This behavior requires a complex phase space with many traps, which can arise due to strong cooperative effects. As a demonstration, we numerically analyze a system of many coupled degrees of freedom, which is externally driven and subject to a white noise perturbation. We find both Poisson statistics for events in log(time) and the $1/f$ power spectrum.

Complexity **10**, 49 (2004)

Evolution in Complex Systems

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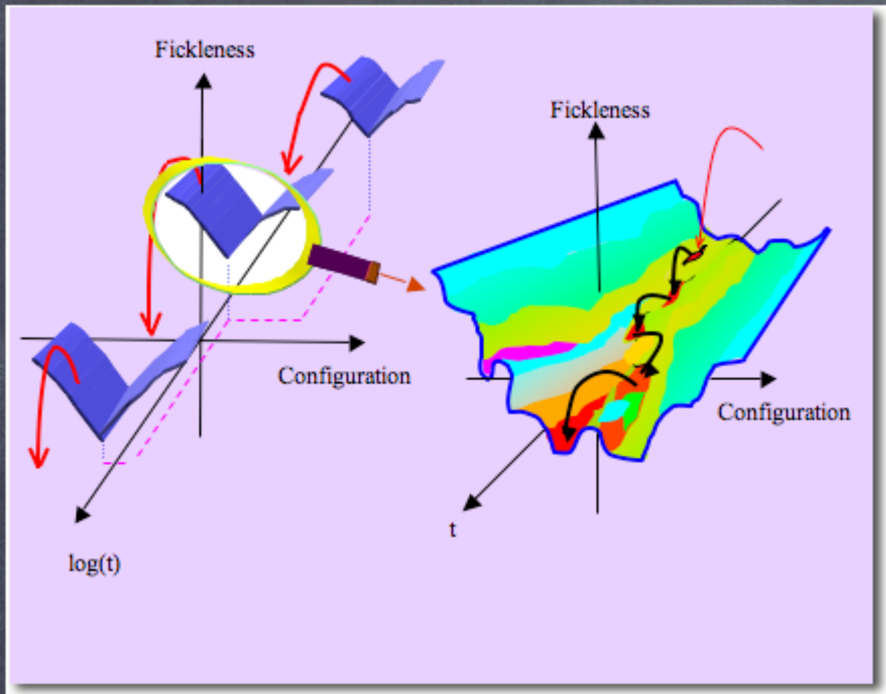
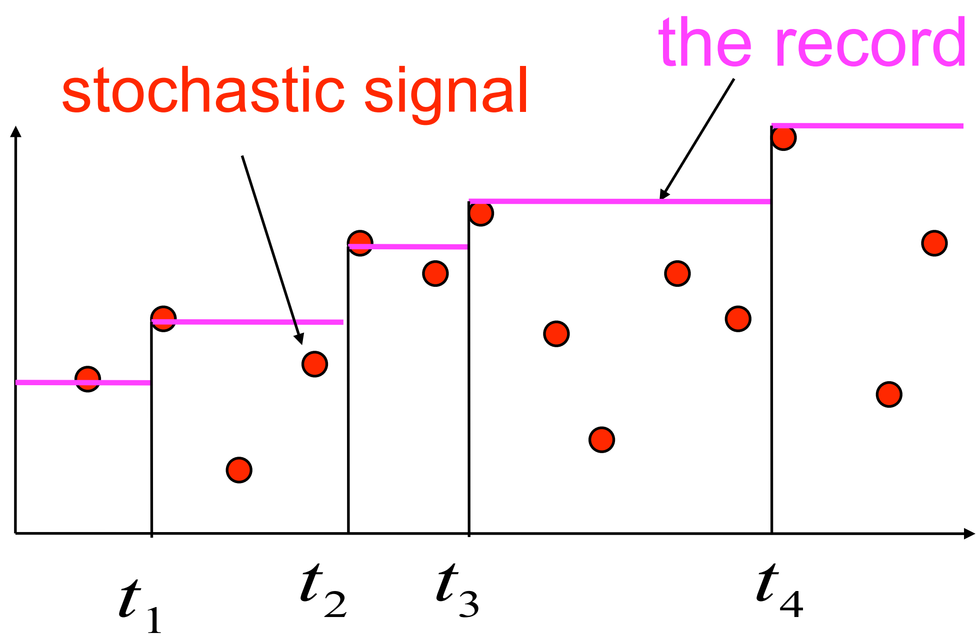
[1]*Department of Mathematics, Imperial College London, South Kensington Campus, London SW7 2AZ, UK;*
and [2]*Department of Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK*

Record dynamics

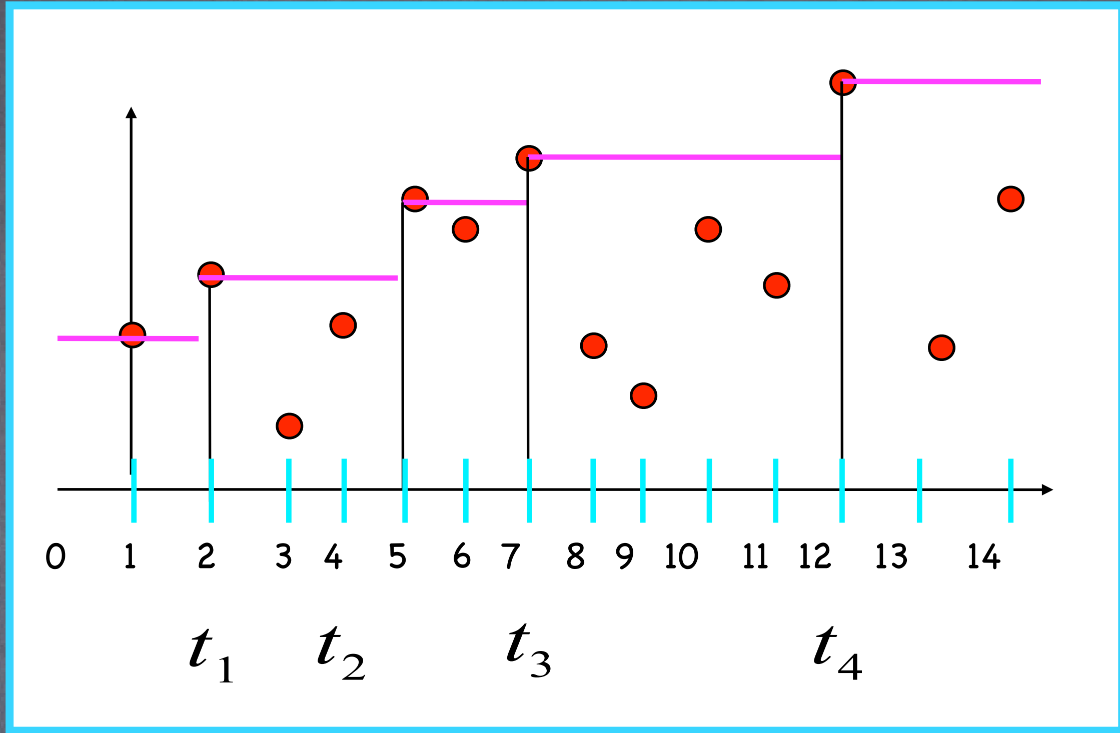
Henrik Jeldtoft Jensen, Imperial College London





$$\tau = \ln(t_k) - \ln(t_{k-1}) = \ln(\frac{t_k}{t_{k-1}})$$ expo. distrib.

Record dynamics



Distribution of the number of records during t time steps independent of the nature of the fluctuating signal:

$$P_1(t) = \frac{1}{t}$$ The first out of t is the biggest

$$P_{(1,m)}(t) = \frac{1}{(m-1)t}$$ Two records during t: one at t=1 with prob (m-1) & one at t=m with prob 1/t.

⇓

$$P_2(t) = \sum_{m=2}^{t} \frac{1}{(m-1)t} \approx \frac{\ln t}{t}$$ Two records during t

⇓

$$P_n(t) \approx \frac{(\ln t)^{n-1}}{(n-1)!}\frac{1}{t} = e^{-}\lambda\frac{\lambda^{n-1}}{(n-1)!}$$ with $\lambda = \ln t$

log Poisson

Henrik Jeldtoft Jensen, Imperial College London

Record dynamics

$$\tau = \ln(t_k) - \ln(k_{k-1}) = \ln(\frac{t_k}{t_{k-1}})$$ exponentially distributed

- Poisson process in logarithmic time
- Mean and variance

$$\langle Q \rangle \propto \ln t \;\text{ and }\; \langle (Q - \langle Q \rangle)^2 \rangle \propto \ln t$$

- Rate of records constant as function of ln(t)
- Rate decreases $\propto 1/t$
- Non-stationary 1/f fluctuations

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Relevance

When systems initially are in a state of high (generalised) internal strain & stress

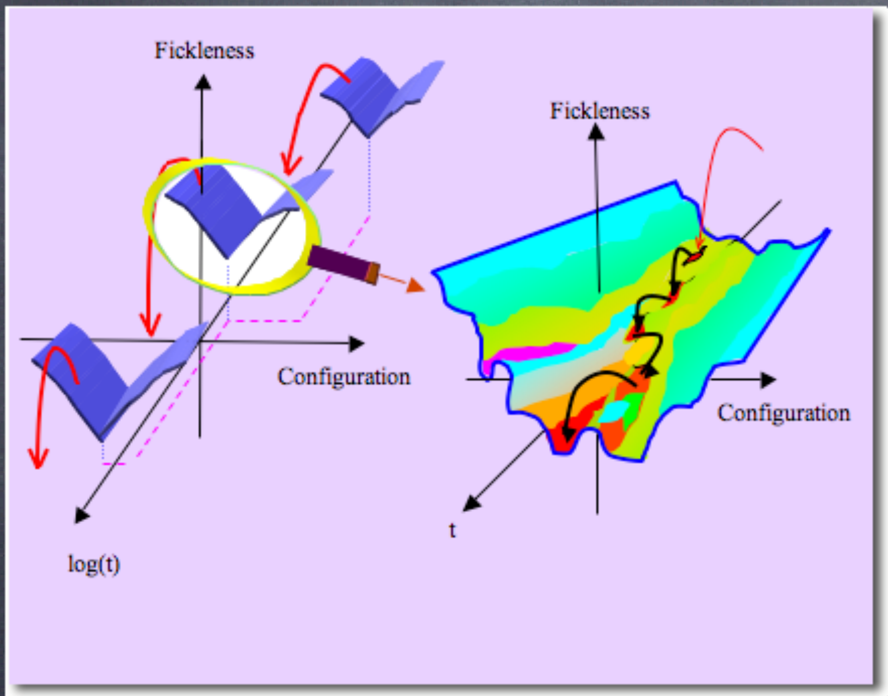
Anderson, Jensen, Oliveira & Sibani, Compelxity, **10**, 49 (2004)

Examples:

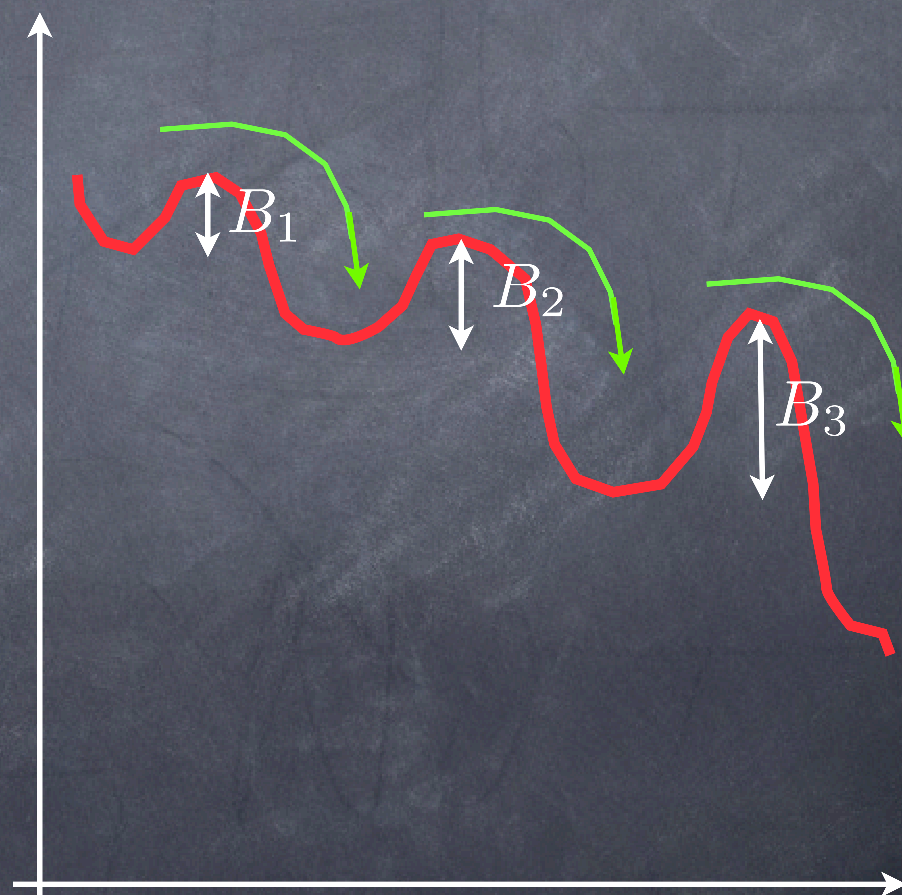
- relaxing spin glass
- magnetic relaxation in superconductors
- evolutionary ecology
- hungry ants
- Omiri after shock law

Record dynamics

The implicit nature of the relevant configuration/state space



$$B_1 \lesssim B_2 \lesssim B_3$$



First Model:

Spin Glass

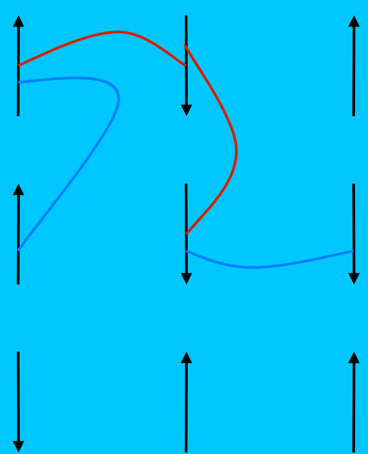
with Sibani

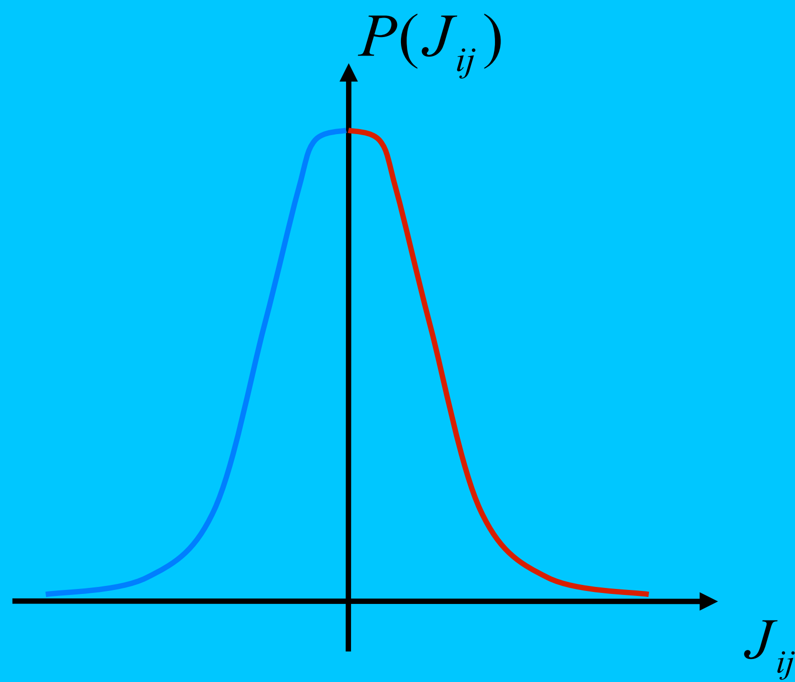
Spin glass

Microscopic magnetic moments – or spins – coupled together with random coupling constants.

The Hamiltonian:

$$H = -\frac{1}{2}\sum_{ij} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j \quad \text{where} \quad \mathbf{S}_i,\ \mathbf{S}_j = \pm 1$$



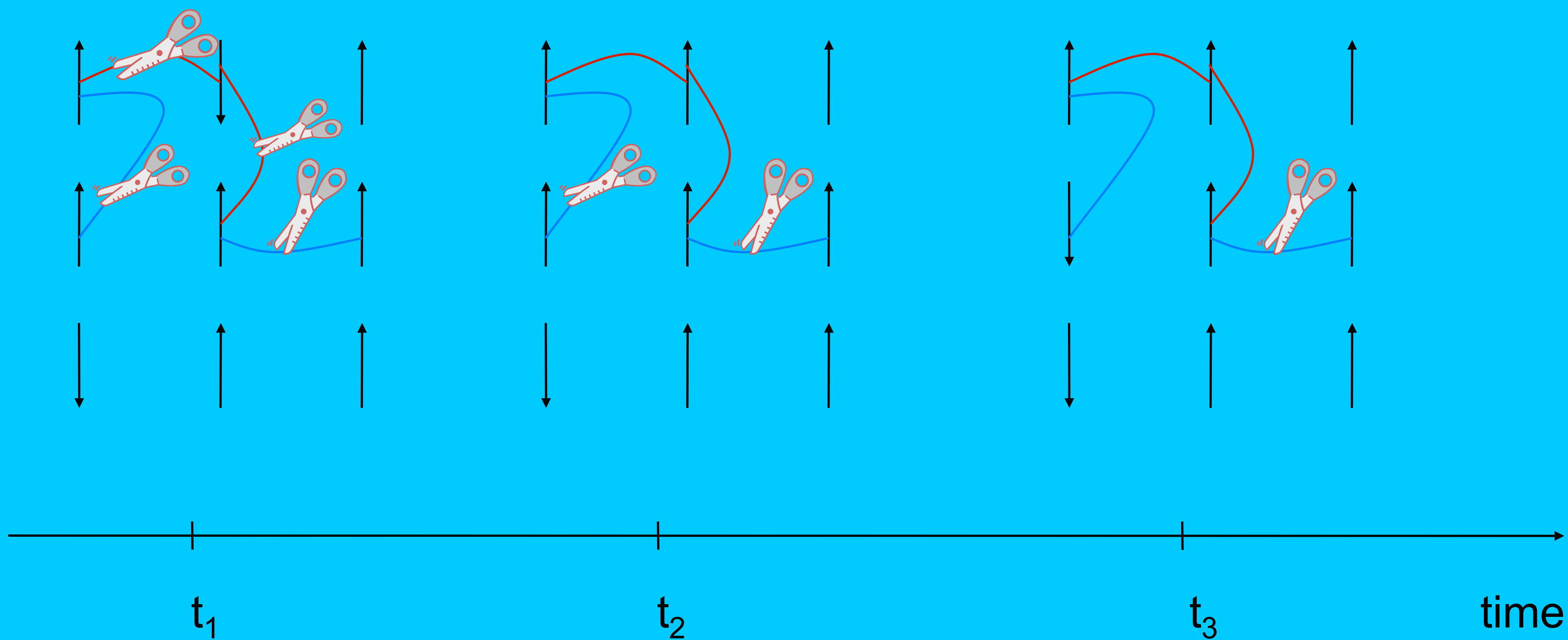


Spin glass

Quench from high temperature:

time < 0: T = high

time > 0: T = very low



Spin glass: heat transfer

Protocol: Quench from high temp. at time t= 0.

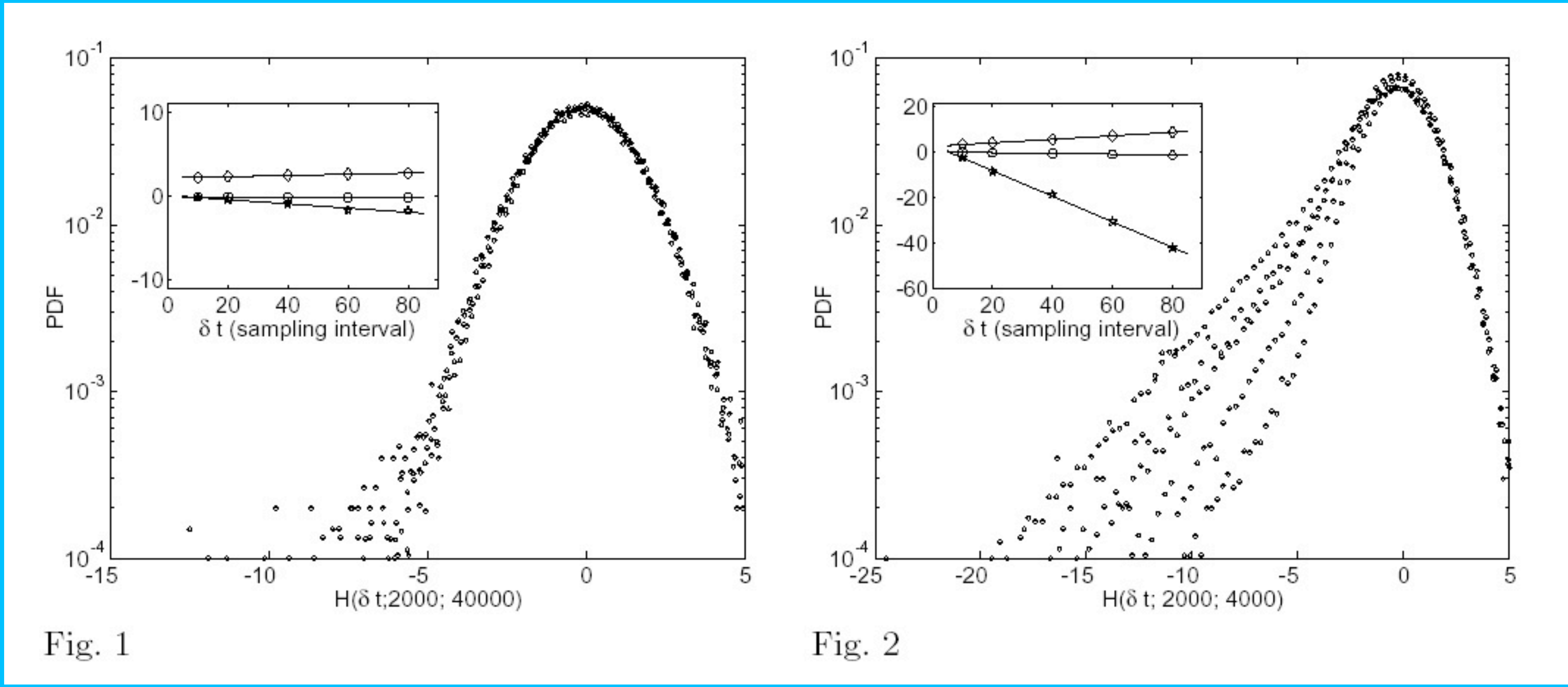
Measure heat transfer, *H*, between spin glass and reservoir during time interval

$$[t_w, t_w + \delta t]$$

- If $\delta t << t_w$ Gaussian p(H)
- If $\delta t \approx t_w$ exponential tail

Spin glass: heat transfer

$\delta t << t_w$ $\delta t \approx t_w$



From Sibani & Jensen, EPL 69, 563 (2005)

Second Model:

Tangled Nature

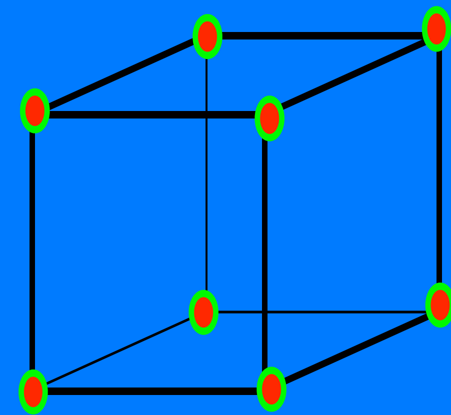
with Anderson, Hall, Sibani

Tangled Nature model of evolution

Definition:

* Individuals $\mathbf{S}^{\alpha} = (S_1^{\alpha}, S_2^{\alpha}, \ldots, S_L^{\alpha})$, where $S_i^{\alpha} = \pm 1$

and $\alpha = 1,2,\ldots,N(t)$



$L = 3$

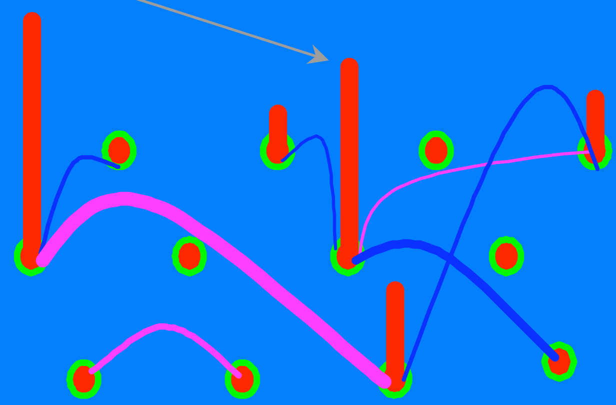
* Dynamics – a time step:
  * **Annihilation:**
    Choose indiv. at random, remove with probability $p_{kill} = const$

☻ Reproduction:

- Choose indiv. at random
- Determine

$$H(\mathbf{S}^{\alpha},t) = \frac{1}{cN(t)} \sum_{\mathbf{S}} J(\mathbf{S}^{\alpha},\mathbf{S}) n(\mathbf{S},t) - \mu N(t)$$

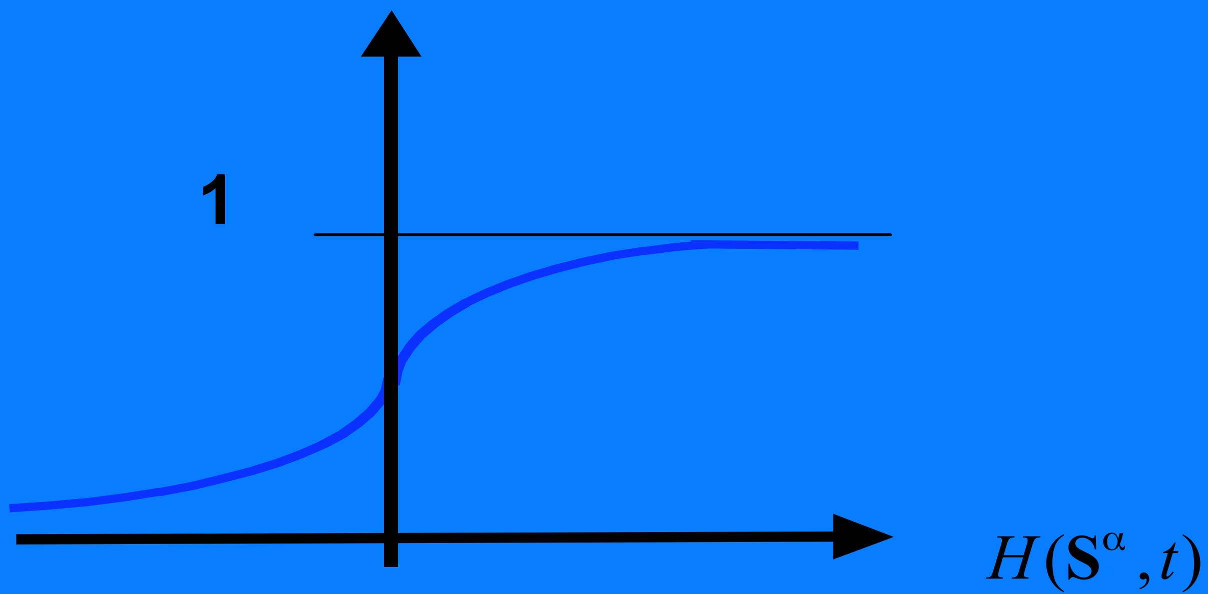
$n(\mathbf{S},t) =$ occupancy at the location $\mathbf{S}$



Wednesday, 5 May 2010

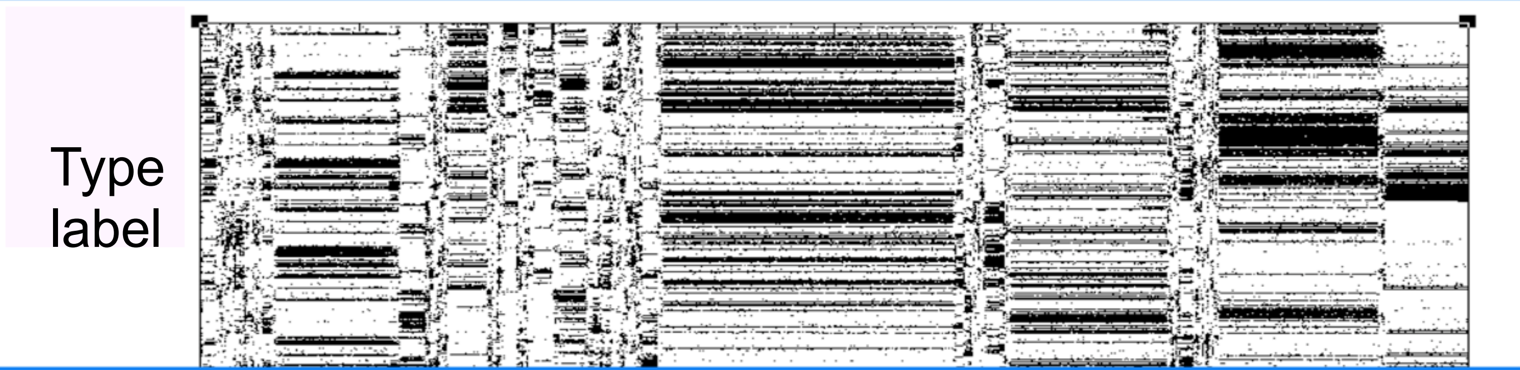
from $H(\mathbf{S}^\alpha, t)$ reproduction probability

$$p_{off}(\mathbf{S}^\alpha, t) = \frac{\exp[H(\mathbf{S}^\alpha, t)]}{1 + \exp[H(\mathbf{S}^\alpha, t)]} \in [0,1]$$



Intermittency at systems level:

Non Correlated



 # generations

1 generation
= $N(t)/p_{kill}$

- Decaying extinction rate
- Realistic species abundance distributions
- Species are laws $S \propto A^z$
- Realistic interaction-diversity relation
- Realistic connectance-diversity relation

For references see: www.ma.ic.ac.uk/~hjjens

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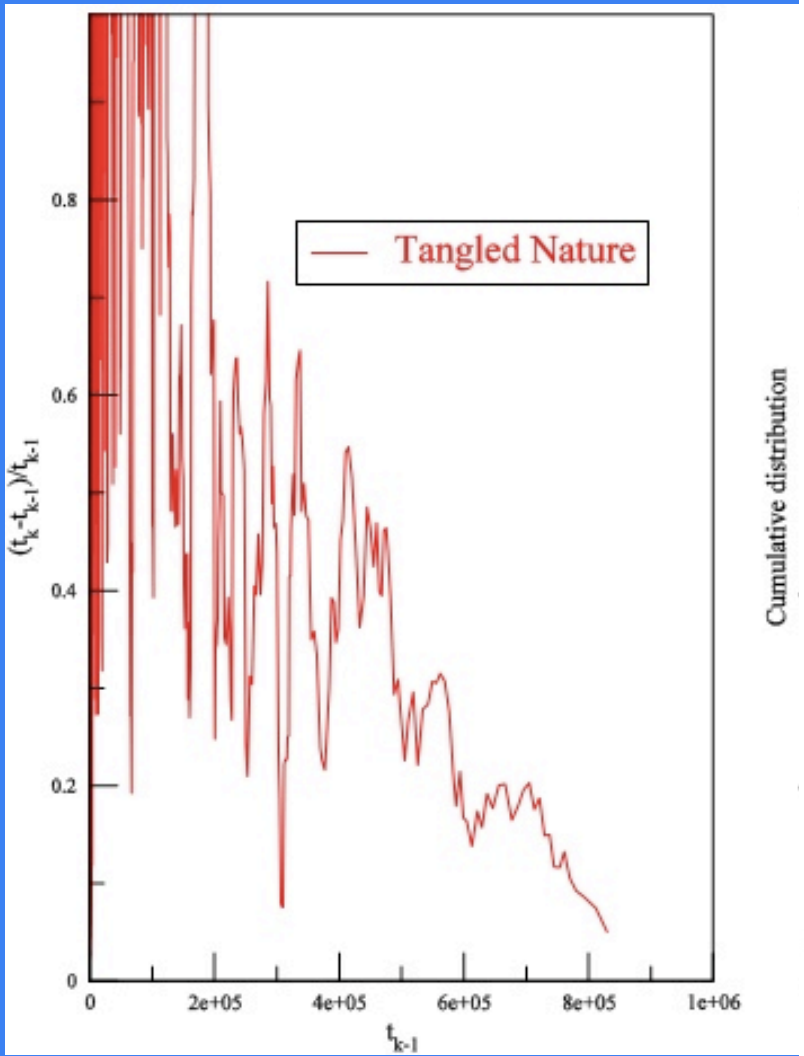
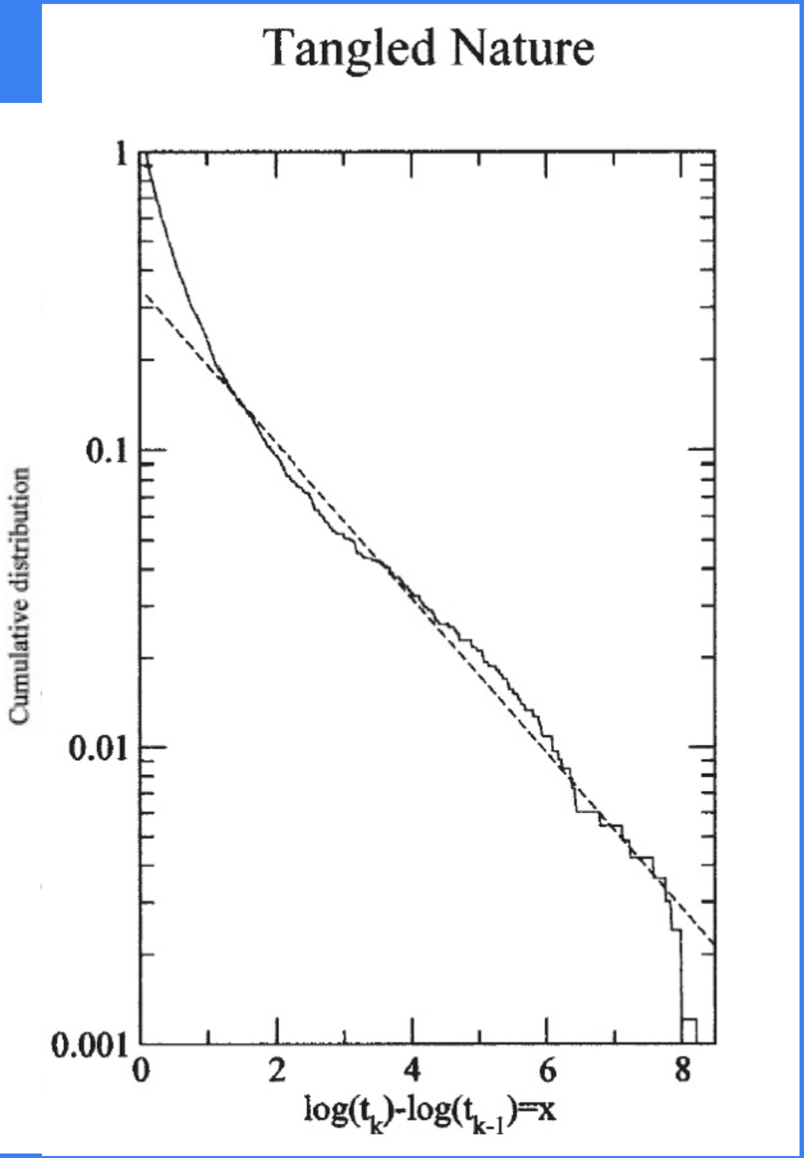
Record dynamics:

Ratio *r* remains non-zero

$$r = (t_k - t_{k-1}) / t_{k-1}$$



Cumulative Distribution



Third System:

Magnetic Relaxation

in

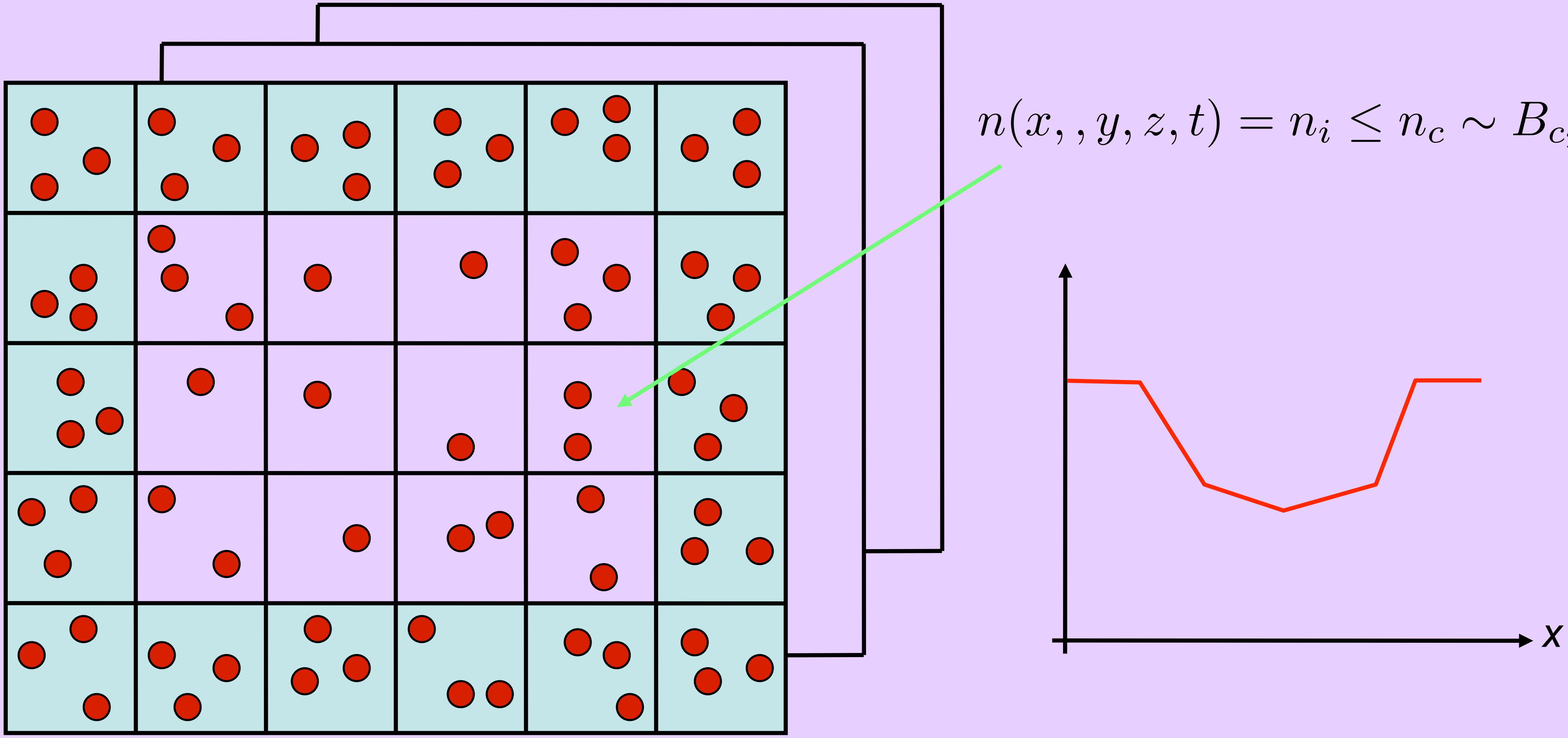
Type II Superconductor

with Nicodemi, Oliveira, Sibani

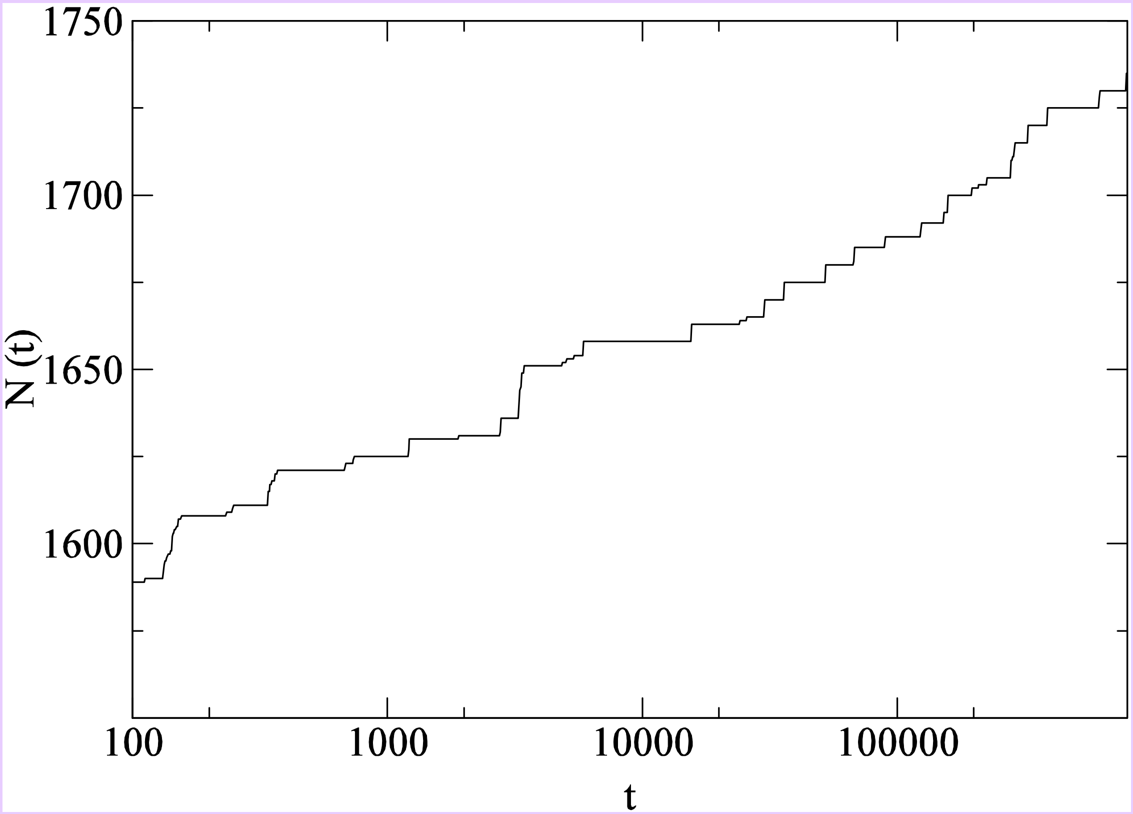
Restricted **O**ccupancy **M**odel

Monte Carlo Kawasaki dynamics on stack of coarse grained superconducting planes

$$n(x,,y,z,t) = n_i \leq n_c \sim B_{c_2}$$

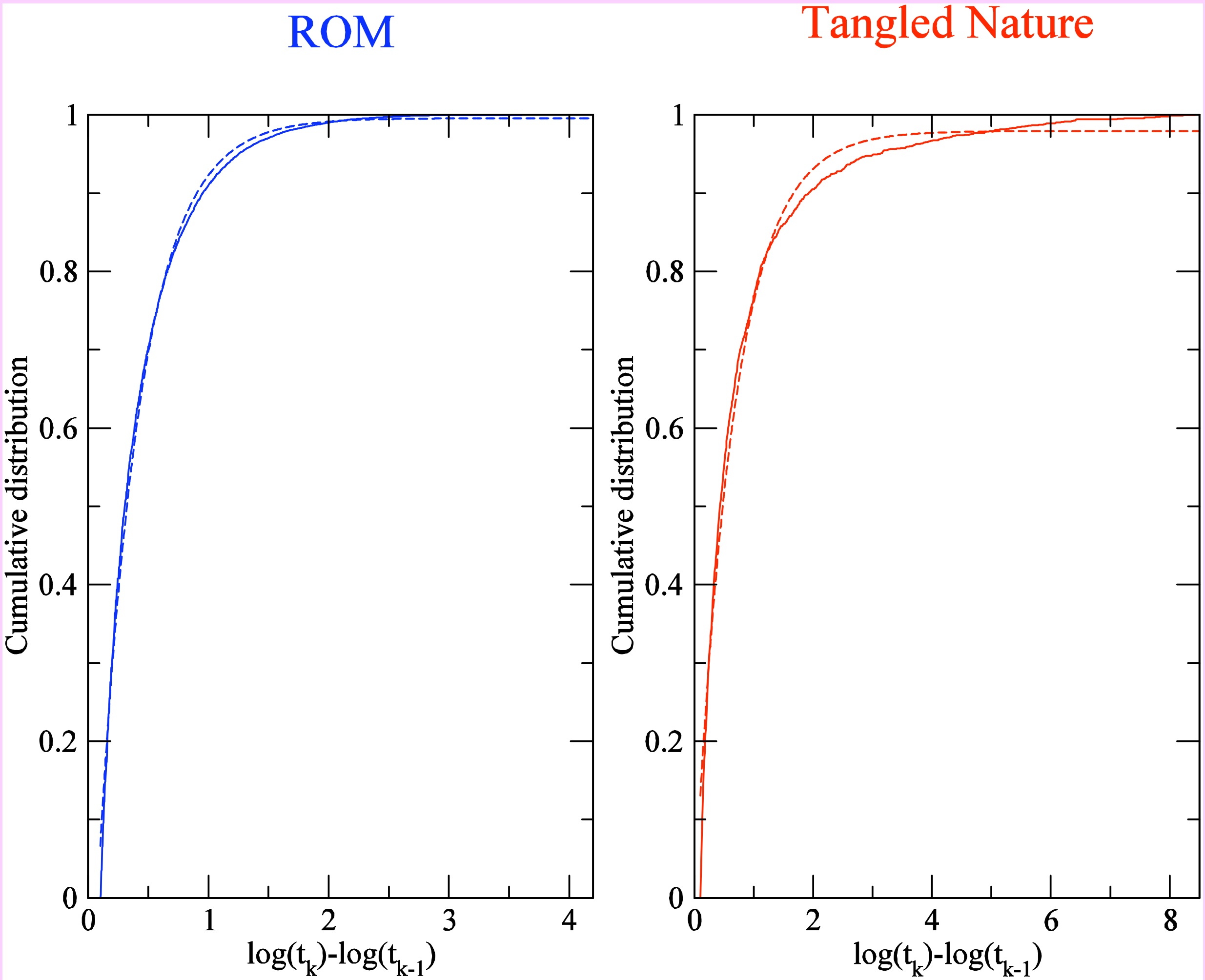


ROM: Temperature independent creep

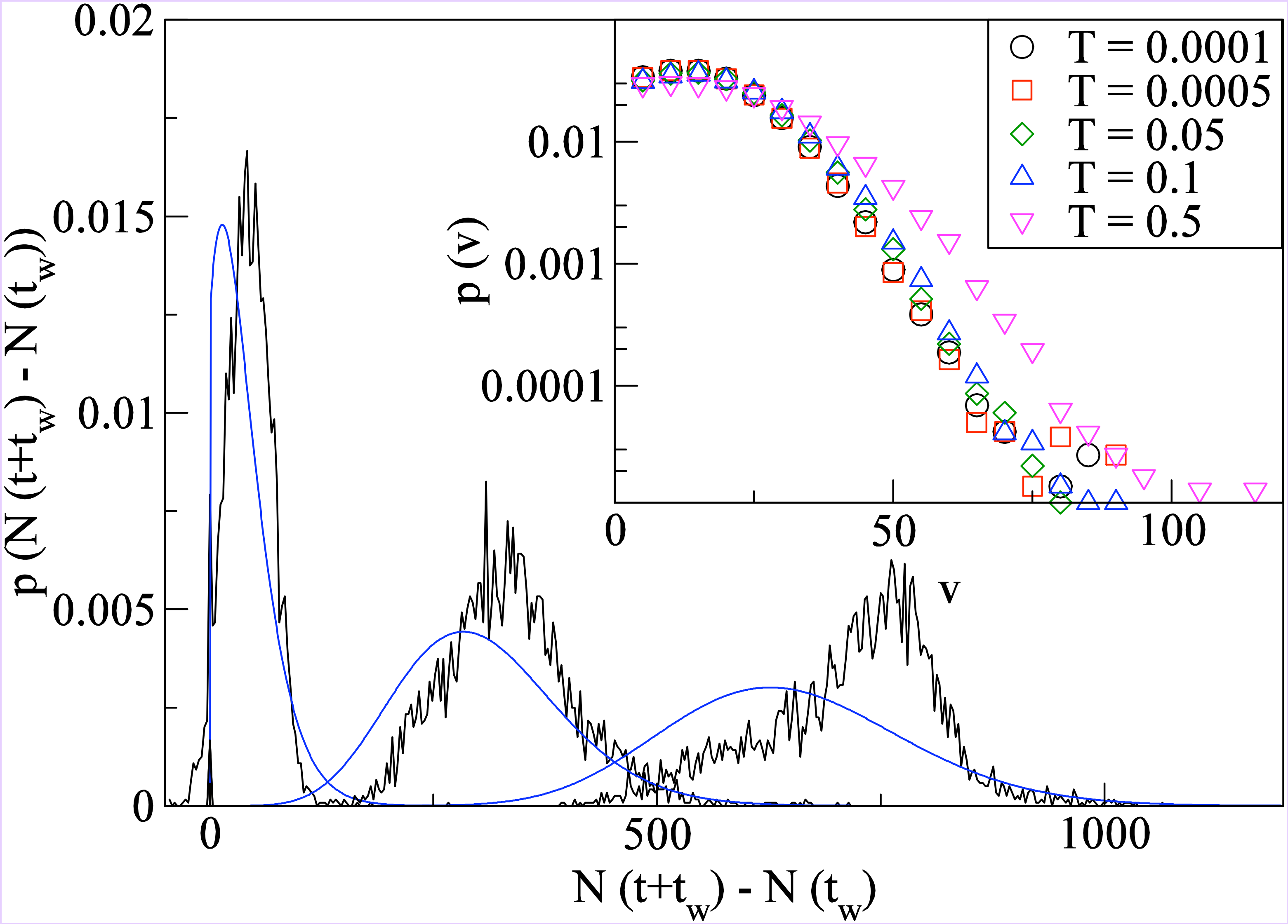


Further evidence

The cumulative distribution of the log waiting times.
Comparison with exponential distribution.



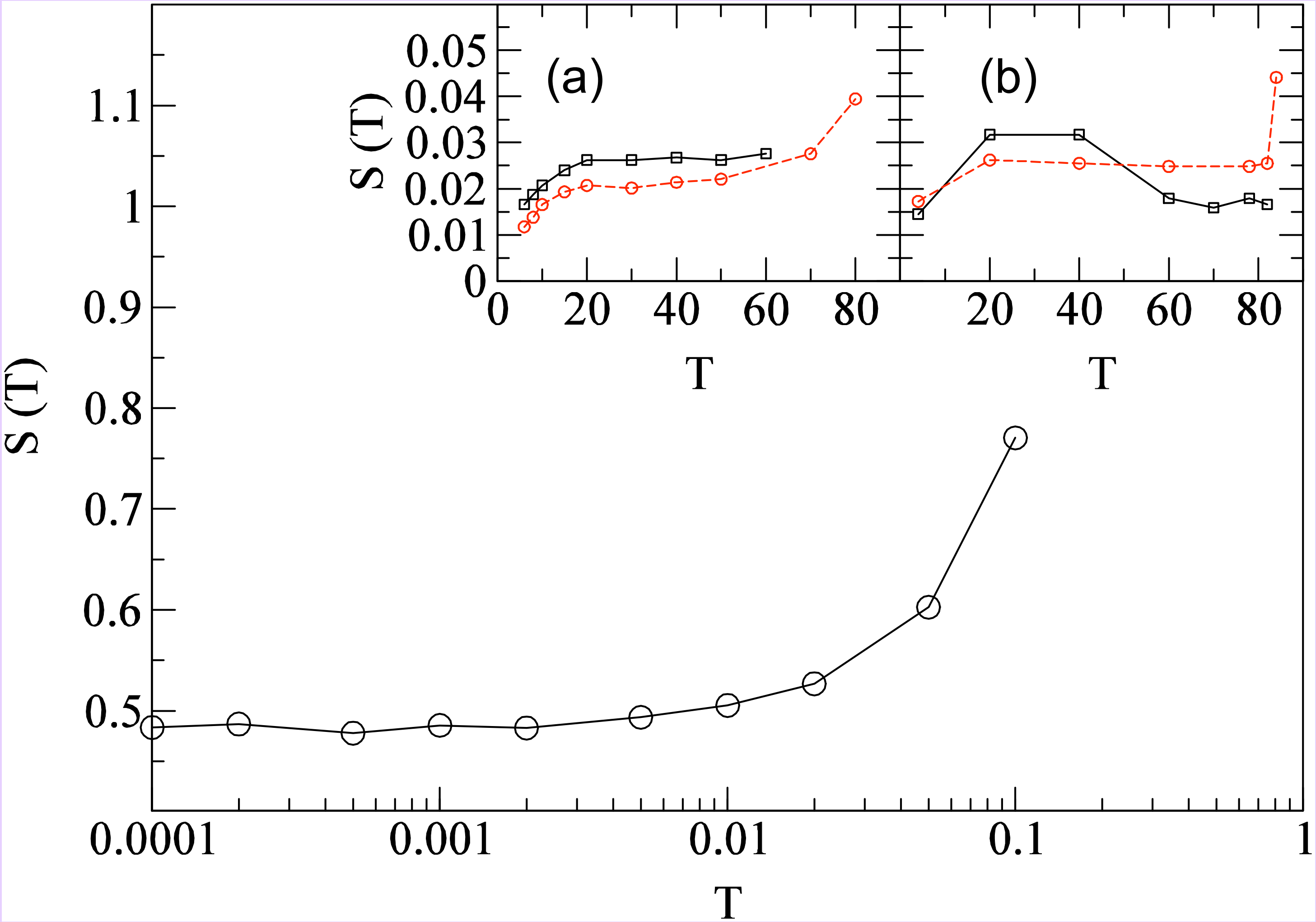
Quake statistics and the total number vortices entering.



The magnetic creep rate:

$$S = \frac{d\ln(M)}{d\ln(t)} \quad \text{where} \quad M(t) = |N(t) - N_{ext}|$$

comparison with experiment



Experim. data from

(a) Civale et al. PRL 65, 1164 (1990)

(b) Kaiser et al. J Cryst Growth 85, 593 (1987)

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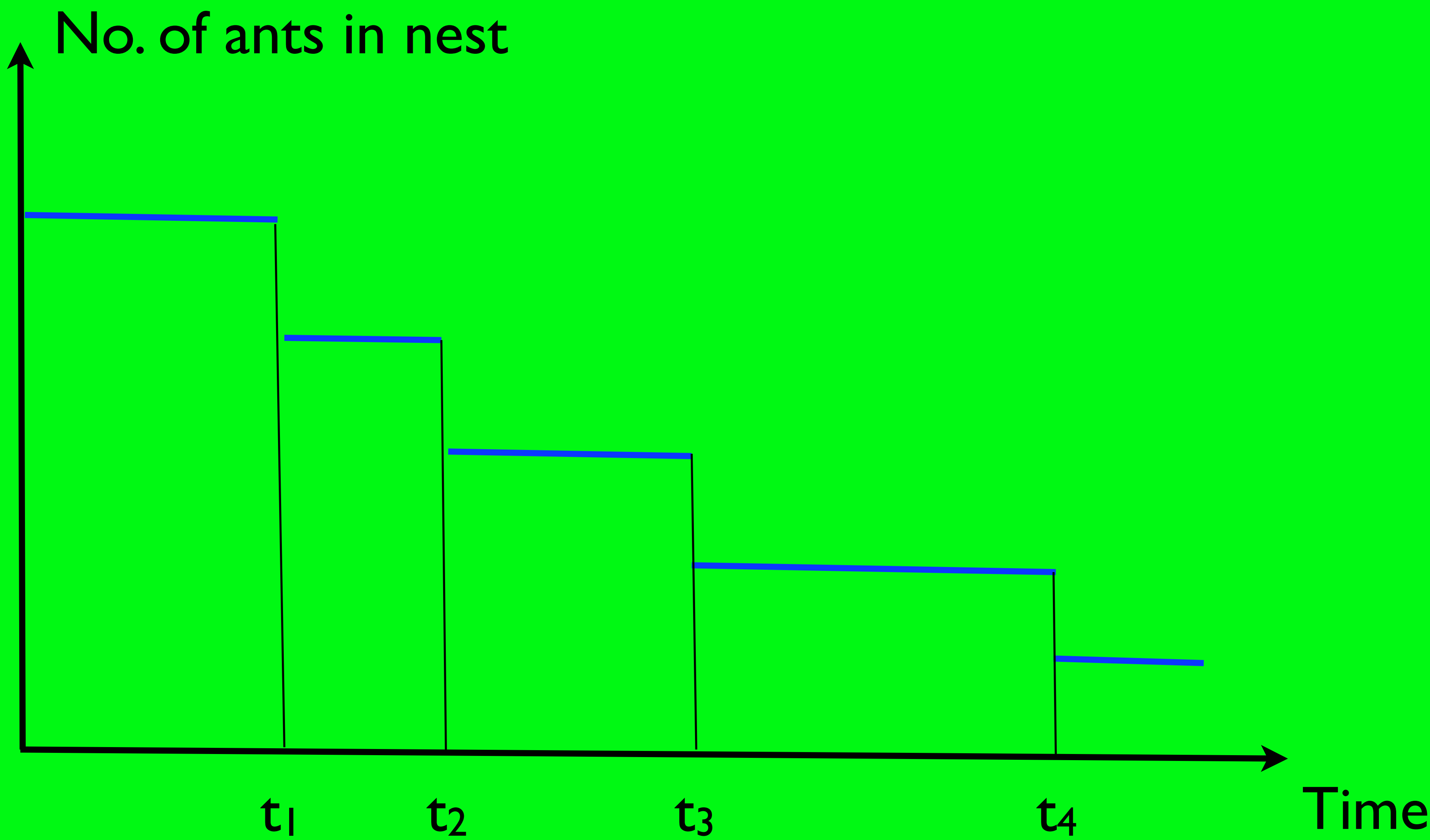
From Oliveira, Jensen, Nicodemi & Sibani PRB 71, 104526 (2005)

Ant colonies

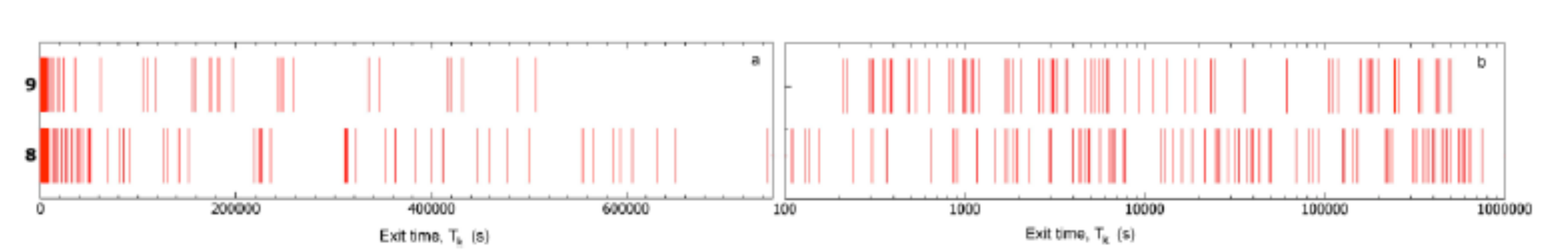
On ants: TO Richardson , EJH Robinson, AB Sendova-Franks,

NR Franks, E Arcaute, K Christensen

Ant colonies



Ant colonies



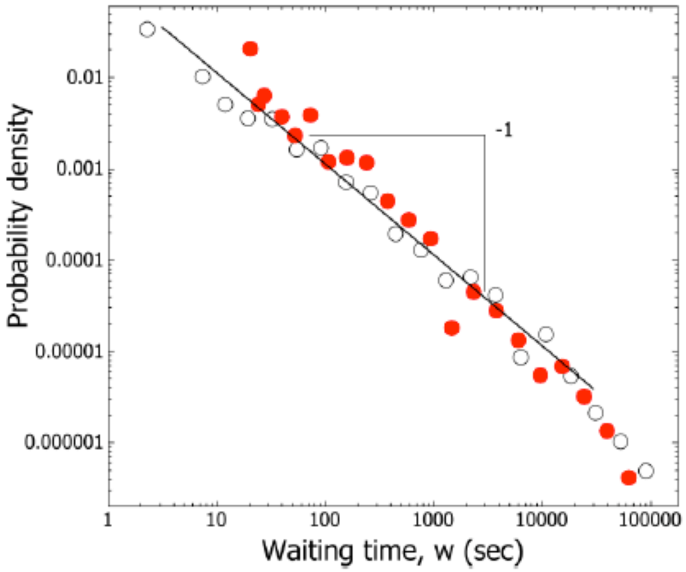


Figure 1. Waiting times between exits are not exponentially distributed.
In both removal (○) and non-removal (●) conditions for the five colonies which underwent both the waiting time probability densities between exits, $w = T_k - T_{k-1}$, follow a heavy-tailed distribution, that is closer to a power-law distribution, $P(w)=w^{-k}$, where k=1, than an exponential, $P(w)=e^{-\lambda x}$, which will not give a straight line on a log-log plot

T.O. Richardson, E.J.H. Robinson, H.J. Jensen, N.R. Franks and A.B. Sendova-Franks, *Record Dynamics in Ants*, PloS One, **5**, e9621 (2010).

Ant colonies

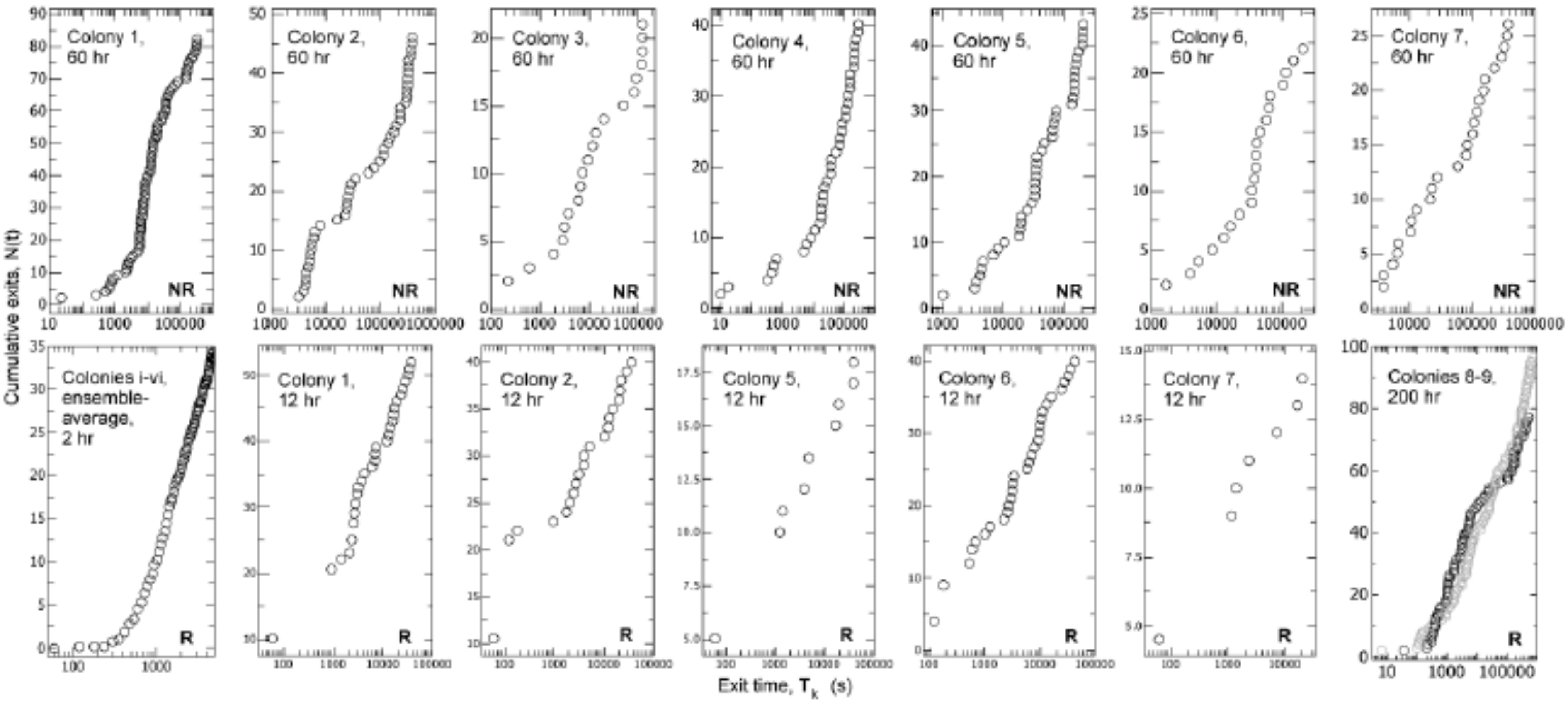


Figure 4. Events occur at random in logarithmic time.
Accumulated number of ant exits over time, $N(t)$ to exit time, T_k. The abscissa is logged to check for constant exit rates in logarithmic time when $T_k >> 1$. NR=non-removal control, R=removal treatment. Colum 1, row 2; Ensemble average, $\langle N(t) \rangle$ for six colonies (i-vi) undergoing 2 hours of external worker removal.

Earthquakes

The quakes are instantaneous on the time scale of the driving.

↓

after shocks relaxation under fixed boundary conditions

↓

Omori 1/t independent of ‘everything’

Earthquakes shocks - aftershocks

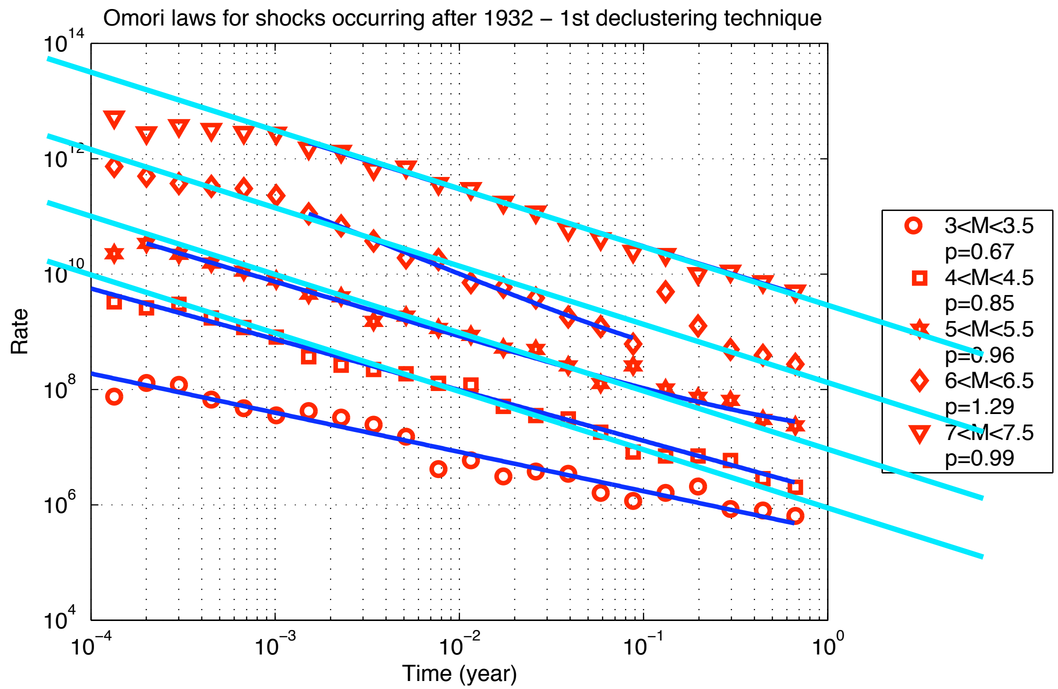


Figure 5. Seismic decay rates of stacked sequences for several magnitude intervals of the mainshocks, for the period from 1932 to 2003 when using the first declustering technique.

From Ouillon and Sornette
Magnitude-depended Omori law, J Geophys Res 111, B04306 (2005)

Record dynamics - and 1/f noise

Short time correlations in logarithmic τ=ln(t)

$$\mathrm{C}(\tau) \simeq \exp(-\lambda\tau) = t^{-\lambda}$$

suggesting a power spectrum (for the non-stationary process) behaving like

$$\mathrm{S}(\omega) \propto 1/\omega^{1-\lambda}$$

with $\lambda = 1/ln(T_0) \sim 1$

Sibani & Littlewood, PRL 71, 1482 (1993)

Record dynamics

Questions:

Which fluctuating quantity undergoes records?

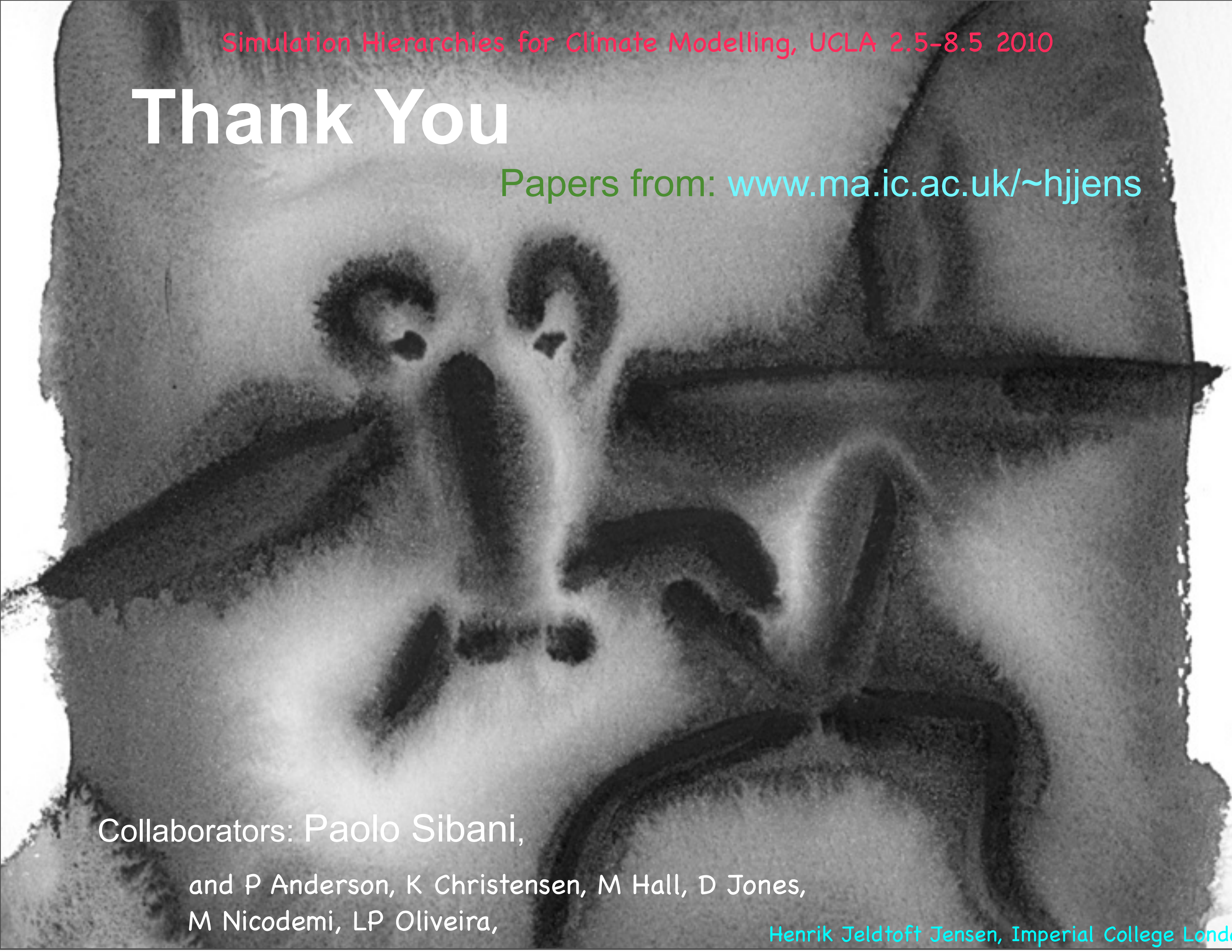
or in other words: the "after shocks" are related to which records?

1/f noise

- Not noise - but fluctuations (deterministic lattice gas)
- Always indicative of long time correlations
- Several mechanisms possible

Consequences of record dynamics.

- Statistics of quake times independent of underlying "noise mechanism"
- Deceasing rate of events
- Likely to show 1/f behaviour


Simulation Hierarchies for Climate Modelling, UCLA 2.5-8.5 2010
Thank You
Papers from: www.ma.ic.ac.uk/~hjjens
Collaborators: Paolo Sibani,
and P Anderson, K Christensen, M Hall, D Jones,
M Nicodemi, LP Oliveira,
Henrik Jeldtoft Jensen, Imperial College London