

# CONVECTIVELY COUPLED WAVES IN A SHEARED BACKGROUND

Boualem Khouider and Ying Han  
University of Victoria

IPAM, Equations Hierarchies, March 22-26, 2010

Acknowledgment: Natural Sciences and Engineering Research Council of Canada (NSERC) and The Canadian Foundation for Climate and Atmospheric Sciences (CFCAS)

# Papers on Multi-cloud Model

- K. and Majda, J. Atmos. Sci. 2006: Linear theory
- K. and Majda, J. Atmos. Sci. 2007: Non-linear simulations
- K. and Majda, J. Atmos. Sci, 2008: Enhanced congestus heating
- K. and Majda, J. Atmos. Sci., 2008: Equatorial CCW's
- Waite and K., J. Atmos., 2009: Boundary layer dynamics
- K., Biello, and Majda: Stochastic multcloud model, CMS 2009
- Han and K., J. Atmos., 2010: CCW's in sheared environment
- K., St Cyr, Majda, and Tribbia, J. Atmos. Sci. 2010 (submitted): MJO and CCW's in GCM
- Slides of Lectures (and 3 videos) on tropics: Banff workshop, April 2009. [www.birs.ca](http://www.birs.ca)



# 09w5073 Multiscale Processes in the Tropics

**Arriving Sunday, April 26 and departing Friday, May 1, 2009**

**Organizers:** Boualem Khouider (University of Victoria), Andrew Majda (New York University - Courant Institute of Mathematical Sciences), Norman McFarlane (Canadian Centre for Climate Modelling and Analysis), Adam Monahan (University of Victoria).

[Confirmed Participants](#)

[Press Release: Multiscale Processes in the Tropics](#)

[Information for Participants](#)

[Schedule and Abstracts \(PDF file\)](#)

[Mailing List](#)

[Workshop Files](#)

[Final Report \(PDF file\)](#)

[Workshop Videos](#)

# Outline

- Motivation: Tropical waves in non-homogeneous background
- Short review of multicloud model and linear equations in background shear
- Results for Case of Meridional Shear: Non Kelvin aspects of Kelvin waves and emergence of Rossby and MRG wave instabilities in multicloud model
- Case of Vertical Shear: Preferred direction of CCW's depends on wind direction at low- and high-level
- Concluding remarks

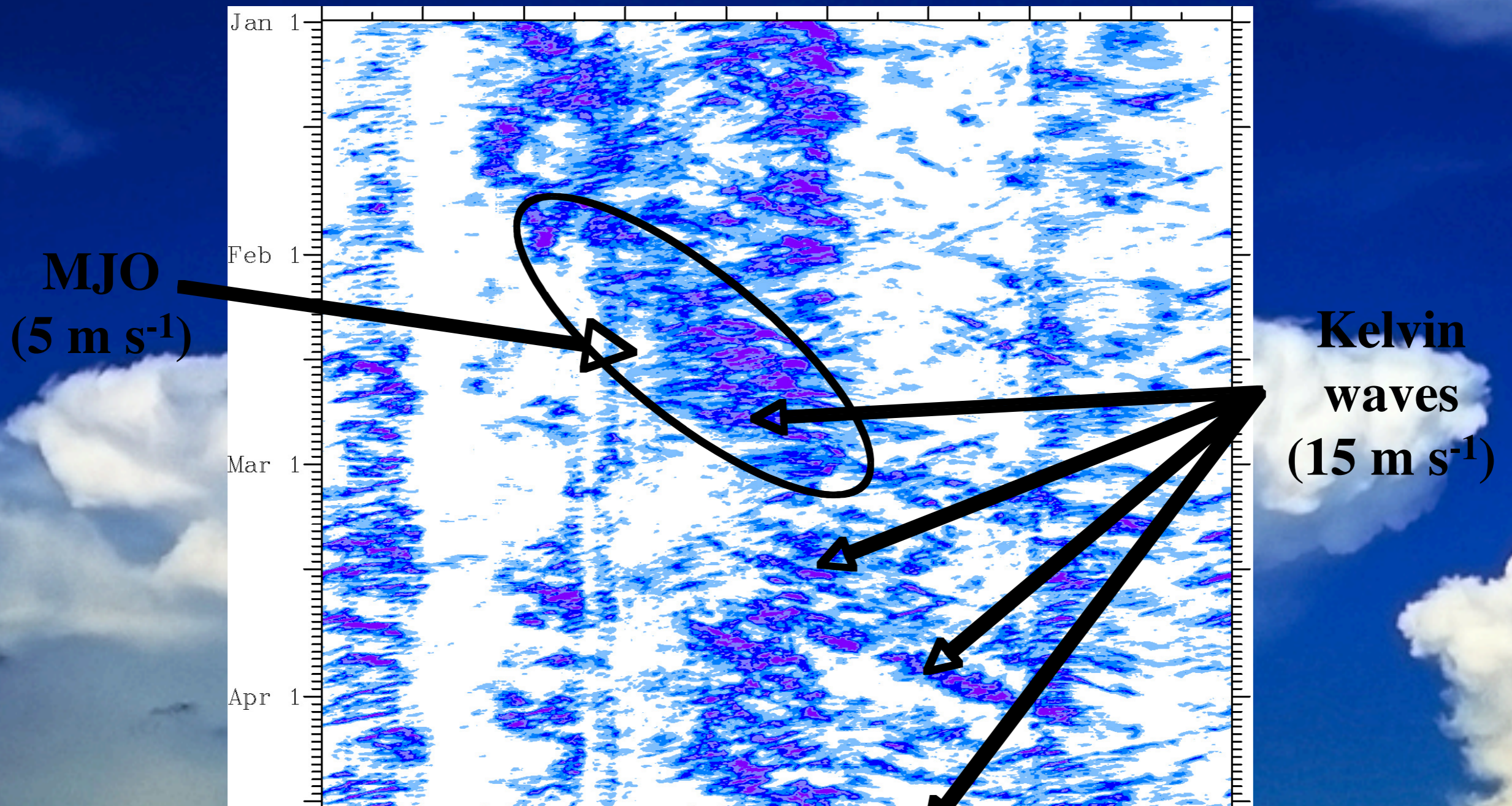
# Motivation

- Equatorially trapped waves are linear wave solutions to the equatorial beta-plane equations (Matsuno 1966)
- Richer spectrum of waves compared to midlatitudes
- EW's modulate and interact with tropical convection on various time and spatial scales (Takayabu 1994, Wheeler and Kiladis 1999, etc.)
- Obs revealed that background meridional flows as well as the MJO envelope affect propagation and morphology of tropical waves (Kiladis et al. 2005, Roundy 2008, Kiladis et al. 2009)
- Non-Kelvin aspects of Convectively-Coupled Kelvin Waves (Kiladis, 2009)... such as meridional outflow at high levels, in regions of deep convection ... that are more prominent during strong MJO events (Roundy 2008).
- “There are strong associations between Kelvin wave activity and extra-tropical Rossby wave activity” G. Kiladis.



# OBSERVATIONS OF KELVIN WAVES AND THE MJO

Time–longitude diagram of CLAU S Tb (2.5S–7.5N), January–April 1987



“Straub and Kiladis (2003a) showed that the number of westward propagating MRG and easterly waves, along with tropical depressions, is increased within the MJO convective region over the western Pacific. The MJO also modulates Kelvin wave activity (eastward propagating superclusters) within its envelope over the Indian Ocean, but well to the east of its location once it propagates into the western Pacific” Kiladis et al. 2005)

# Kelvin Events during Different Phases of the Indian Ocean MJO ... several differences

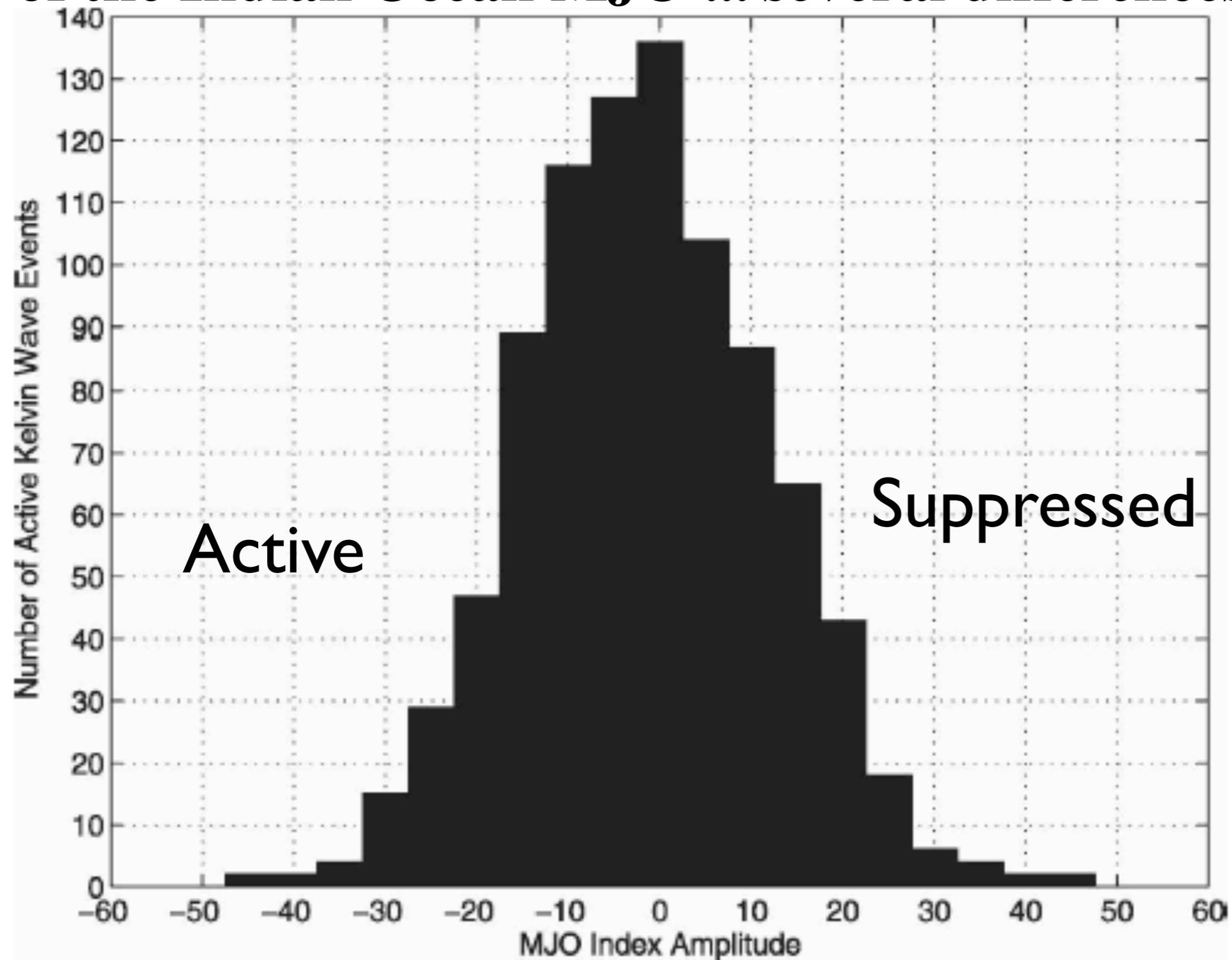
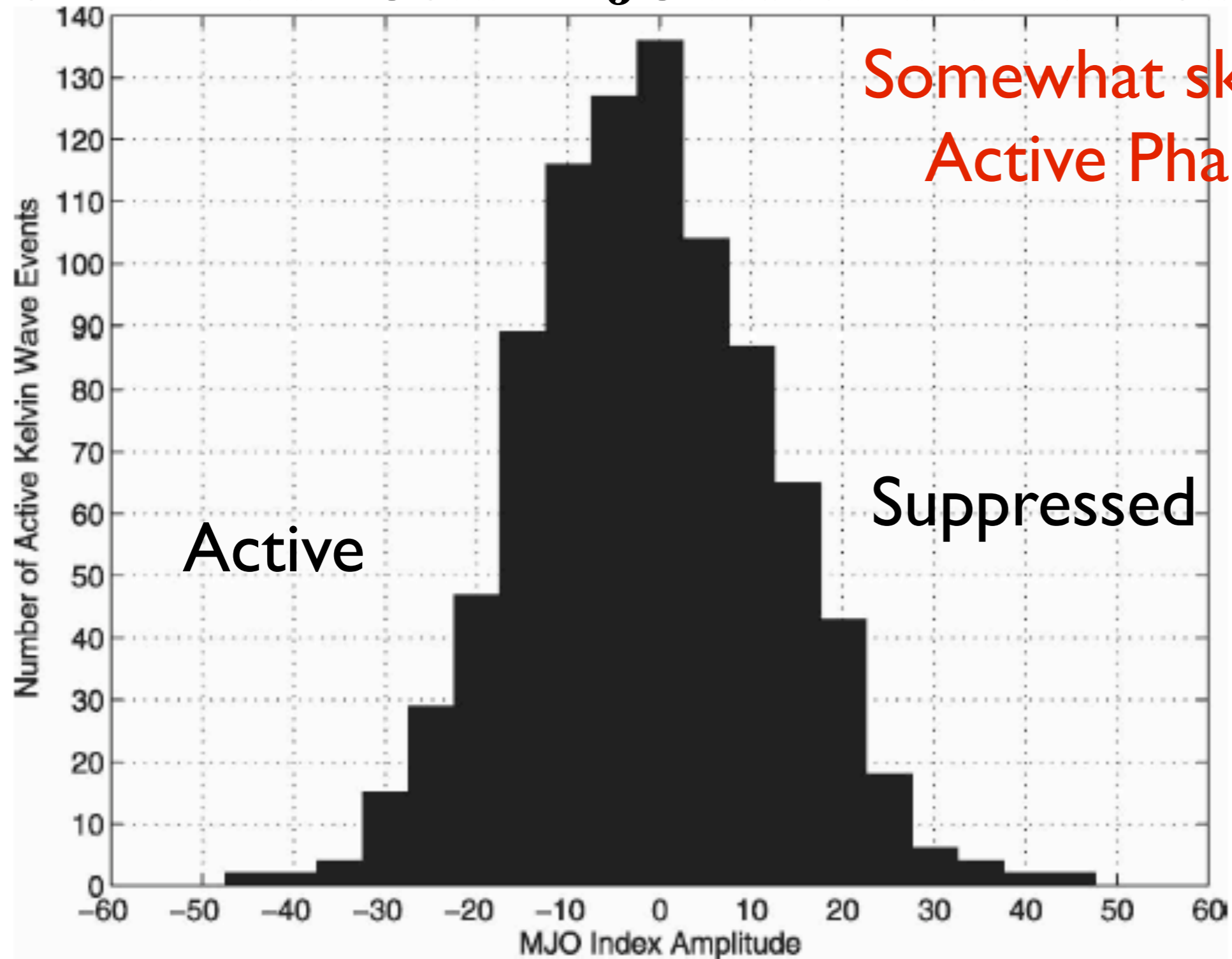


FIG. 2. Histogram of OLR anomalies ( $\text{W m}^{-2}$ ) in the equatorial MJO index (from  $80^\circ\text{E}$ ) on the primary list of dates of Kelvin wave active convective events at  $80^\circ\text{E}$ . The bin width is  $5 \text{ W m}^{-2}$ .

# Kelvin Events during Different Phases

of the Indian Ocean MJO ... several differences



Somewhat skewed toward  
Active Phase (by 24%)

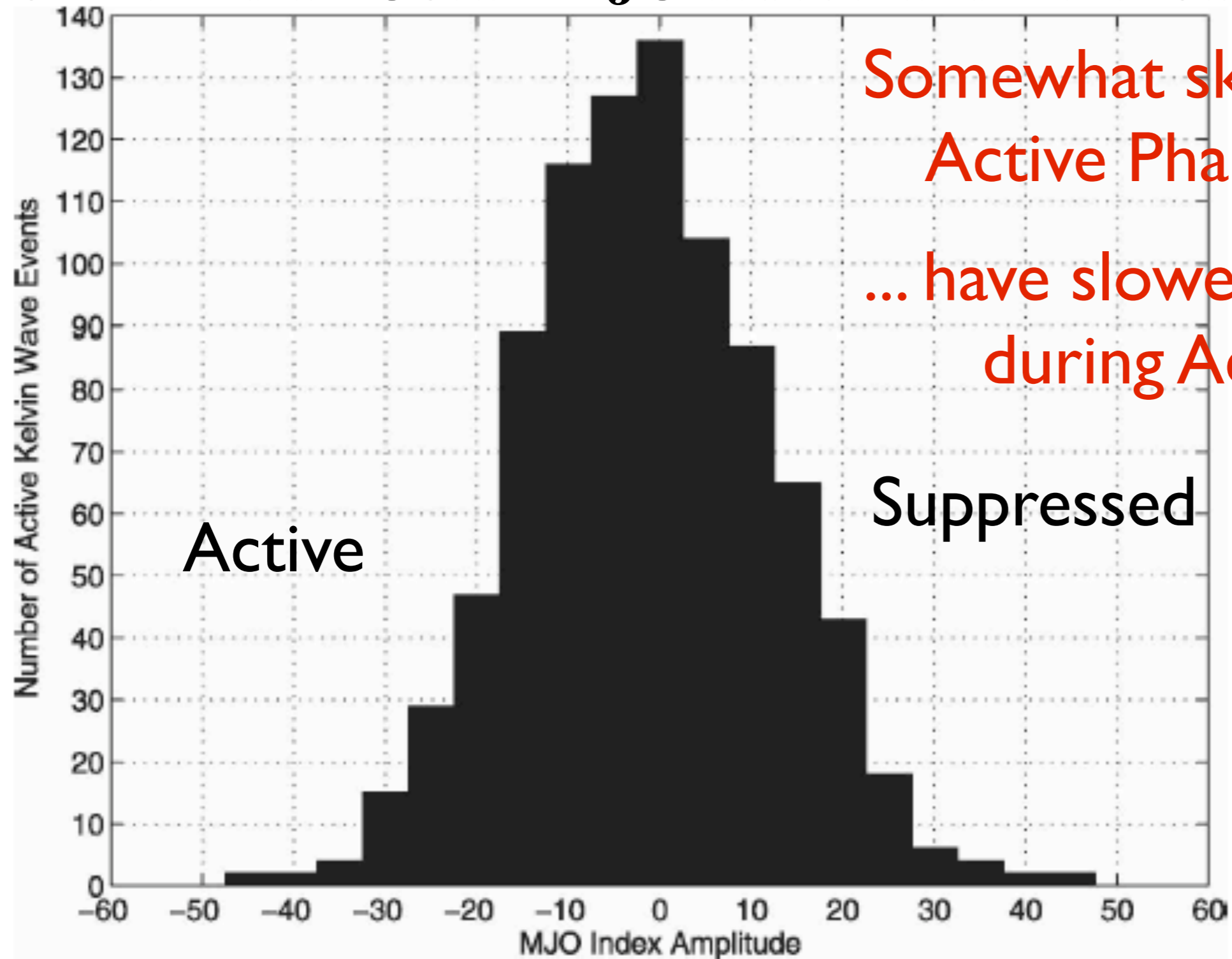
Active

Suppressed

FIG. 2. Histogram of OLR anomalies ( $\text{W m}^{-2}$ ) in the equatorial MJO index (from  $80^\circ\text{E}$ ) on the primary list of dates of Kelvin wave active convective events at  $80^\circ\text{E}$ . The bin width is  $5 \text{ W m}^{-2}$ .

# Kelvin Events during Different Phases

of the Indian Ocean MJO ... several differences



Somewhat skewed toward Active Phase (by 24%)

... have slower phase speeds during Active Phase

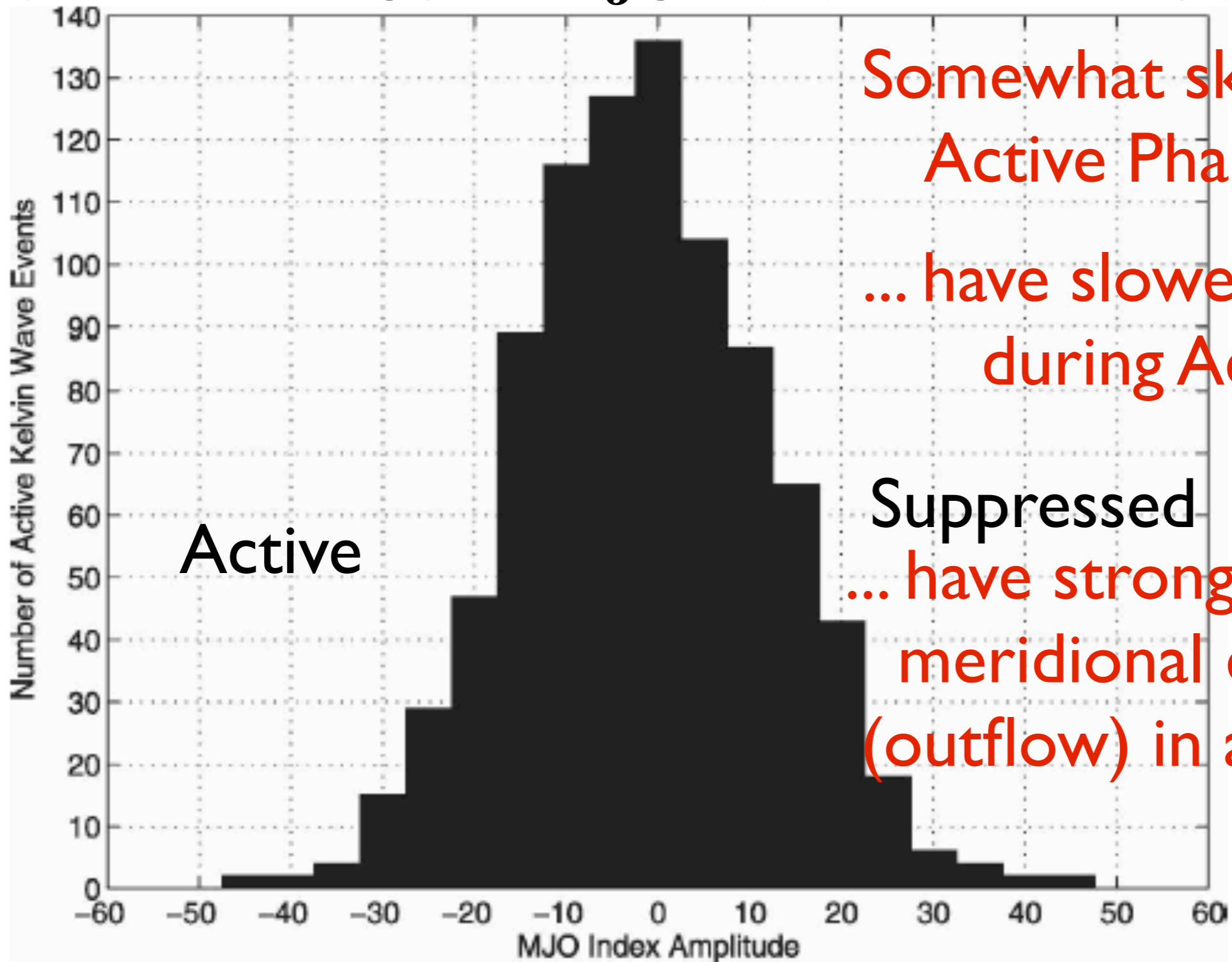
Suppressed

Active

FIG. 2. Histogram of OLR anomalies ( $\text{W m}^{-2}$ ) in the equatorial MJO index (from  $80^\circ\text{E}$ ) on the primary list of dates of Kelvin wave active convective events at  $80^\circ\text{E}$ . The bin width is  $5 \text{ W m}^{-2}$ .

# Kelvin Events during Different Phases

of the Indian Ocean MJO ... several differences



Somewhat skewed toward Active Phase (by 24%)

... have slower phase speeds during Active Phase

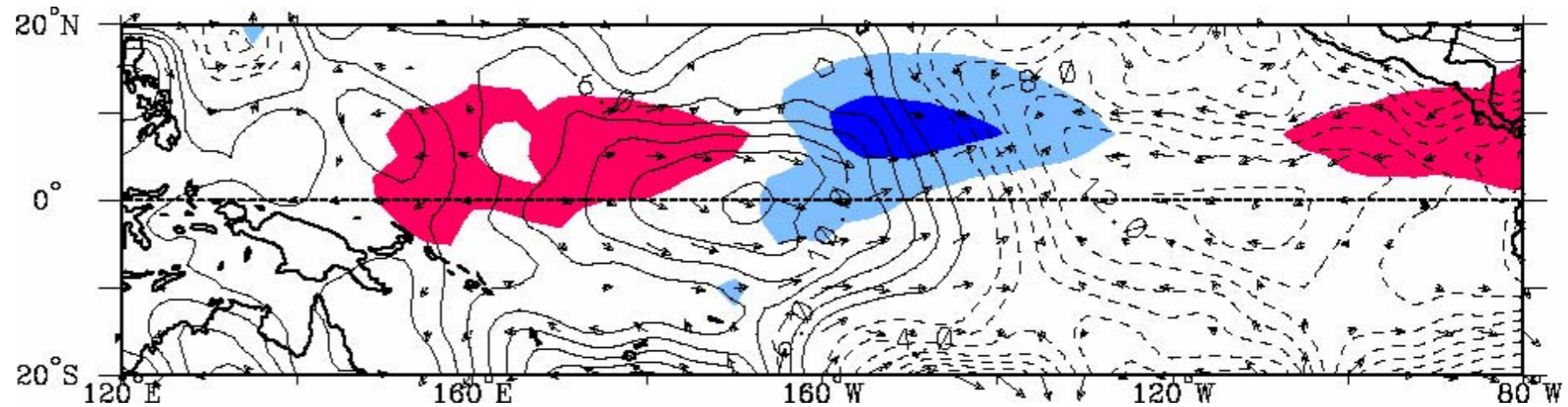
Suppressed

... have stronger high-level meridional divergence (outflow) in active phase

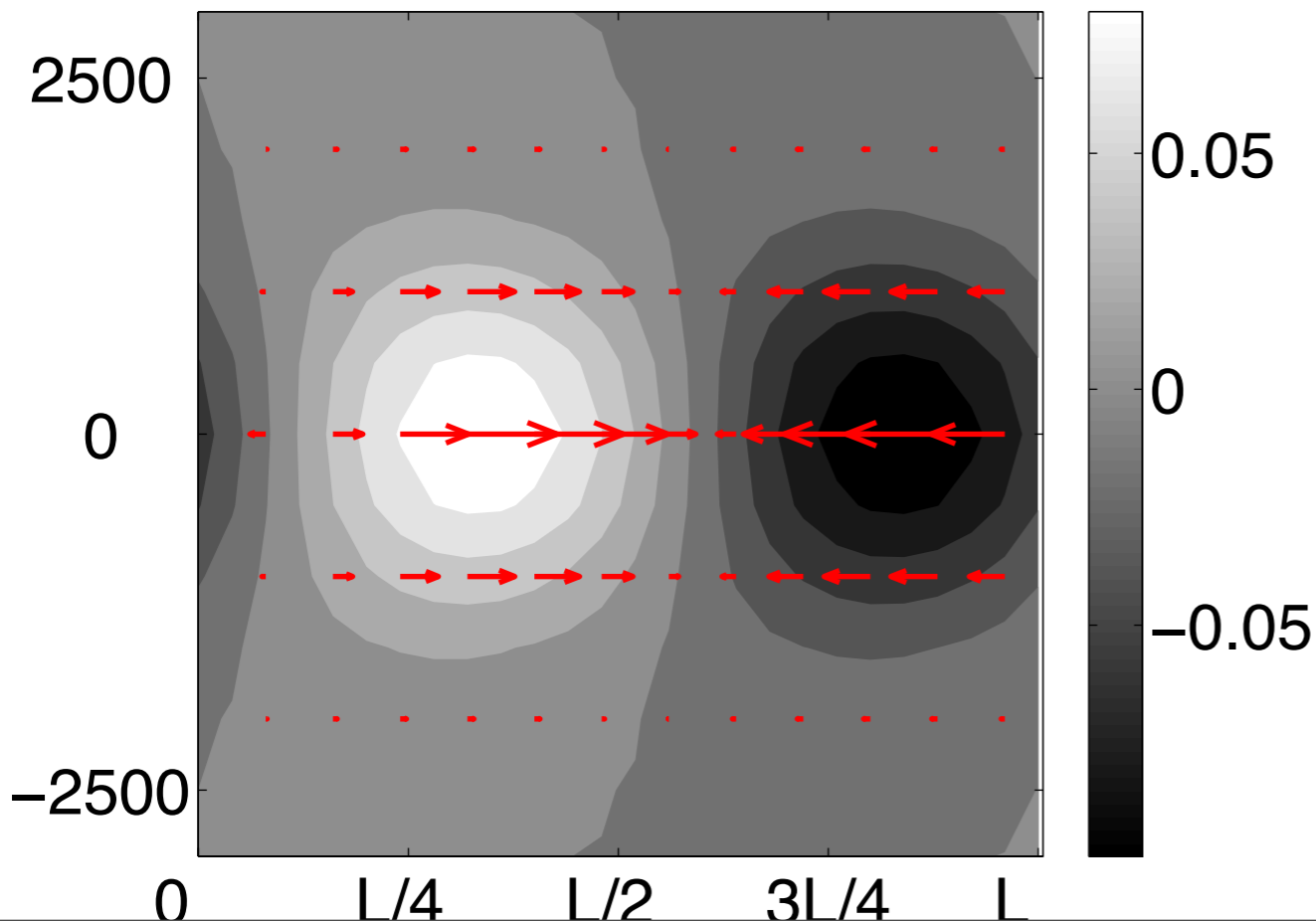
FIG. 2. Histogram of OLR anomalies ( $\text{W m}^{-2}$ ) in the equatorial MJO index (from  $80^\circ\text{E}$ ) on the primary list of dates of Kelvin wave active convective events at  $80^\circ\text{E}$ . The bin width is  $5 \text{ W m}^{-2}$ .

OLR and 1000 hPa Flow Regressed against Kelvin-filtered OLR  
(scaled  $-20 \text{ W m}^2$ ) at  $10^\circ\text{N}$ ,  $150^\circ\text{W}$  for June-Aug. 1979-2004

Day 0

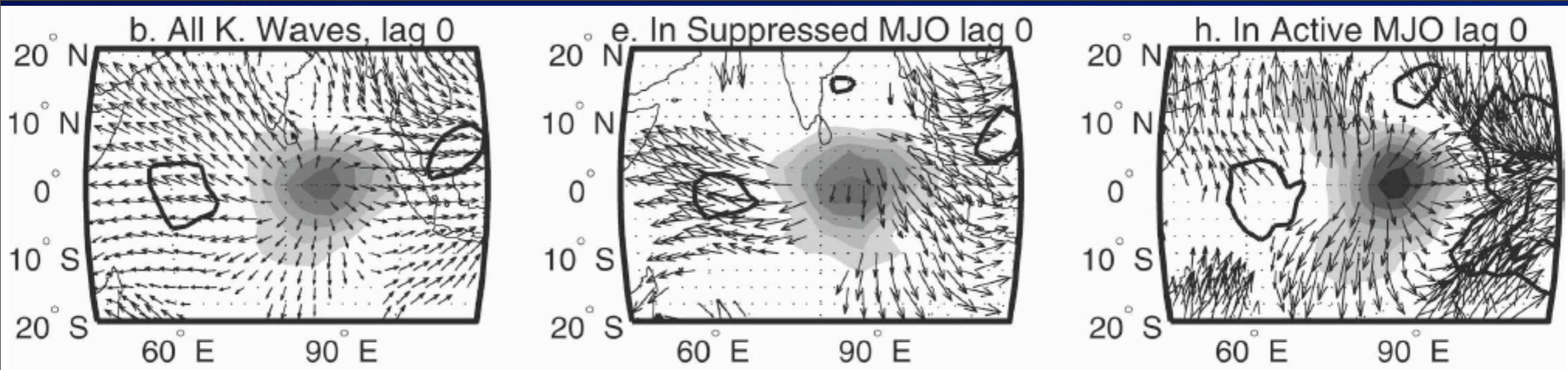


Pressure, 0 km Elevation

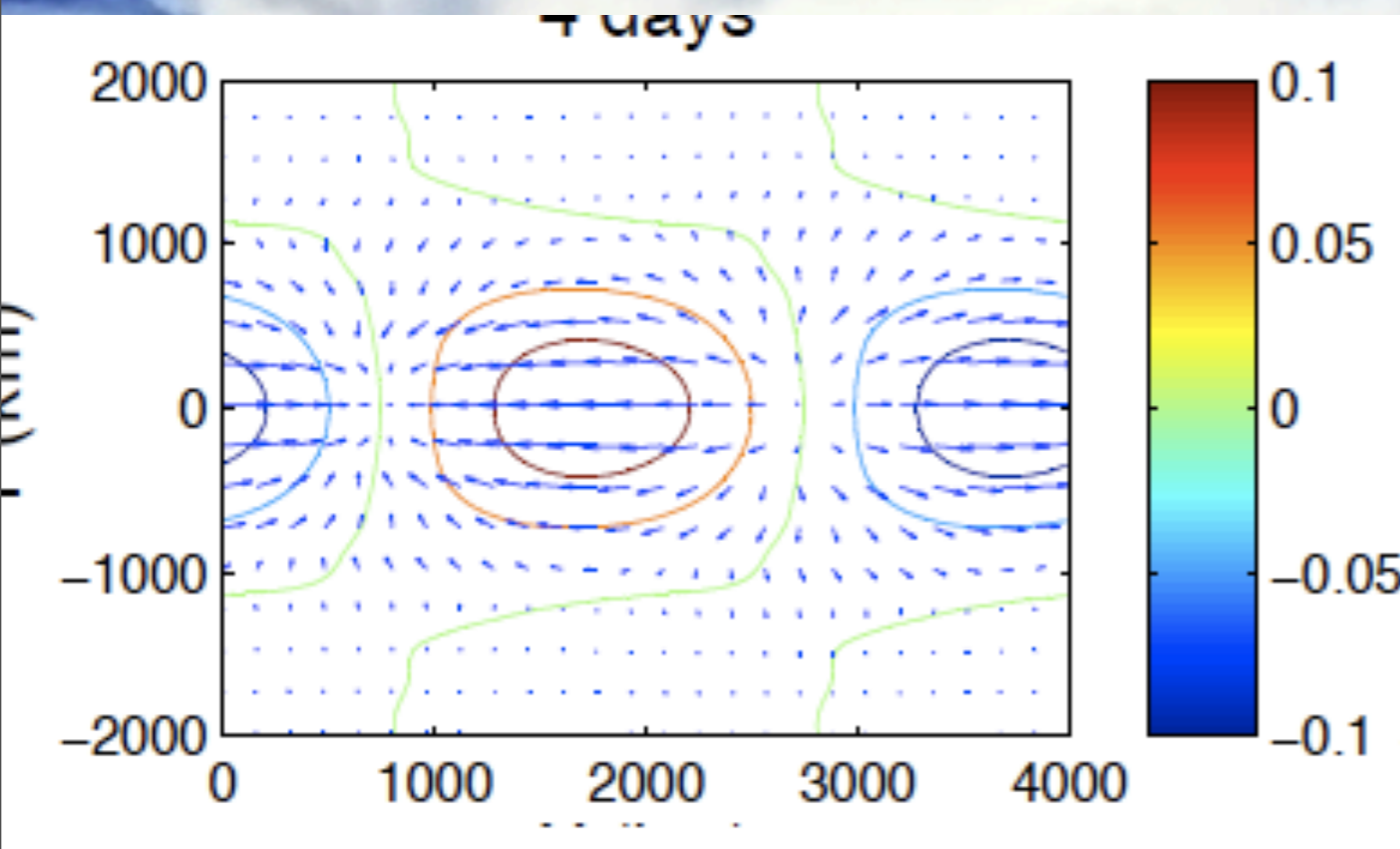


Geopotential Height (contours 2 m)  
Wind (vectors, largest around  $5 \text{ m s}^{-1}$ )  
OLR (shading starts at  $\pm 6 \text{ W s}^{-2}$ ), negative blue

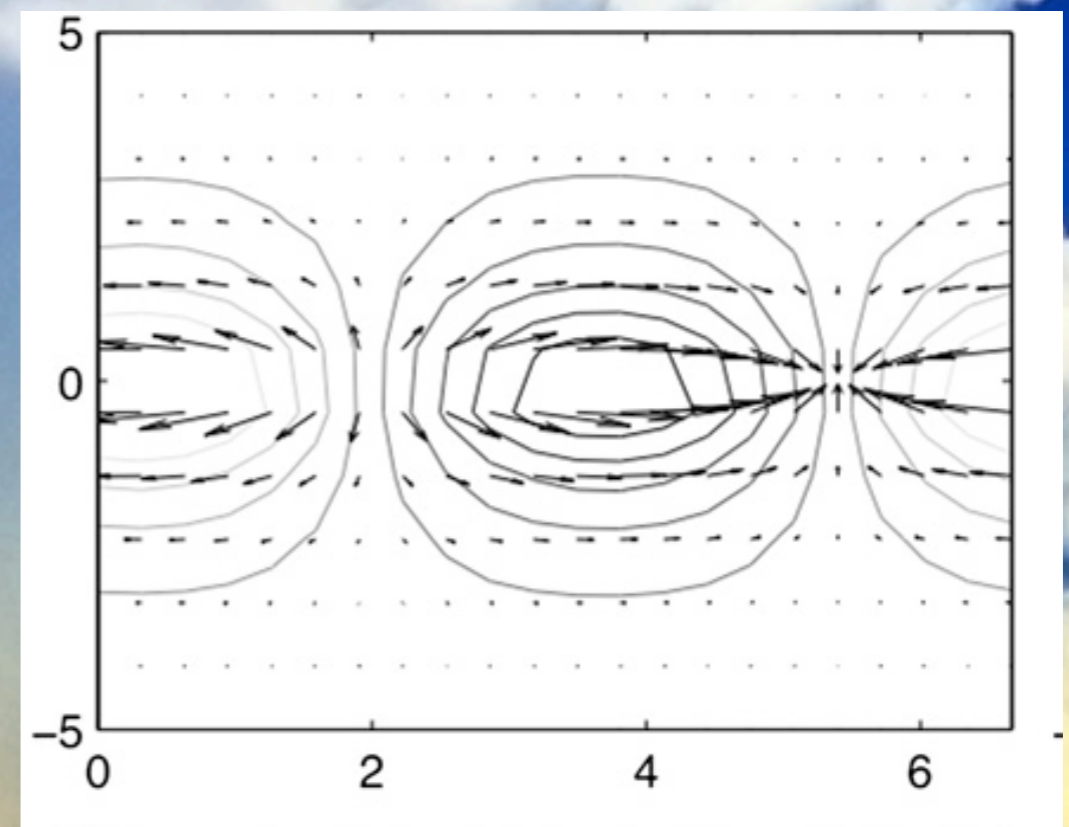
G. Kiladis



## Kelvin wave composites from Roundy 2008

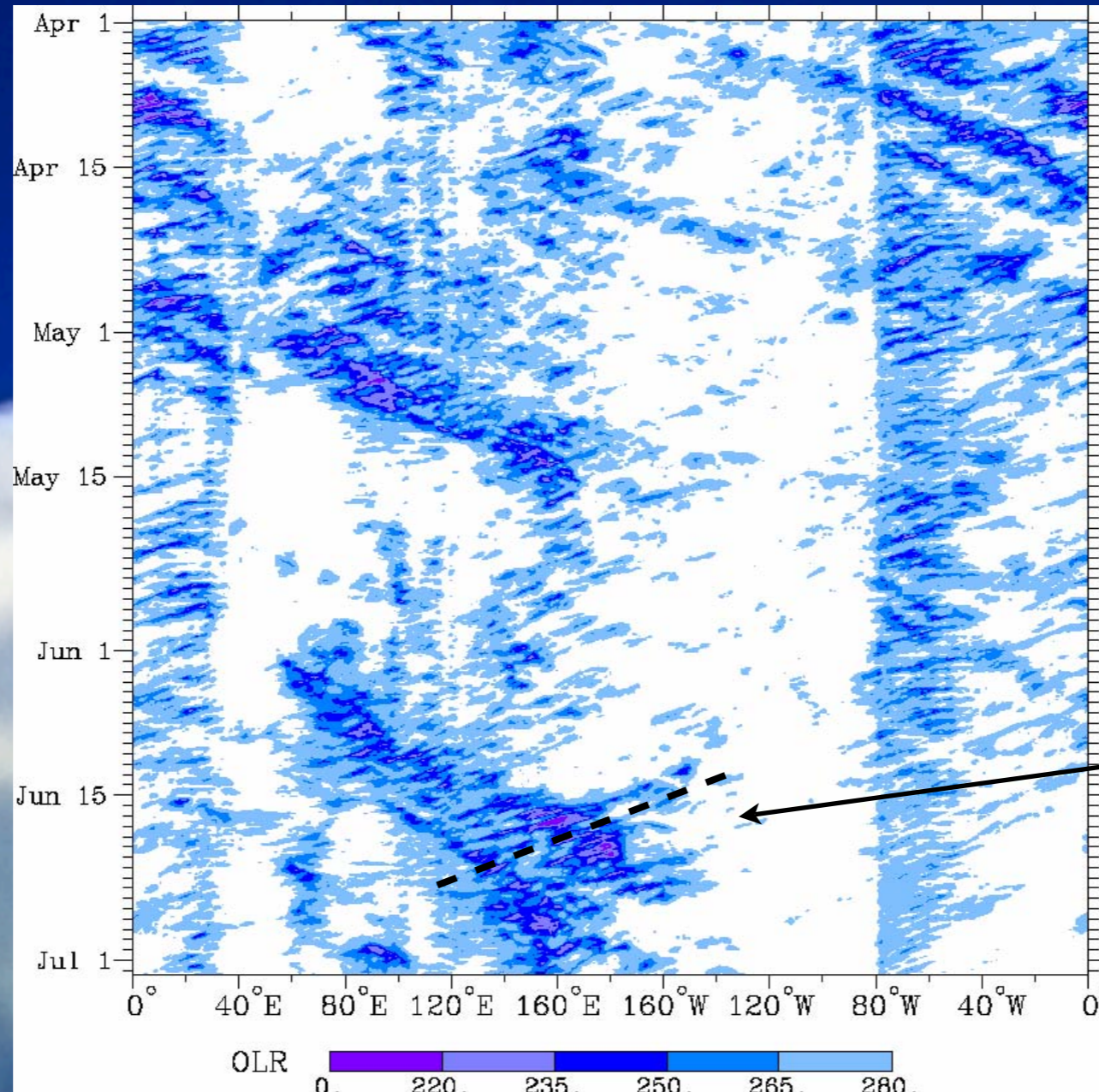


Dry Kelvin in a meridional shear background  
(Ferguson, K. and Namazi 2009)



(Dry) Kelvin propagating through an idealized  
ICTZ (Dias and Pauluis 2009)

# Westward Inertio-Gravity Waves (2-day waves)

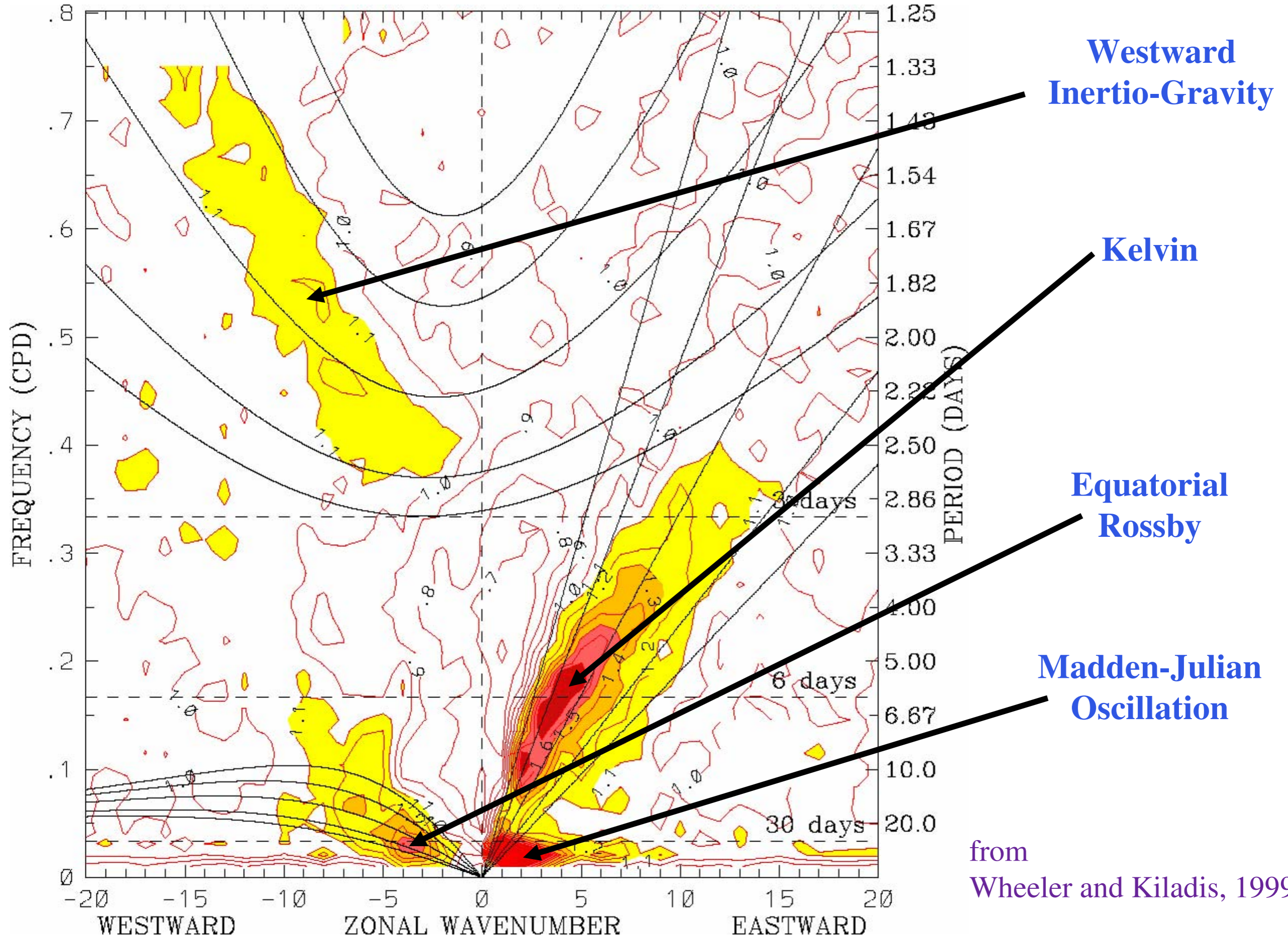


-16 m/s

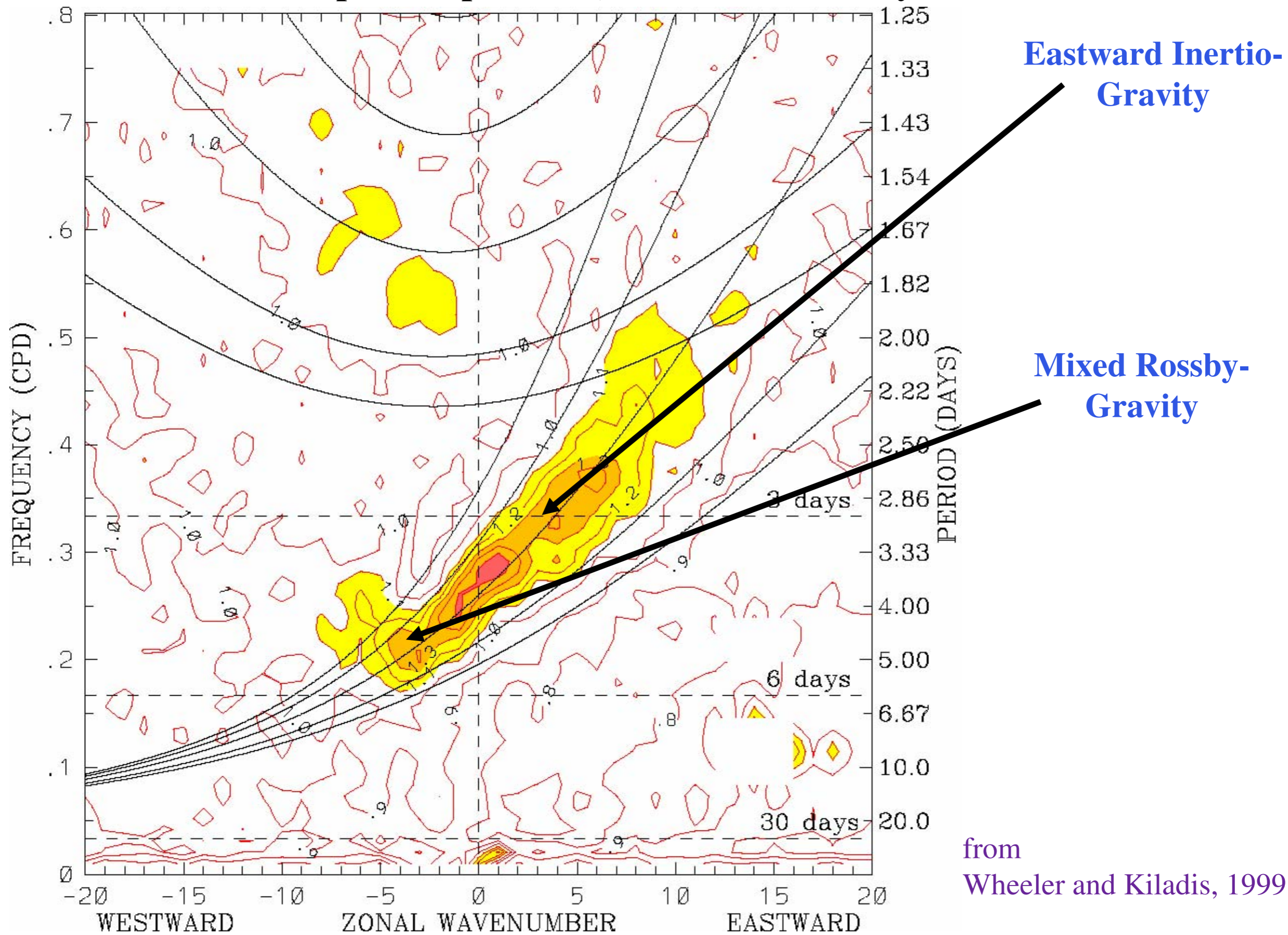
G. Kiladis

Often Associated with MJO and have phase speeds ranging from 10 to  $>30 \text{ m s}^{-1}$  Kiladis et al. (2009)

# OLR power spectrum, 1979–2001 (Symmetric)

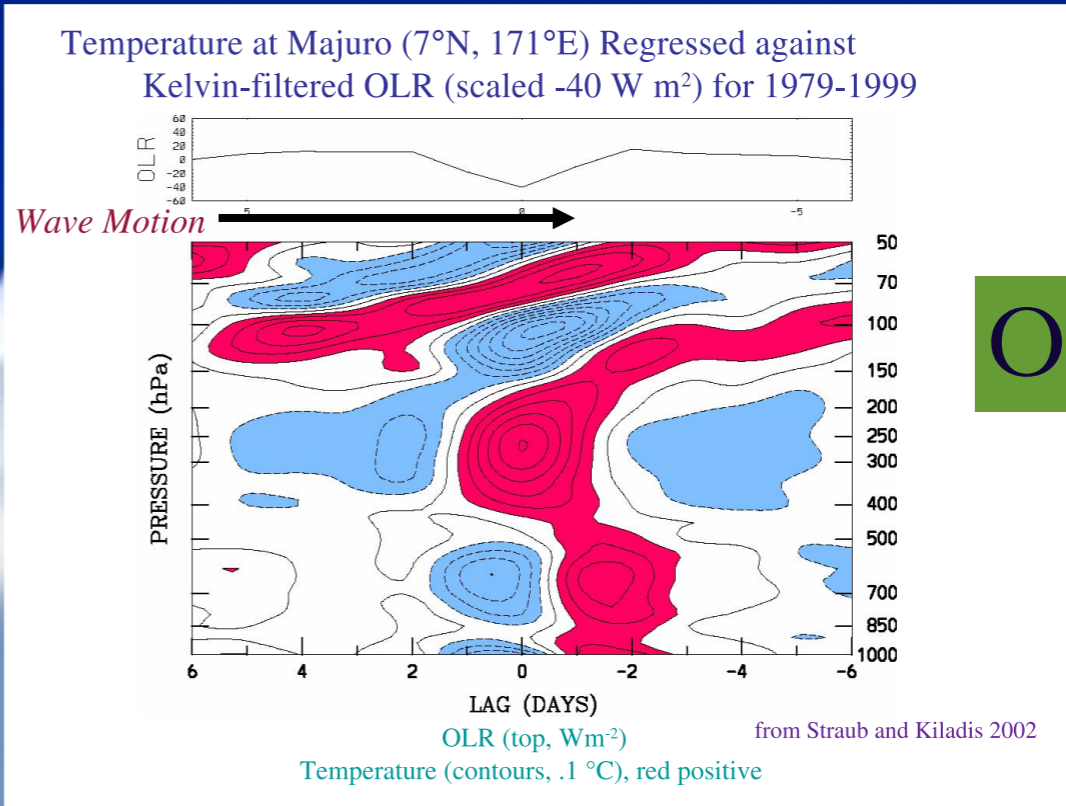
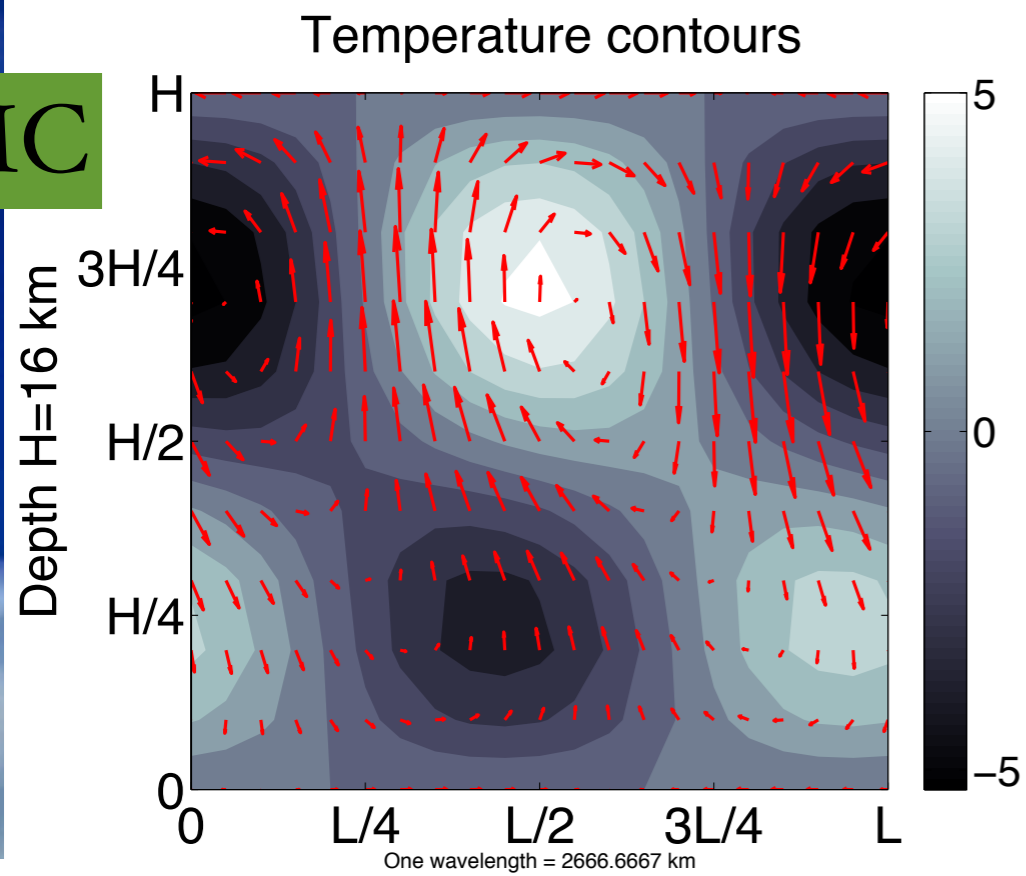


# OLR power spectrum, 1979–2001 (Antisymmetric)

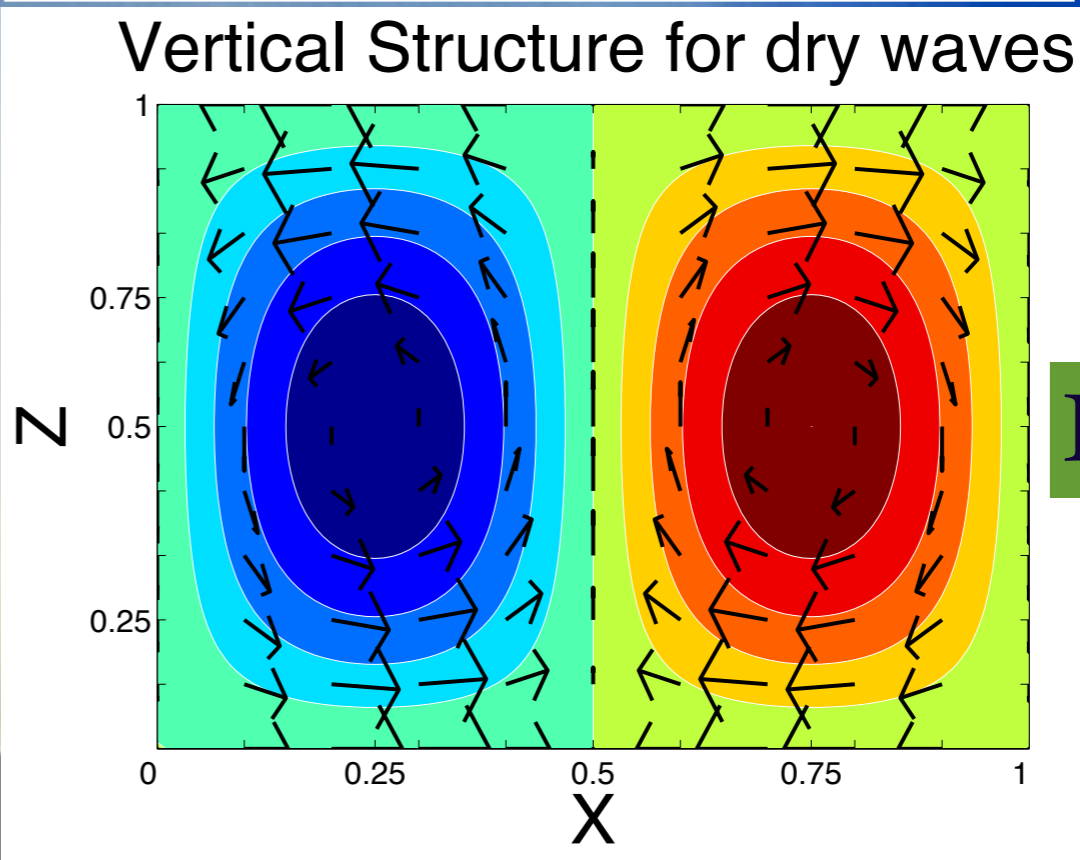
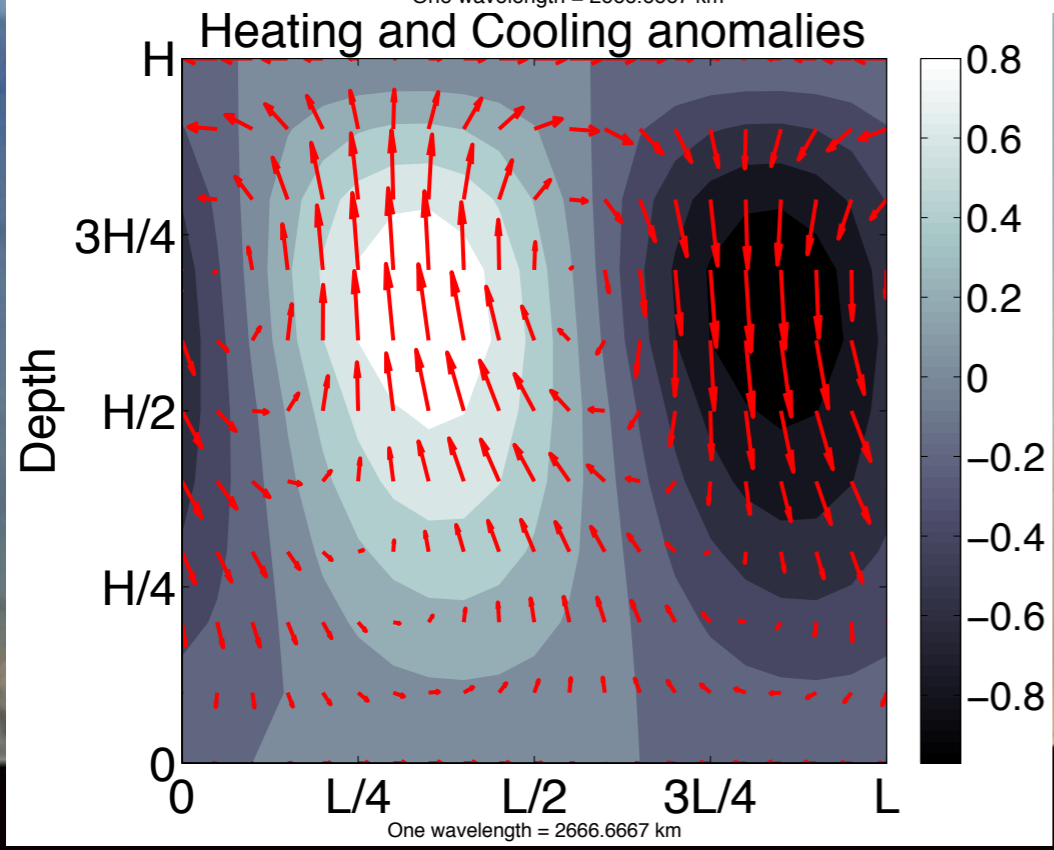


# Vertical Structure of Kelvin Wave

MC



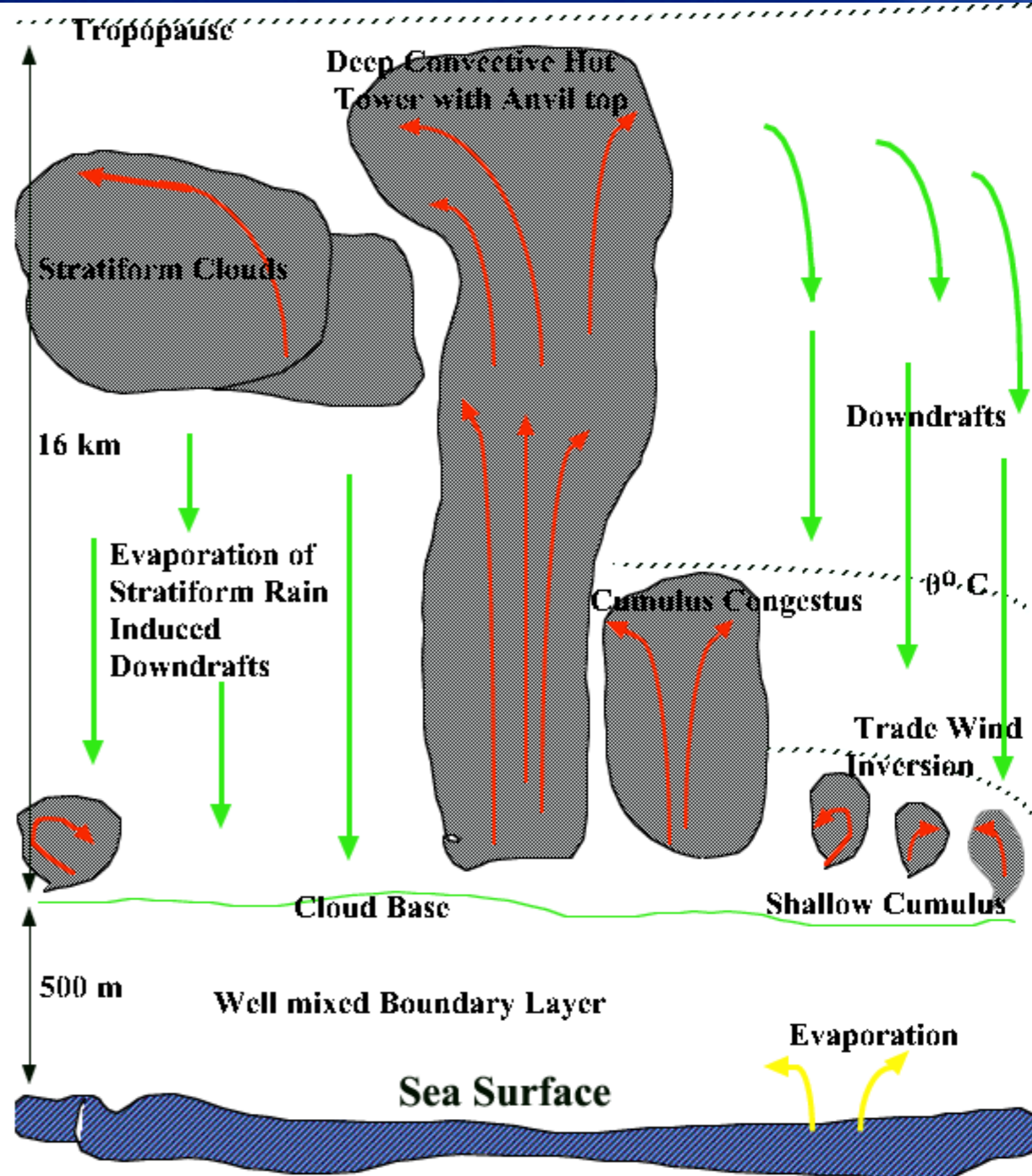
Obs



Dry

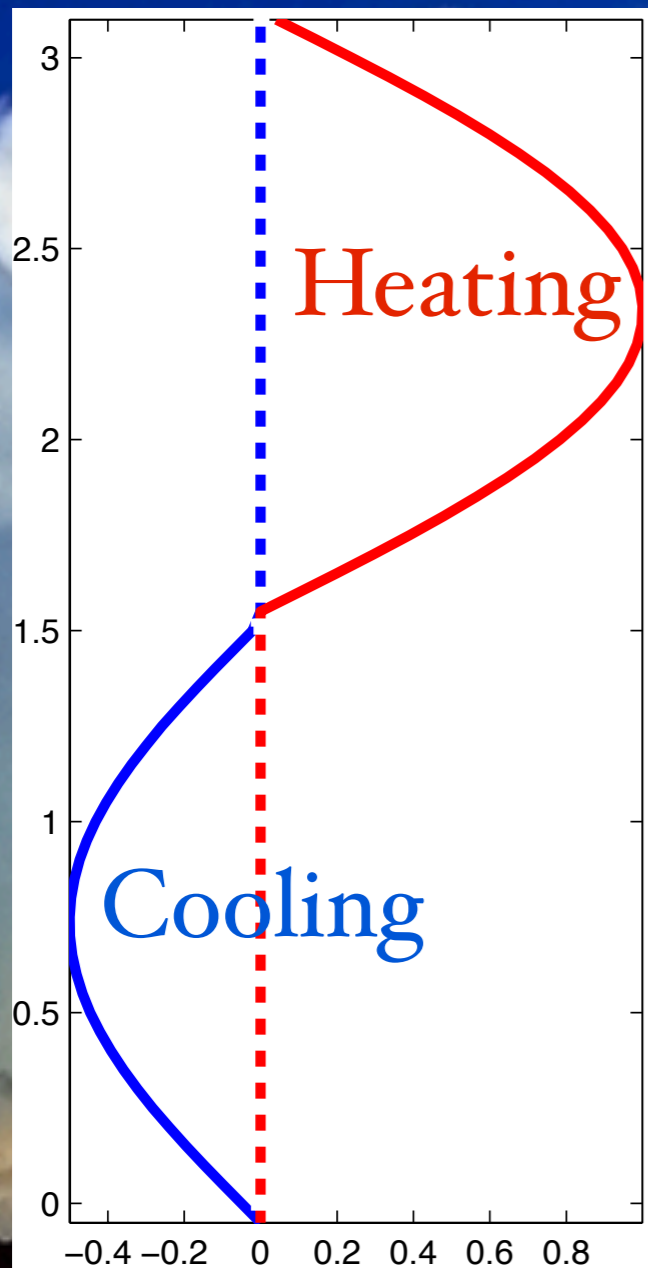
# THE THREE CLOUD MODEL: AN IDEALIZED PICTURE

Self-similar structure of organized tropical convective systems: form the individual convective cell (represented here) to the meso-scale clusters (squall lines) to the synoptic scale super-clusters (Kelvin, 2 day waves) to the planetary/ intra-seasonal envelopes (MJO)

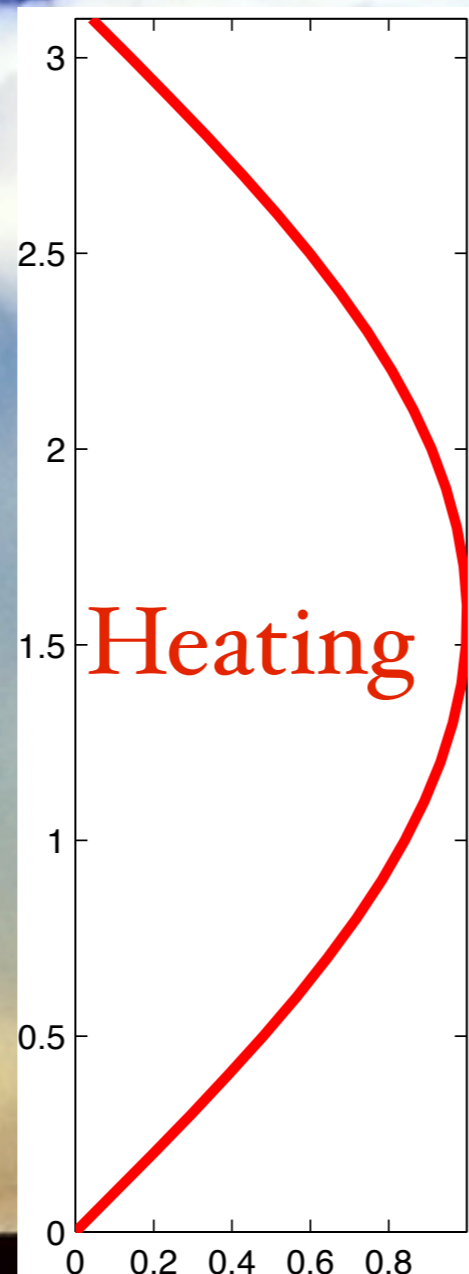


# Idealized heating profiles

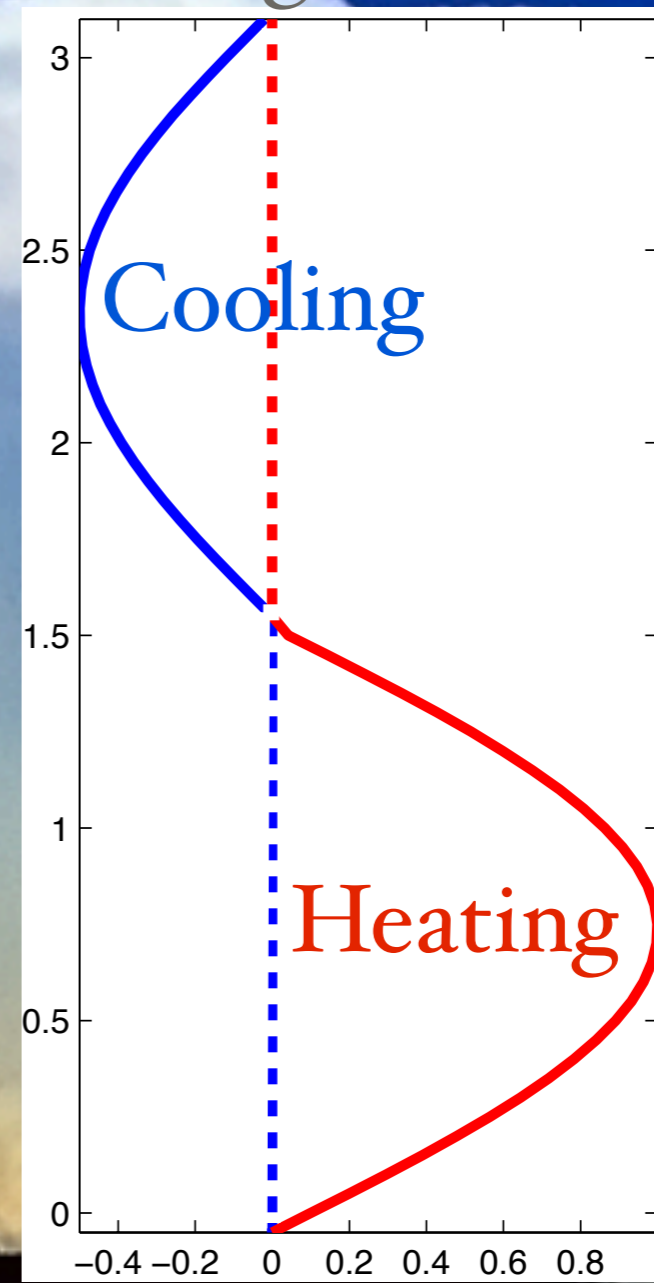
Stratiform



Deep



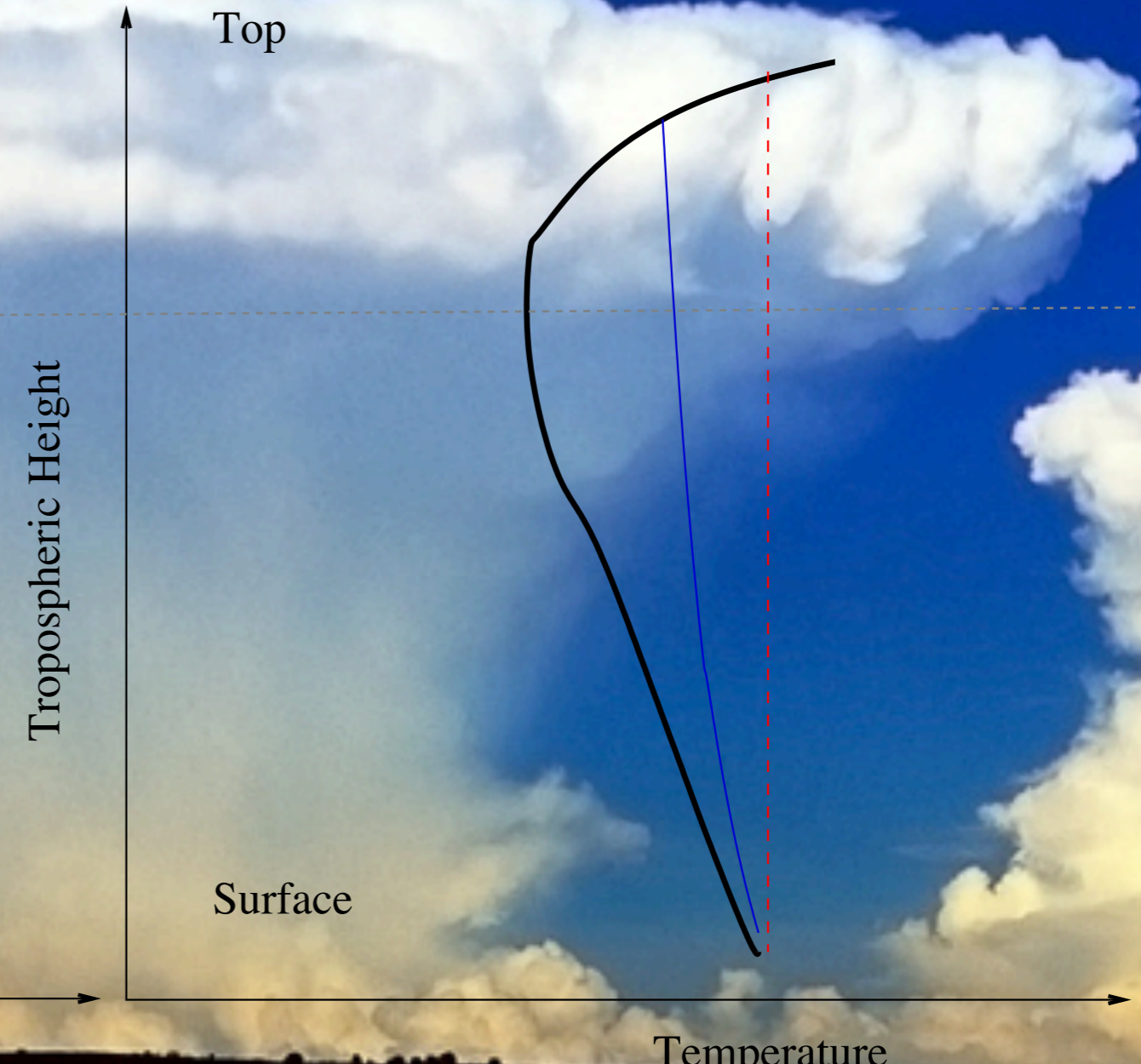
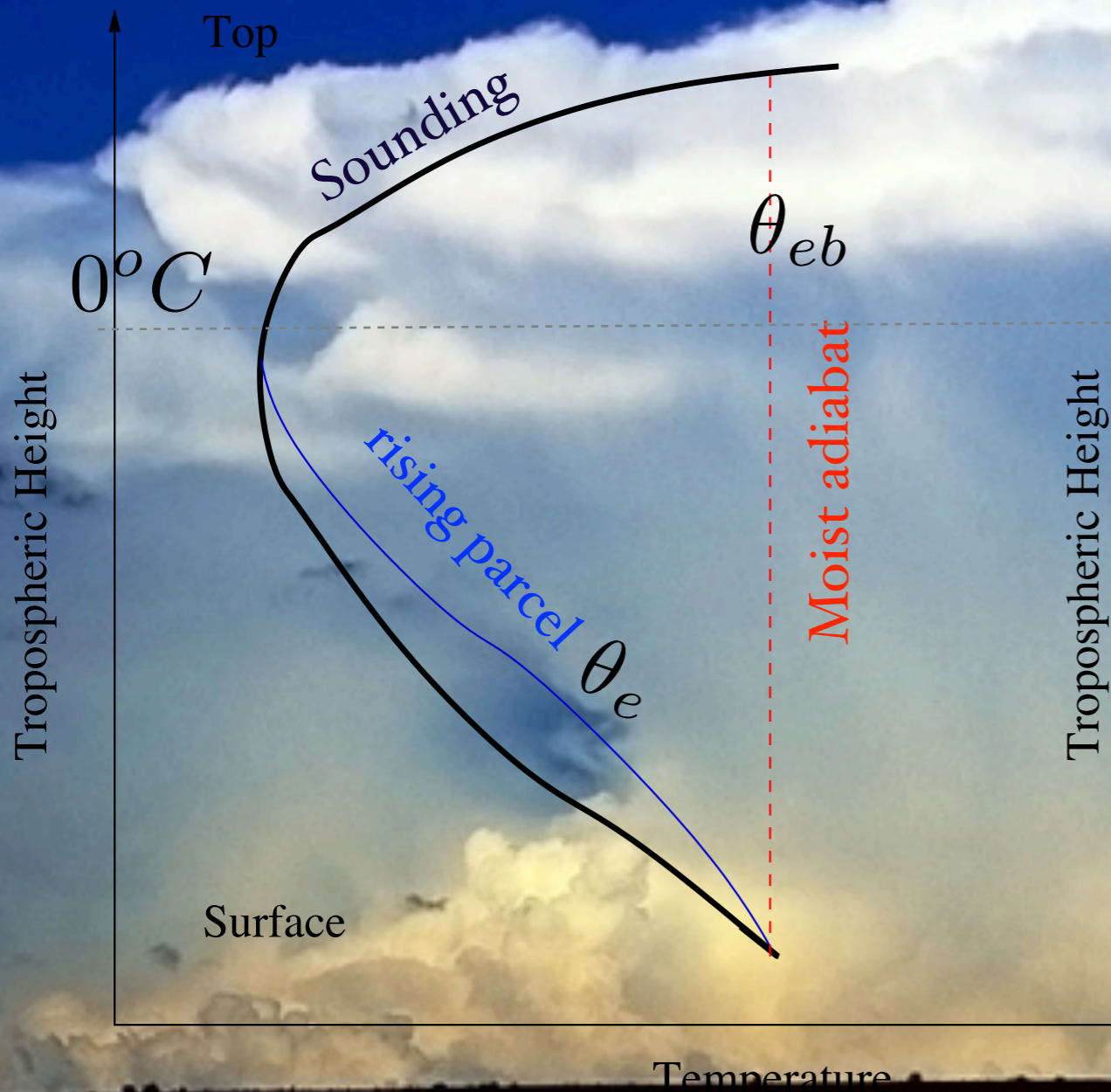
Congestus



# Dilute parcel lifting

Dry environment

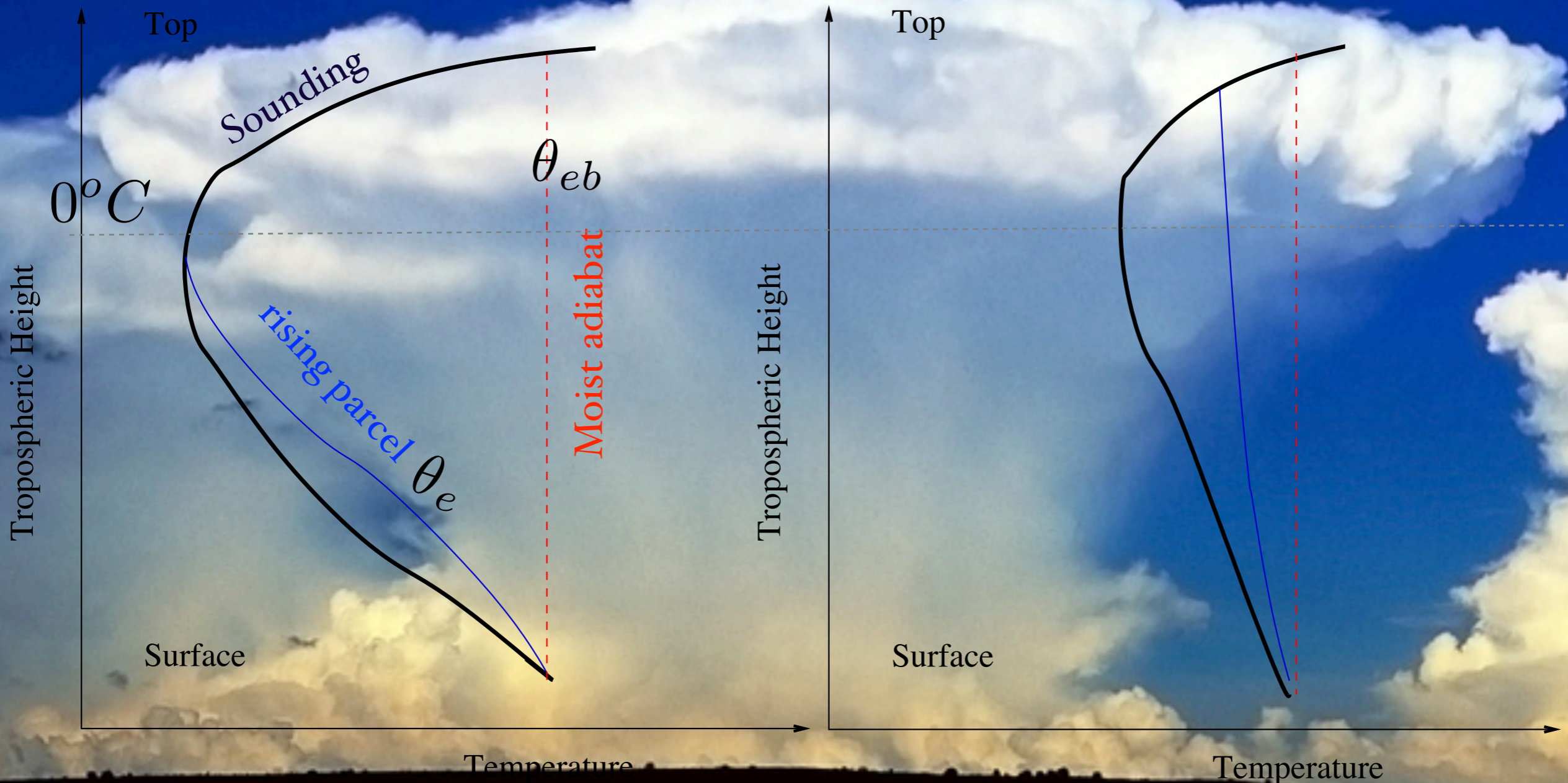
Moist environment



# Dilute parcel lifting

Dry troposphere with positive CAPE will favor congestus clouds

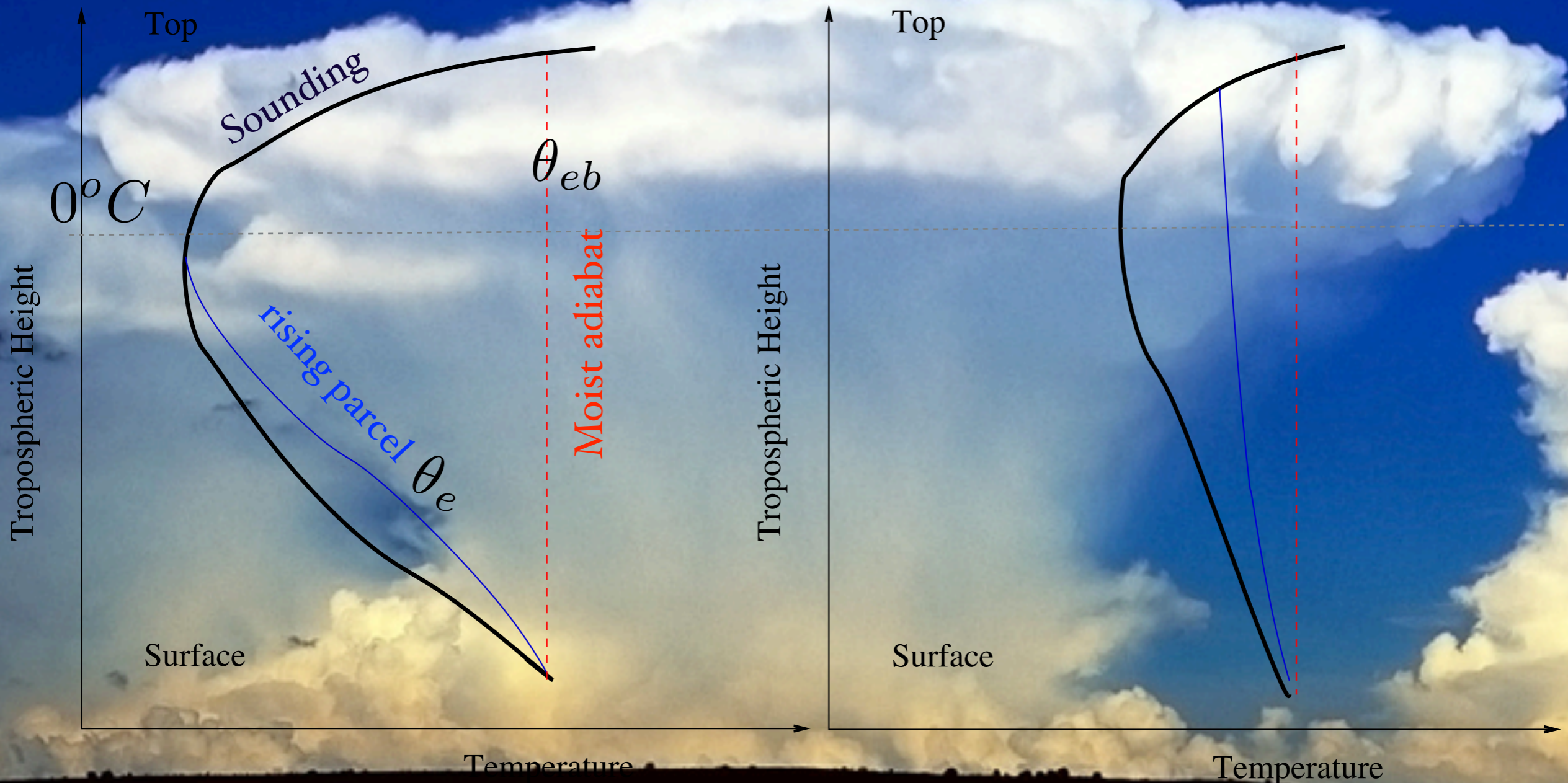
Moist environment



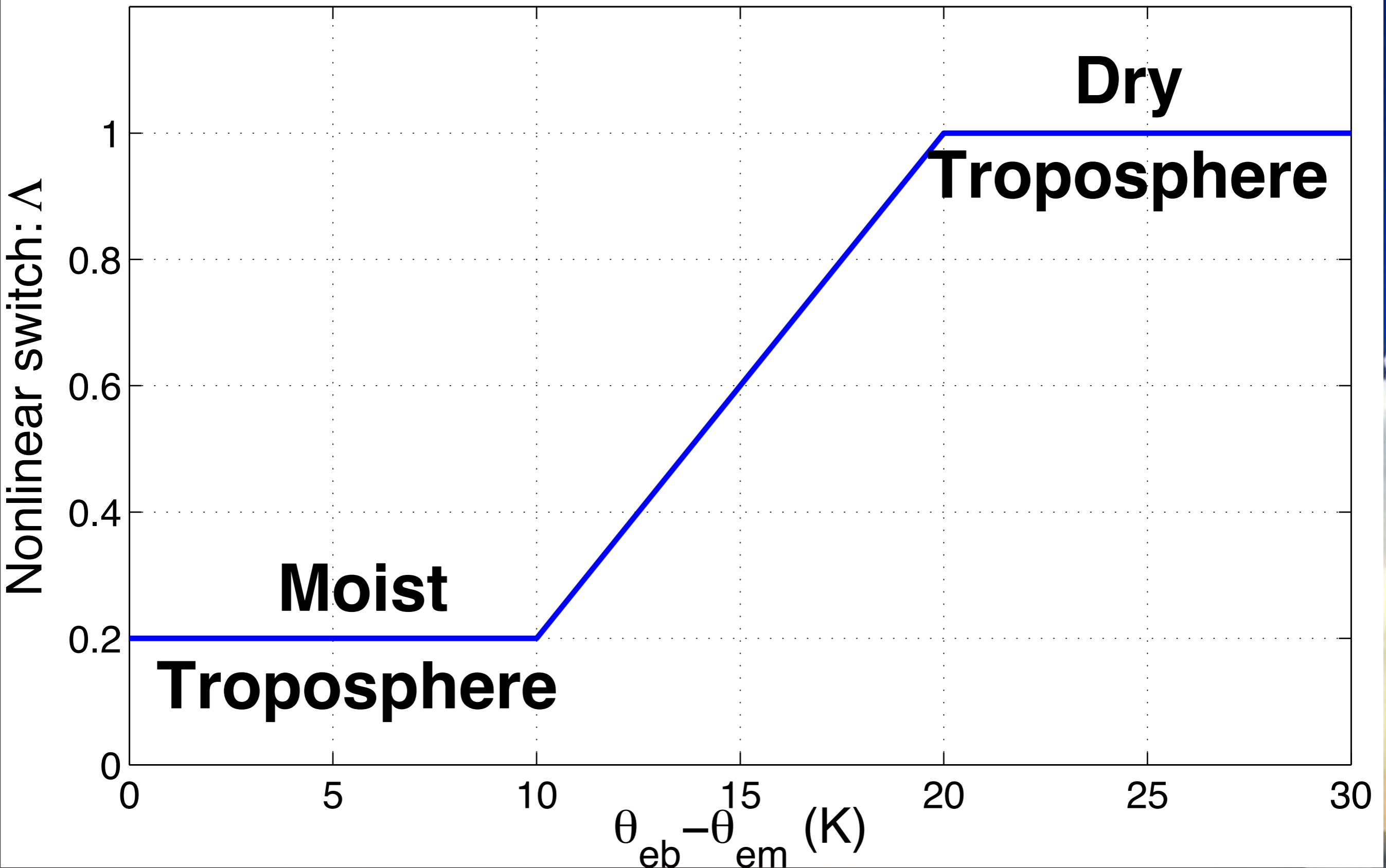
# Dilute parcel lifting

Dry troposphere with positive CAPE will favor congestus clouds

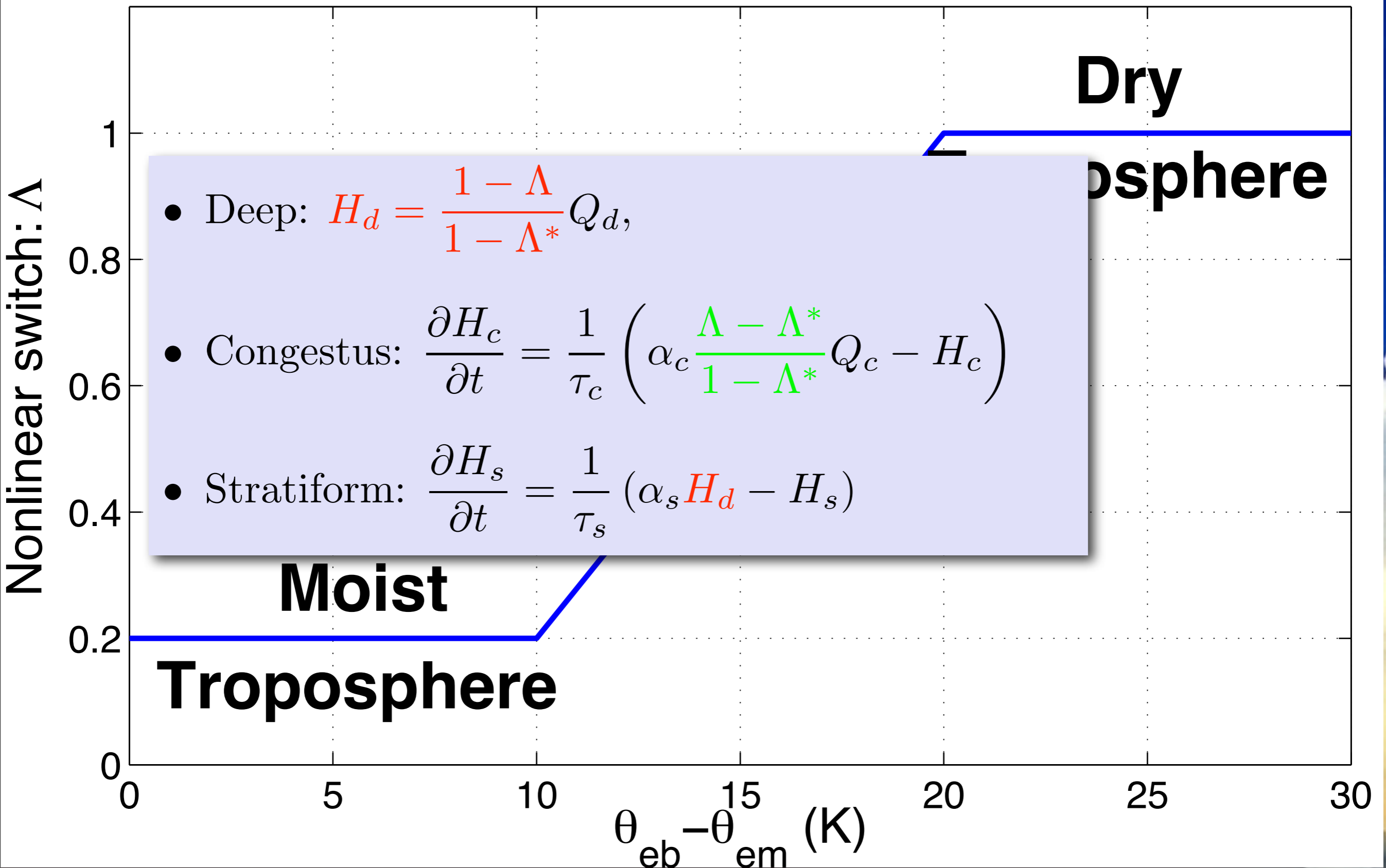
Deep convection is allowed (beyond freezing level) when troposphere is moist



# Moist Switch Based convective parametrization



# Moist Switch Based convective parametrization



# Crude vertical approximation of primitive equations: 2 first modes

- Horizontal Velocity:

$$\mathbf{V} = \bar{\mathbf{U}} + \sqrt{2} \cos\left(\frac{\pi z}{H_T}\right) \mathbf{v}_1 + \sqrt{2} \cos\left(\frac{2\pi z}{H_T}\right) \mathbf{v}_2$$

- Vertical velocity:

$$w = -\frac{H_T}{\pi} \sqrt{2} \left[ \sin\left(\frac{z\pi}{H_T}\right) \operatorname{div} \mathbf{v}_1 + \frac{1}{2} \sin\left(\frac{2\pi z}{H_T}\right) \operatorname{div} \mathbf{v}_2 \right]$$

- Potential temperature:

$$\Theta = z + \sqrt{2} \sin\left(\frac{\pi z}{H_T}\right) \theta_1 + 2\sqrt{2} \sin\left(\frac{2\pi z}{H_T}\right) \theta_2.$$

# Linear Eqns in Shear Background

$$\bar{U}(x, y, z) = \bar{u}_0(y) + \bar{u}_1 \sqrt{2} \cos(z) + \bar{u}_2 \cos(2z).$$

## Dynamical Core

$$\frac{\partial u_1}{\partial t} + \bar{u}_0(y) \frac{\partial u_1}{\partial x} + v_1 \frac{\partial \bar{u}_0(y)}{\partial y} - \frac{\partial \theta_1}{\partial x} - yv_1 = -\frac{C_d u_0}{h_b} u_1 - \frac{1}{\tau_D} u_1 - \frac{\sqrt{2}}{2} \left( \frac{3}{2} \bar{u}_1 \frac{\partial u_2}{\partial x} + 3\bar{u}_2 \frac{\partial u_1}{\partial x} + \frac{1}{2} \bar{u}_1 \frac{\partial v_2}{\partial y} + 2\bar{u}_2 \frac{\partial v_1}{\partial y} \right)$$

$$\frac{\partial u_2}{\partial t} + \bar{u}_0(y) \frac{\partial u_2}{\partial x} + v_2 \frac{\partial \bar{u}_0(y)}{\partial y} - \frac{\partial \theta_2}{\partial x} - yv_2 = -\frac{C_d u_0}{h_b} u_2 - \frac{1}{\tau_D} u_2 - \frac{\sqrt{2}}{2} \left( -\bar{u}_1 \frac{\partial v_1}{\partial y} \right)$$

$$\frac{\partial v_1}{\partial t} + \bar{u}_0(y) \frac{\partial v_1}{\partial x} - \frac{\partial \theta_1}{\partial y} + yu_1 = -\frac{C_d u_0}{h_b} v_1 - \frac{1}{\tau_D} v_1 - \frac{\sqrt{2}}{2} \left( \bar{u}_1 \frac{\partial v_2}{\partial x} + \bar{u}_2 \frac{\partial v_1}{\partial x} \right)$$

$$\frac{\partial v_2}{\partial t} + \bar{u}_0(y) \frac{\partial v_2}{\partial x} - \frac{\partial \theta_2}{\partial y} + yu_2 = -\frac{C_d u_0}{h_b} v_2 - \frac{1}{\tau_D} v_2 - \frac{\sqrt{2}}{2} \left( \bar{u}_1 \frac{\partial v_1}{\partial x} \right)$$

$$\frac{\partial \theta_1}{\partial t} + \bar{u}_0(y) \frac{\partial \theta_1}{\partial x} - \left( \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \right) = S_1 - \frac{\sqrt{2}}{2} \left( 2\bar{u}_1 \frac{\partial \theta_2}{\partial x} - \bar{u}_2 \frac{\partial \theta_1}{\partial x} \right)$$

$$\frac{\partial \theta_2}{\partial t} + \bar{u}_0(y) \frac{\partial \theta_2}{\partial x} - \frac{1}{4} \left( \frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} \right) = S_2 - \frac{\sqrt{2}}{4} \left( \bar{u}_1 \frac{\partial \theta_1}{\partial x} \right)$$

# Moisture and Convective variables

$$\frac{\partial q}{\partial t} + \bar{u}_0(y) \frac{\partial q}{\partial x} = -\tilde{Q} \left( \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \right) - \tilde{\lambda} \tilde{Q} \left( \frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} \right) - P + \frac{1}{H_T} D - (\bar{u}_1 + \tilde{\alpha} \bar{u}_2) \frac{\partial q}{\partial x}$$

$$\frac{\partial \theta_{eb}}{\partial t} + \bar{u}_0(y) \frac{\partial \theta_{eb}}{\partial x} = \frac{1}{\tau_e} (\theta_{eb}^* - \theta_{eb}) - \frac{D}{h_b} - \sqrt{2} (\bar{u}_1 \frac{\partial \theta_{eb}}{\partial x} + \bar{u}_2 \frac{\partial \theta_{eb}}{\partial x})$$

$$\frac{\partial H_s}{\partial t} + \bar{u}_0(y) \frac{\partial H_s}{\partial x} = \frac{1}{\tau_s} [\alpha_s H_d - H_s] + \frac{32\sqrt{2}}{15\pi} (\bar{u}_1 \frac{\partial H_s}{\partial x}) *$$

$$\frac{\partial H_c}{\partial t} + \bar{u}_0(y) \frac{\partial H_c}{\partial x} = \frac{1}{\tau_c} [\alpha_c \frac{\Lambda - \Lambda^*}{1 - \Lambda^*} Q_c - H_c] - \frac{32\sqrt{2}}{15\pi} (\bar{u}_1 \frac{\partial H_c}{\partial x}) *$$

\* Based on heating-only (red) basis functions from previous slide

$$P = -\frac{2\sqrt{2}}{\pi} (H_d + \xi_s H_s + \xi_c H_c)$$

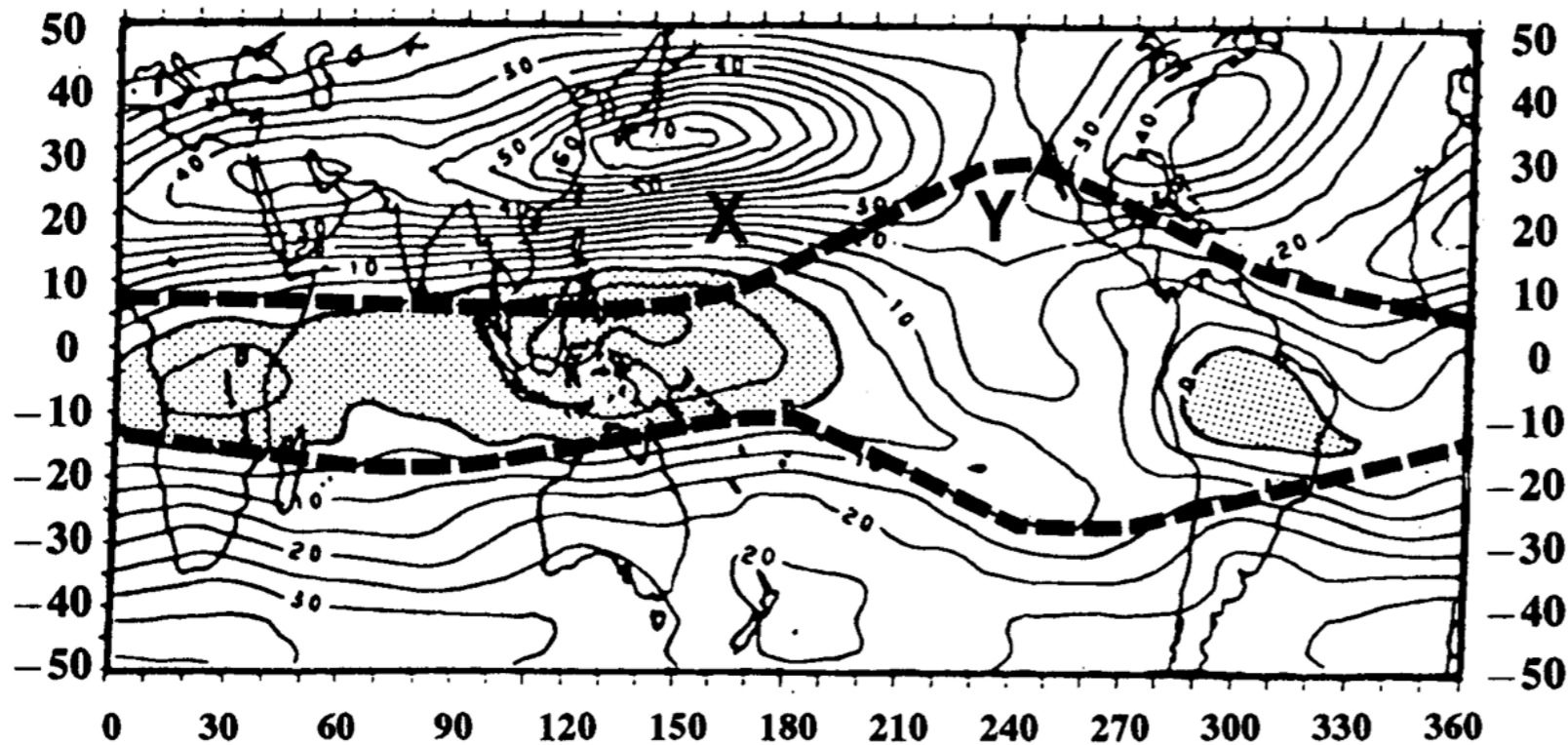
# Linear solution strategy

- Galerkin projection in meridional direction using parabolic-cylinder functions

$$U(x, y, t) \approx \sum_{n=0}^N U_n(x, t) \phi_n(y), N = 15$$

- Plus a radiation condition to eliminate spurious waves
- Wave solution reduces to eigenvalue problem
- Eigenvalues: growth rate and phase speed

# Background meridional shear and tropical-extratropical interactions



Zhang and Webster (1989)

FIG. 25. The distribution of the turning latitude of an equatorially trapped Rossby wave as a function of longitude and latitude in the two-dimensional basic boreal winter 200 mb zonal wind shown in Fig. 11. Letters X and Y denote forcing locations. Forcing located at X where  $y > y_i$  will not project significantly on equatorially trapped waves. Forcing located at Y where  $y < y_i$  will force trapped waves.

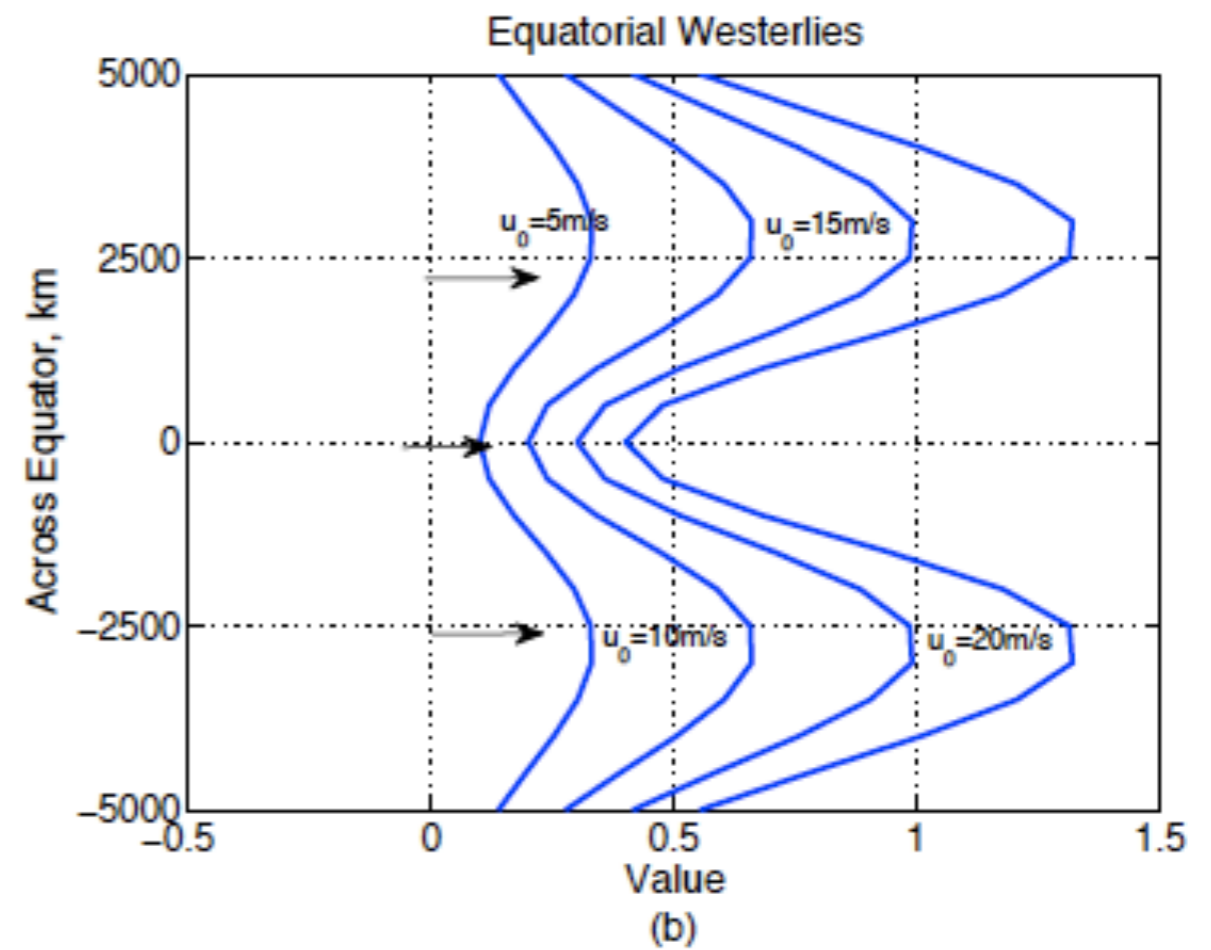
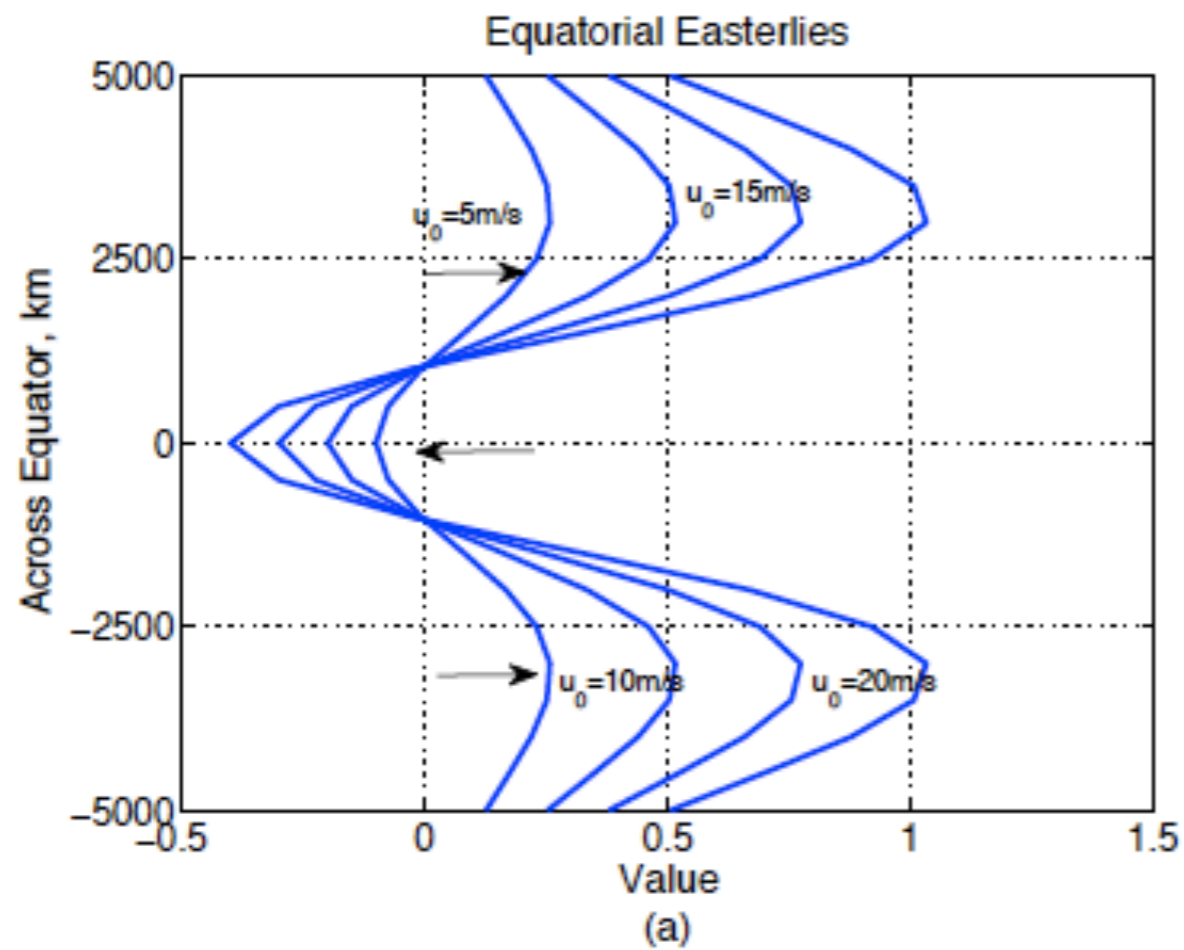
E. Rossby waves are more trapped during E. Easterlies and less-trapped during E. westerlies.

Tropical-Extratropical interactions favored during E. westerly period (Zhang and Webster).

Open Gate for Extra-tropical R. waves to propagate towards the equator (Kiladis).

Questions we want address: Which shear profile favors E. Rossby in first place?  
Effect of background shear on CCW in general?

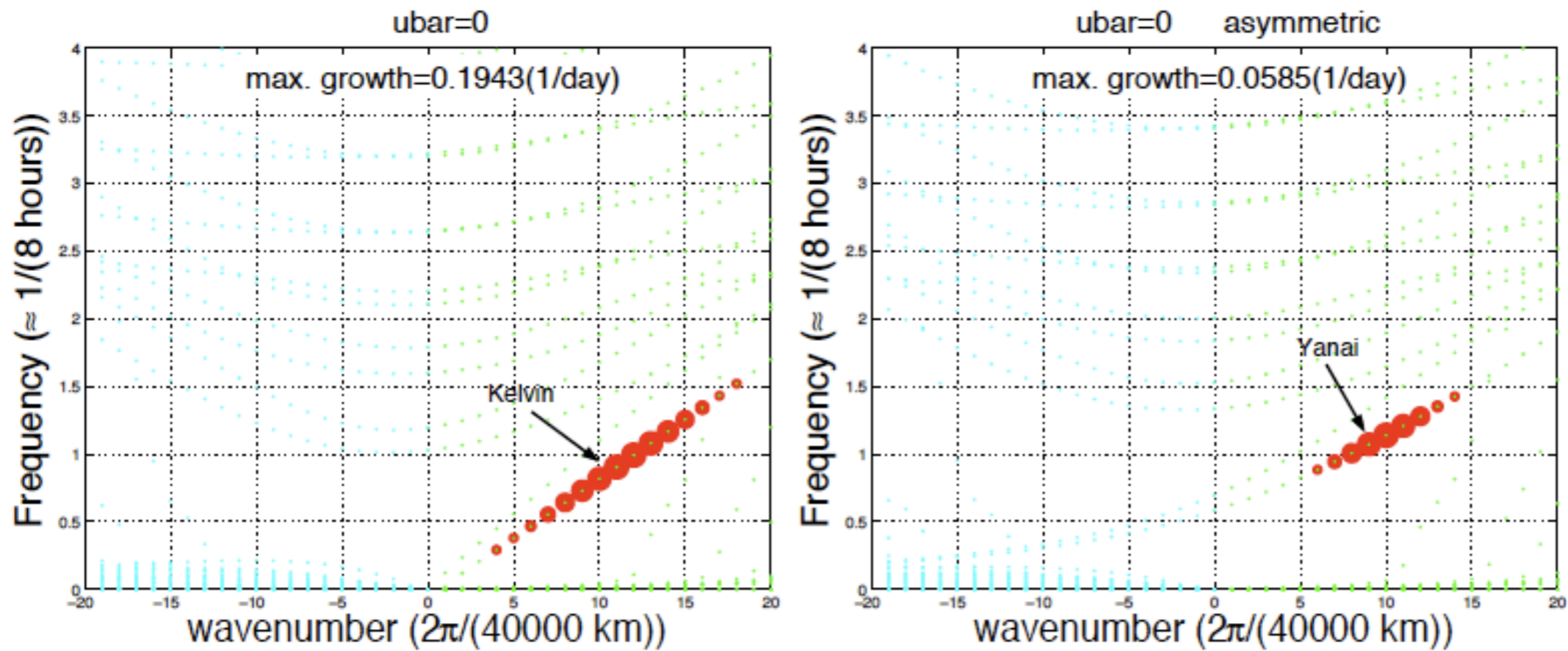
# Meridional Shear



# Shear-free case

- Fix multicloud parameters so that in shearless case only Kelvin and  $n=0$  EIG are unstable.
- By “tuning” some key parameters, other higher frequency gravity waves (e.g 2-day waves) can be destabilized
- But Rossby & (westward) MRG waves remain always stable

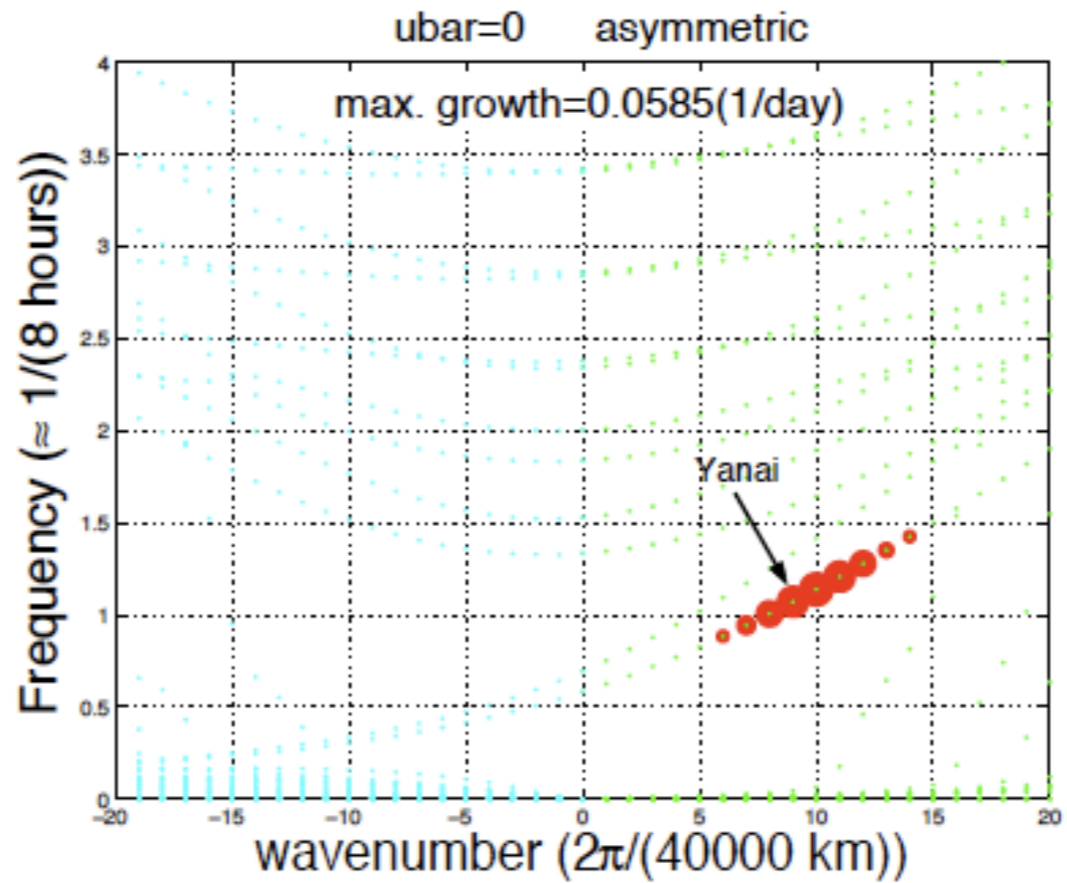
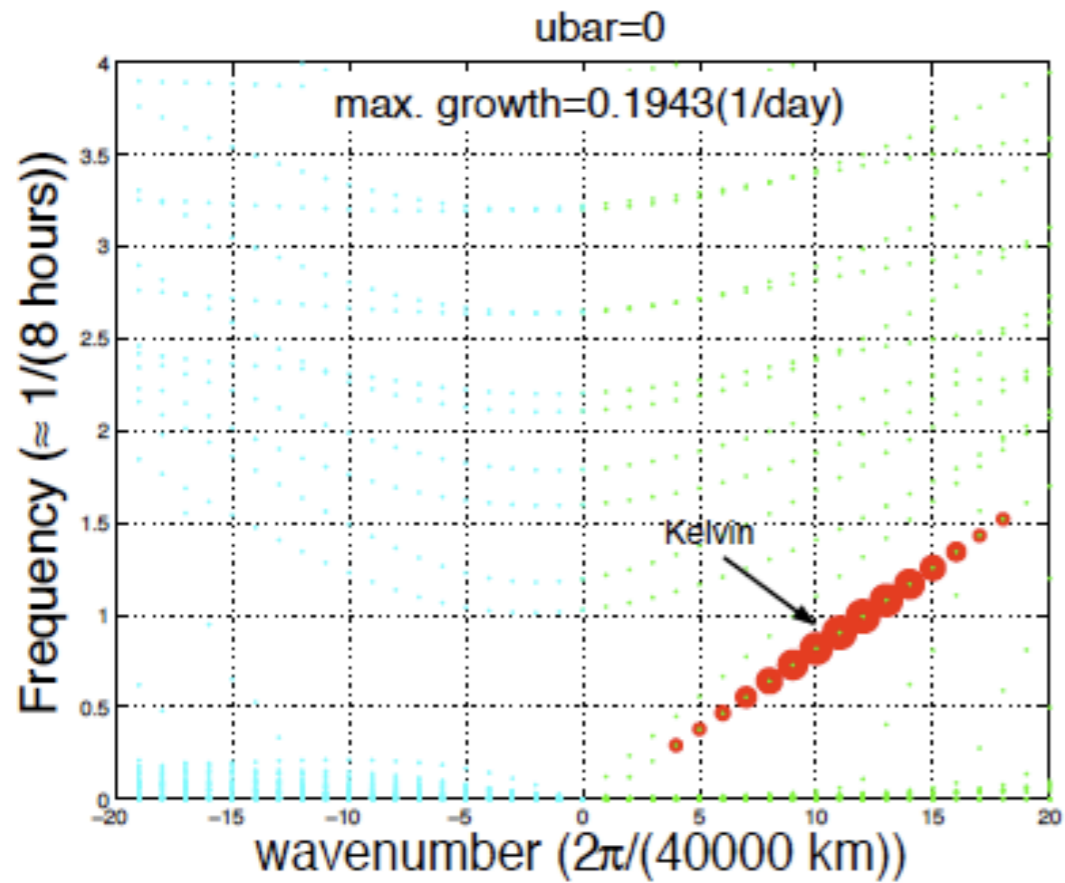
# Shear-free case



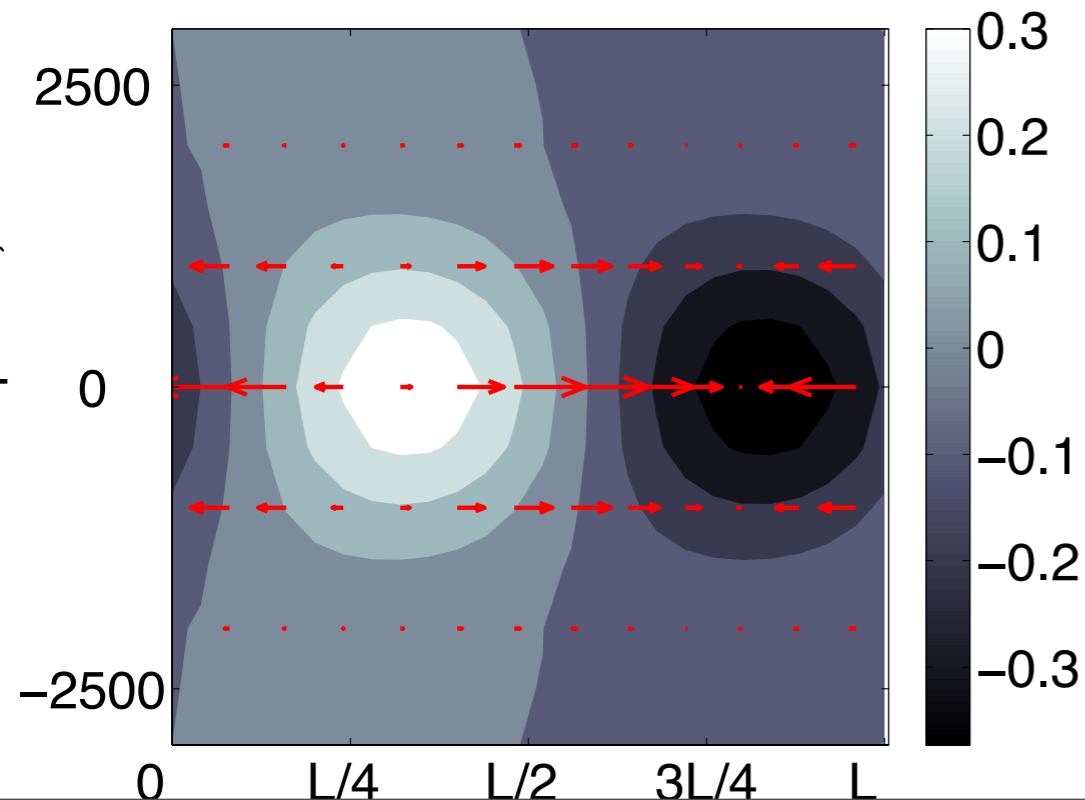
waves) can be destabilized

- But Rossby & (westward) MRG waves remain always stable

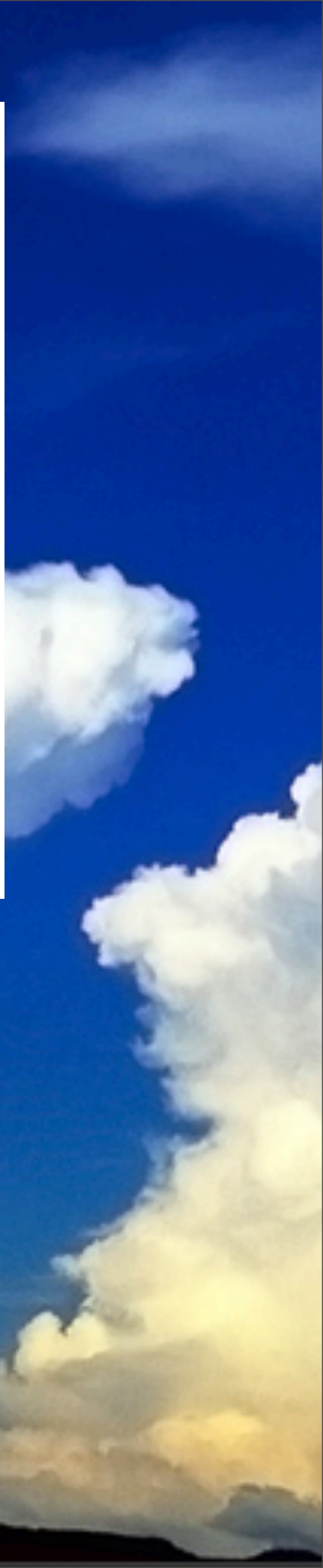
# Shear-free case



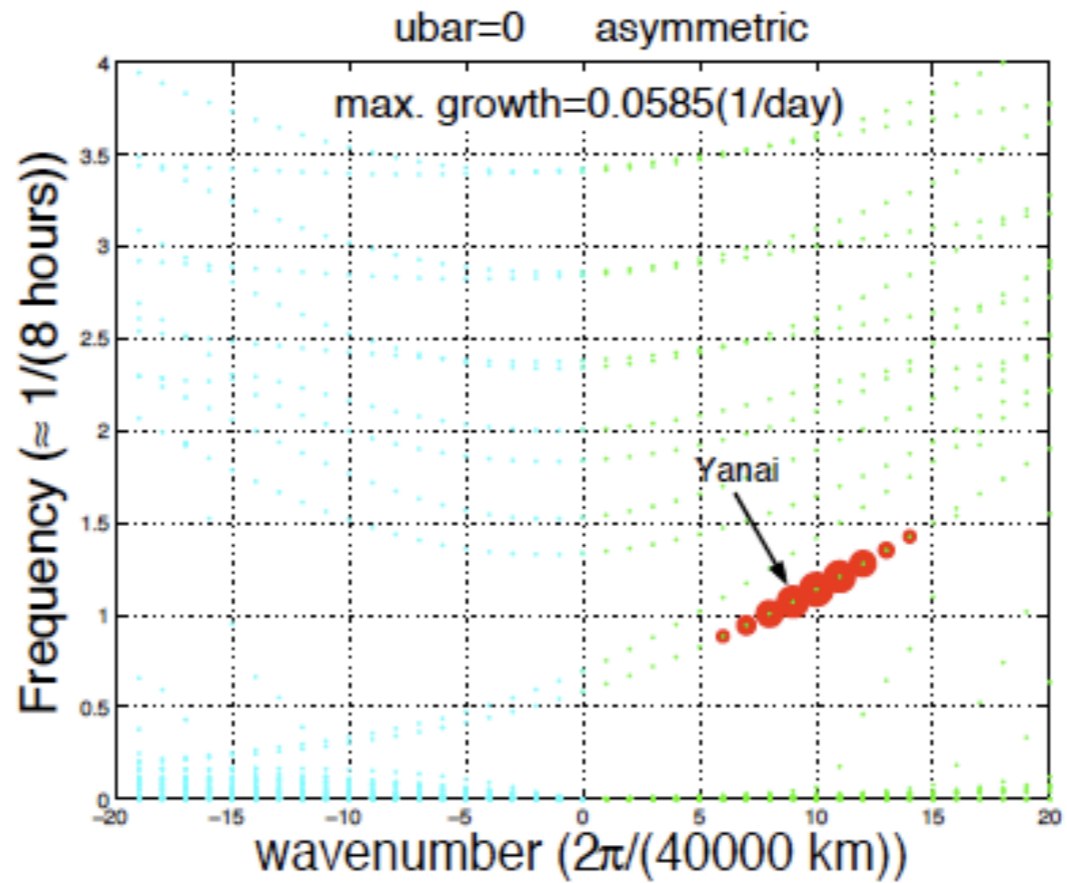
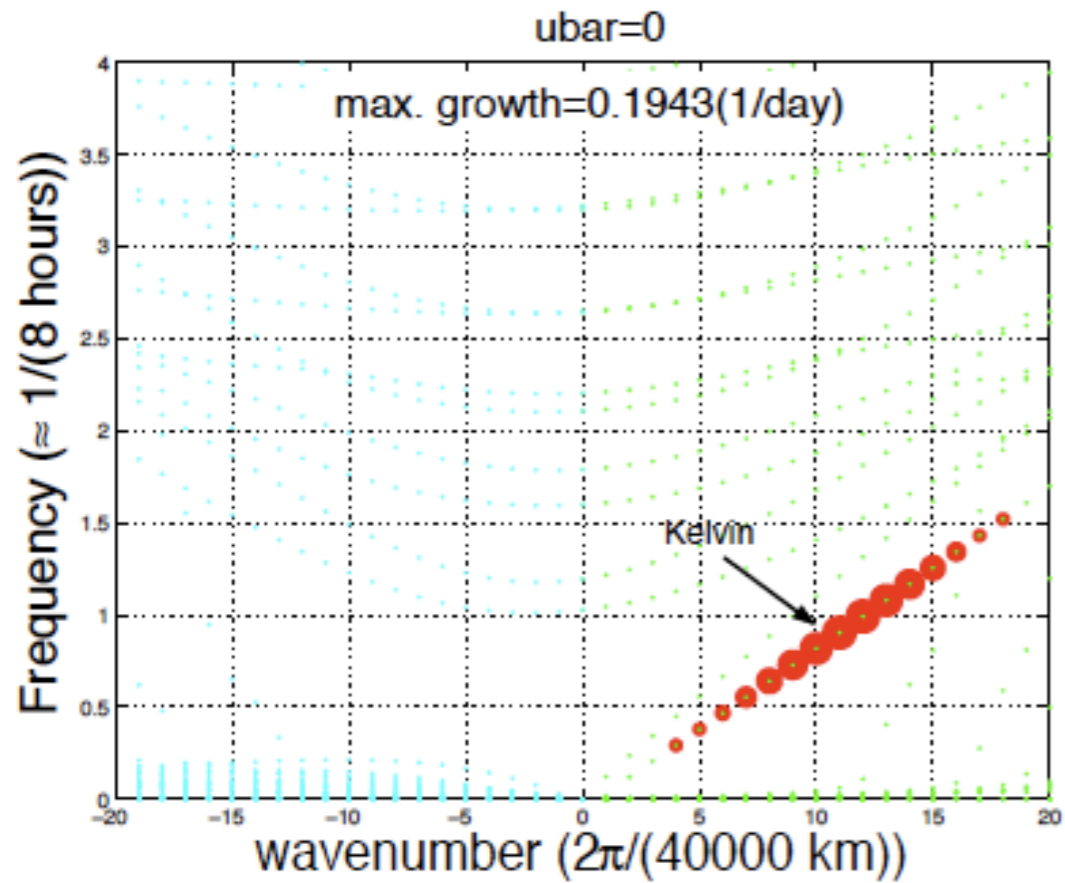
Heating and Cooling, 15 km Elevation



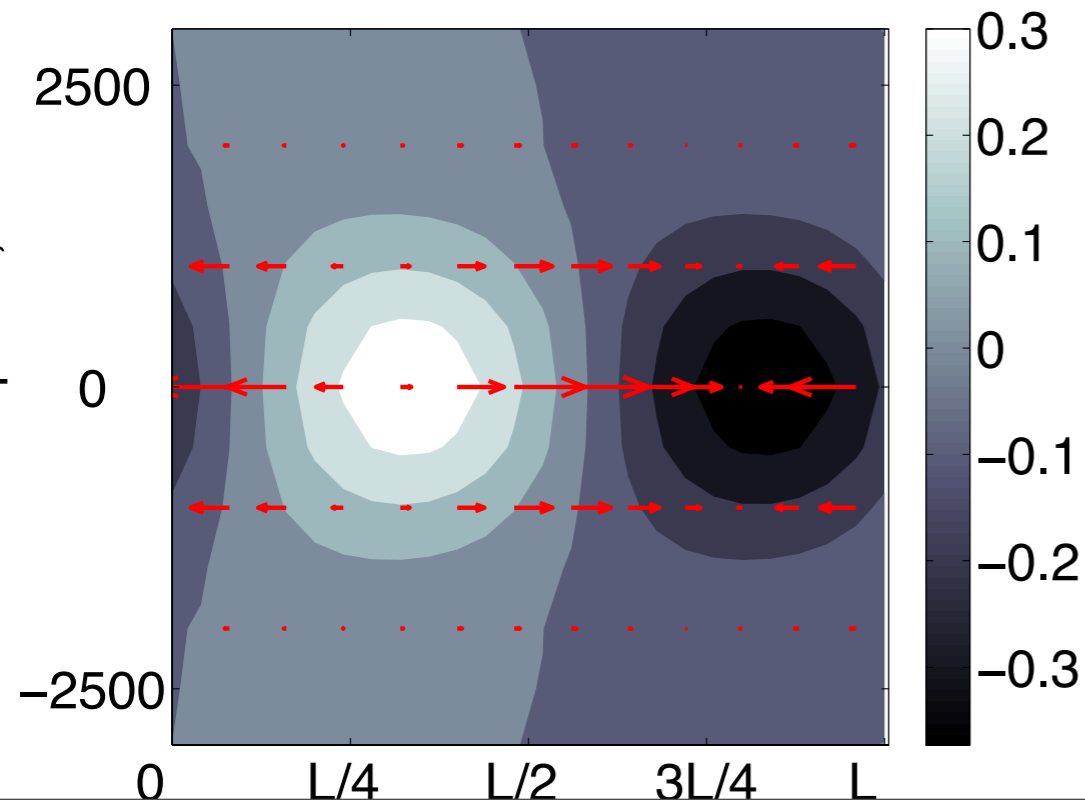
ized  
(ward) MRG waves



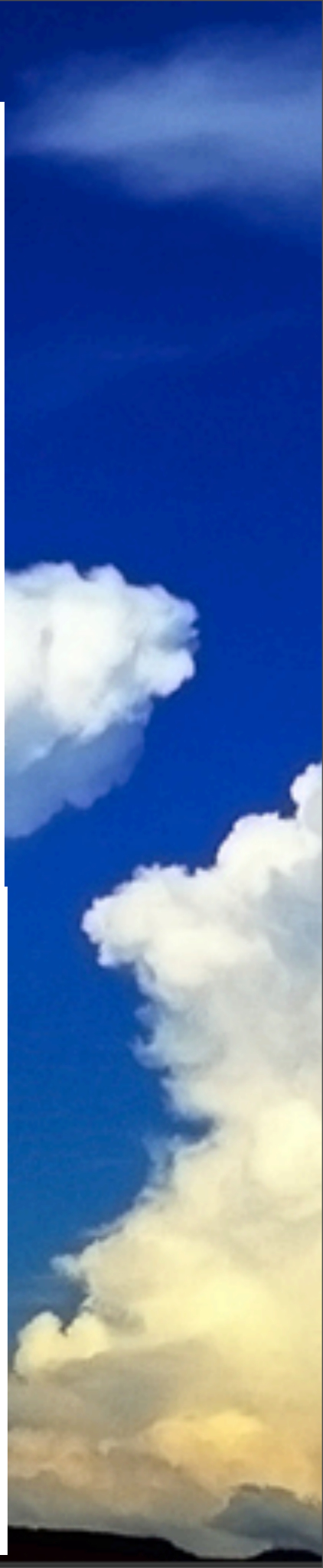
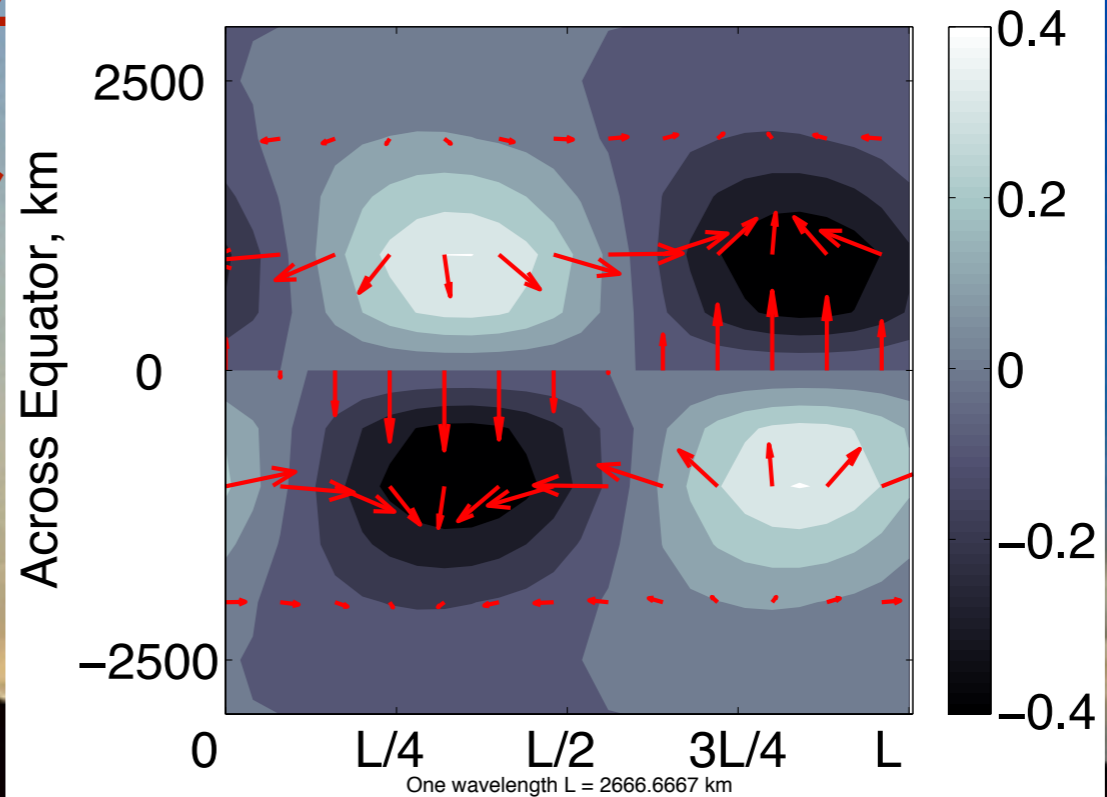
# Shear-free case



Heating and Cooling, 15 km Elevation



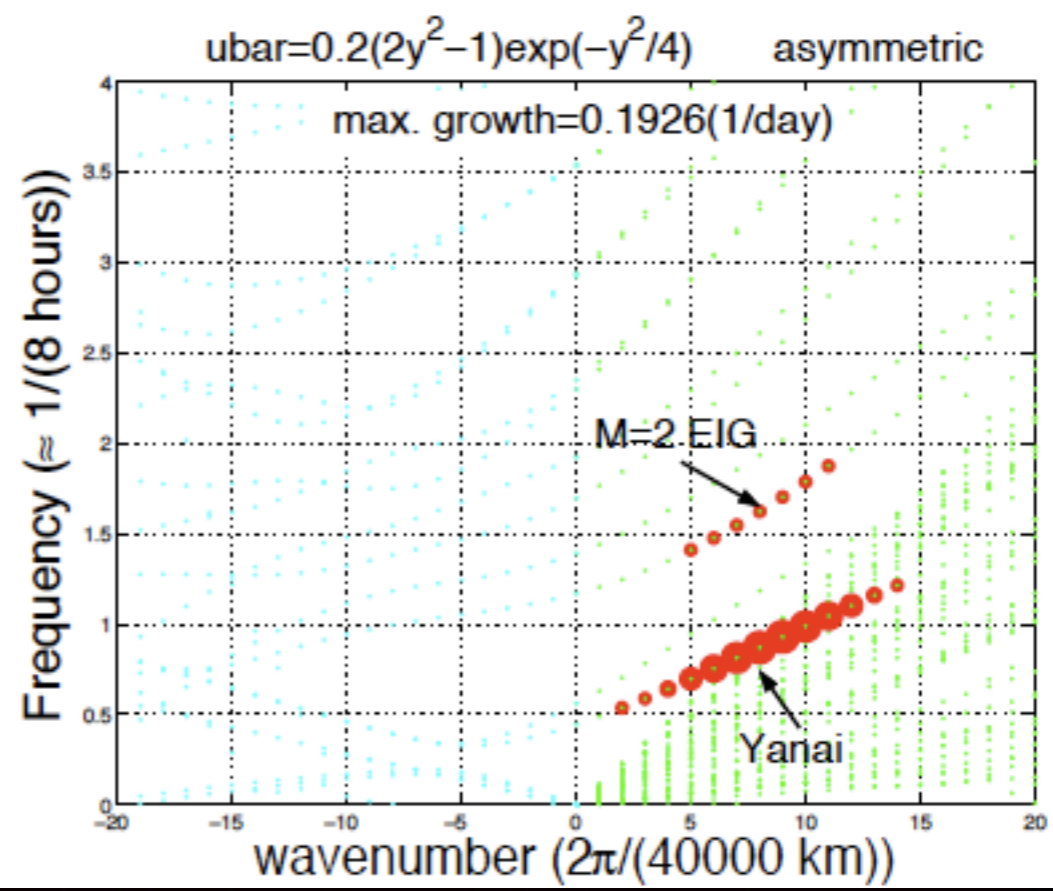
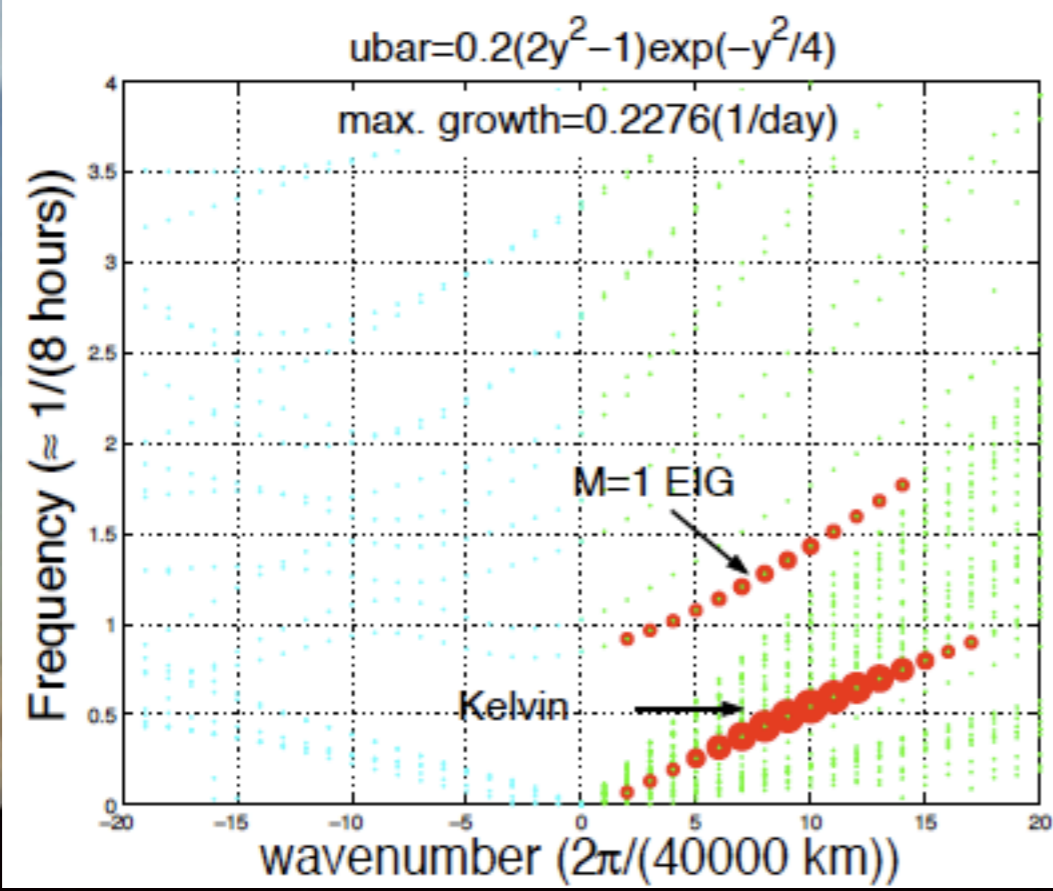
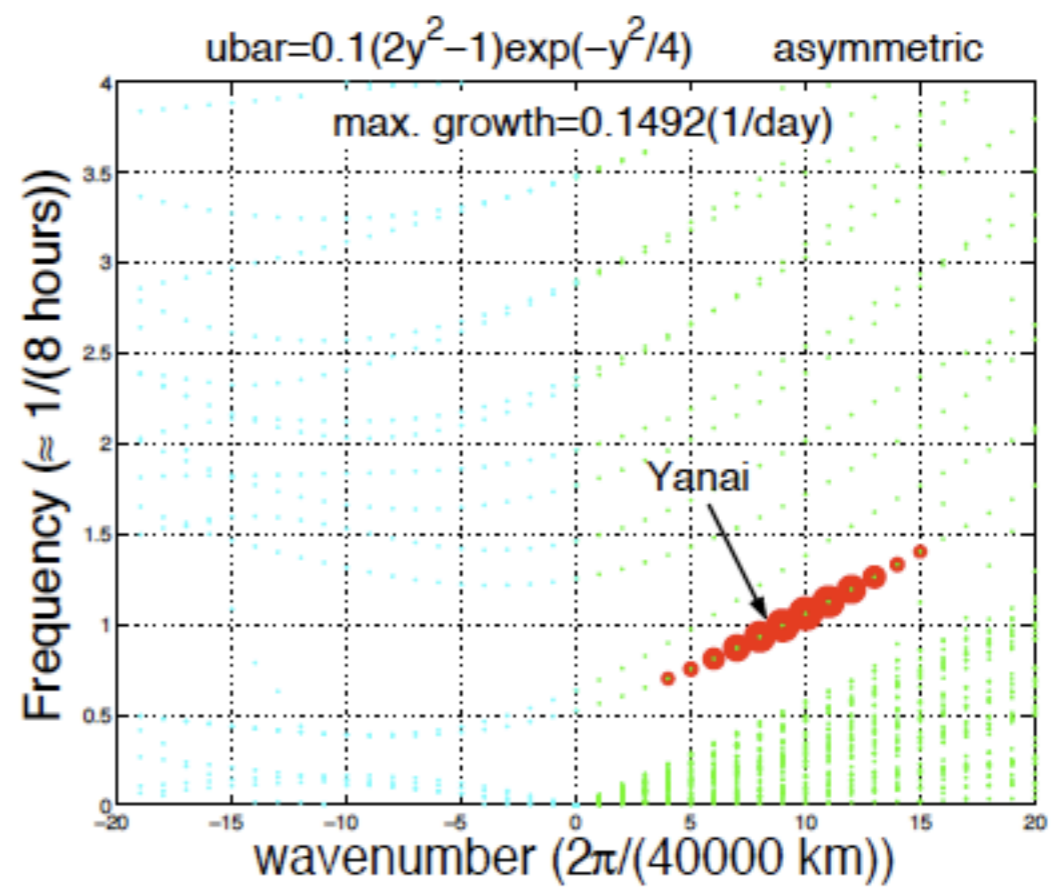
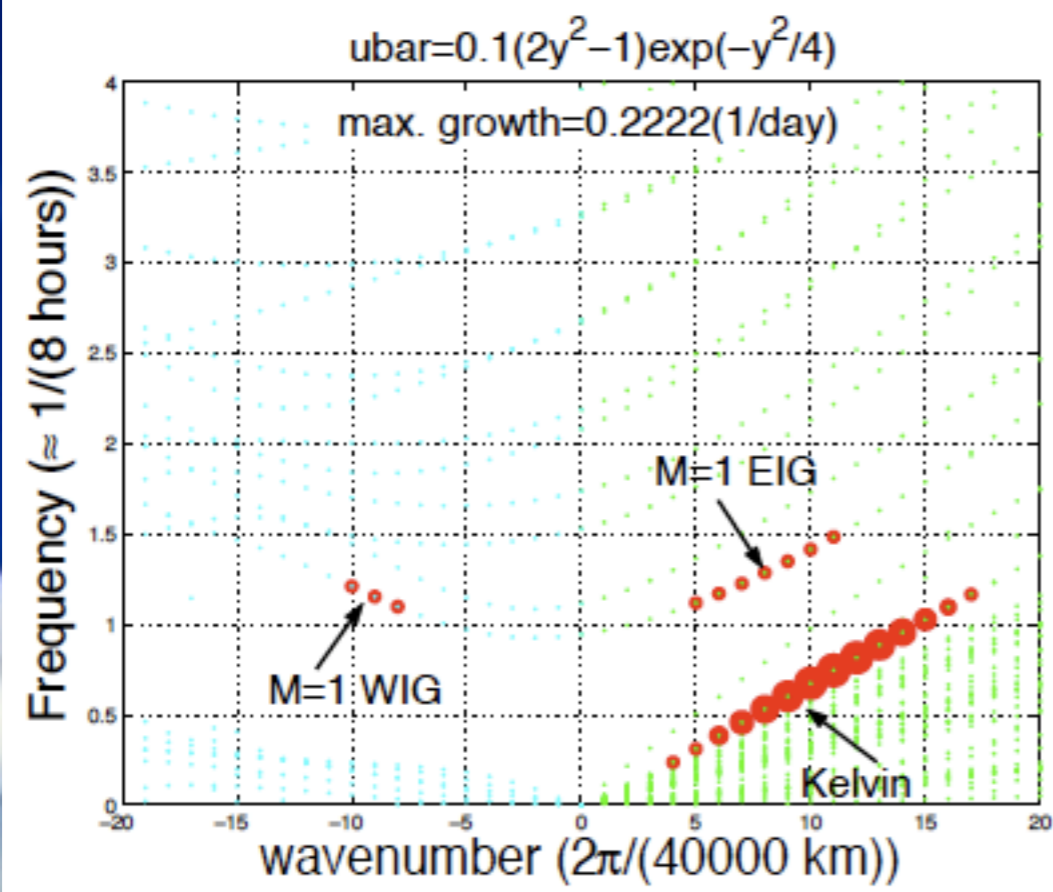
Heating and Cooling, 15 km Elevation



# Weak Equatorial Easterly Wind

$u_0 = -5m/s$

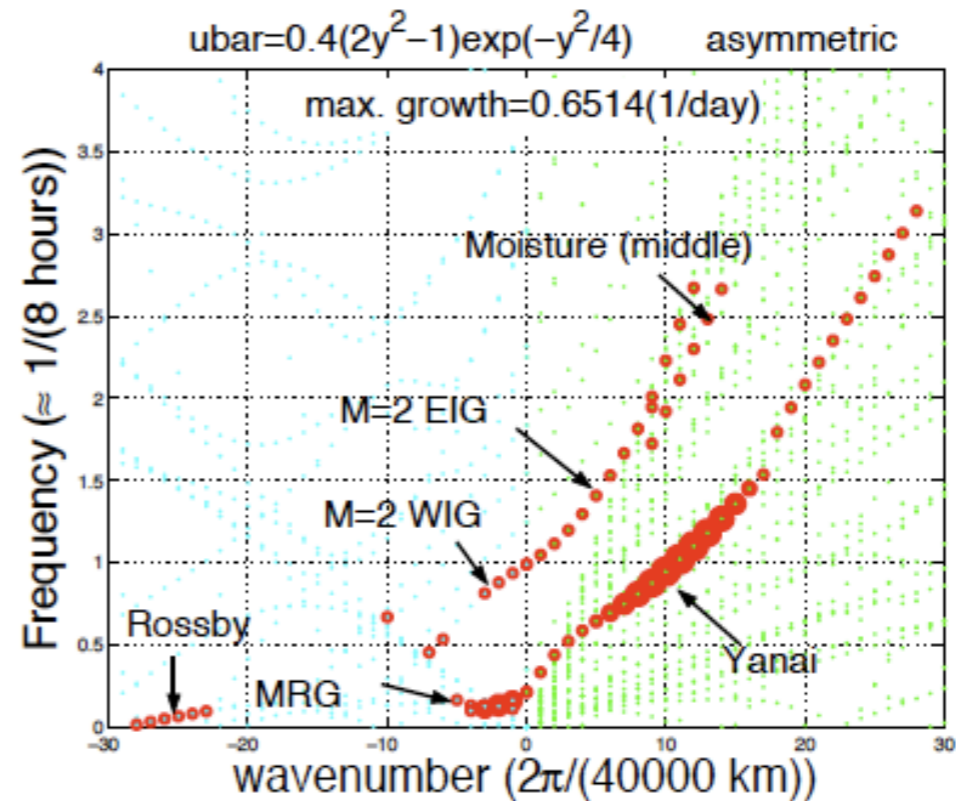
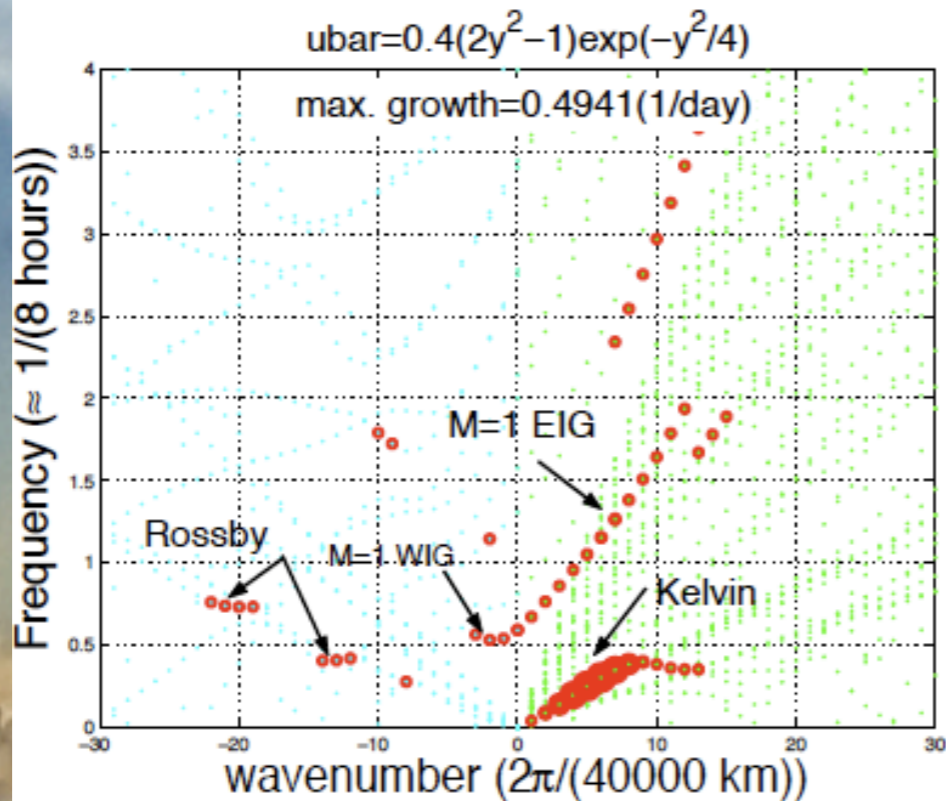
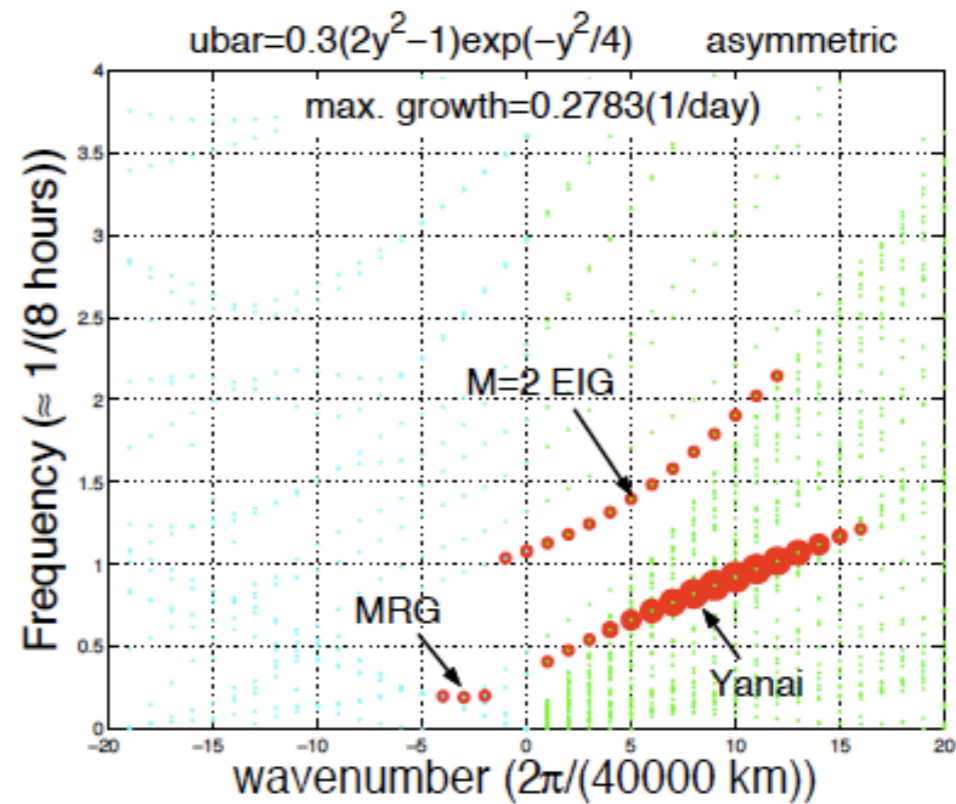
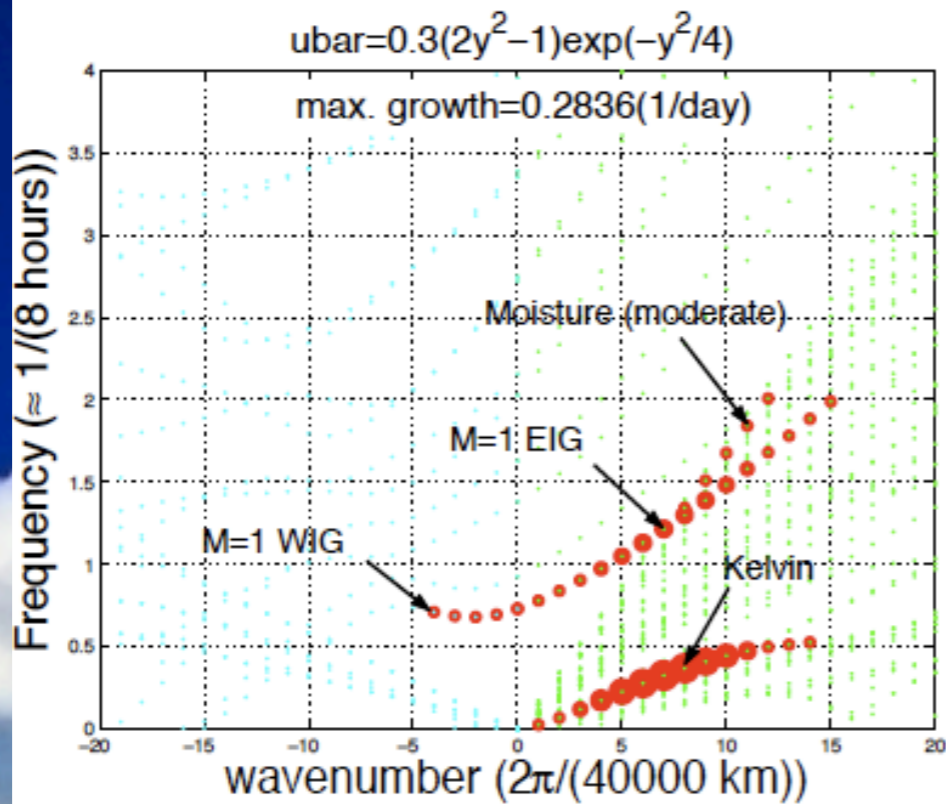
$u_0 = -10m/s$



# Strong Eq. Easterly

$u_0 = -15m/s$

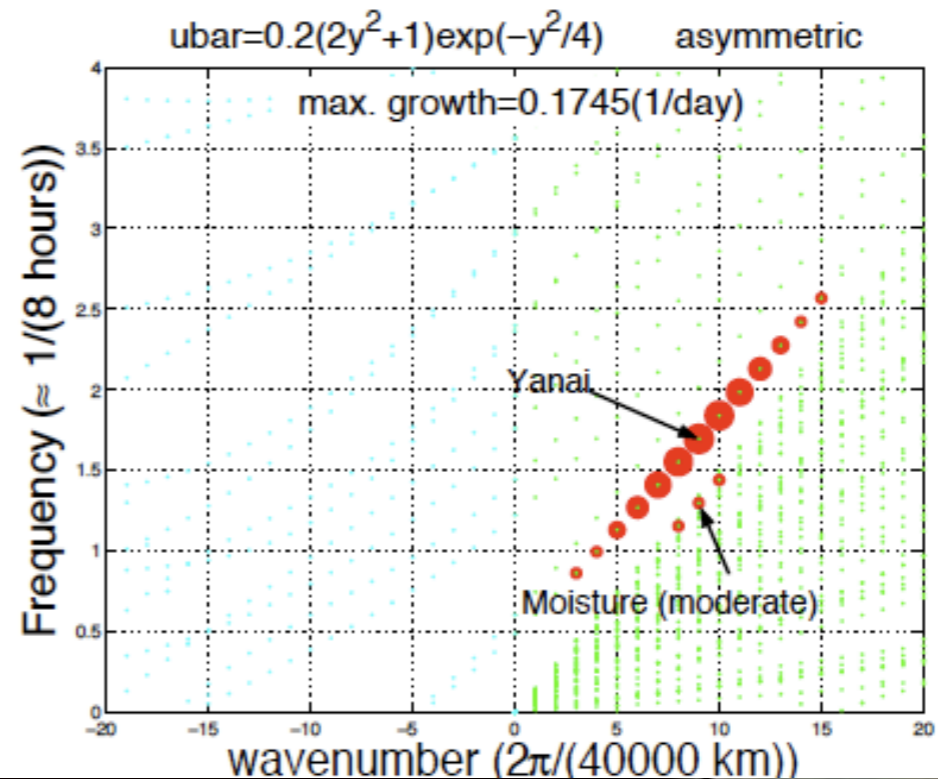
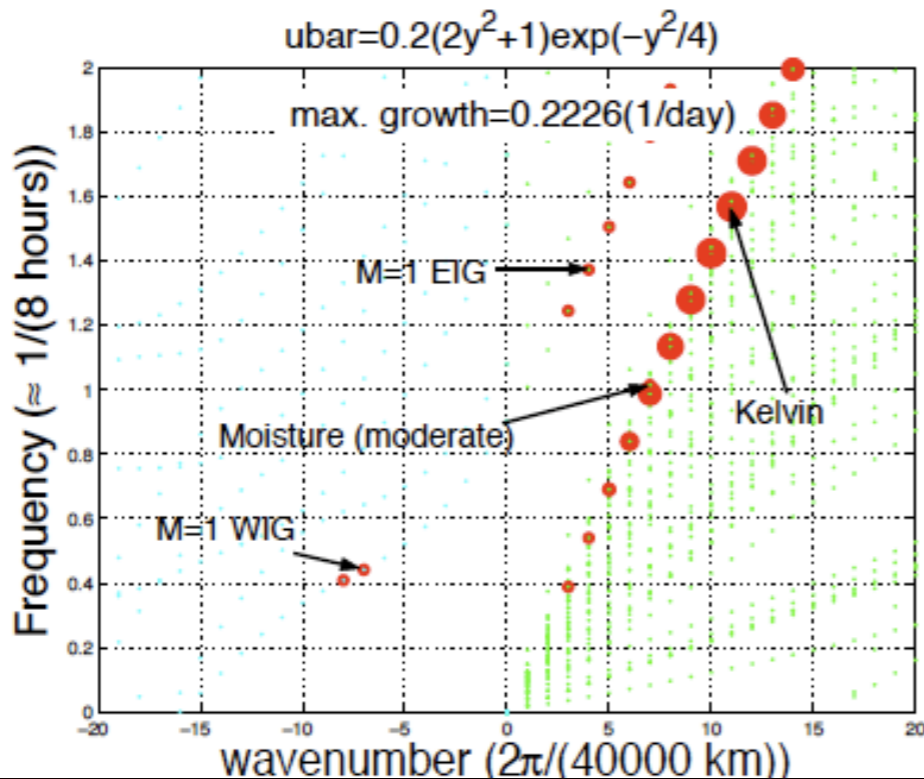
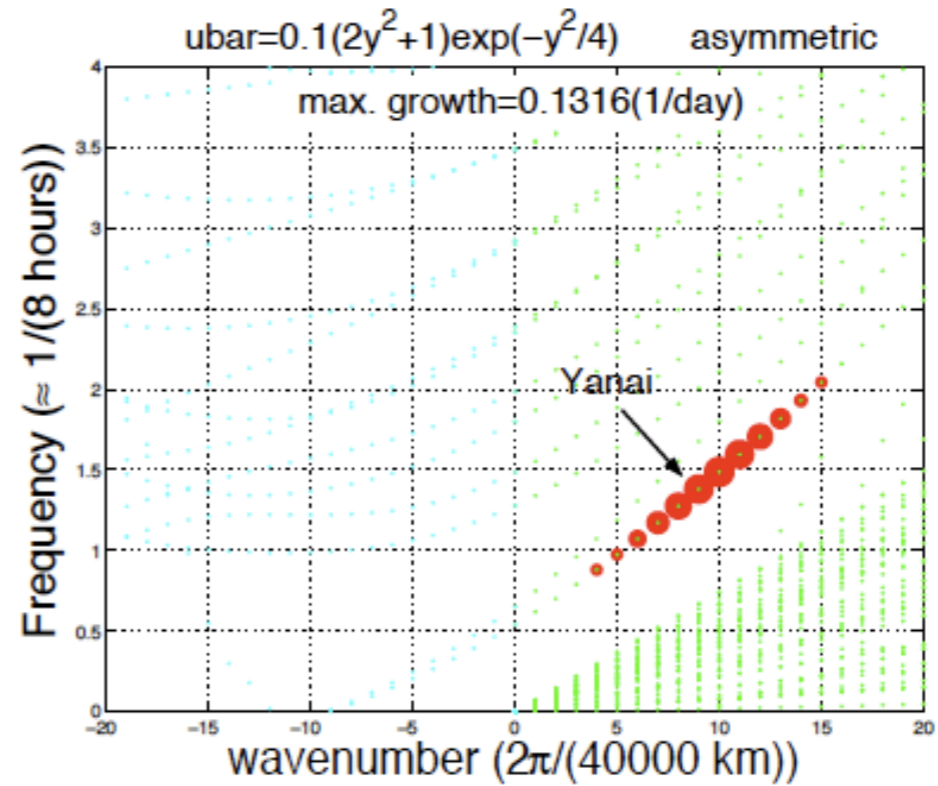
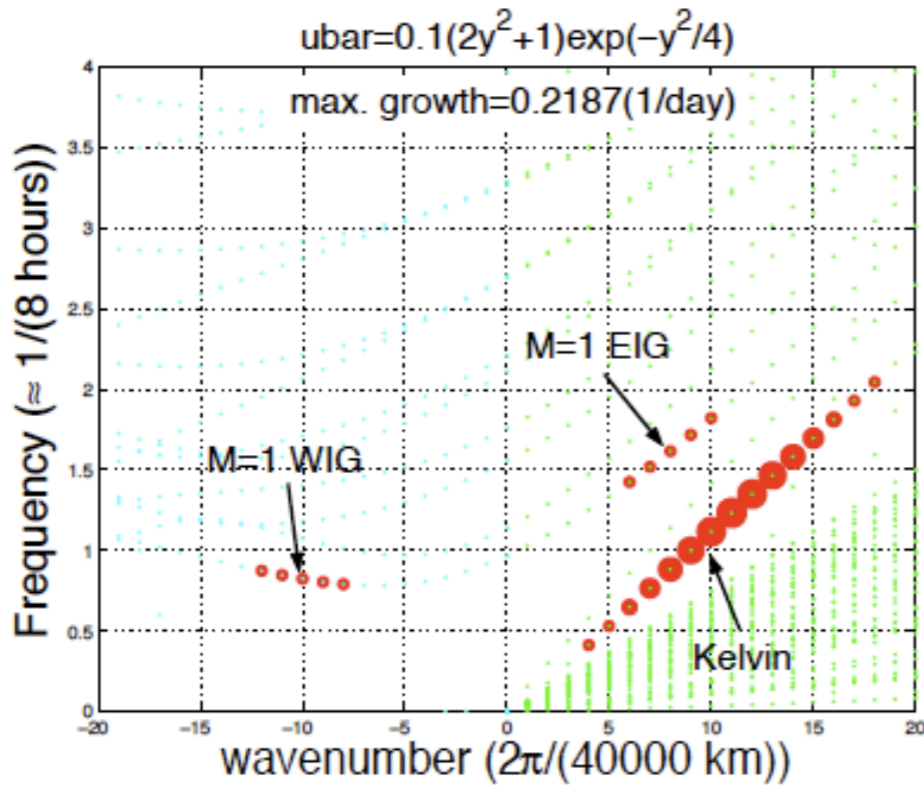
$u_0 = -20m/s$



# Equatorial Westerly wind

$u_0 = 5m/s$

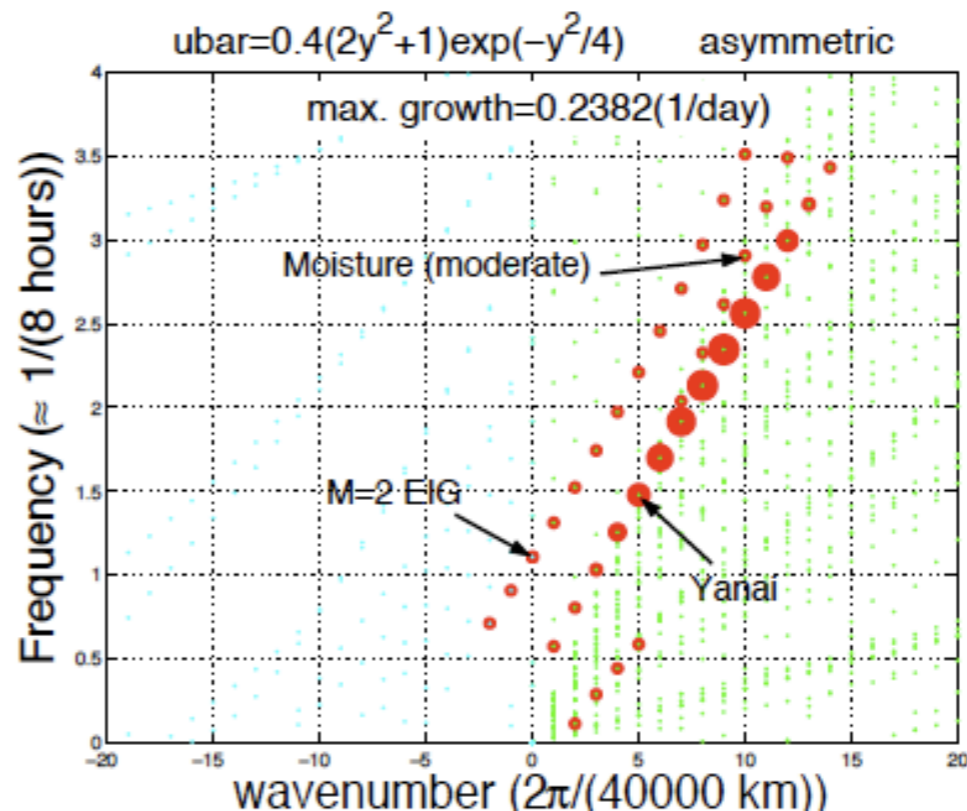
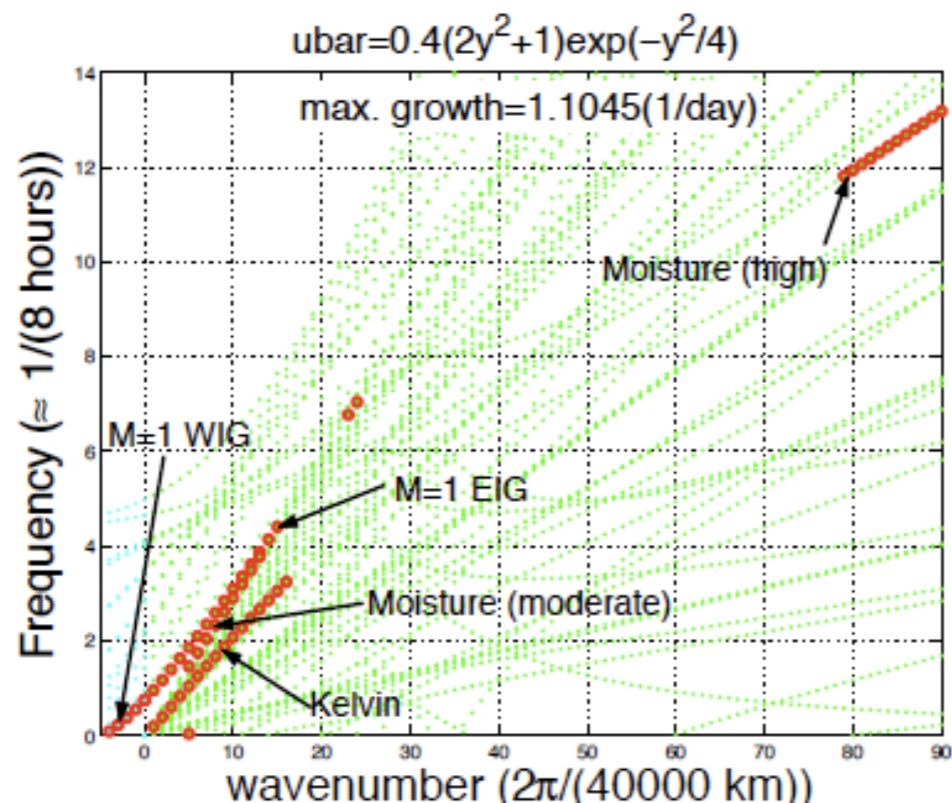
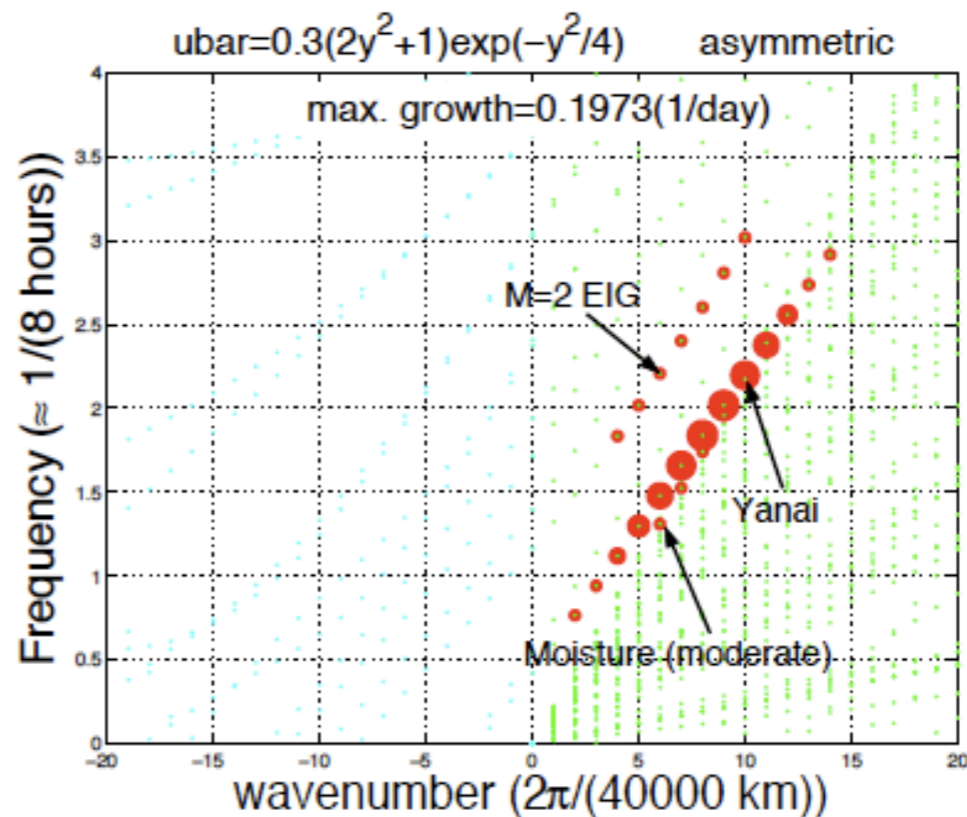
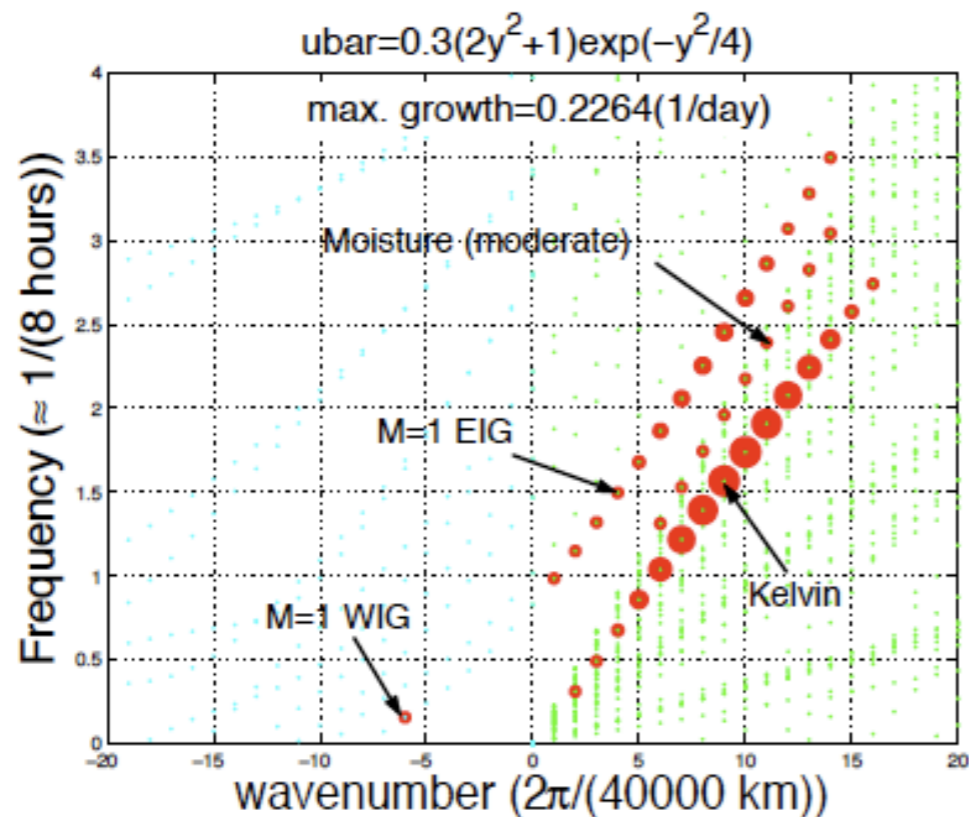
$u_0 = 10m/s$



# Unrealistic wind values yield catastrophic instabilities of moisture modes, though **no Rossby or MRG wave instabilities**

$u_0 = 15m/s$

$u_0 = 20m/s$



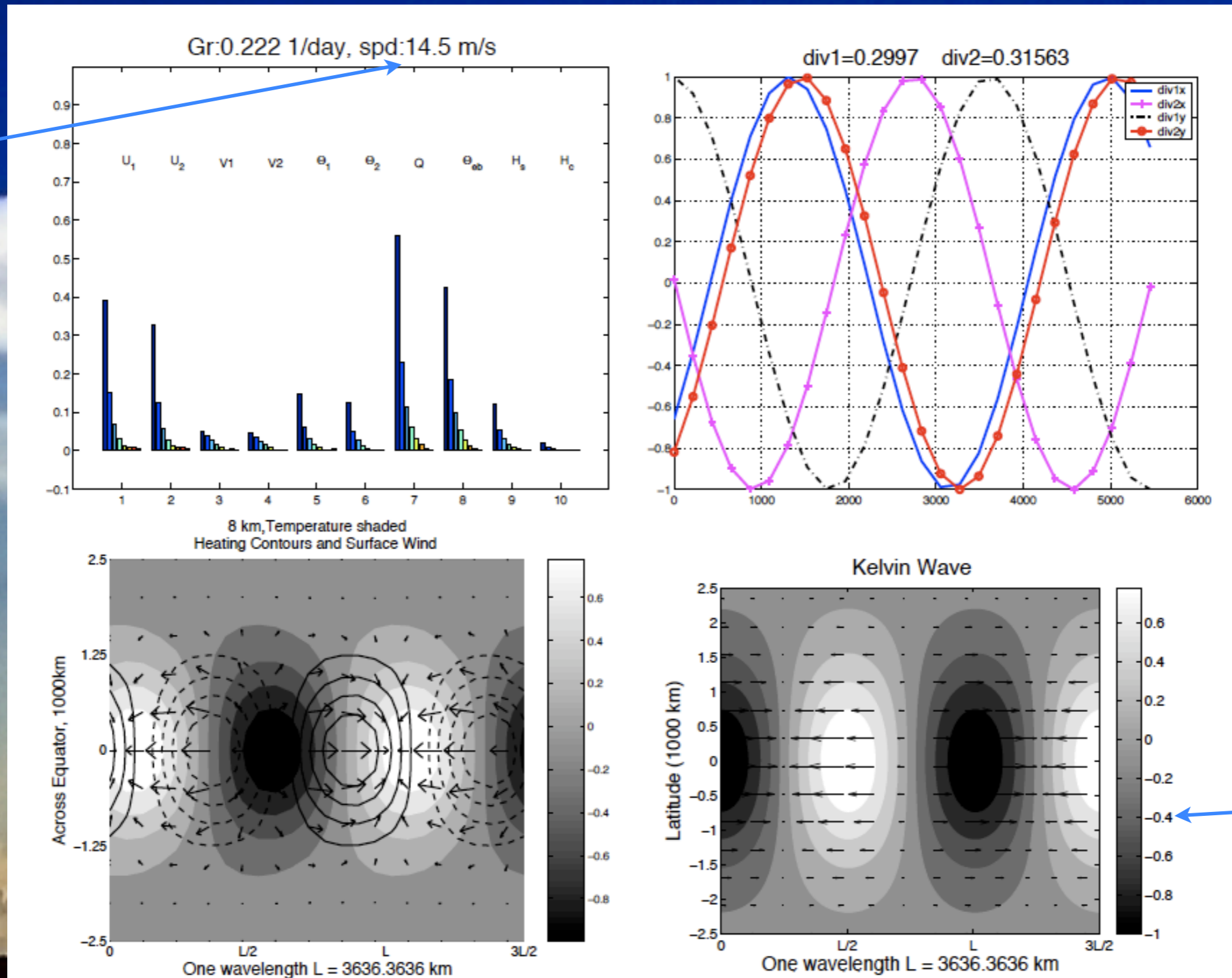
# Effect of Meridional Shear on Kelvin wave:

Induces weak meridional wind with significant contribution (30%) to horizontal convergence and slightly more trapped.

Meridional convergence lags zonal convergence ==> growth.

Doppler shift

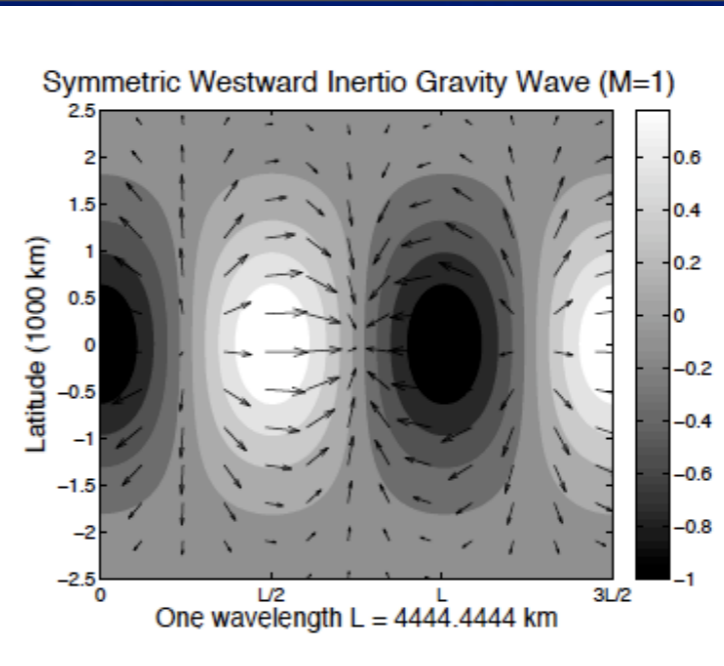
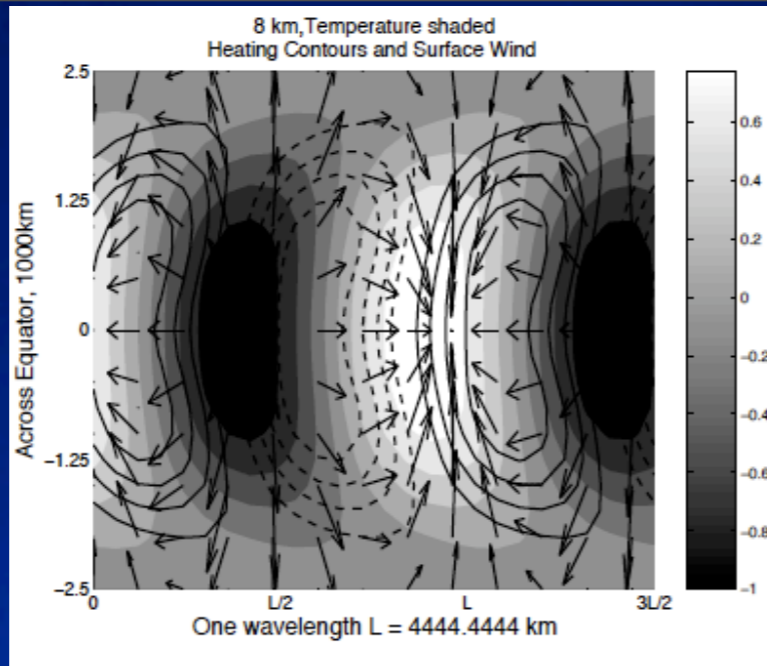
Easterly:  $w_0 = -5 \text{ m/s}$



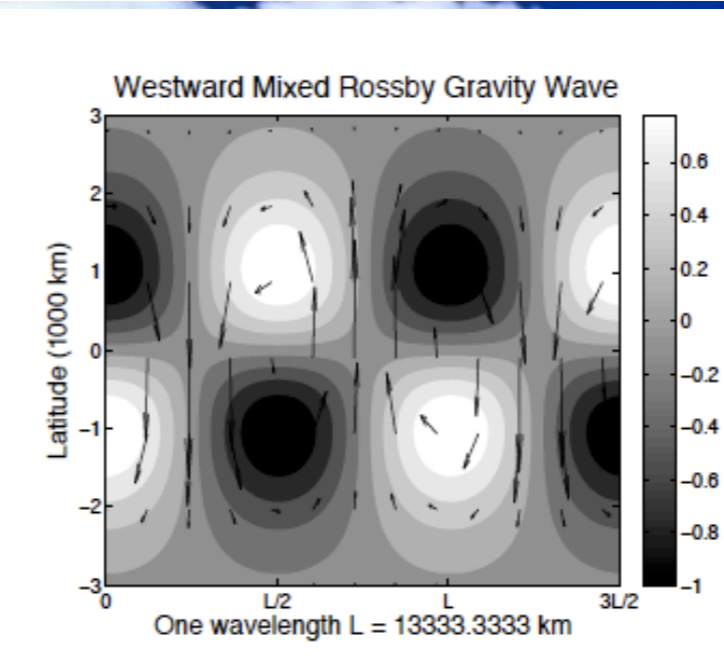
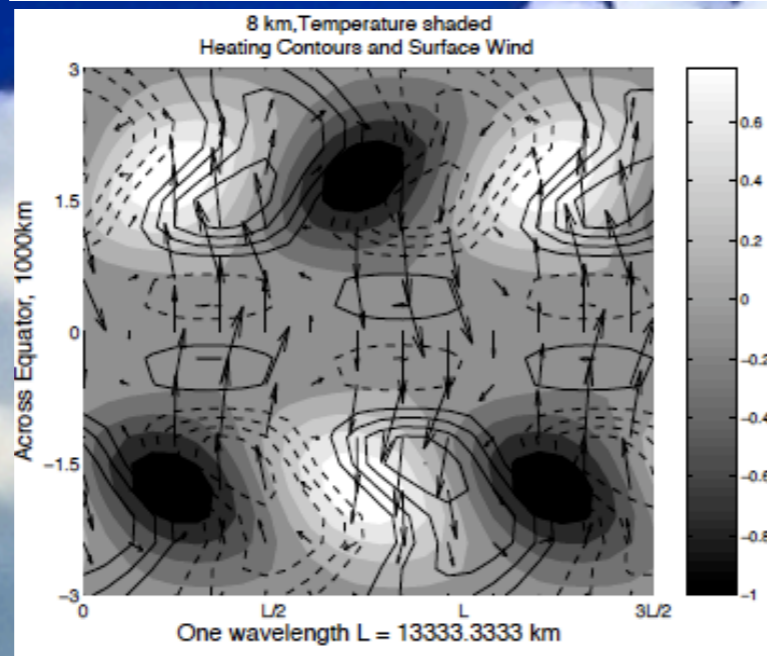
2nd Baroclinic Dry Kelvin wave

potential temperature (shaded), heating and cooling anomalies (contours)

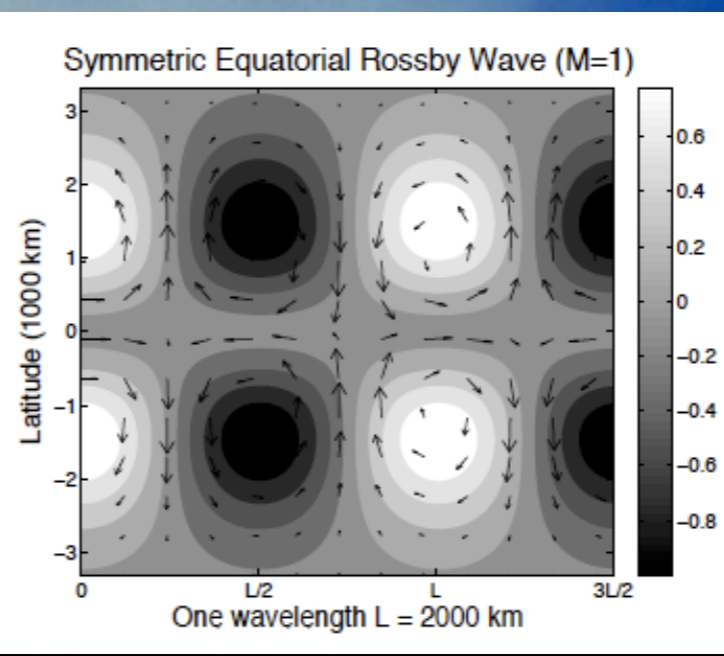
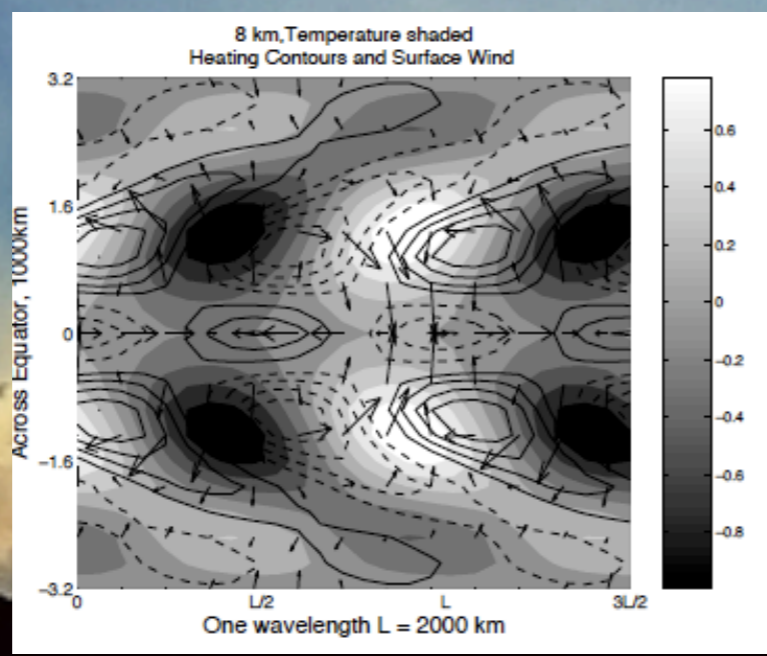
N=1 WIG



MRG

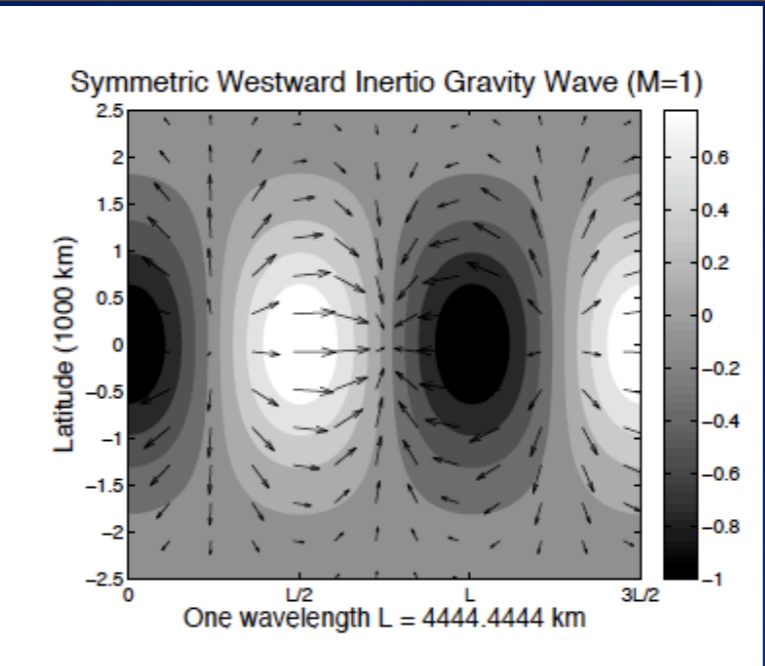
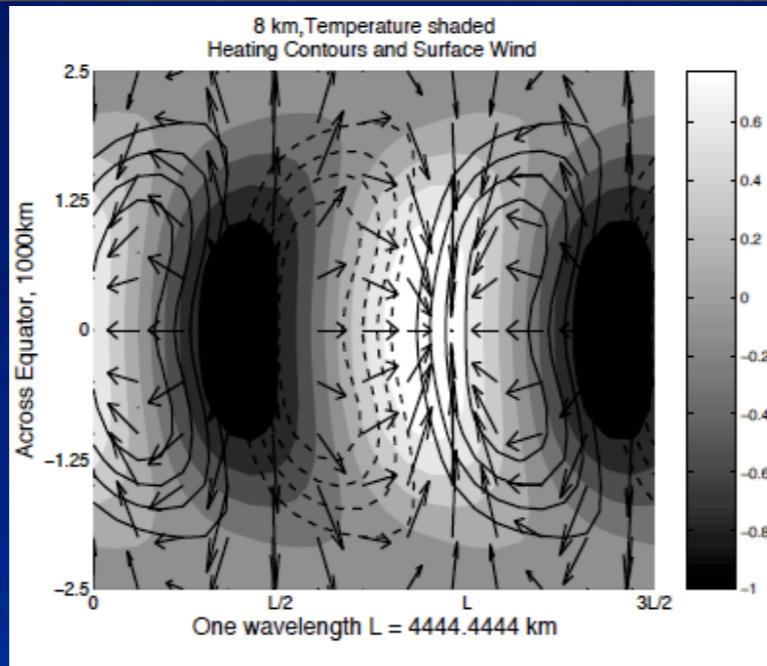


N=1  
Rossby

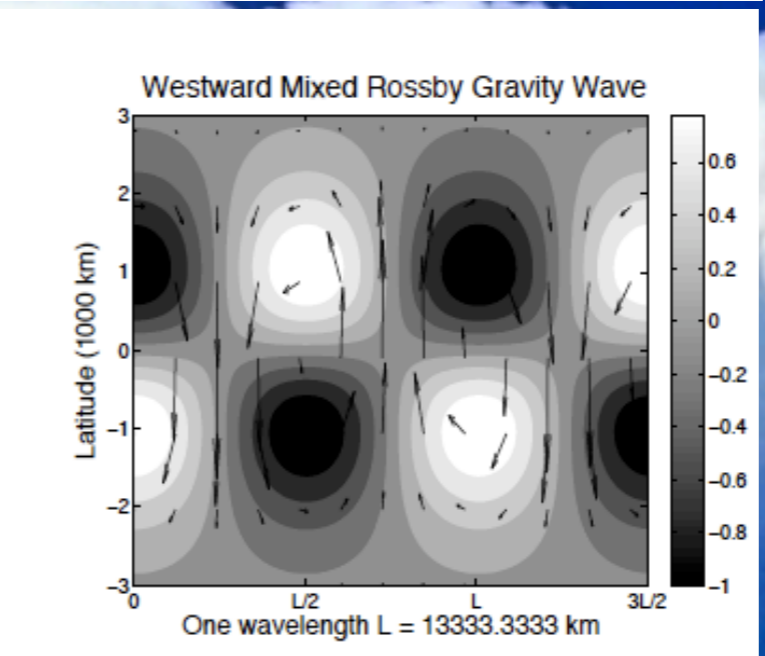
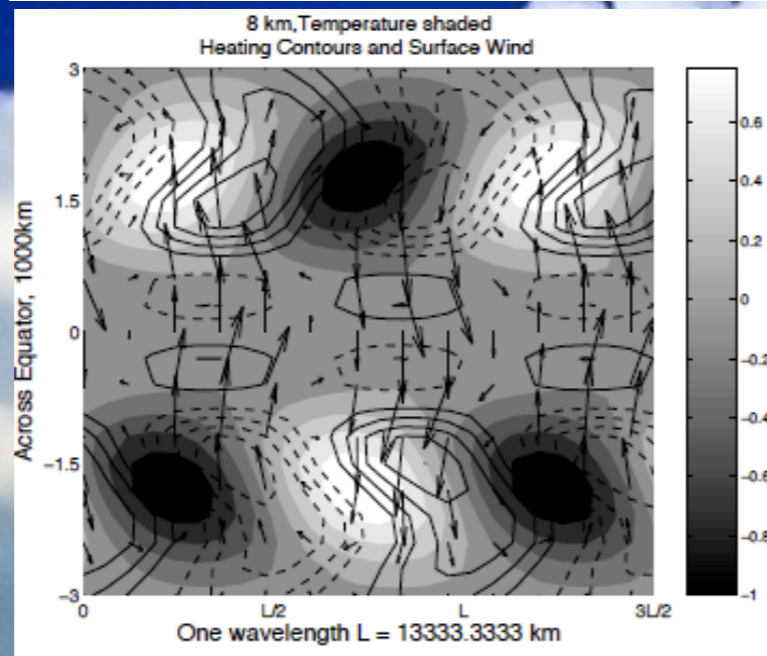


Dry 2nd Baroclinic equivalents

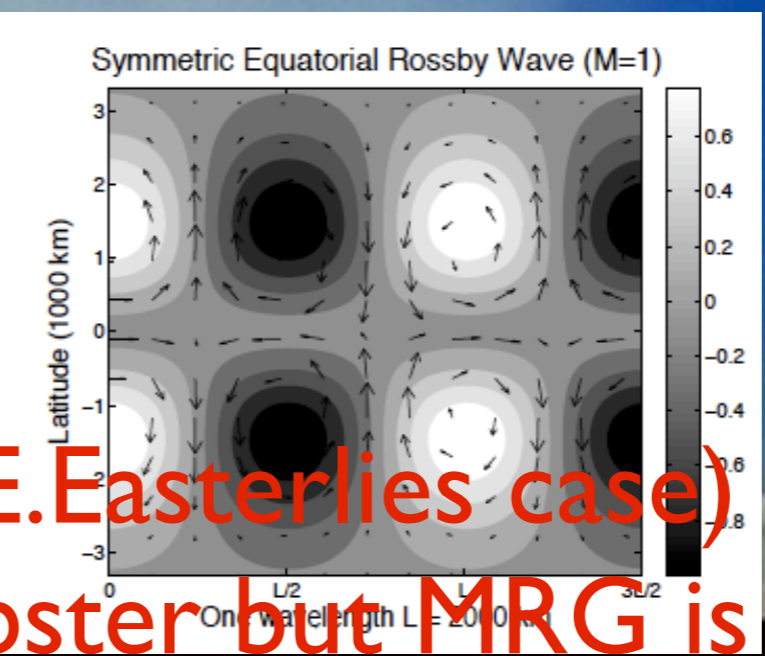
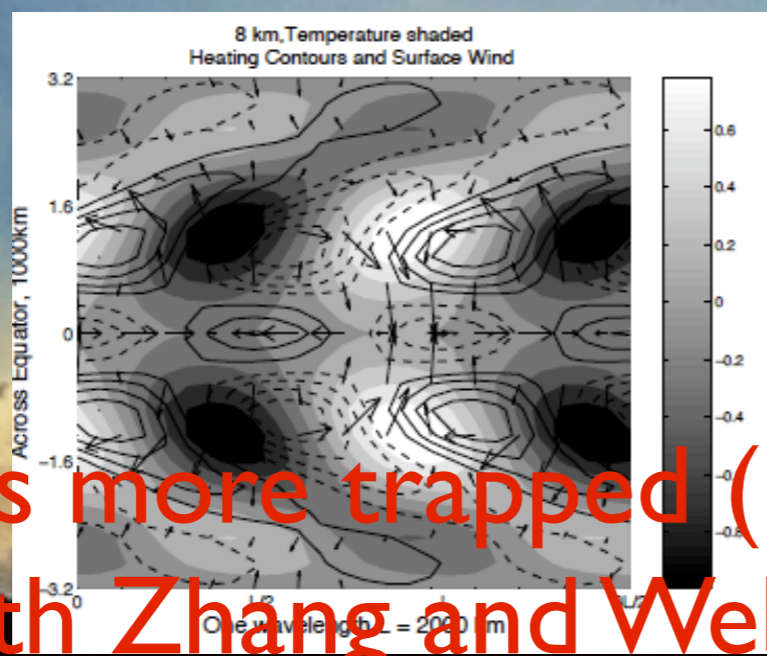
N=1 WIG



MRG



N=1  
Rossby



Rossby is more trapped (E.Easterlies case) consistent with Zhang and Webster but MRG is not!

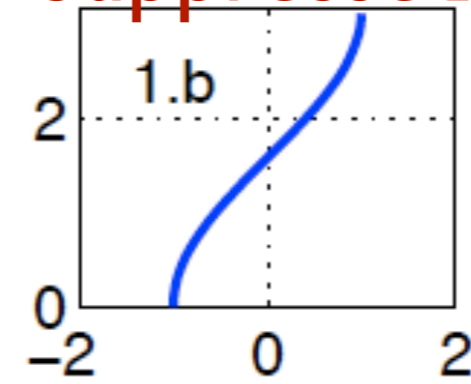
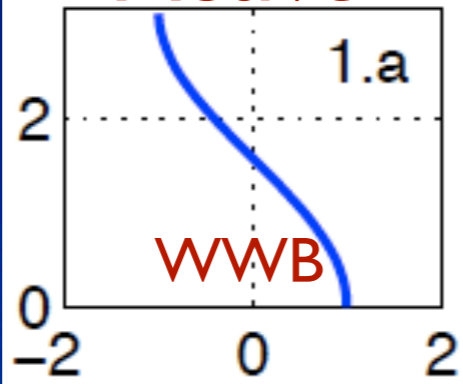
Dry 2nd Baroclinic equivalents

# Vertical Shear Profiles

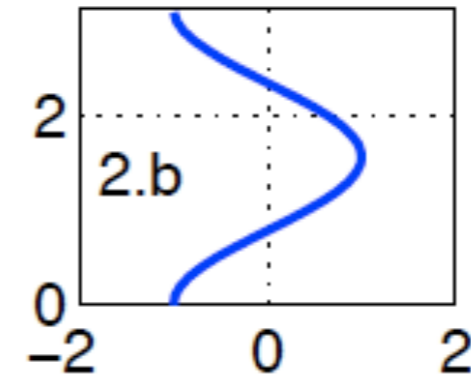
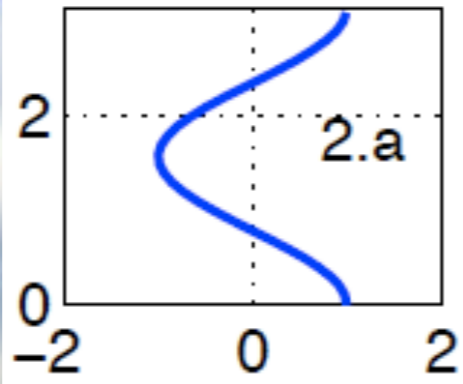
Active

Suppressed

$$\bar{u}_1 > 0$$



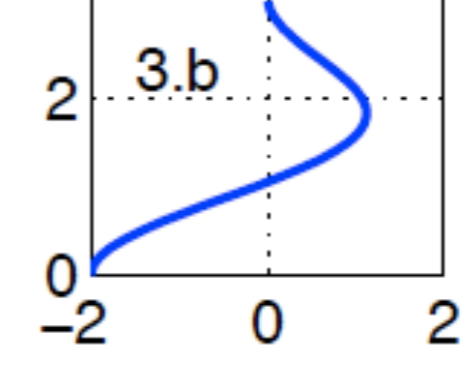
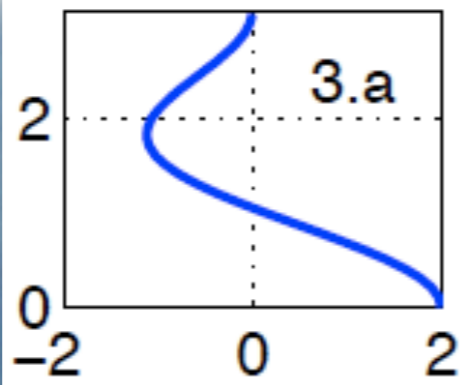
$$\bar{u}_2 > 0$$



$$\bar{u}_1 < 0$$

$$\bar{u}_2 < 0$$

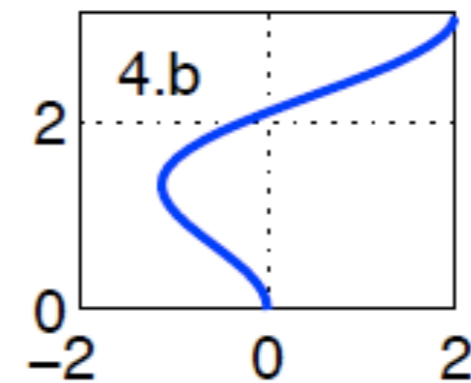
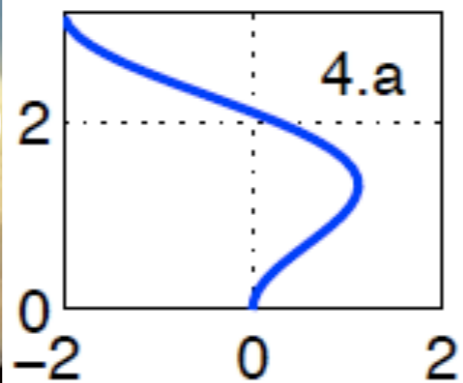
$$\bar{u}_1 = \bar{u}_2 > 0$$



$$\bar{u}_1 = \bar{u}_2 < 0$$

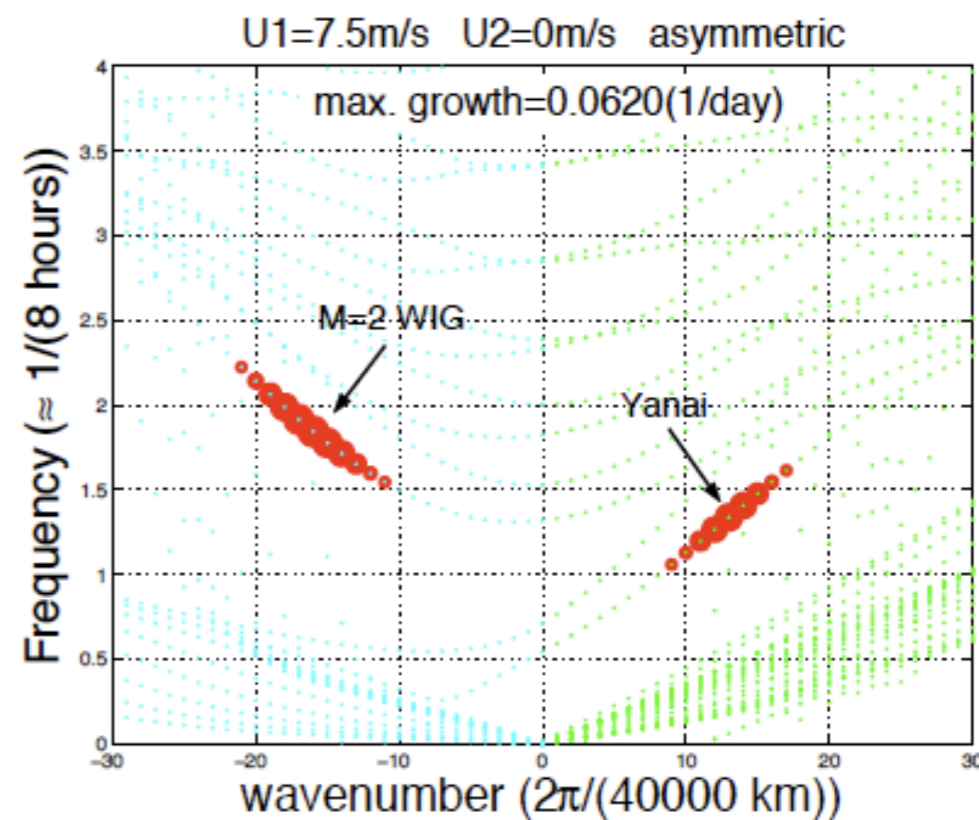
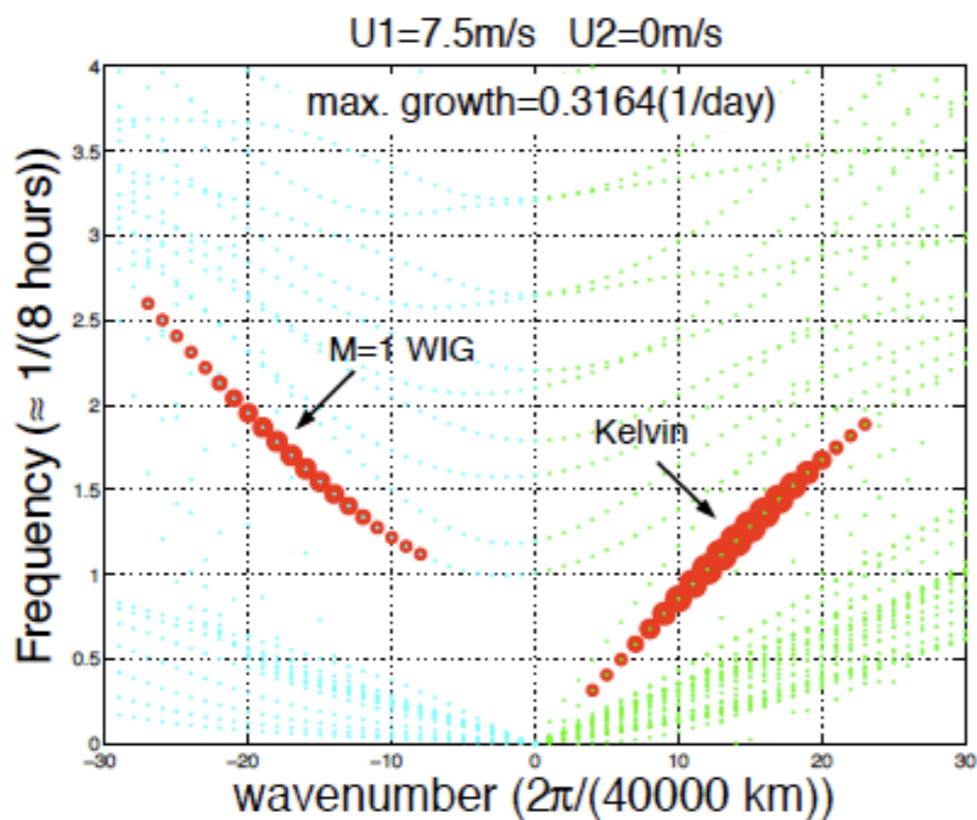
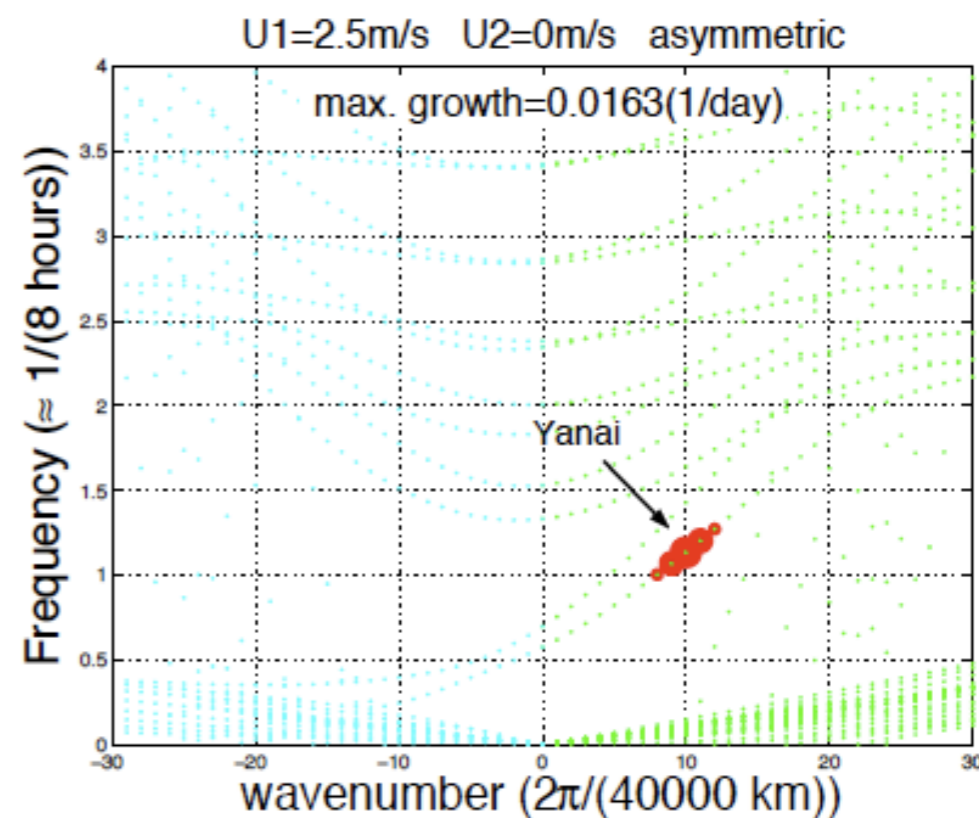
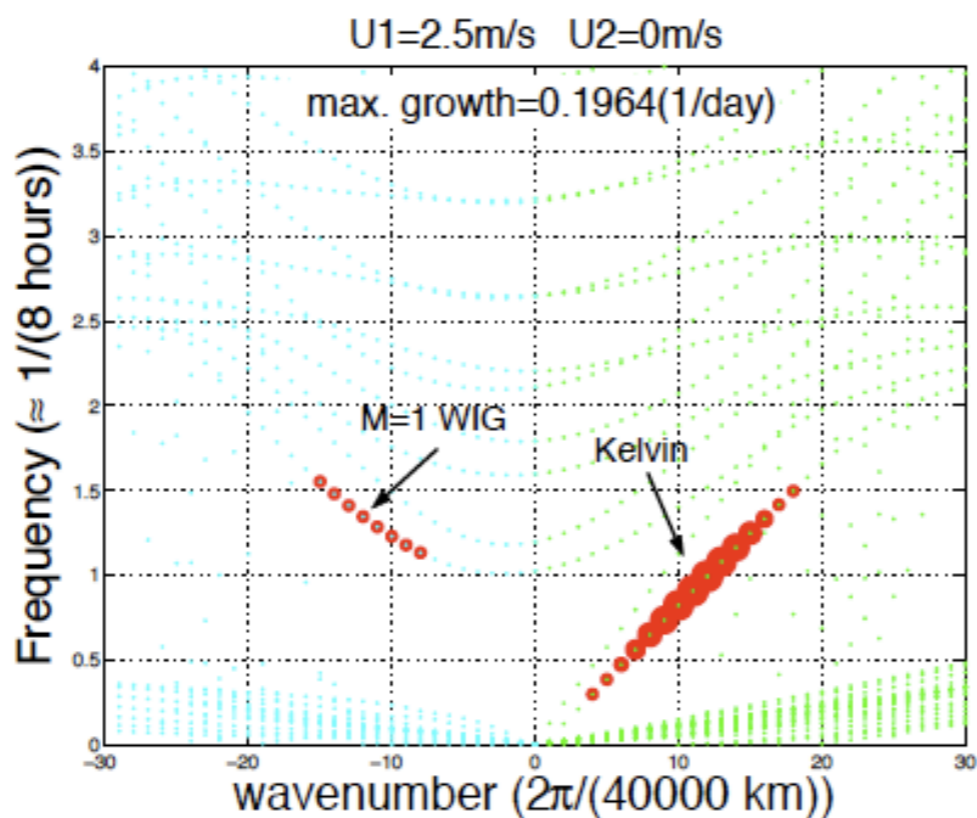
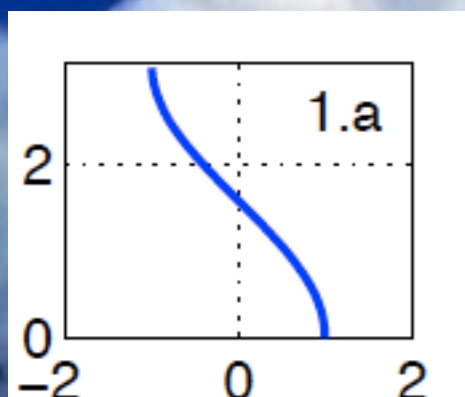
Not physical!

$$\bar{u}_1 = -\bar{u}_2 > 0$$

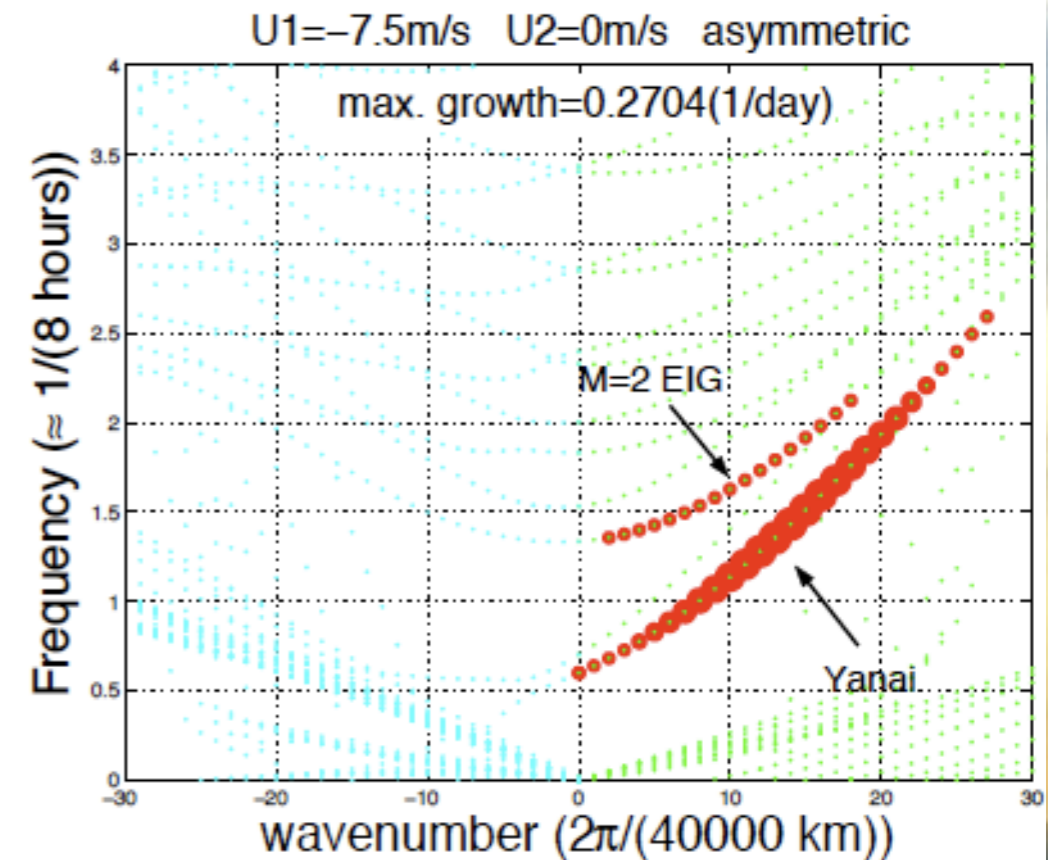
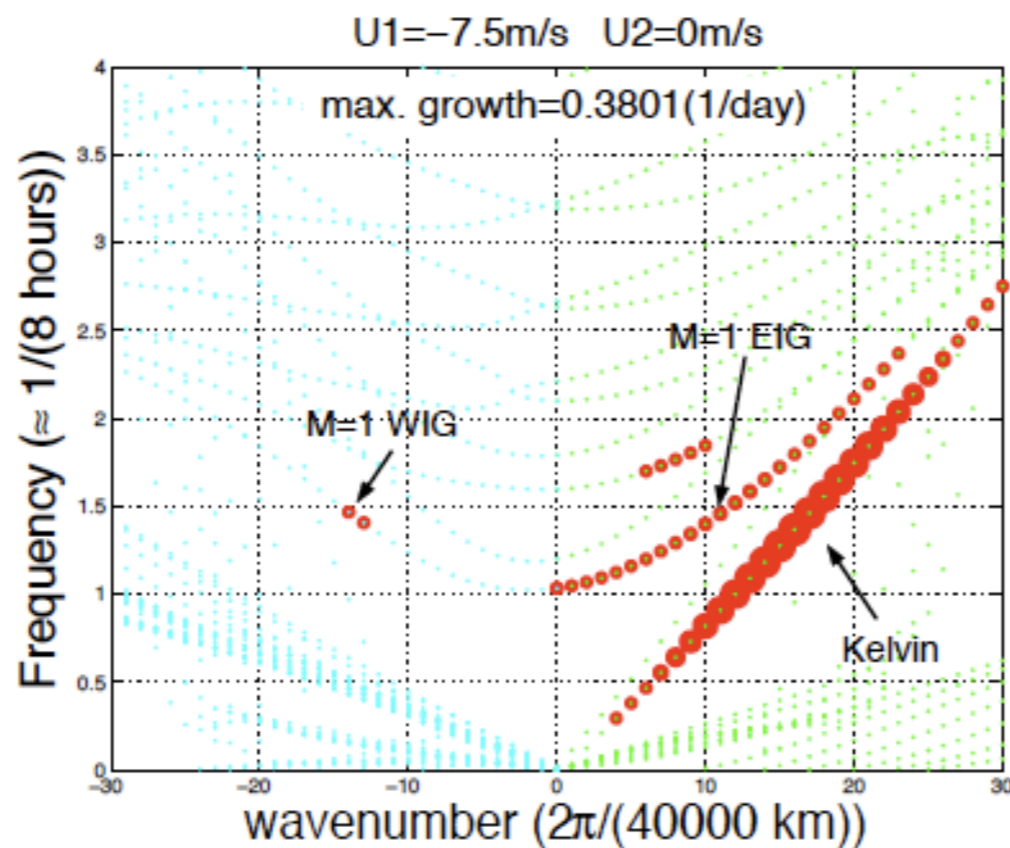
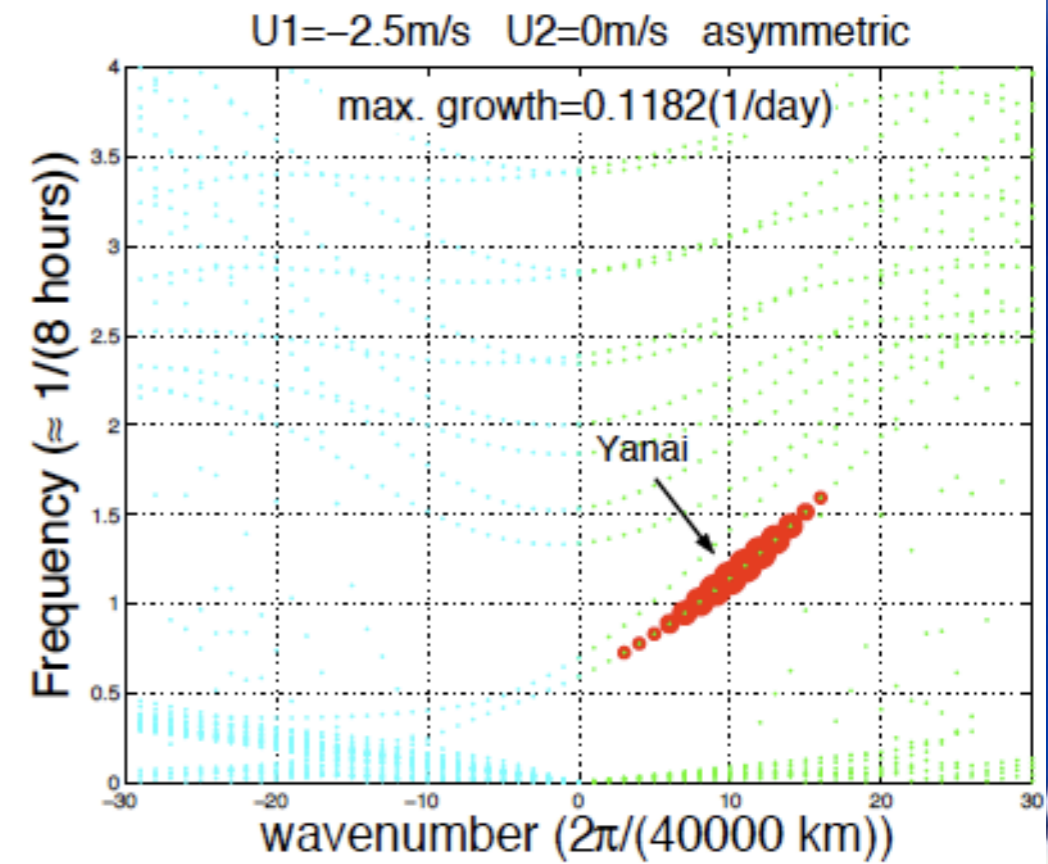
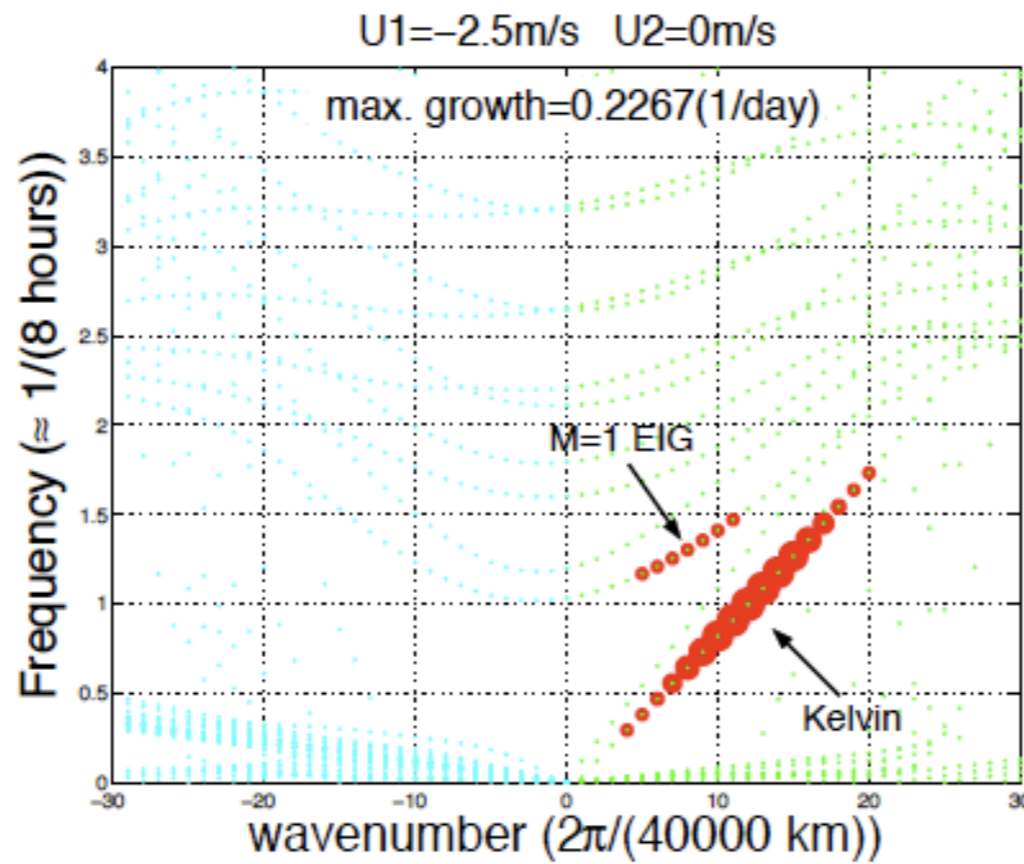
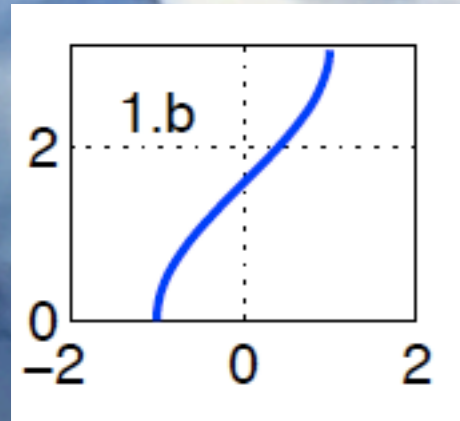


$$\bar{u}_1 = -\bar{u}_2 < 0$$

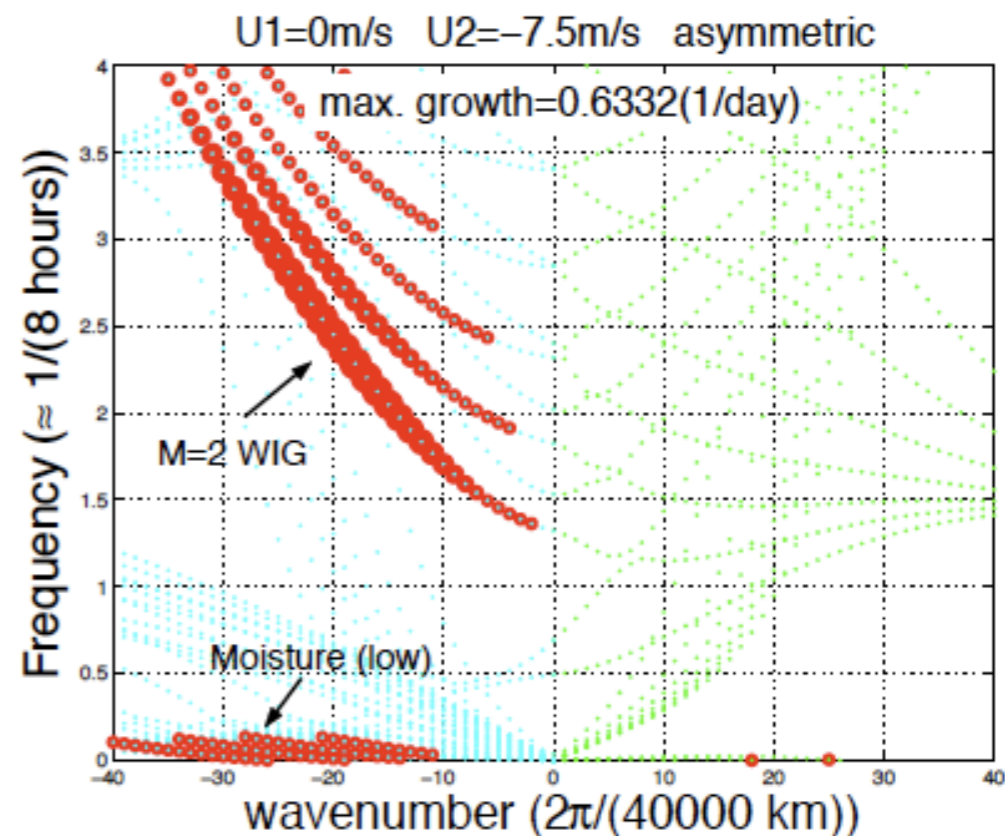
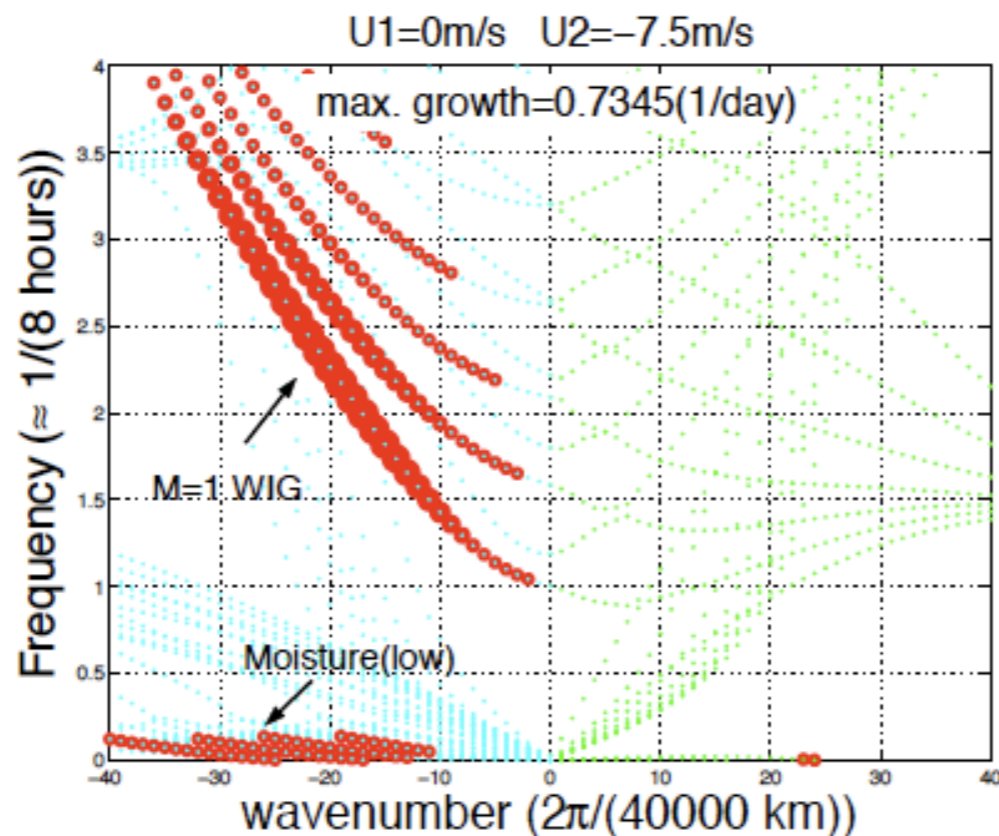
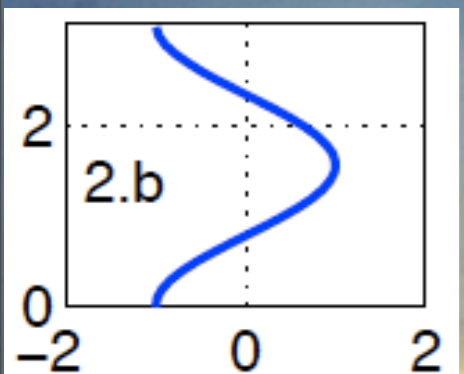
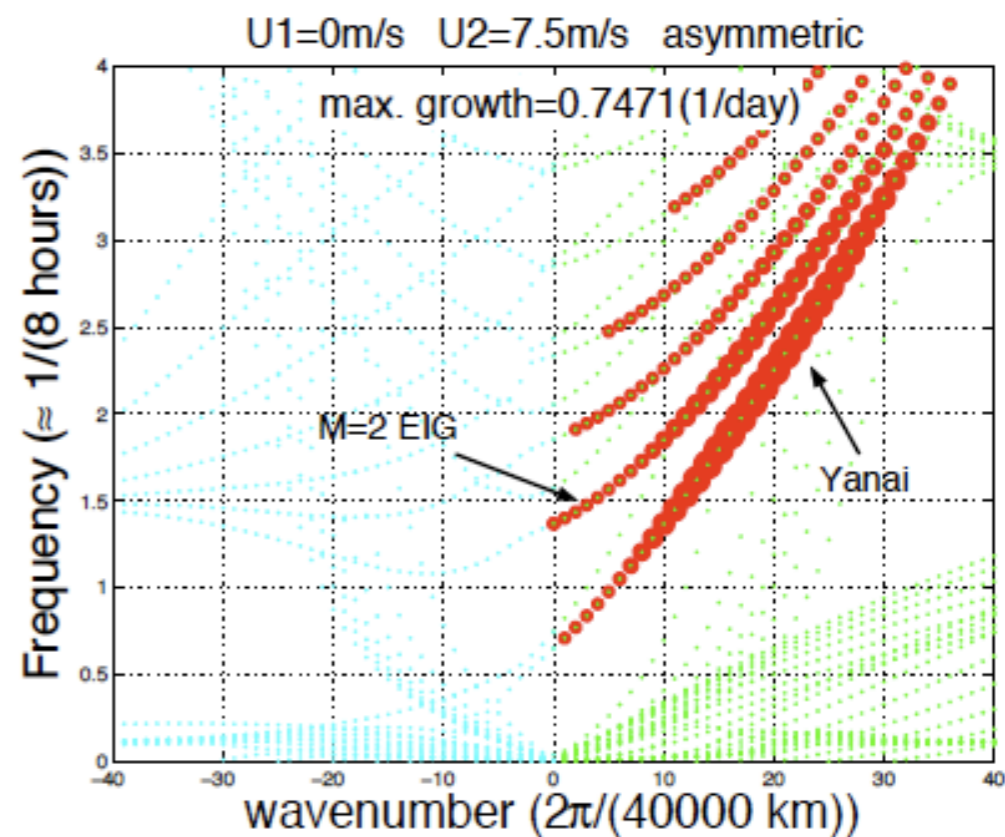
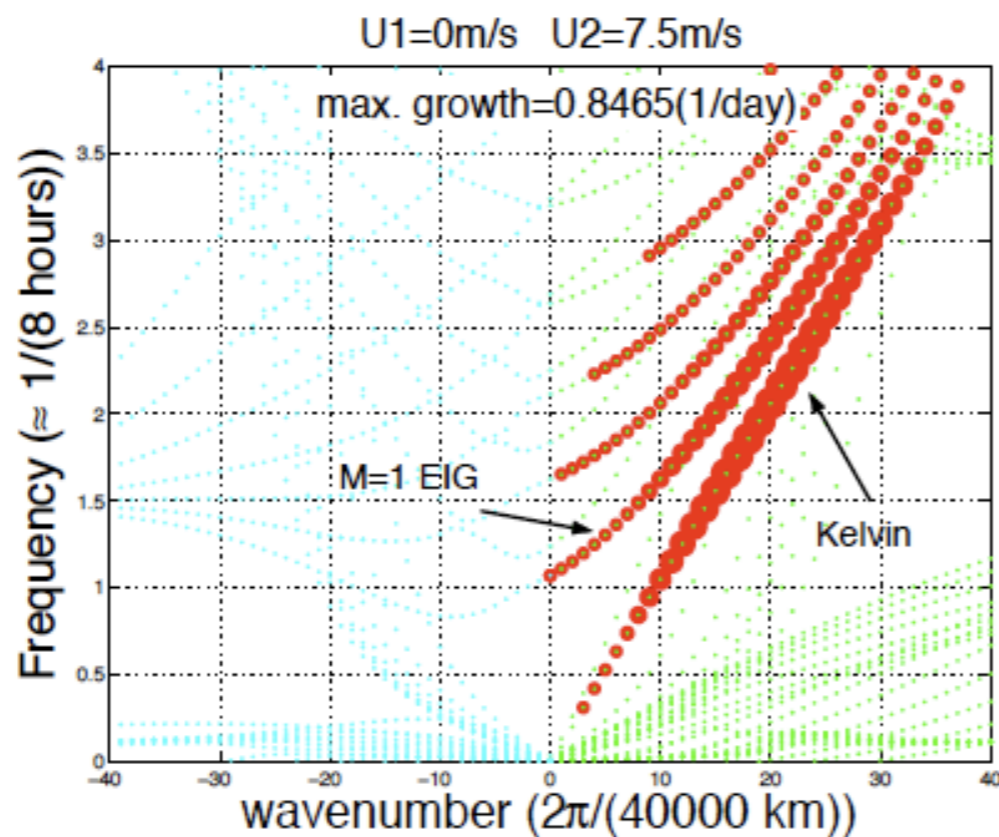
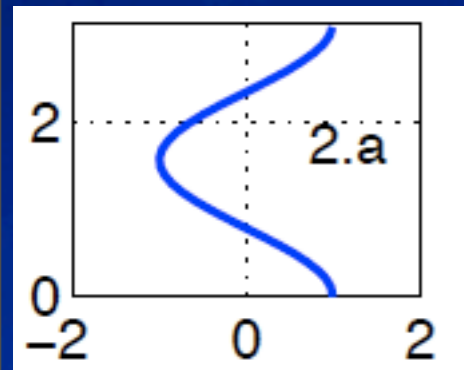
# 1st Baroclinic Shear With high level **easterly** wind



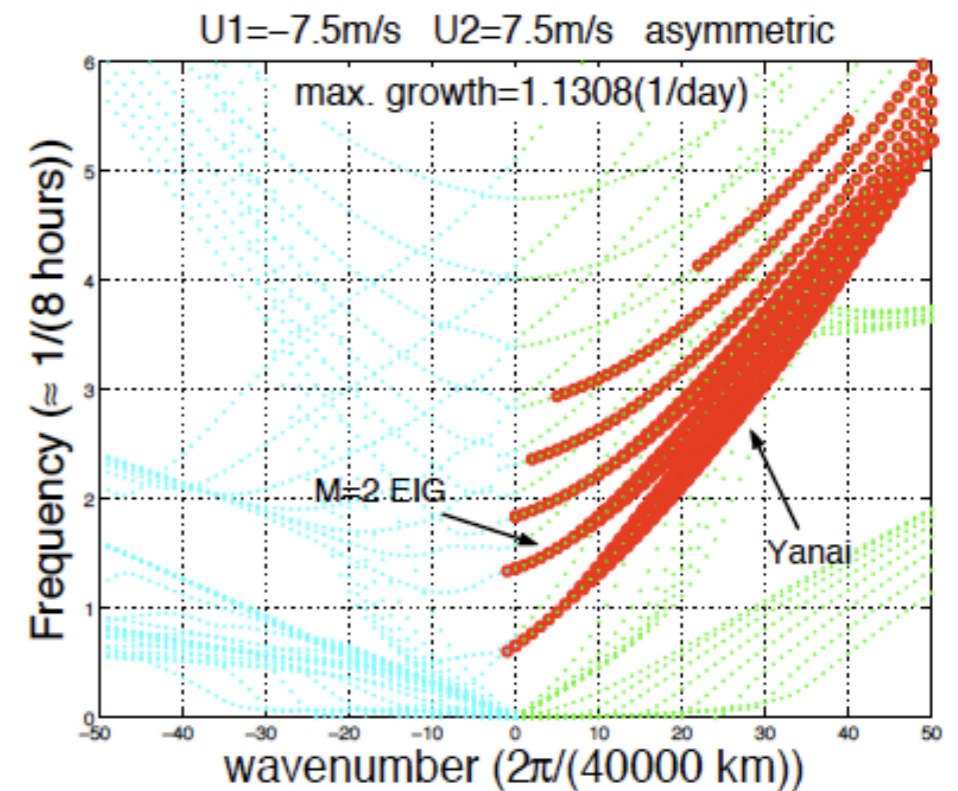
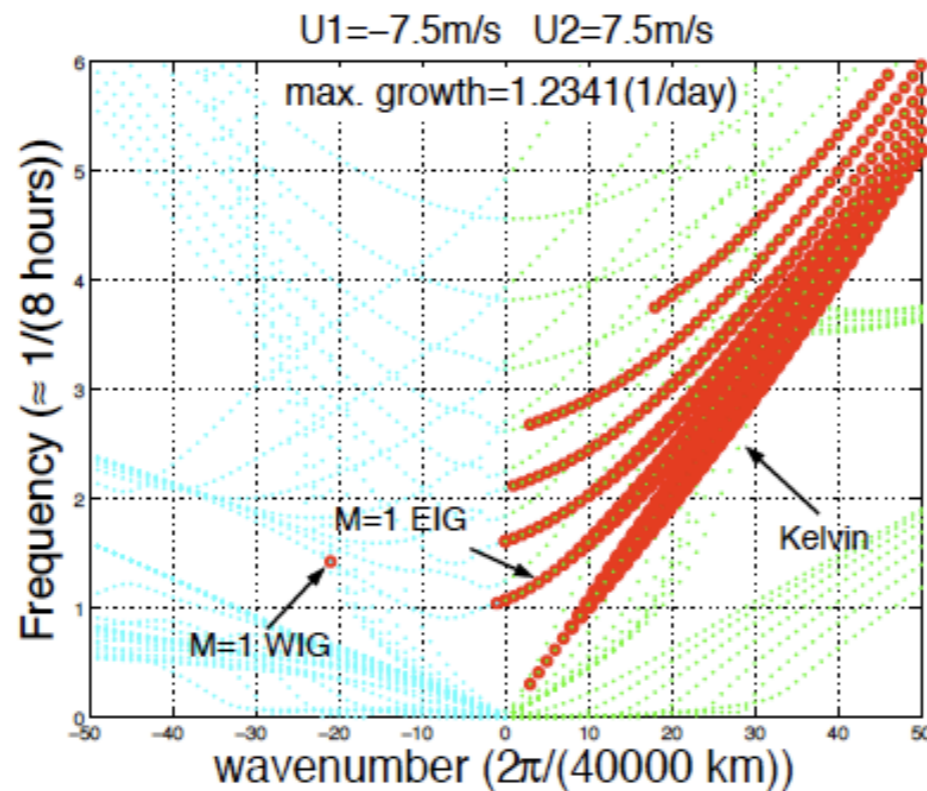
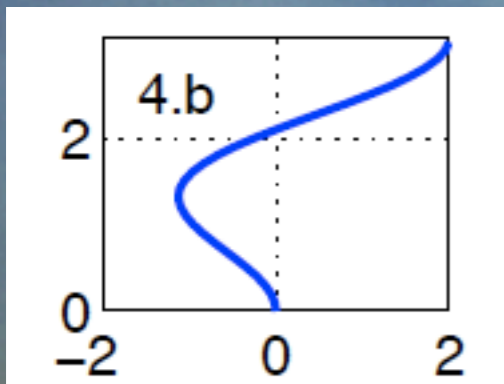
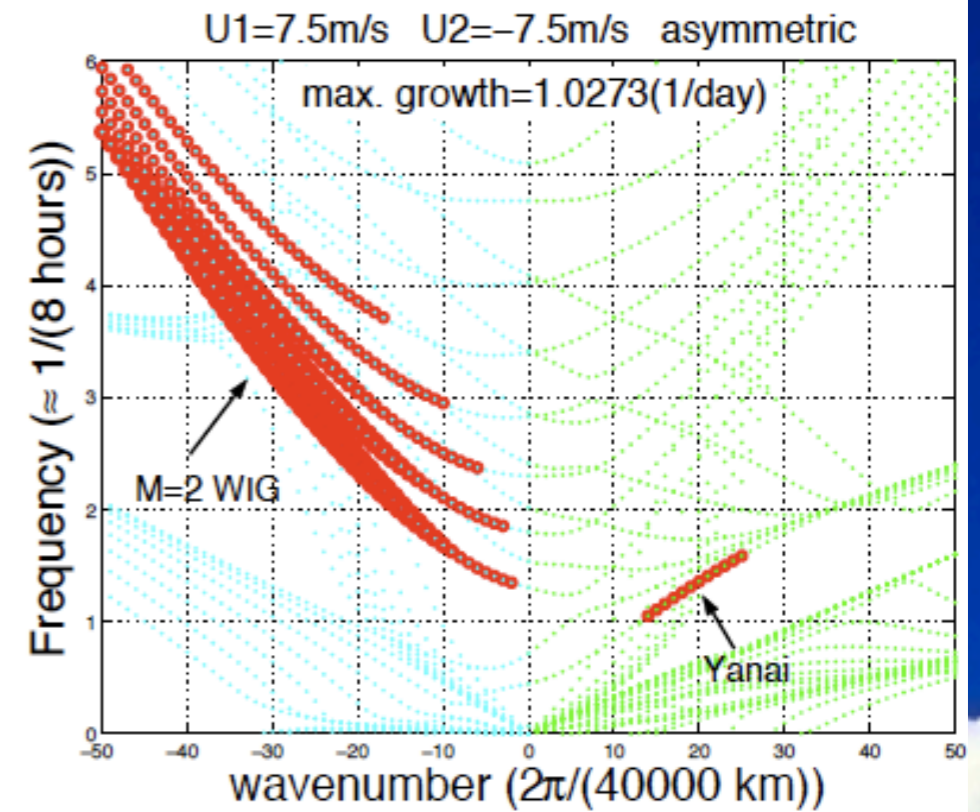
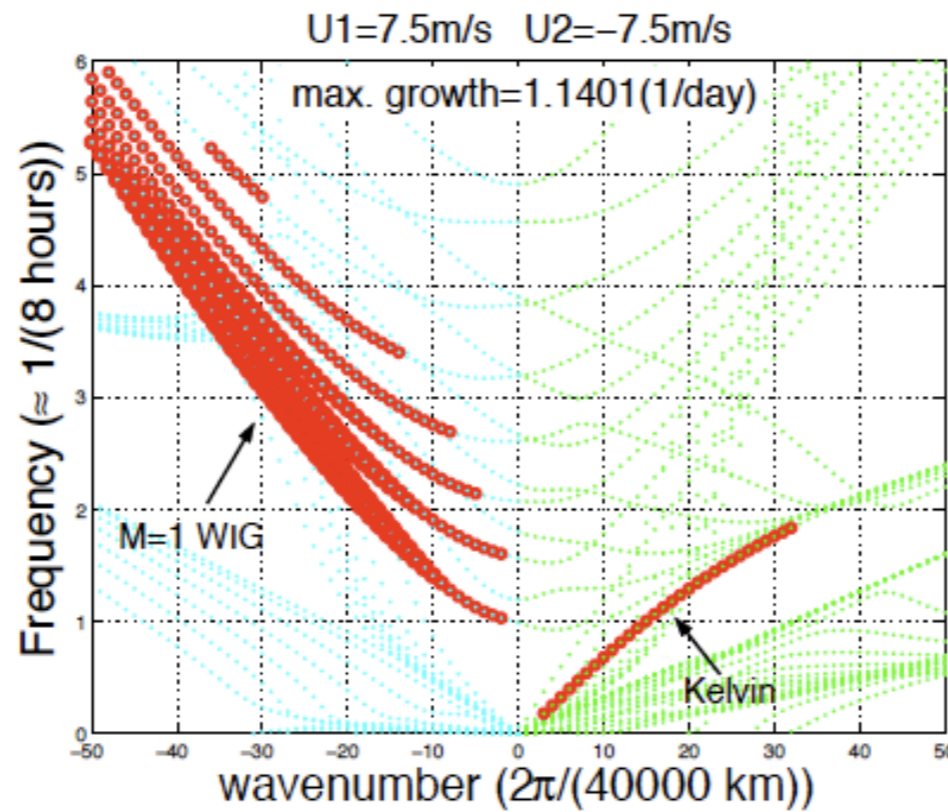
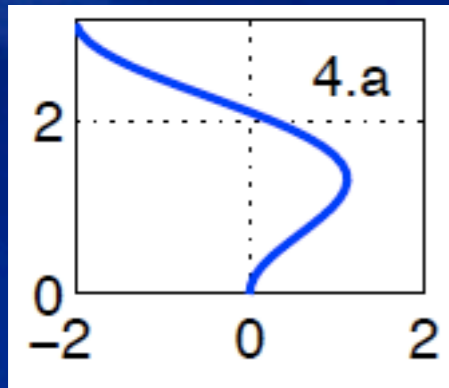
# 1st Baroclinic Shear With high level **westerly** wind



# 2nd Baroclinic Shear (mid-level jet)

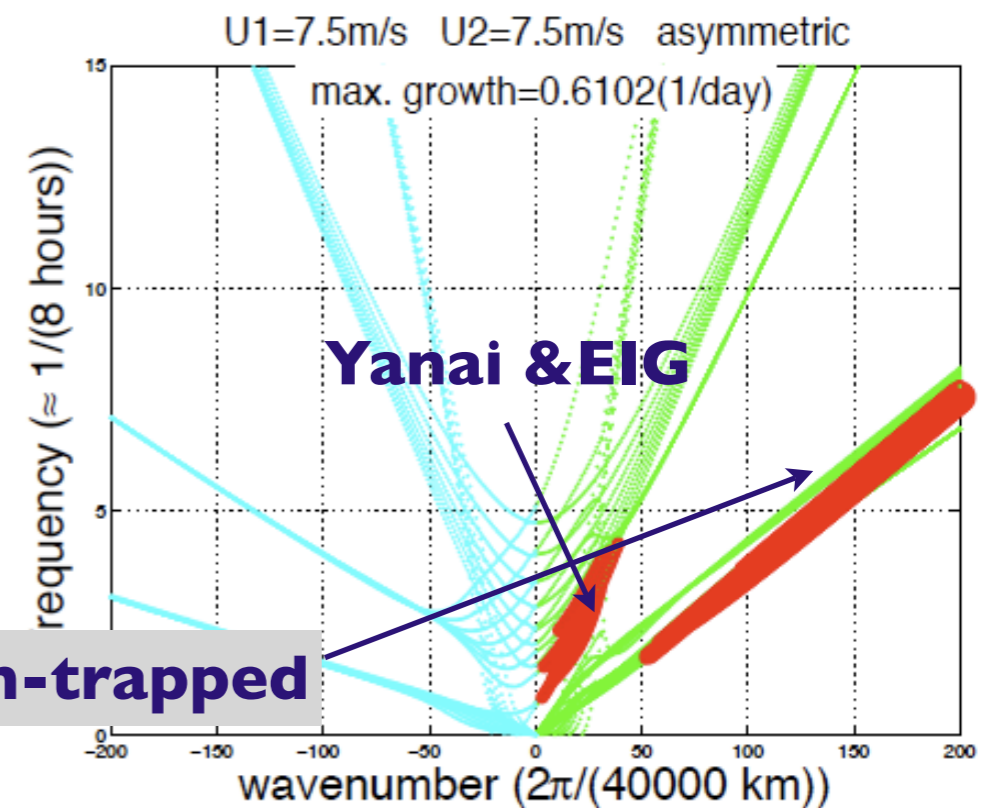
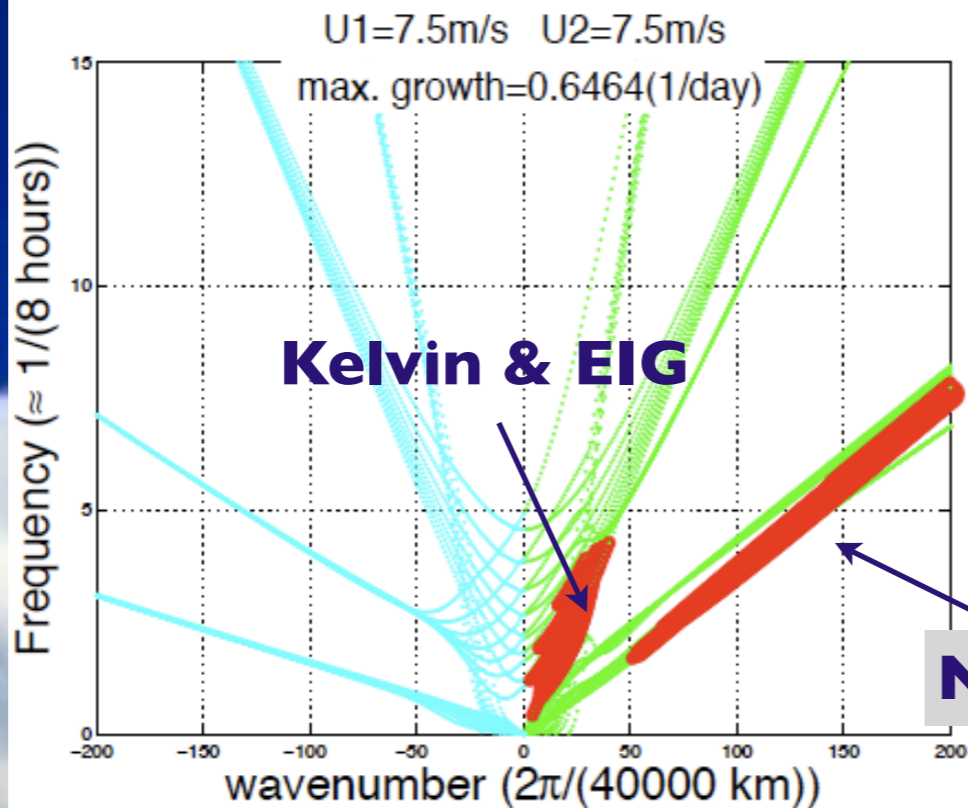
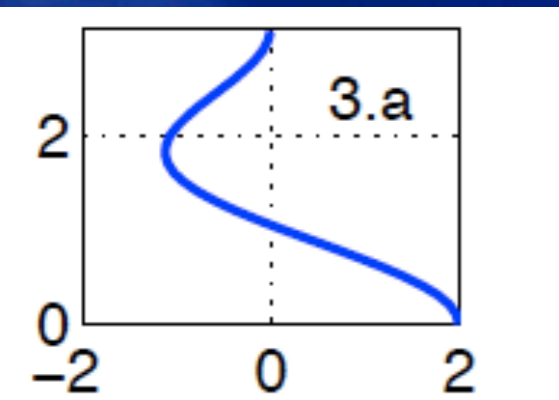


# High level winds

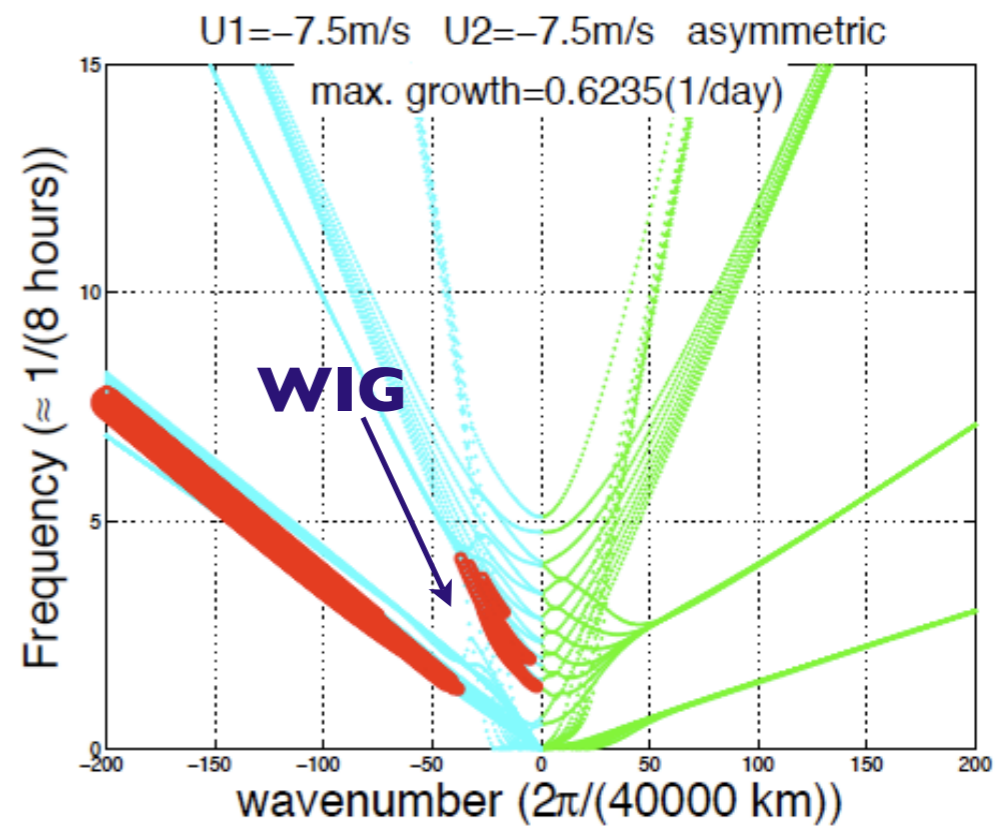
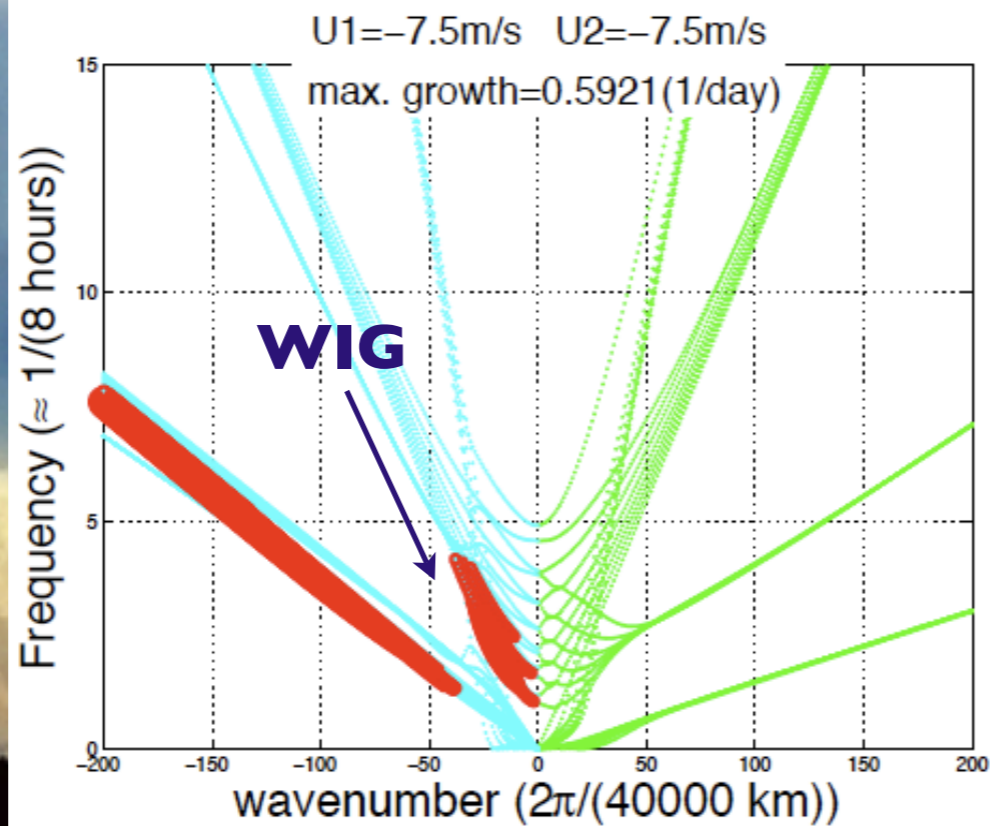
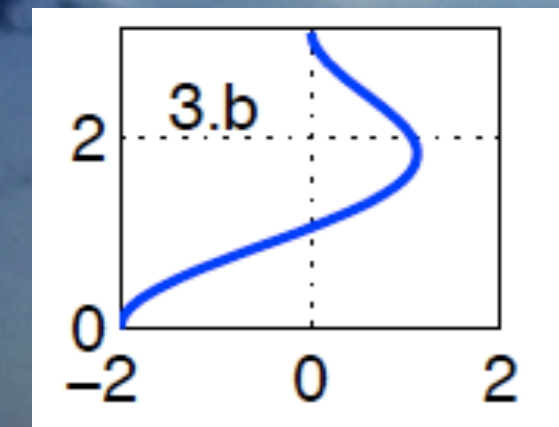


# Low level winds

Instability of Non-trapped (moisture) mode at small scale.



**Non-trapped**



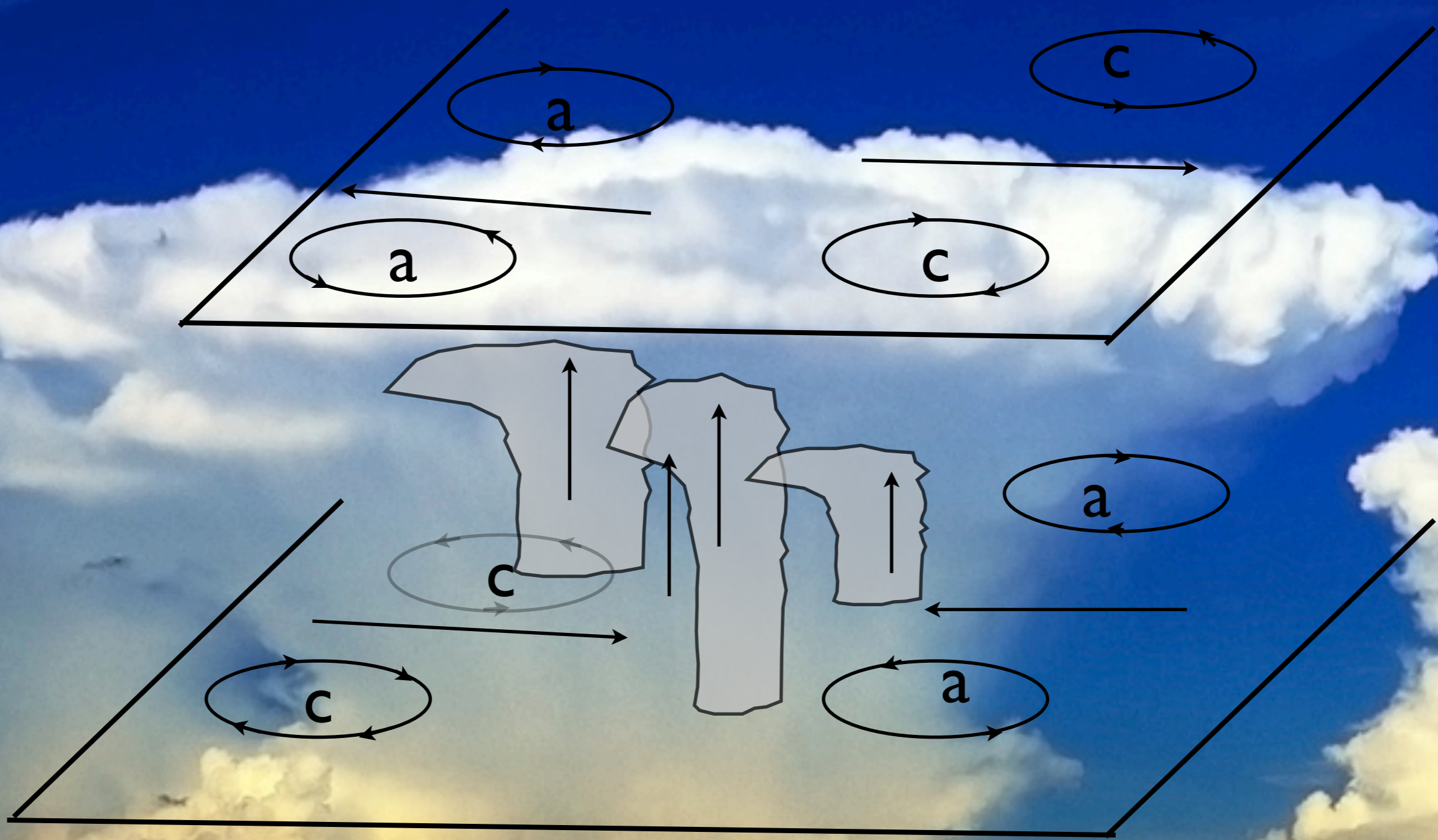
# Remarks and Conclusions

- Meridional/barotropic shear induces
  - ✓ evident doppler shift
  - ✓ increase growth rates and destabilizes higher waves
  - ✓ significant distortion of wave structure (consistent with dry-dynamics studies)
  - ✓ non-Kelvin aspects of Kelvin waves: non-trivial meridional wind and convergence for Kelvin wave (consistent with obs)
  - ✓ Strong equatorial easterly winds: instability of Rossby and MRG waves. Purely dry-shear instability confirmed. Consistent with observation showing Rossby and MRG waves evolve in a barotropic environment. Also GCM simulations with multcloud model.
  - ✓ E. Easterly: trapping of CCWs. Not so significant effect in E. westerly except for highly large wind values that lead to instability of small scale moisture mode... supports the theses that tropical-extratropical interactions is due to intrusion of extratropical Rossby waves.

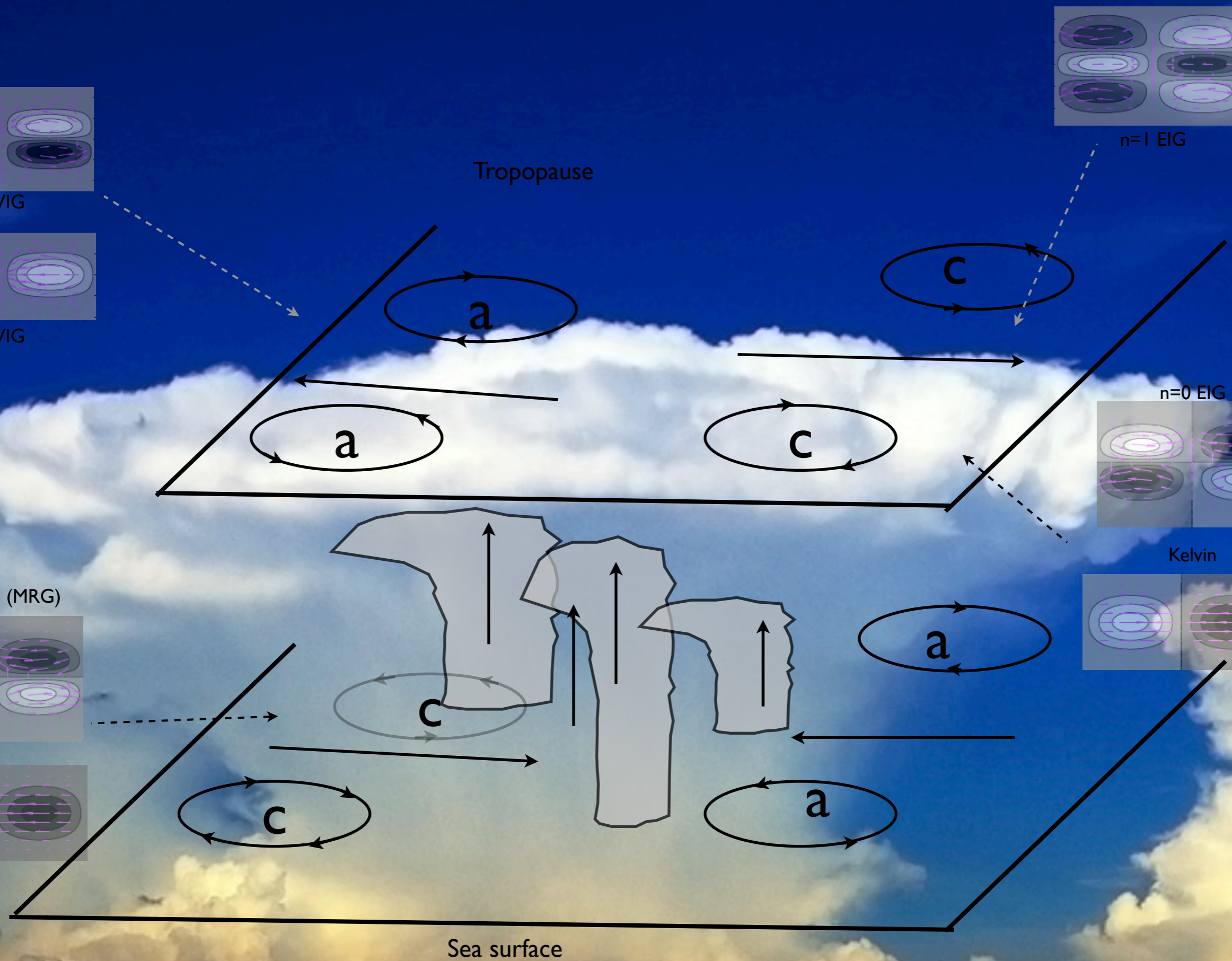
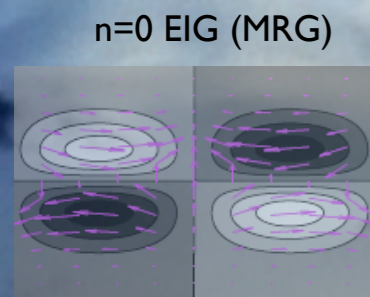
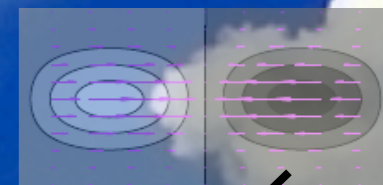
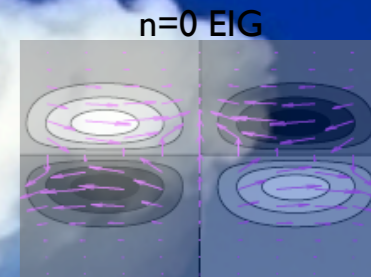
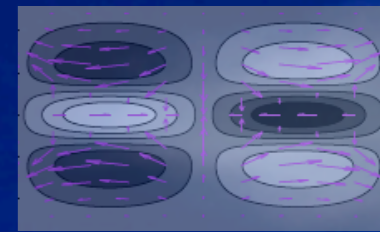
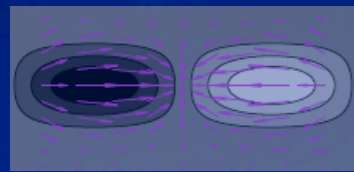
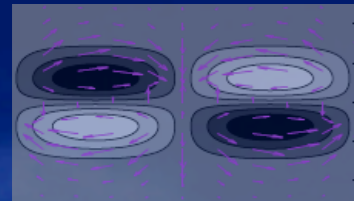
## ● Vertical Shear

- ✓ doppler shift not evident for CCWs
- ✓ overall increase of growth rates and destabilization of higher waves with increasing wind speeds but No Rossby or MRG waves.
- ✓ High-level easterlies and/or low-level westerlies destabilize both west. and east. G. waves while high-level westerlies and/or low-level easterlies destabilize east. G. waves and stabilize west. G. waves.
- ✓ Westerlies at both high- and low-levels induce east. G. waves while Easterlies induce west. G. waves.
- ✓ In the absence of high level winds, strong low level winds induce catastrophic instabilities of non-equatorially trapped (moisture) modes.
- ✓ These results make physical sense since the moisture and convective variables see only either the low-level ( $q$ ,  $t_{eb}$ ,  $H_c$ ) or the high-level ( $H_s$ ) wind.
- ✓ Results can be used to interpret observations of CCW's in MJO envelope.

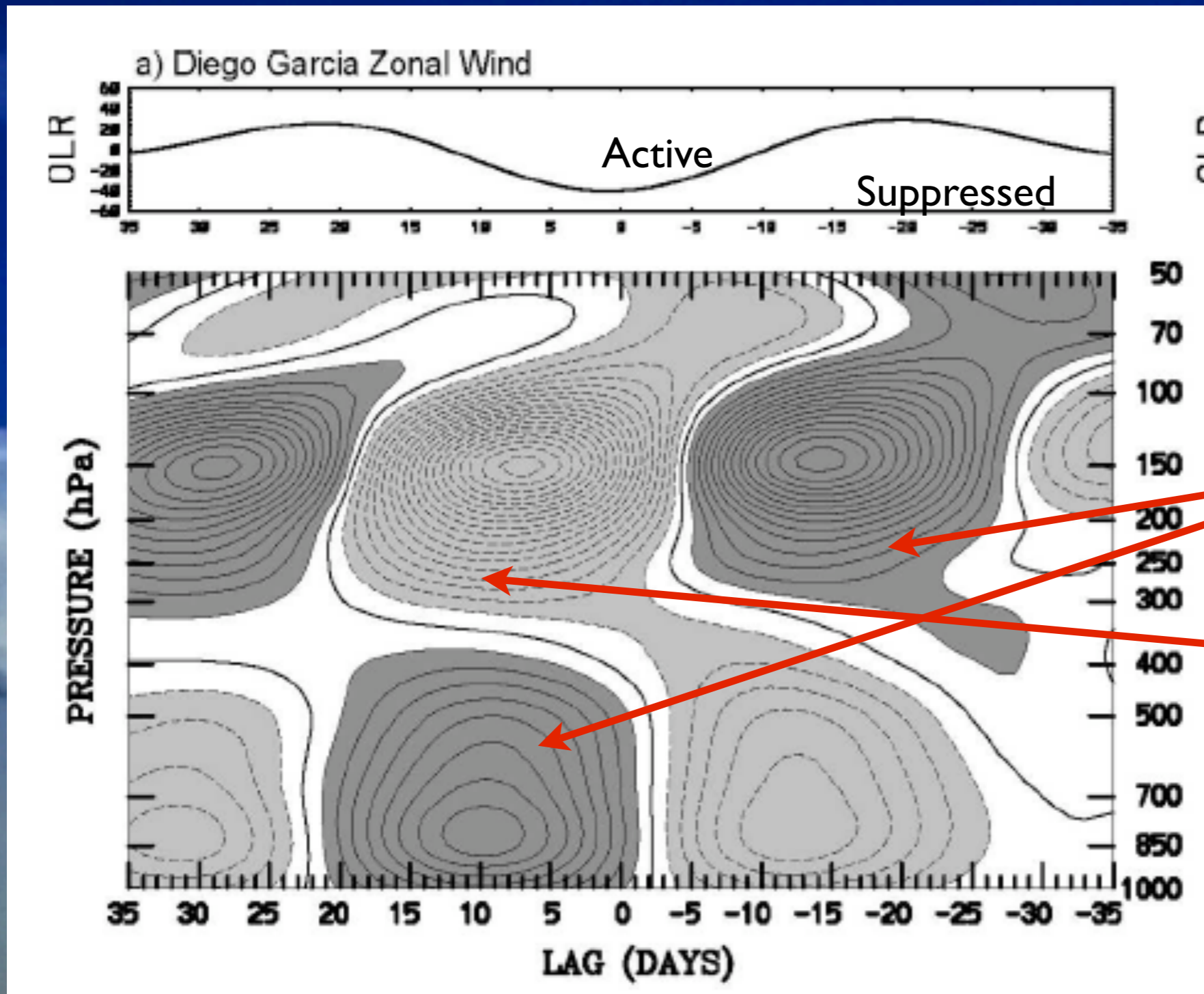
Tropopause



Sea surface



# Convectively Coupled waves in MJO envelope



Kelvin waves in both active and suppressed phase as suggested by Roundy (2008)

$N=1$  WIG (2-day) waves in active phase

Kiladis et al. 2005

However, 2-day waves can also exist in the active phase of CC Kelvin and/or Rossby waves ... GCM simulations using multicloud model.