Models of Clouds and Atmospheric Waves in the Tropics

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work with Sam Stechmann (UCLA)

Workshop on Equation Hierarchies for Climate Modeling

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Multiscale clouds and waves in the tropics

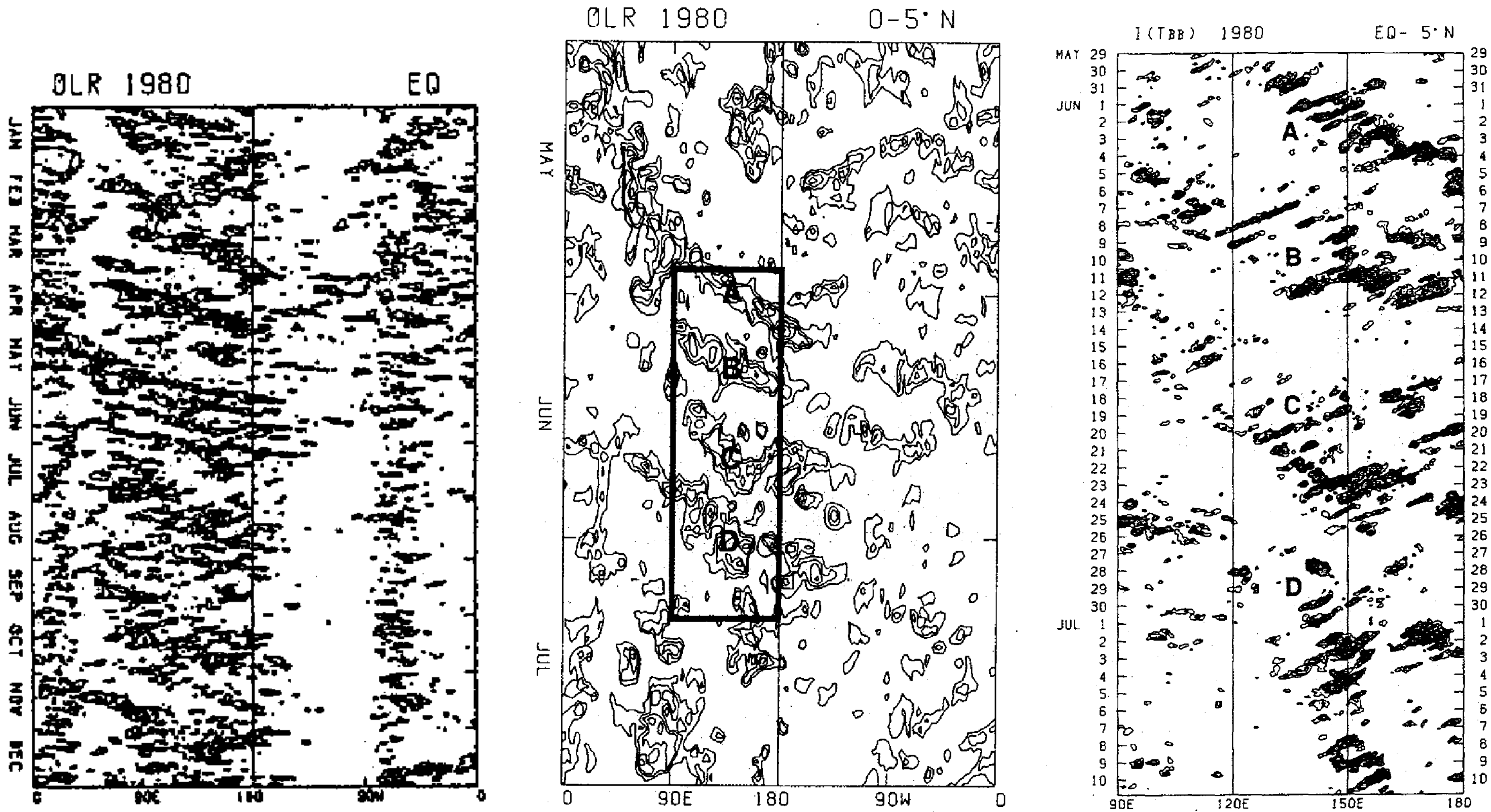
Clouds in the tropics are organized across vast length and time scales

- Cloud systems
- Wave trains of cloud systems
  called convectively coupled waves
- Envelopes of convectively coupled waves
  called the Madden–Julian oscillation (MJO)

Global climate models (GCMs)

- use grid spacings of ≈ 100 km
- must represent clouds as a sub-grid scale process
- can hope to resolve convectively coupled waves
- but do not adequately capture convectively coupled waves

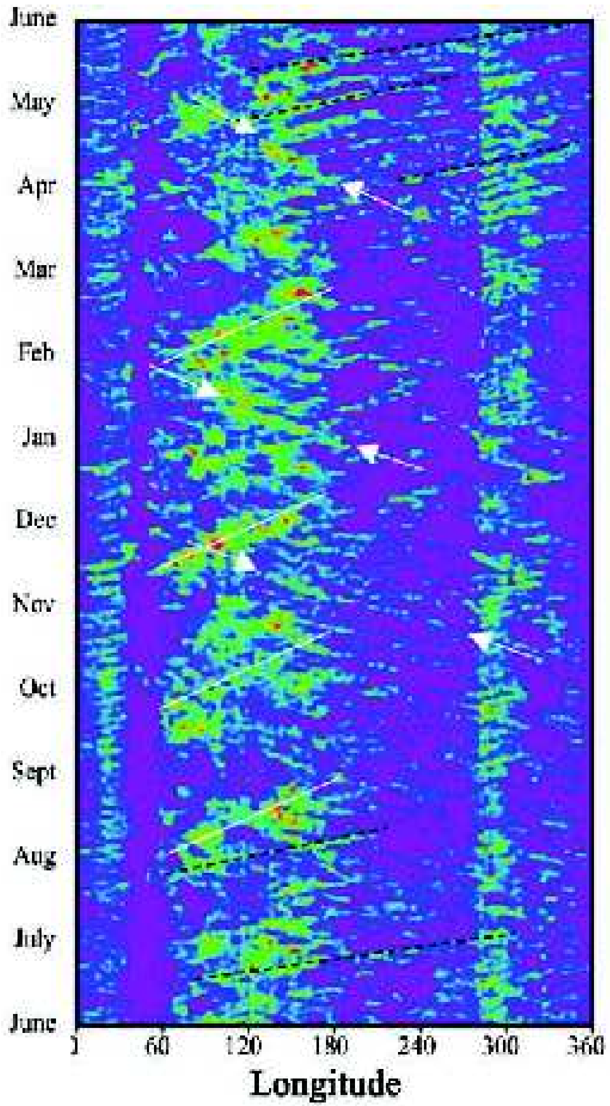
Multiscale clouds and waves in the tropics



from Nakazawa (1988)

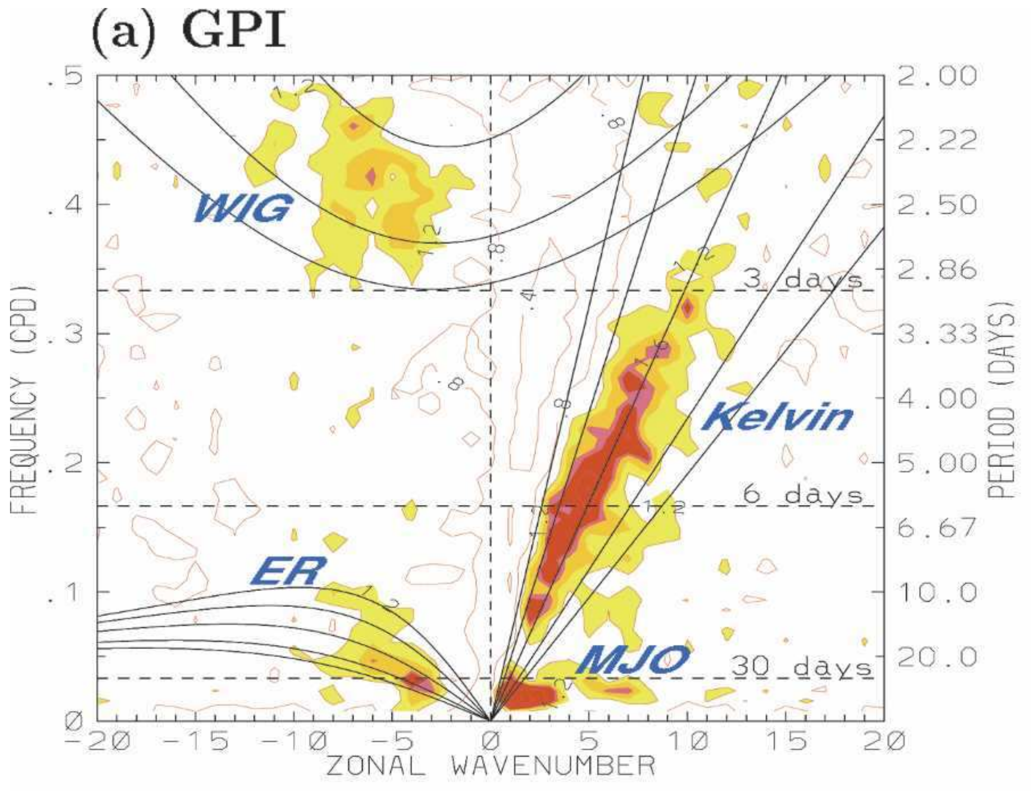
Multiscale clouds and waves in the tropics

Precipitation



2000–2001 (from Zhang 2005)

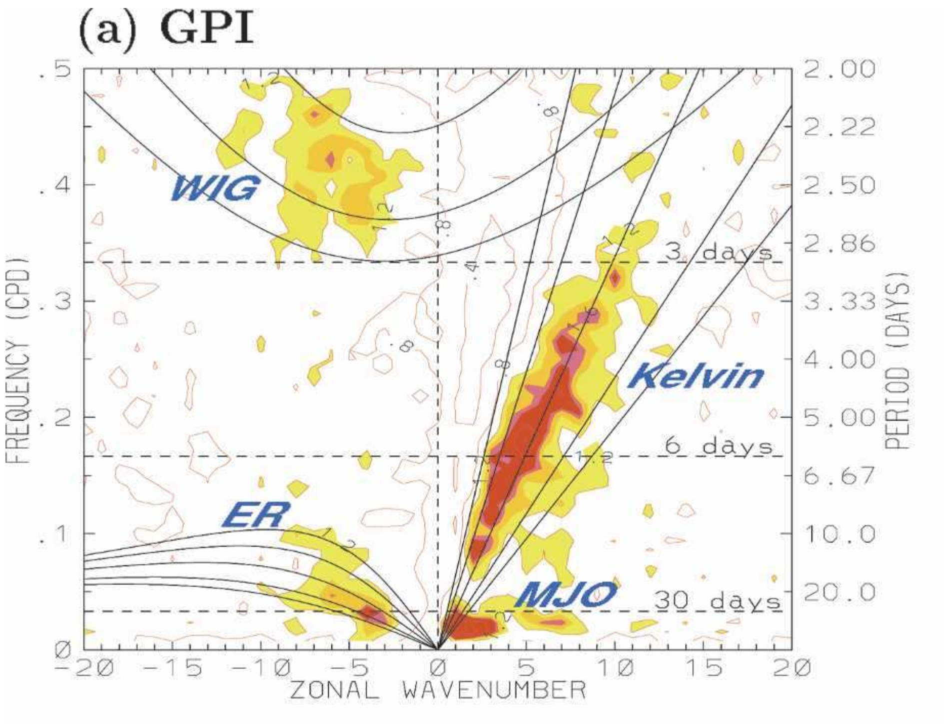
Spectral Power



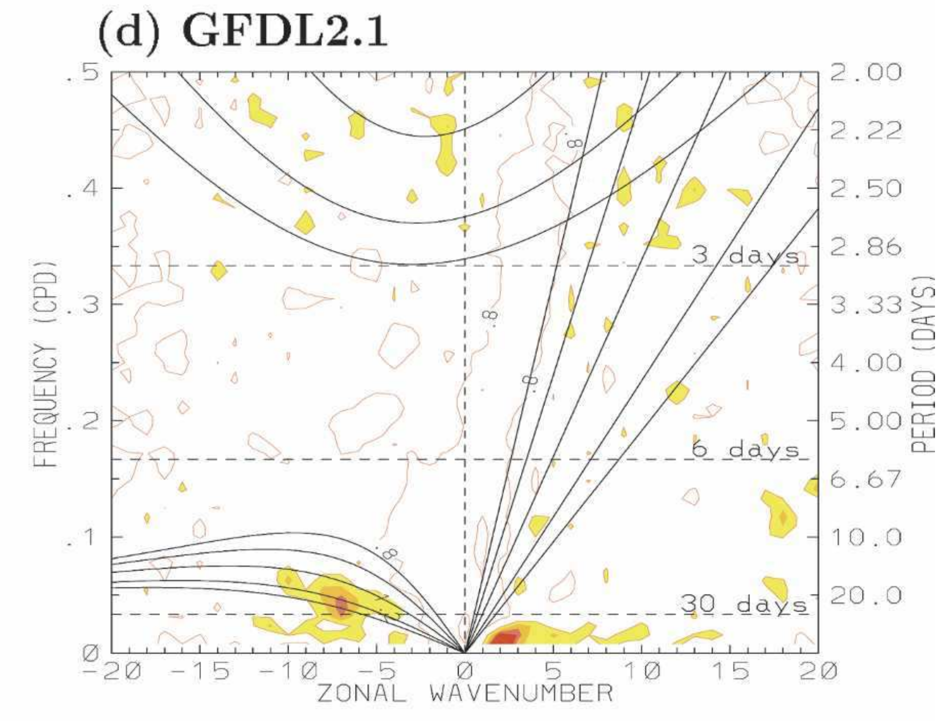
from Lin et al. 2006

Multiscale clouds and waves in the tropics

Observations



Global Climate Model (GCM)



from Lin et al. (2006)

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Multiscale clouds and waves in the tropics

Key questions:

- What causes cloud systems to organize into wave trains?

- What are the physical mechanisms of convectively coupled waves?
- How can the sub-grid scale representation of clouds in GCMs be changed to better represent these waves?

- What are the physical mechanisms of the Madden–Julian oscillation?
- How can GCMs be changed to better represent the MJO?

Outline

1. What are the fundamental physical mechanisms of the Madden–Julian oscillation?

   - A simple model for the MJO's "skeleton"

2. What causes cloud systems to organize into wave trains?

   - Gravity waves in shear
   - Nonlinear PDE model

3. How does the MJO envelope interact with the convectively coupled waves embedded within it?

   - Convectively coupled wave–mean flow interaction
   - Multi-scale asymptotic model
   - Convective momentum transport and the MJO's "muscle"

The Skeleton of Tropical Intraseasonal Oscillations

Andrew J. Majda Samuel N. Stechmann

to appear in *Proc. Natl. Acad. Sci.*, 2009

- New minimal dynamical model for the MJO
- Recovers robustly, for the first time, its fundamental features (i.e., its "skeleton") on intraseasonal/planetary scales:
  - peculiar dispersion relation of $d\omega/dk \approx 0$
  - slow phase speed of roughly 5 m/s
  - horizontal quadrupole vortex structure

Fluid dynamics of the tropical atmosphere

Fluid dynamics of the tropical atmosphere

$$\frac{Du}{Dt} - yv = -\frac{\partial p}{\partial x}$$

$$\frac{Dv}{Dt} + yu = -\frac{\partial p}{\partial y}$$

$$0 = -\frac{\partial p}{\partial z} + g\frac{\theta}{\theta_{ref}}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{D\theta}{Dt} + w\frac{d\bar{\Theta}}{dz} = 0$$

$(u, v) =$ horizontal velocity

$w =$ vertical velocity

$p =$ pressure

$\theta =$ temperature

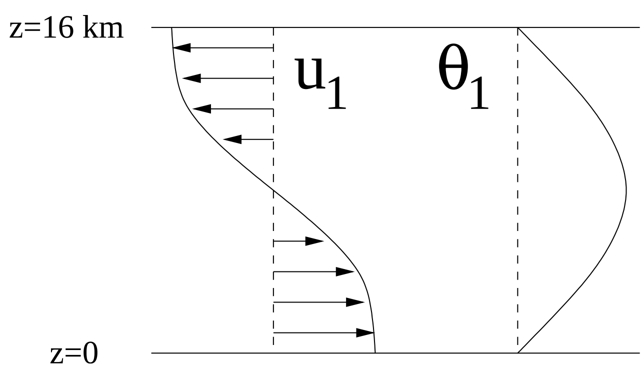
Vertical modes: Equatorial shallow water equations

Linear waves

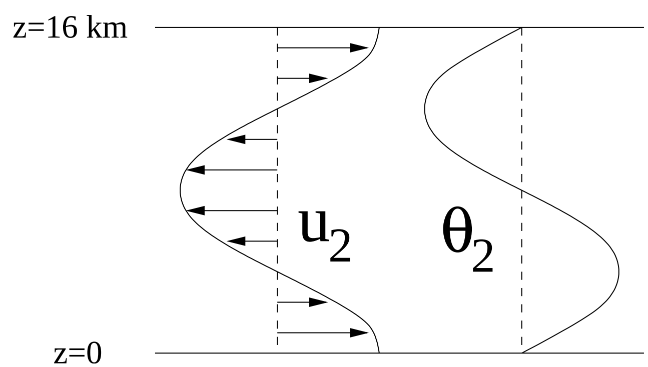
- Vertical modes: $u(x,y,z,t) = \sum_j u_j(x,y,t)\cos jz$, etc.
- Equatorial shallow water system for each vertical mode j:

$$\frac{\partial u_j}{\partial t} - y v_j - \frac{\partial \theta_j}{\partial x} = 0$$
$$\frac{\partial v_j}{\partial t} + y u_j - \frac{\partial \theta_j}{\partial y} = 0$$
$$\frac{\partial \theta_j}{\partial t} - \frac{1}{j^2}\left(\frac{\partial u_j}{\partial x} + \frac{\partial v_j}{\partial y}\right) = 0$$

- Gravity wave speed $\propto 1/j$



50 m/s



25 m/s

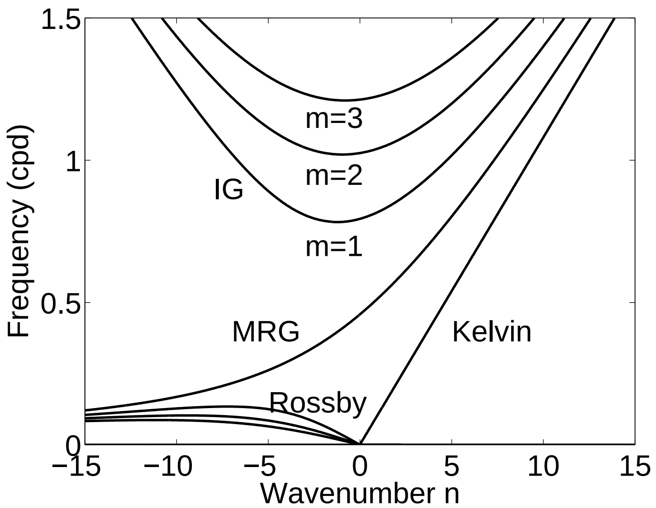
Equatorially trapped, dispersive waves

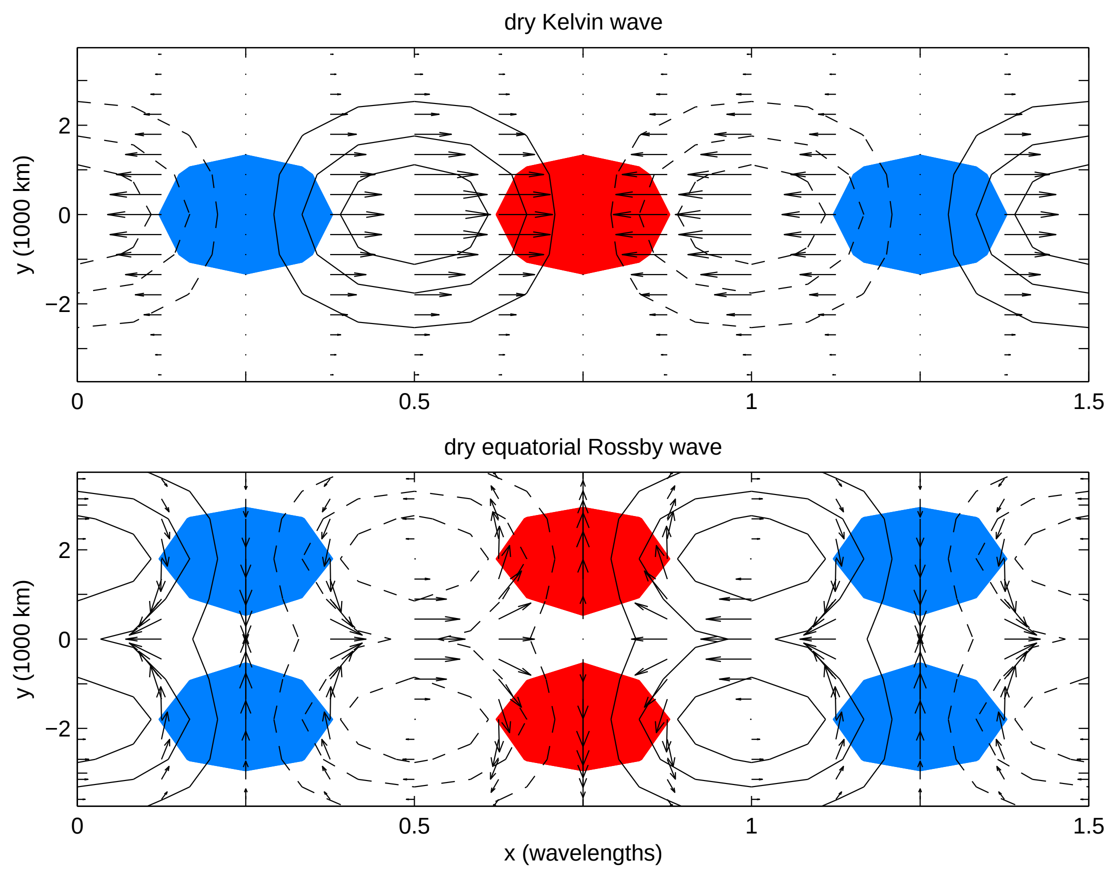
Equatorial shallow water equations have a natural basis in y

Expand in y direction using parabolic cylinder functions $\phi_m(y)$:

$$\phi_0(y) \propto \exp\left(-\frac{y^2}{2}\right), \quad \phi_1(y) \propto y \exp\left(-\frac{y^2}{2}\right), \quad \phi_2(y) \propto (2y^2-1)\exp\left(-\frac{y^2}{2}\right)$$

Dispersion curves for equatorial shallow water eqns.





Fluid dynamics of the tropical atmosphere

Summary

- Primitive equations $\longrightarrow$ equatorial shallow water equations
  - Vertical modes $\cos jz$, gravity wave speeds $\propto 1/j$
- Equatorial shallow water equations
  - Expand in y using parabolic cylinder functions $\phi_m(y)$
  - Equatorially trapped, dispersive waves (Kelvin, Rossby, etc.)

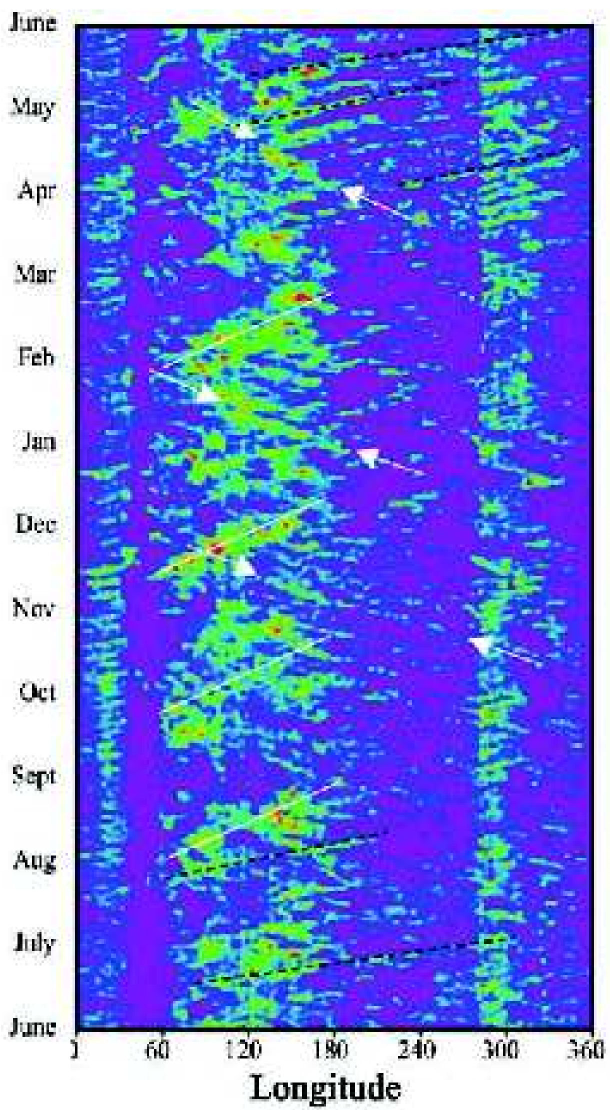
The Madden–Julian Oscillation (MJO)

in the Observational Record

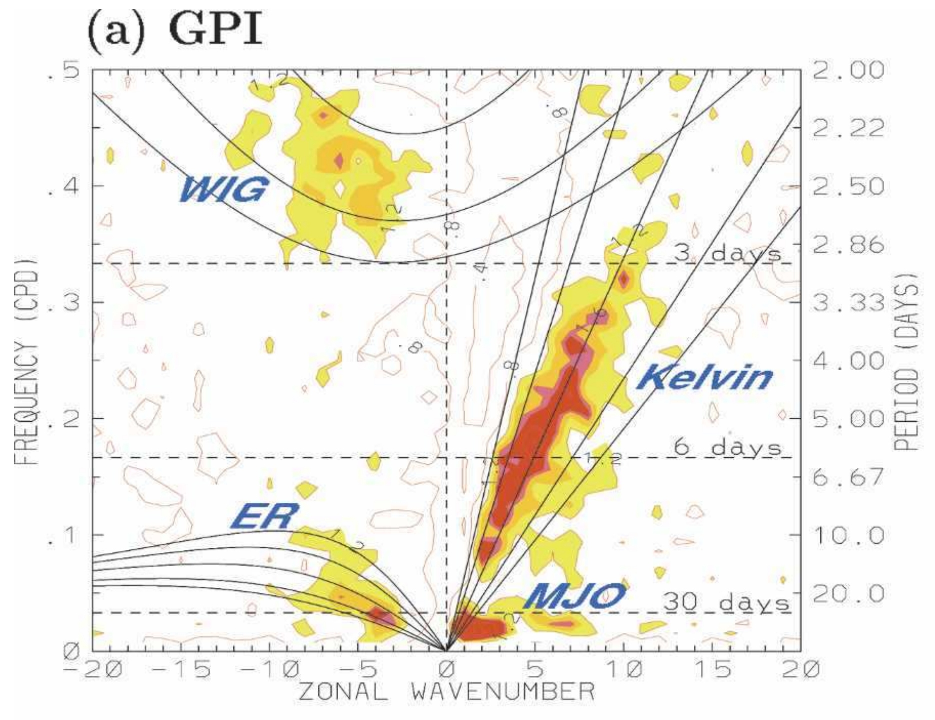
Observations of the MJO

Precipitation

2000–2001 (from Zhang 2005)



Spectral Power



from Lin et al. 2006

MJO: peculiar dispersion relation $\frac{d\omega}{dk} \approx 0$

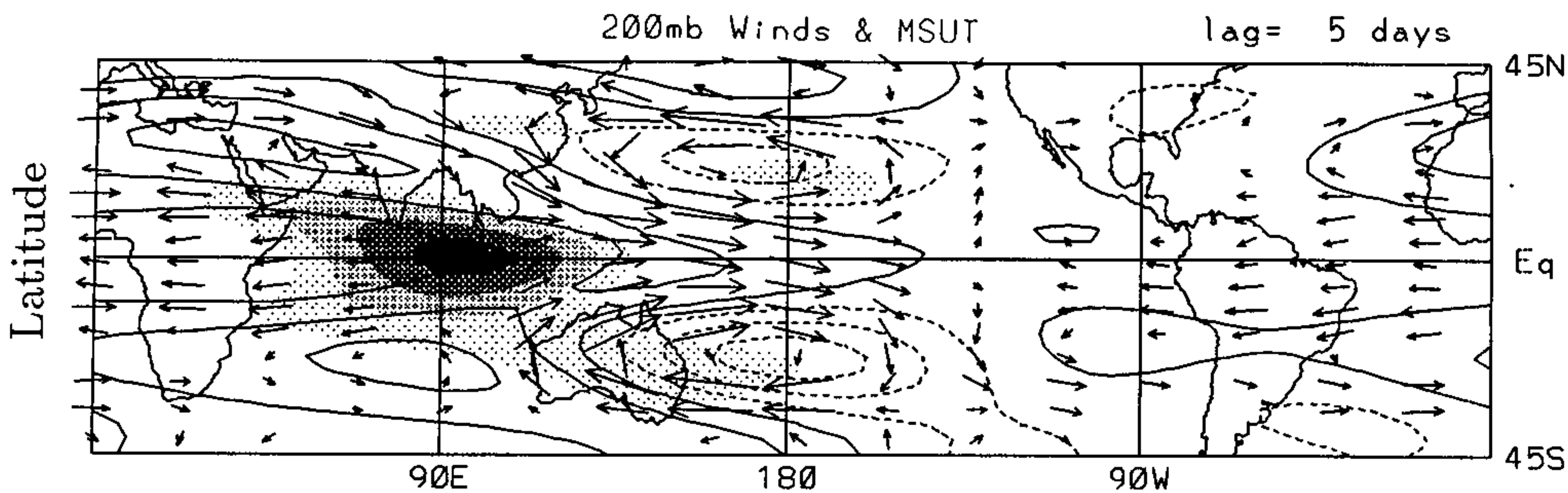
MJO: eastward propagation

MJO is envelope of smaller-scale convectively coupled waves

Horizontal structure of MJO

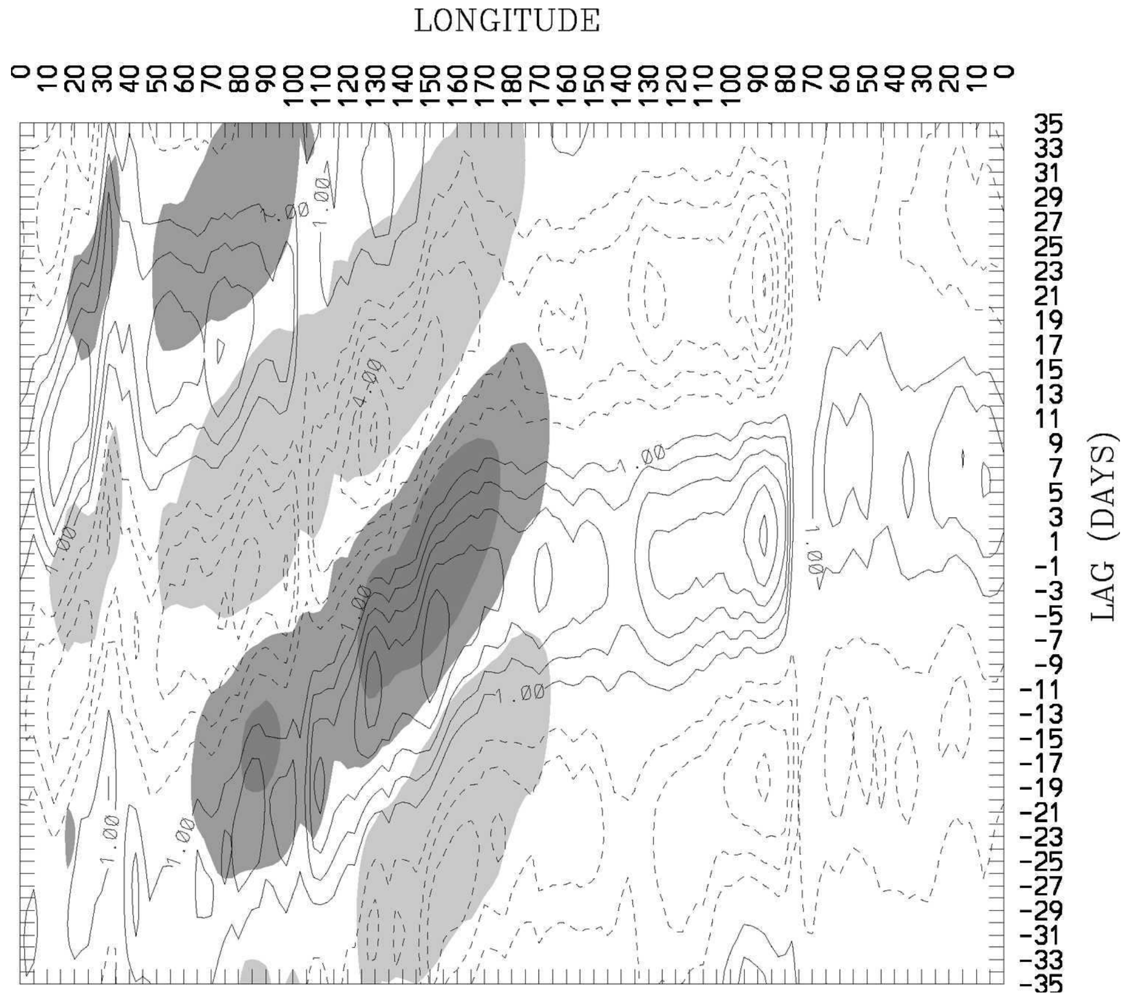
Quadrupole vortices:

Hendon and Salby (1994)



Moisture preconditioning in the MJO

Kiladis et al (2005)



Moisture (contours) leads enhanced convection (dark shading)

Previous attempts at a theory for the MJO

MJO originally discovered in 1971

Previous theories emphasized planetary scale instability mechanisms such as

- evaporation–wind feedback
- boundary layer frictional convective instability
- stochastic linearized convection
- radiation instability
- ...

But all these theories are at odds with observational record in various crucial ways

No theory for the MJO has yet been generally accepted

- “Search for the Holy Grail of tropical atmospheric dynamics”

A new model for the MJO

Fundamental mechanism proposed for MJO skeleton

Neutrally stable interactions between

1. planetary scale, lower tropospheric moisture
2. synoptic scale convectively coupled wave activity

- Tacit assumption: primary instabilities/damping occur on synoptic scales

- MJO “muscle” from other potential upscale transport effects from synoptic scales
  - convective momentum transports from synoptic scale waves
  - variations in surface heat fluxes

Minimal dynamical model

$$u_t - yv = -p_x$$
$$yu = -p_y$$
$$0 = -p_z + \theta$$

$$u_x + v_y + w_z = 0$$

$$\theta_t + w = \bar{H}a$$
$$q_t - \tilde{Q}w = -\bar{H}a$$
$$a_t = \Gamma q(\bar{a} + a)$$

Momentum equations:

- Equatorial long-wave scaling
- Coriolis term: equatorial β-plane approx.
- Hydrostatic balance

Thermodynamic equations:

- q: lower tropospheric moisture
- a: amplitude of wave activity envelope

Key mechanism: positive q creates a tendency to enhance wave activity a

Minimal number of parameters: $\tilde{Q}, \Gamma, \bar{a}$

Minimal dynamical model

(vertical truncation)

$$
\begin{aligned}
u_t - yv - \theta_x &= 0 \\
yu - \theta_y &= 0 \\
\theta_t - u_x - v_y &= \bar{H}a \\
q_t + \tilde{Q}(u_x + v_y) &= -\bar{H}a \\
a_t &= \Gamma \bar{a} q
\end{aligned}
$$

- Truncate at first vertical baroclinic mode
- Matsuno–Gill-like model
  without dissipative mechanisms
  but with
  - lower tropospheric moisture, q
  - envelope of synoptic scale wave activity, a, provides dynamic planetary-scale heating

Minimal dynamical model

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Simplified case: 1D dynamics above the equator with perfect east–west symmetry

Exact solution: $$2\omega^2 = \Gamma \bar{R} + k^2 \pm \sqrt{(\Gamma \bar{R} + k^2)^2 - 4\Gamma \bar{R} k^2 (1 - \tilde{Q})}$$

Approx. solution: $$\omega \approx \sqrt{\Gamma \bar{R}(1 - \tilde{Q})}$$

Model recovers peculiar dispersion relation $d\omega/dk \approx 0$

Minimal dynamical model

(vertical and meridional truncation)

$$K_t + K_x = -\frac{1}{\sqrt{2}}\bar{H}A$$

$$R_t - \frac{1}{3}R_x = -\frac{2\sqrt{2}}{3}\bar{H}A$$

$$Q_t + \frac{1}{\sqrt{2}}\tilde{Q}K_x - \frac{1}{6\sqrt{2}}\tilde{Q}R_x = \left(-1 + \frac{1}{6}\tilde{Q}\right)\bar{H}A$$

$$A_t = \Gamma\bar{a}Q$$

Meridional structures:

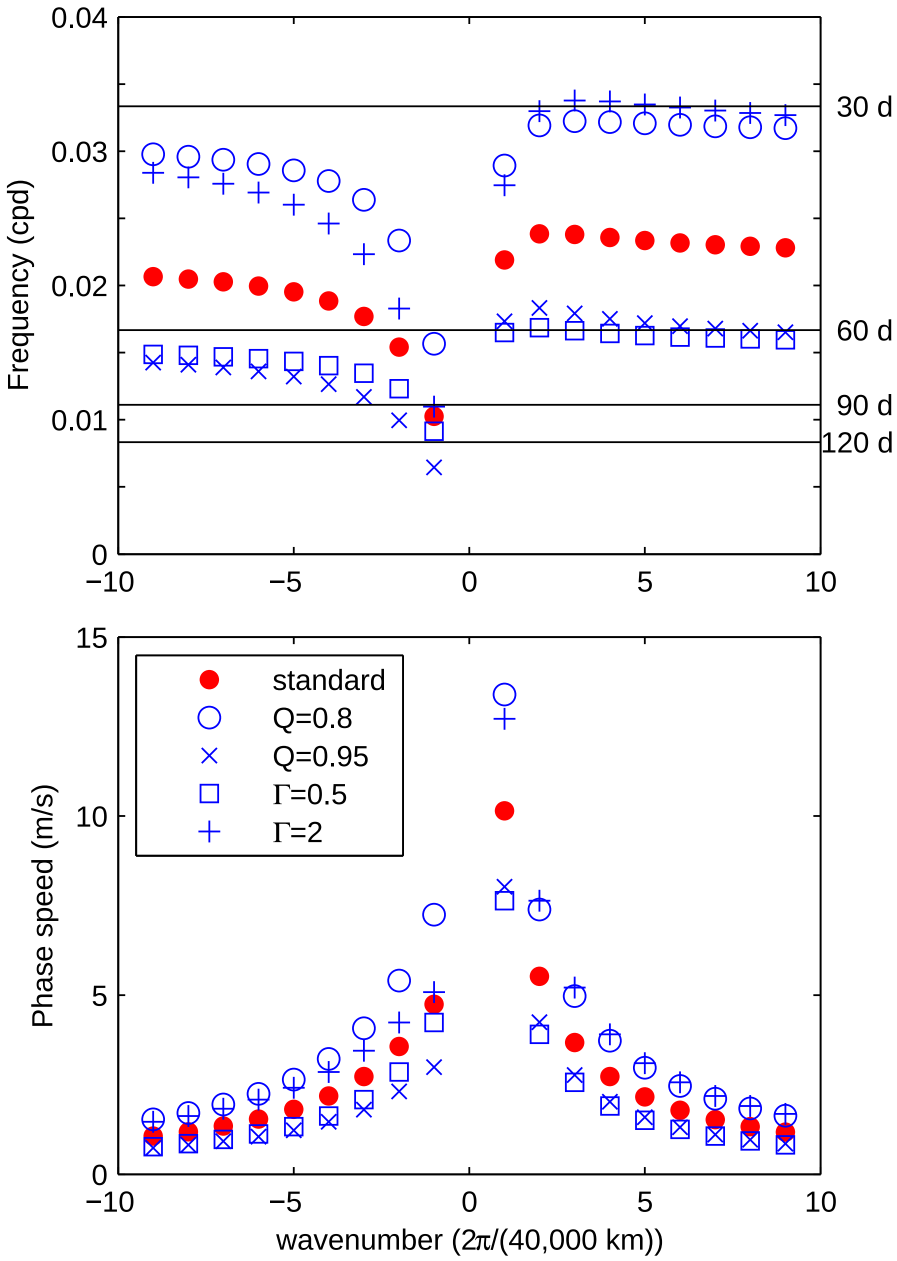
K: Kelvin wave

R: first symmetric equatorial Rossby wave

Q: $\exp(-y^2/2)$

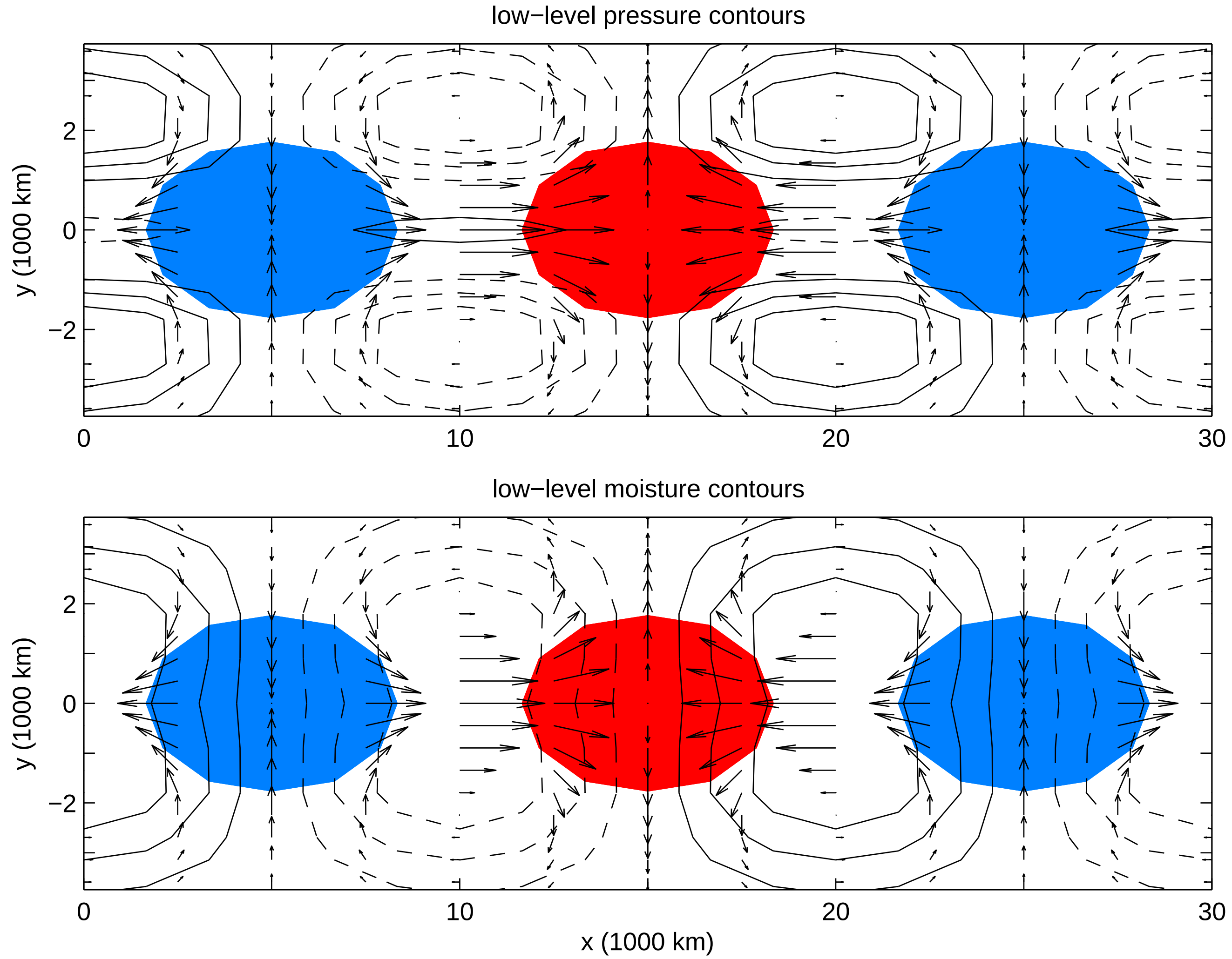
A: $\exp(-y^2/2)$

Oscillation frequency and phase speed



- Eastward MJO branch: $\frac{d\omega}{dk} \approx 0$ on *intraseasonal* time scales
- Westward branch: *seasonal* time scales for wavenumbers 1 and 2
- Phase speeds of roughly 5 m/s
- Results robust over parameter space

Physical structure of MJO skeleton



suppressed convection ($A < 0$) enhanced convection ($A > 0$)

- horizontal quadrupole vortices
- moisture leads convection
- Kelvin wave structure on equator
- off-equatorial quadrupole Rossby gyres

Summary

- New minimal dynamical model for the MJO
- Recovers robustly, for the first time, its fundamental features (i.e., its "skeleton") on intraseasonal/planetary scales:
  - peculiar dispersion relation of $d\omega/dk \approx 0$
  - slow phase speed of roughly 5 m/s
  - horizontal quadrupole vortex structure
- Simple formula for MJO oscillation frequency: $\omega \approx \sqrt{\Gamma \bar{R}(1 - \tilde{Q})}$
- Explanation of preferred eastward propagation of intraseasonal variability
- Neutrally stable model on planetary/intraseasonal scales
  - Tacit assumption: primary instabilities on synoptic scales
- "Muscle" of MJO provided by other upscale transports from synoptic scales

Majda and Stechmann (2009) *Proc. Natl. Acad. Sci.*

Outline

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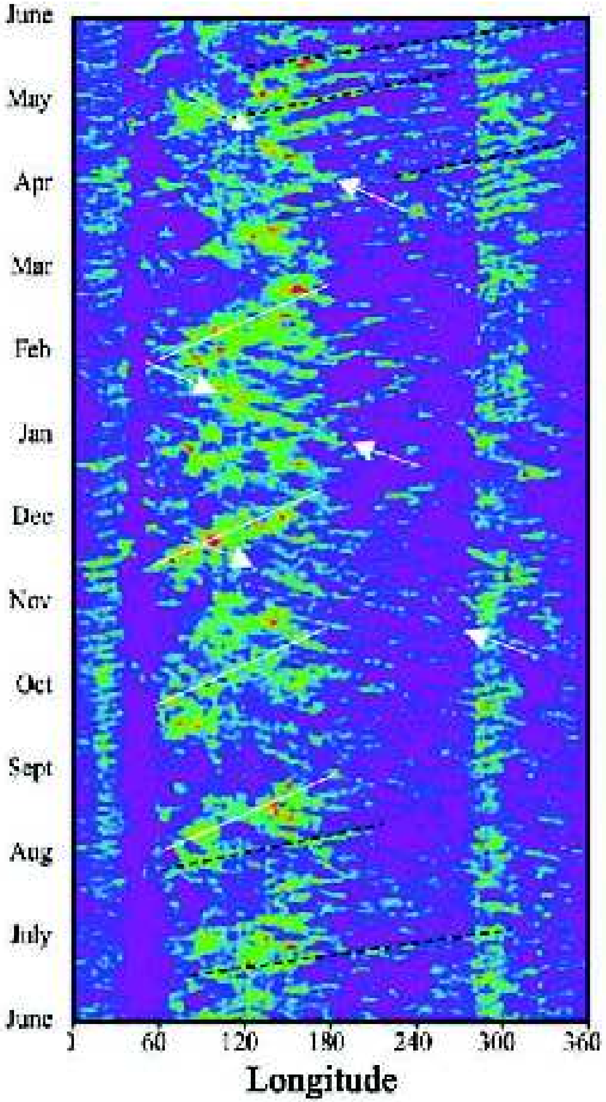


from Zhang (2005)

The Madden–Julian oscillation (MJO) is an envelope of convectively coupled waves

What are the physical mechanisms of the MJO?

- Does the MJO regulate the waves?
- Do the waves drive the MJO?
- Is there cooperative interaction between the MJO and waves?

What is the missing physics of the MJO in GCMs?

Hypothesis:

1. Proper representation of convectively coupled waves
2. Proper representation of interactions between convectively coupled waves and the larger-scale environment

Can features of the MJO be captured by a simplified model that has the two ingredients above?

1. Multicloud model of Khouider and Majda (2006) as model for convectively coupled waves
2. Multiscale asymptotic model for convectively coupled wave–mean flow interaction

Majda and Stechmann (2009) *J. Atmos. Sci.*

Dynamic model for convective wave–mean interaction

$$\frac{\partial \bar{U}}{\partial T} + \frac{\partial}{\partial z}\langle \overline{w'u'} \rangle = 0$$

$$\frac{\partial u'}{\partial t} + \bar{U}\frac{\partial u'}{\partial x} + w'\frac{\partial \bar{U}}{\partial z} + \frac{\partial p'}{\partial x} = S'_{u,1}$$

(with similar equations for other variables)

Key features of the model:

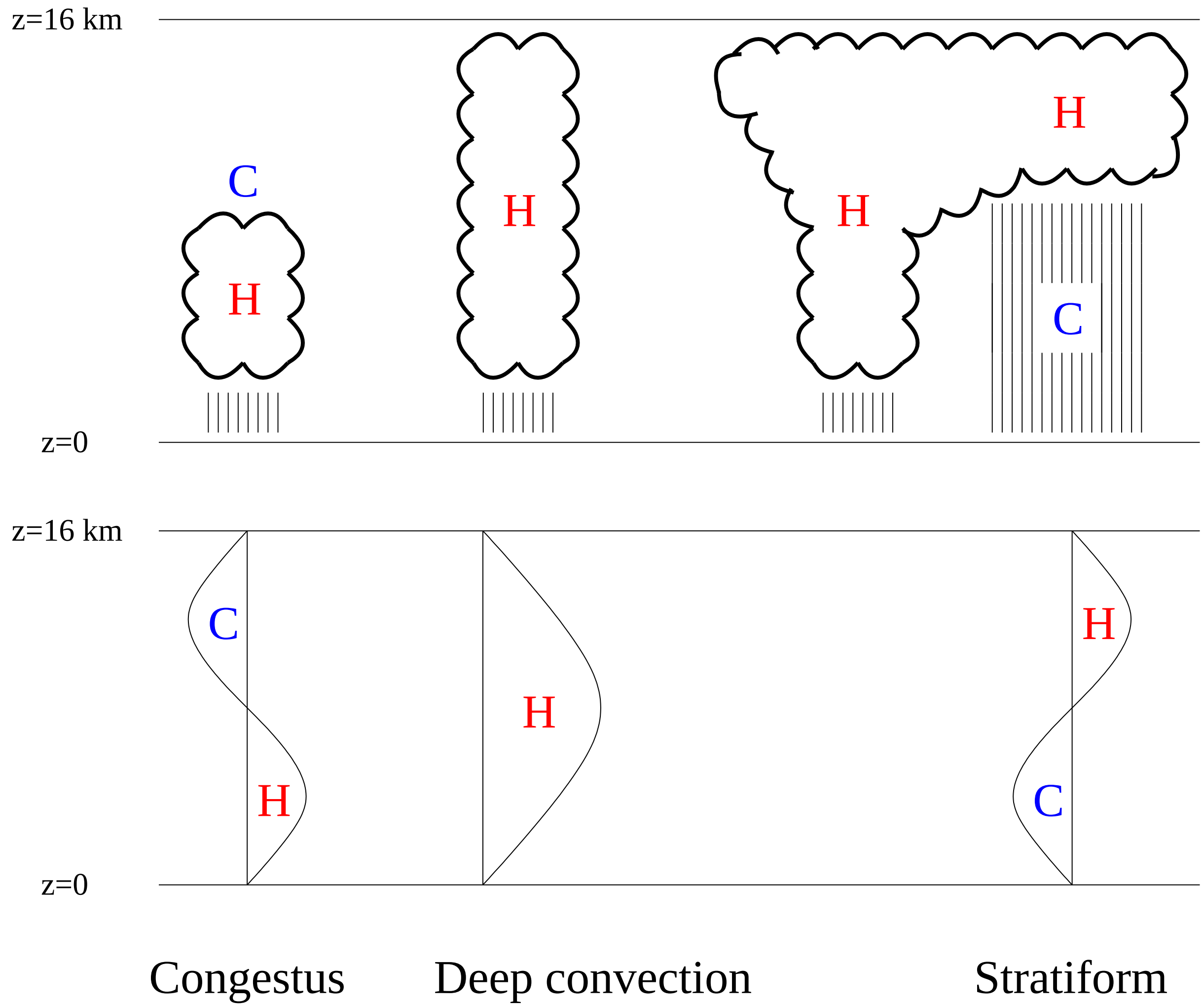
- Eddy flux convergence of wave momentum, $\partial_z \langle \overline{w'u'} \rangle$, feeds the mean flow $\bar{U}$
- Advection of the waves u' by the mean flow $\bar{U}$
- Mean flow time scale $T = \epsilon^2 t$ is longer than that for the waves

Multiscale asymptotic derivation of model

Use multicloud model of Khouider and Majda (2006) as model for convectively coupled waves u', θ', etc.

The Multicloud Model (Khouider and Majda 2006)

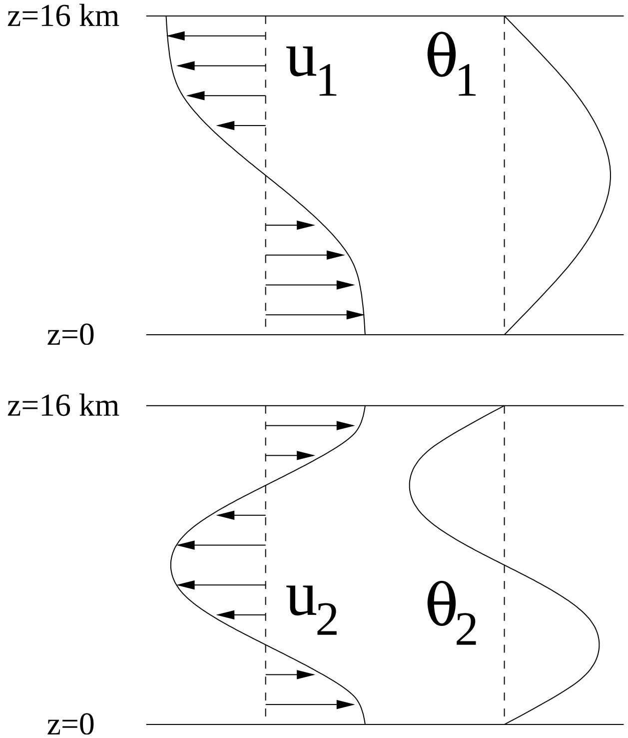
(a model for convectively coupled waves)



- Three cloud types
- Two vertical modes

Equations of the multicloud model

Two linear shallow water systems, coupled through nonlinear source terms:



$$
\begin{cases}
\dfrac{\partial u_1}{\partial t} - \dfrac{\partial \theta_1}{\partial x} = -\dfrac{1}{\tau_u} u_1 \\
\dfrac{\partial \theta_1}{\partial t} - \dfrac{\partial u_1}{\partial x} = H_d - R_1
\end{cases}
$$

$$
\begin{cases}
\dfrac{\partial u_2}{\partial t} - \dfrac{\partial \theta_2}{\partial x} = -\dfrac{1}{\tau_u} u_2 \\
\dfrac{\partial \theta_2}{\partial t} - \dfrac{1}{4}\dfrac{\partial u_2}{\partial x} = H_c - H_s - R_2
\end{cases}
$$

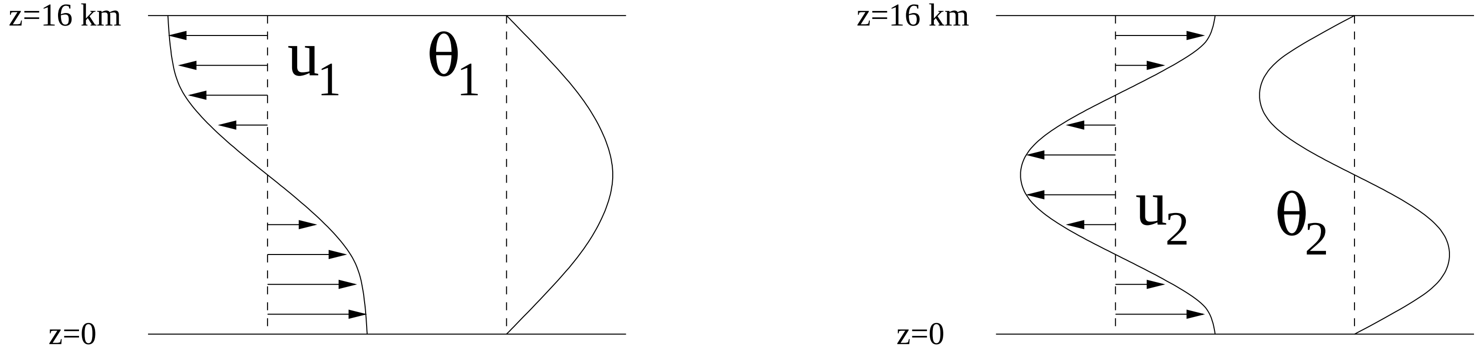
H_d = Deep convective heating $\quad$ H_c = Congestus heating

R = Radiative cooling $\quad$ H_s = Stratiform heating

+ 4 more prognostic equations for θ_{eb}, q, H_s, H_c

+ diagnostic equations for some source terms

Equations of the multicloud model



2-mode shallow water equations with nonlinear source terms

+ 4 more prognostic equations for θ_{eb}, q, H_s, H_c

Mathematical form: nonlinear system of PDE with nonlinear source terms

$$\frac{\partial \mathbf{u}}{\partial t} + A(\mathbf{u})\frac{\partial \mathbf{u}}{\partial x} = \mathbf{S}(\mathbf{u}), \qquad \mathbf{u} = (u_1, \theta_1, u_2, \theta_2, \theta_{eb}, q, H_s, H_c)$$

Source terms are parameterizations of physical processes such as cloud heating, radiative cooling, evaporation, downdrafts

Dynamic model for convective wave–mean interaction

$$\frac{\partial \bar{U}}{\partial T} + \frac{\partial}{\partial z}\langle\overline{w'u'}\rangle = 0$$

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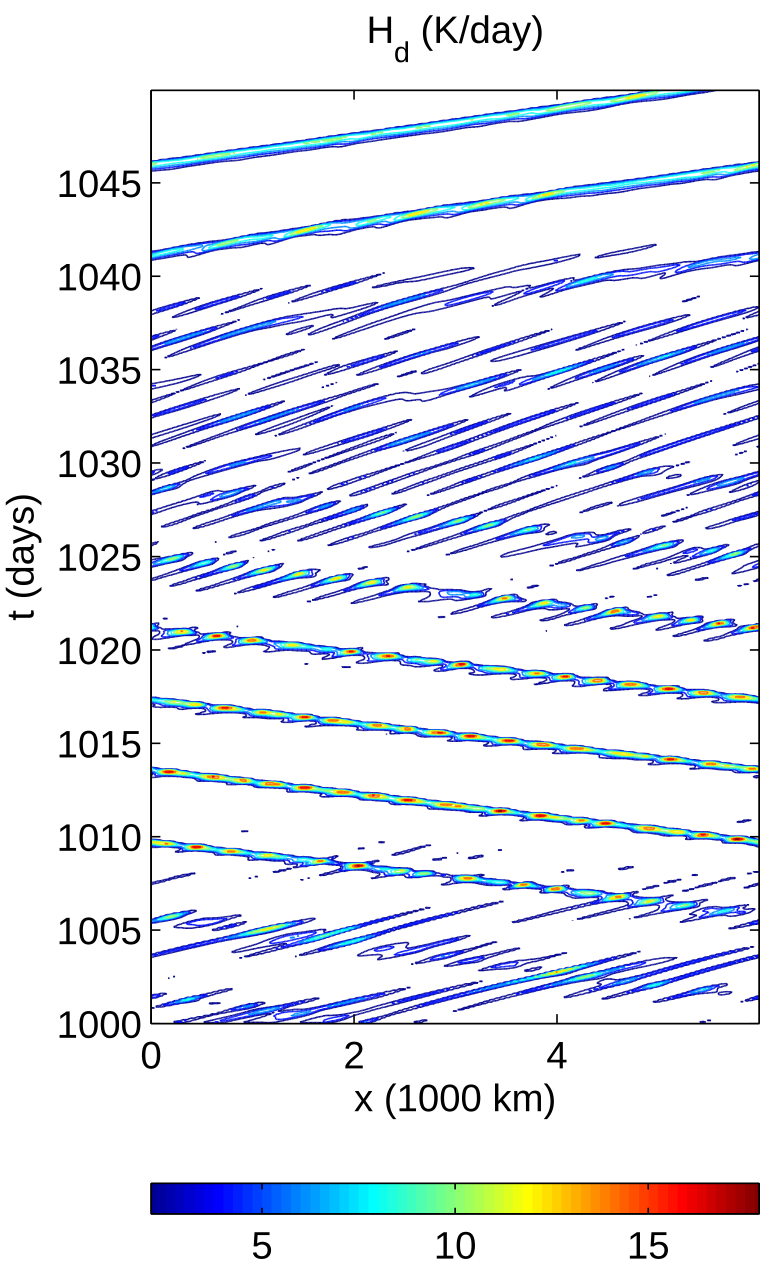
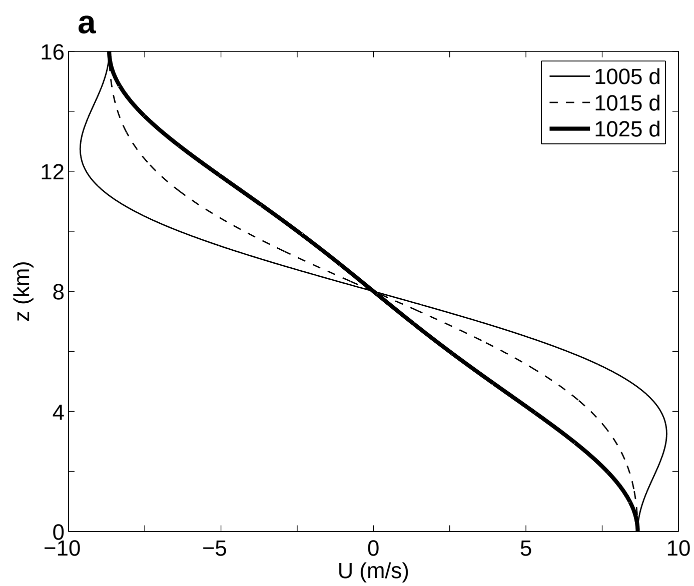
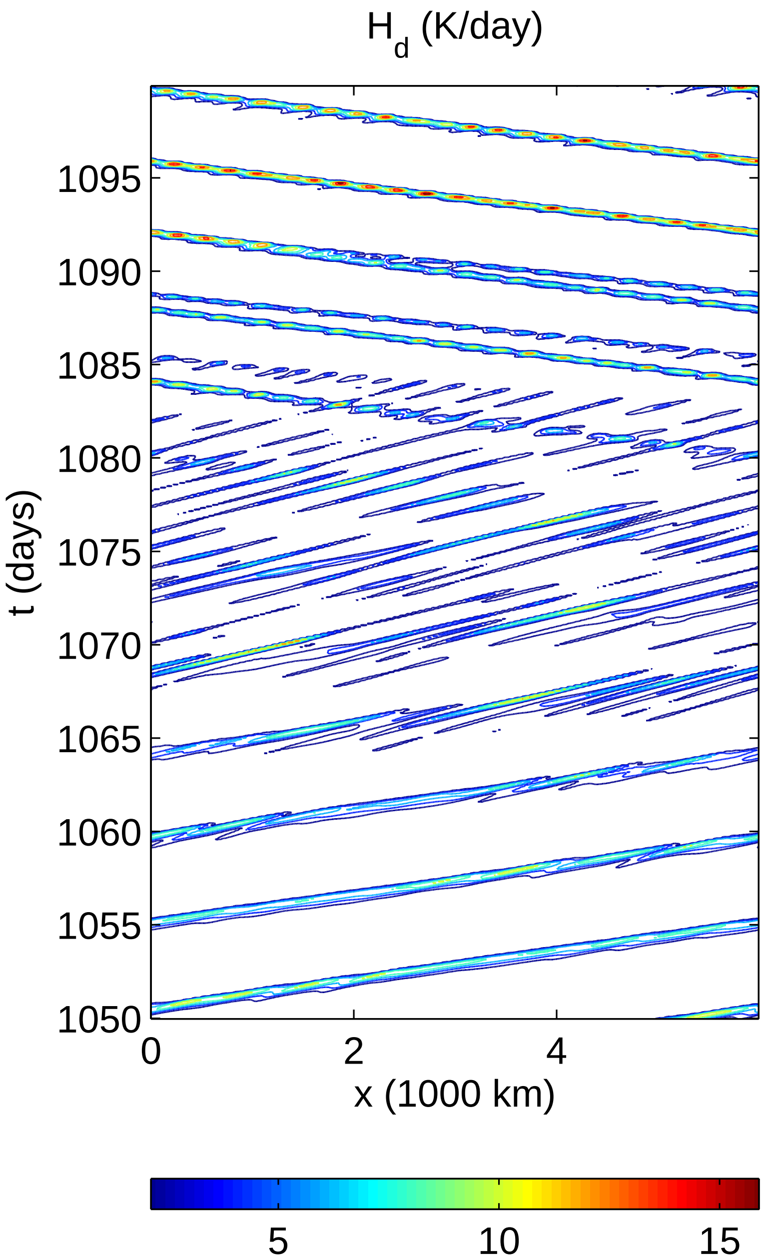
Intraseasonal oscillations and multiscale waves

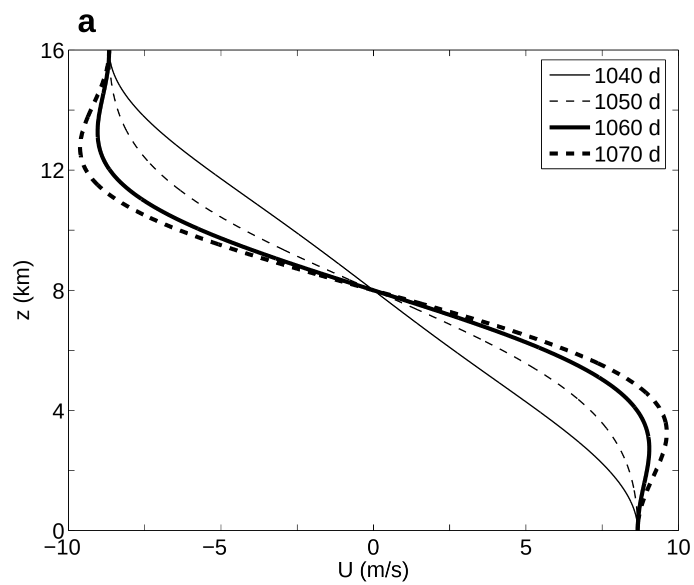




- Either coherent or scattered waves depending on mean wind
- CMT can either accelerate or decelerate mean wind

Westerly wind burst intensification

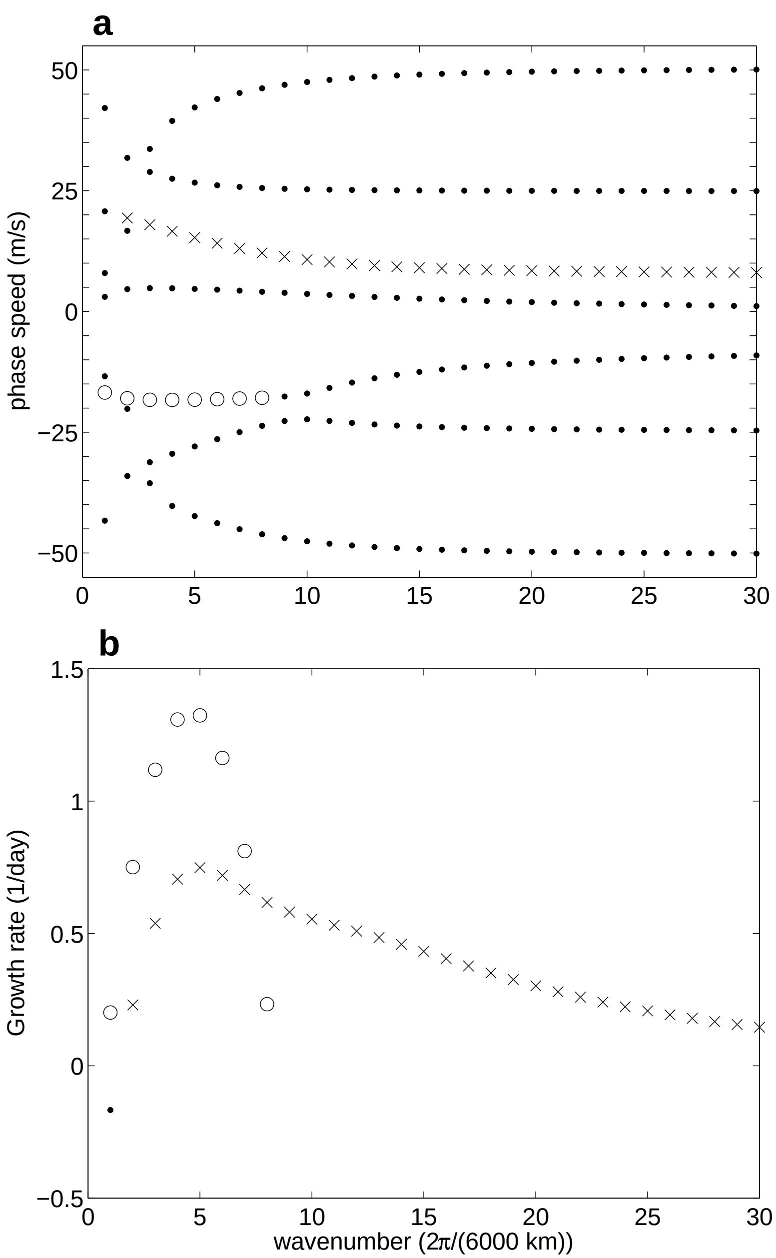


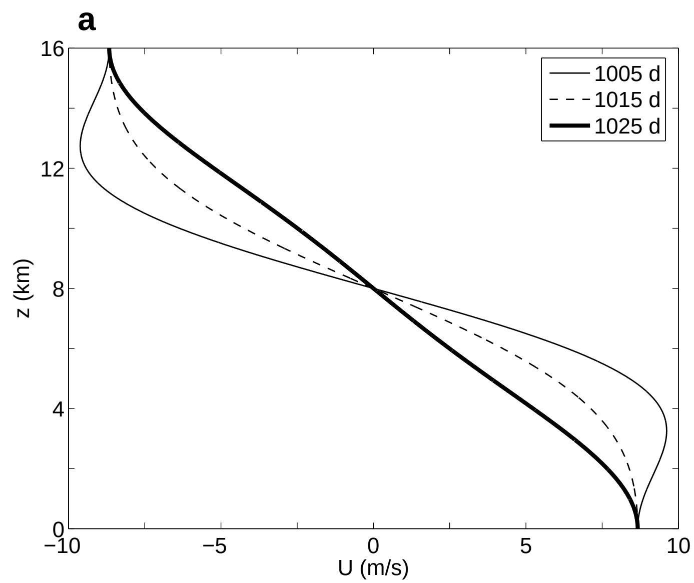


- Convective momentum transport from convectively coupled wave drives a lower tropospheric westerly jet

Linear theory with background wind shear

$t = 1005$ days





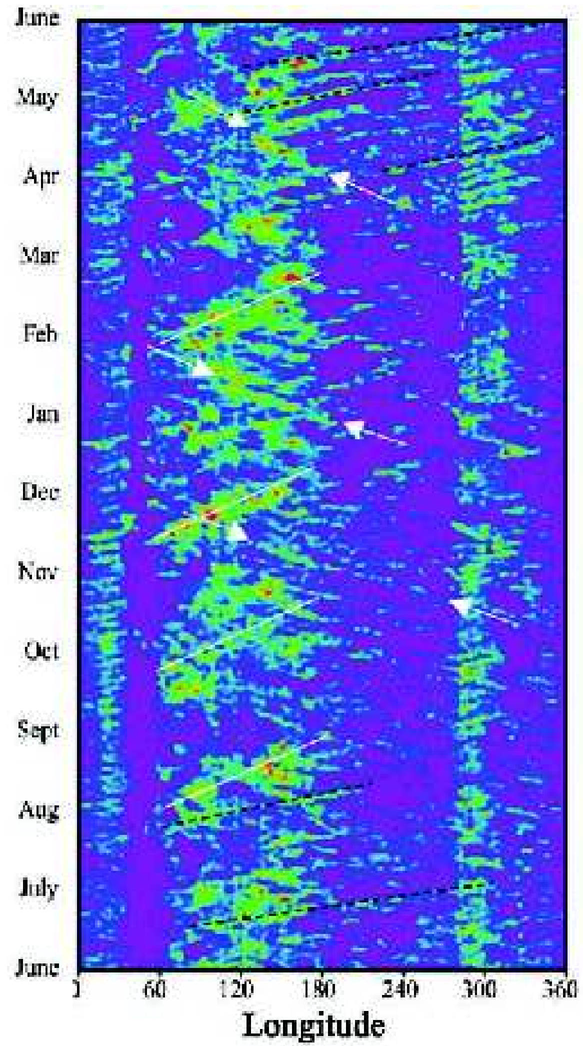
- Westward-propagating CCWs favored at larger scales
- Eastward-propagating CCWs favored at smaller scales

Relevance to the Madden–Julian oscillation

Results suggest cooperative interaction between convectively coupled waves and the MJO

To capture a realistic MJO, global climate models will need

- proper representation of CCWs (including vertical tilts)
- proper representation of interactions between CCWs and larger-scale environment



Zhang (2005)

Cloud-Resolving Model (CRM) simulations of CCWs:

What is the role of CMT from mesoscale convection?

Results vary depending on strength of momentum damping:

$$\frac{\partial u}{\partial t} = -\frac{1}{\tau}u + \cdots$$

- Held et al. (1993): No momentum damping: Long-time oscillation develops
  - Is this due to CMT interactions or stratospheric interactions?
- Grabowski & Moncrieff (2001): Weak momentum damping: CCWs develop with significant CMT
- Tulich et al. (2007): Stronger momentum damping: CCWs develop with little or no CMT
- Held et al. (1993): Intense momentum damping: Convection shut down except at a few grid points

Summary

1. A simple model for the MJO's "skeleton"
   - peculiar dispersion relation of $d\omega/dk \approx 0$
   - slow phase speed of roughly 5 m/s
   - horizontal quadrupole vortex structure

2. What causes cloud systems to organize into wave trains?
   - Nonlinear PDE model for gravity waves in shear
   - Results suggest wind shear can create favorable environment for wave trains of cloud systems

3. How does the MJO envelope interact with the waves embedded within it?
   - Multiscale asymptotic model for convectively coupled wave–mean flow interaction
   - Results suggest cooperative interaction between MJO and convectively coupled waves