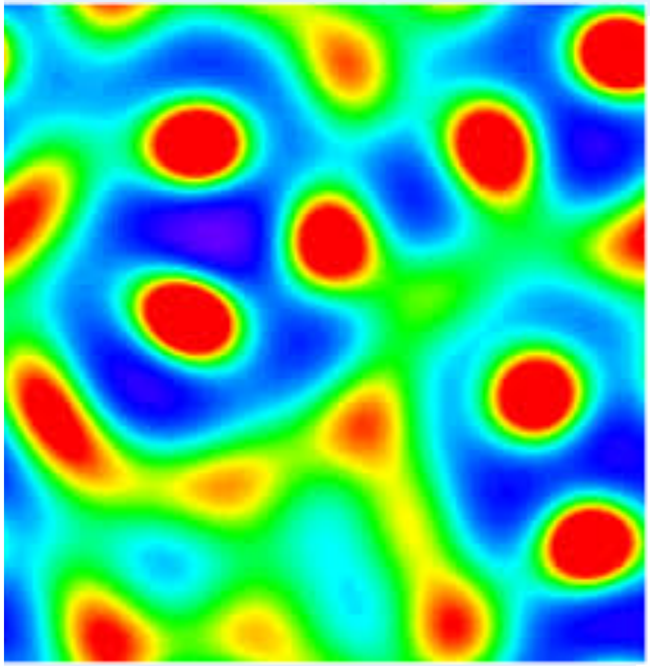
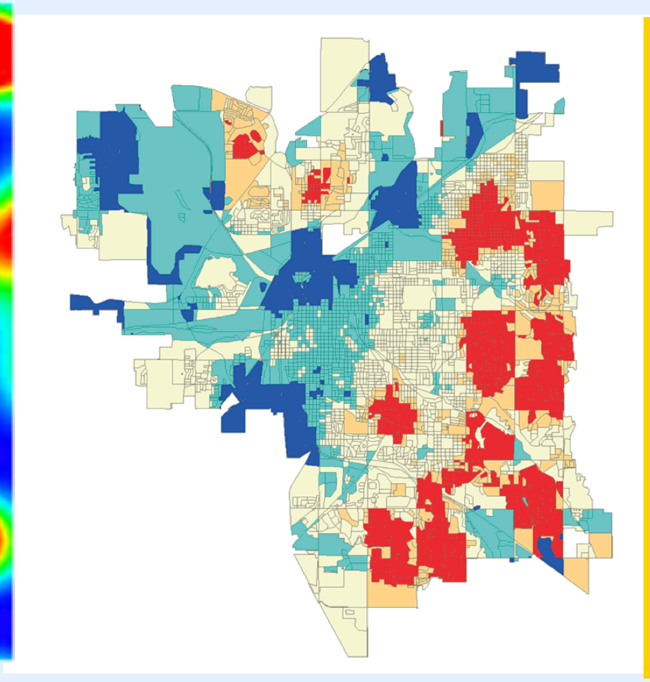
Case studies from mathematical models of criminal behavior

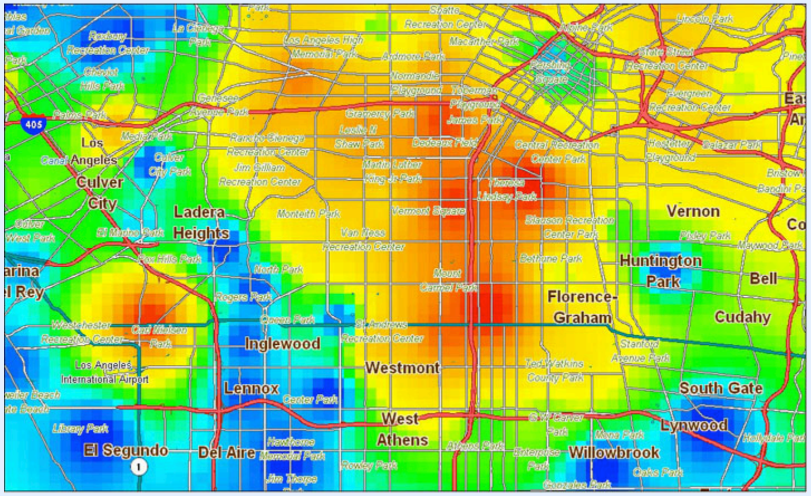




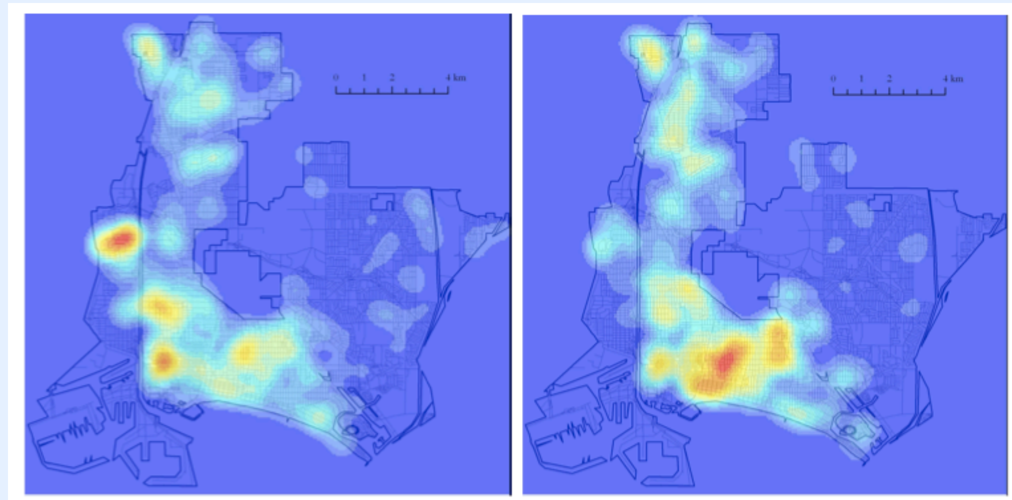
Maria R. D'Orsogna, Dept. Mathematics CSUN

1. Crime hotspots
2. An informant and a criminal
3. Recidivism and rehabilitation policies
4. Criminal networks

1. Crime Hotspots



GTA, Los Angeles



Residential burglary, Long Beach

Spatio-temporal clusters of crime

Burglary, grand theft auto

Time scales: hours to months

Geographical scales: a few city blocks

Broken Windows effect

Crime generates more and worse crime

"Consider a building with a few broken windows. If the windows are not repaired, the even tendency is for vandals to break a few more windows. Eventually, they may even break into the building, and if it's unoccupied, perhaps become squatters or light fires inside. Or consider a sidewalk. Some litter accumulates. Soon, more litter accumulates. Eventually, people start leaving bags of trash from take-out restaurants there or breaking into cars."

James Wilson and George Kelling, Atlantic Monthly 1982

Routine Activity Theory

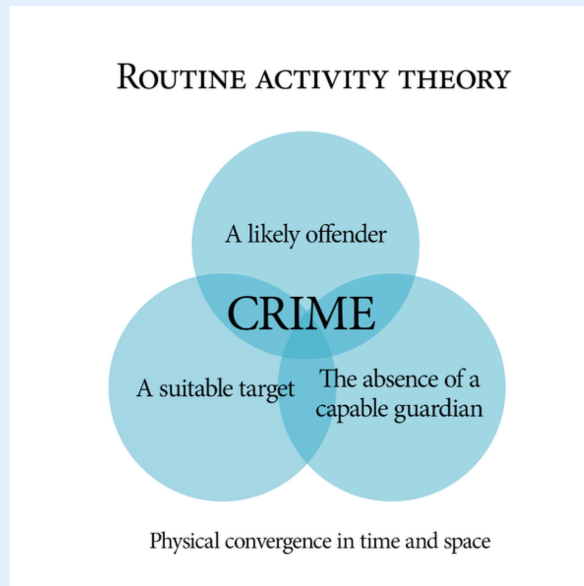
"I always go back to the same places because, one you been there, you know just about when you been there before and when you can go back. And every time I hit a house, it's always on the same day of the week."

SOCIAL CHANGE AND CRIME RATE TRENDS: A ROUTINE ACTIVITY APPROACH*

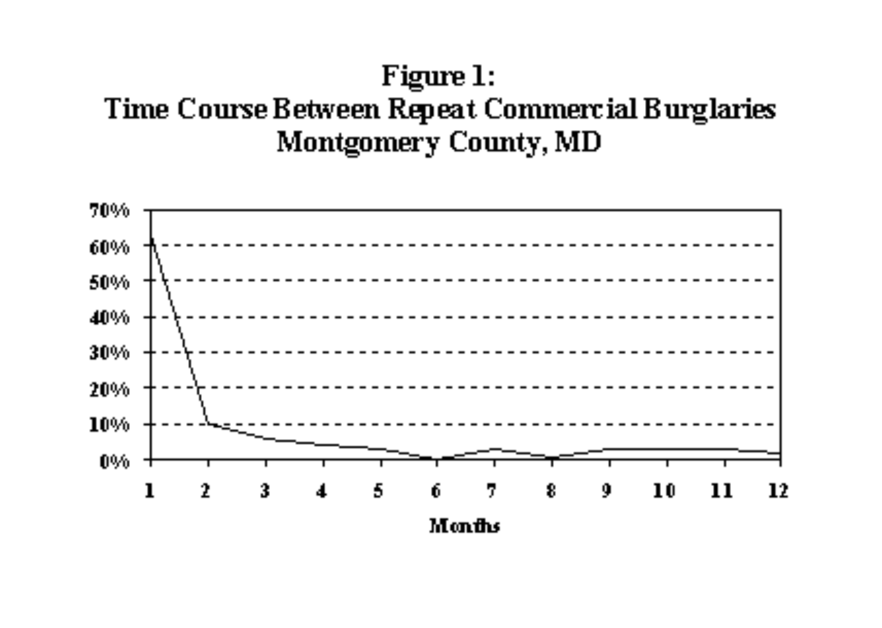
LAWRENCE E. COHEN AND MARCUS FELSON
University of Illinois, Urbana

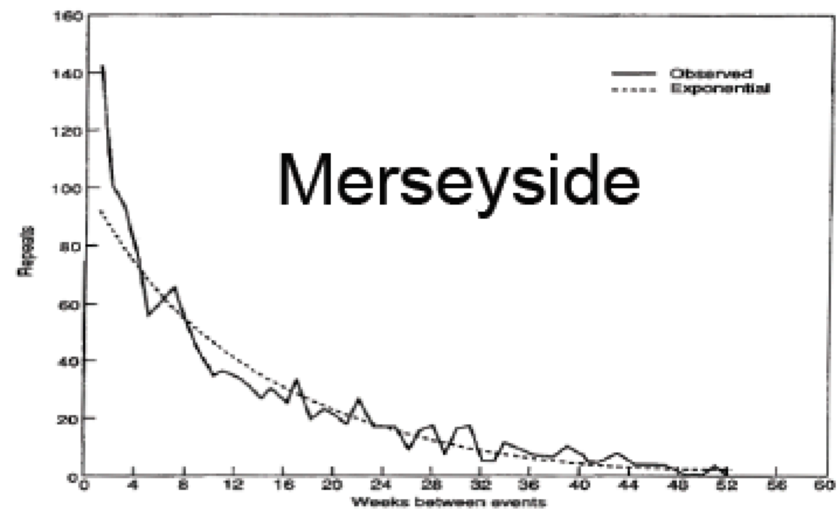
American Sociological Review 1979, Vol. 44 (August):588–608.

In this paper we present a "routine activity approach" for analyzing crime rate trends and cycles. Rather than emphasizing the characteristics of offenders, with this approach we concentrate upon the circumstances in which they carry out predatory criminal acts. Most criminal acts require convergence in space and time of *likely offenders*, *suitable targets* and the *absence of capable guardians* against crime. Human ecological theory facilitates an investigation into the way in which social structure produces this convergence, hence allowing illegal activities to feed upon the legal activities of everyday life. In particular, we hypothesize that the dispersion of activities away from households and families increases the opportunity for crime and thus generates higher crime rates. A variety of data is presented in support of the hypothesis, which helps explain crime rate trends in the United States 1947–1974 as a byproduct of changes in such variables as labor force participation and single-adult households.

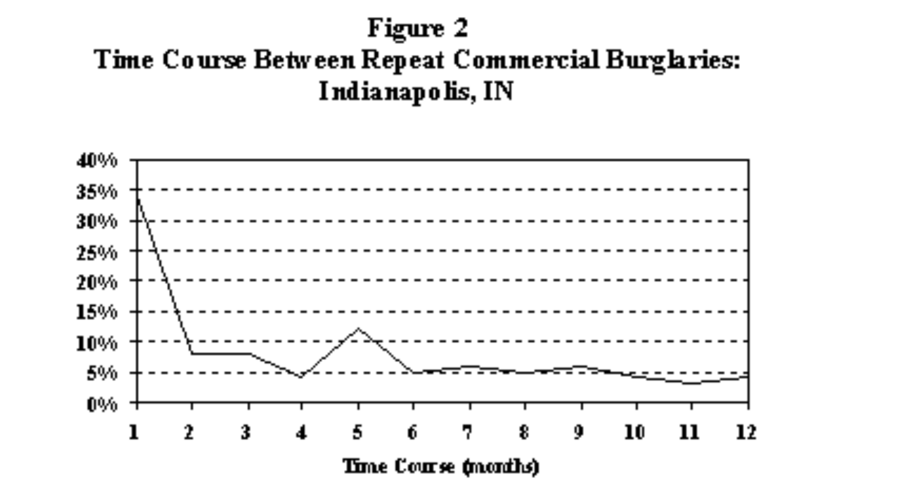


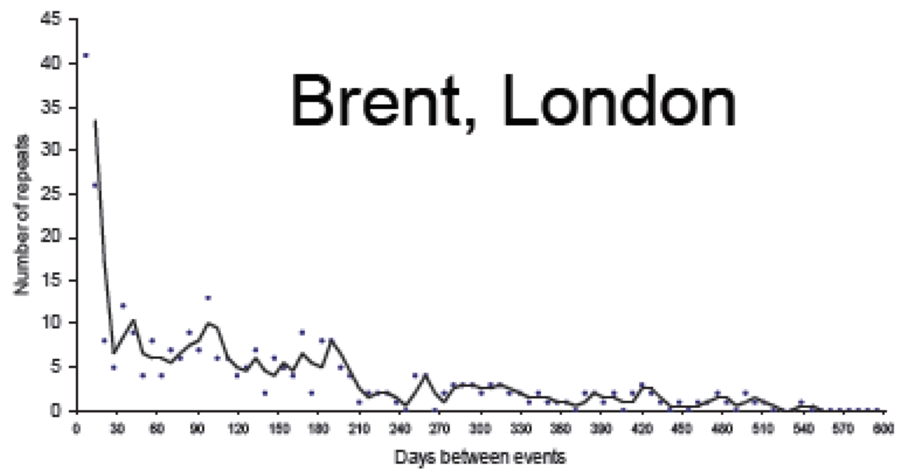
Repeat victimization



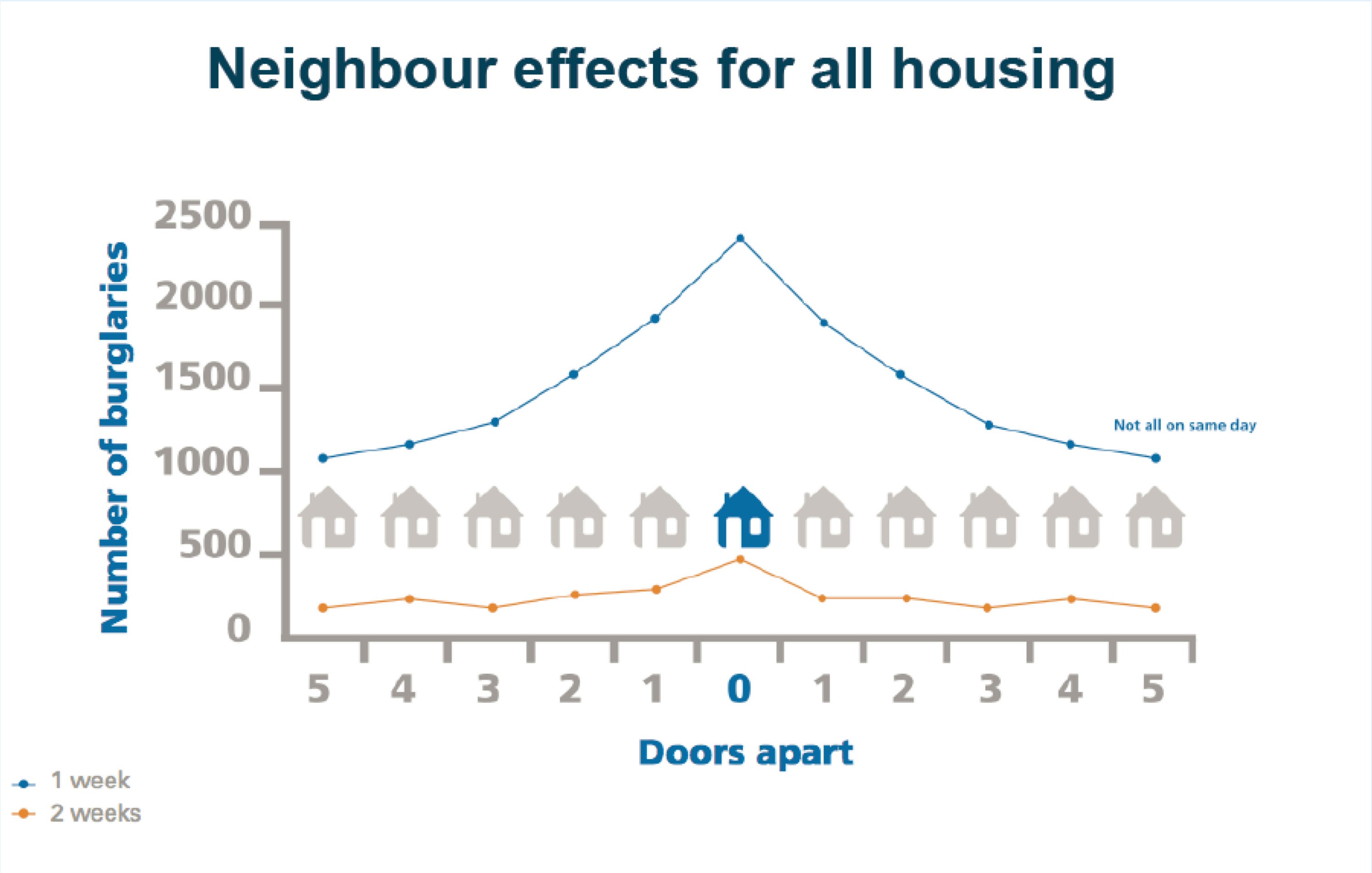


Temporal decay of residential burglary revictimisation over a 19 month period.



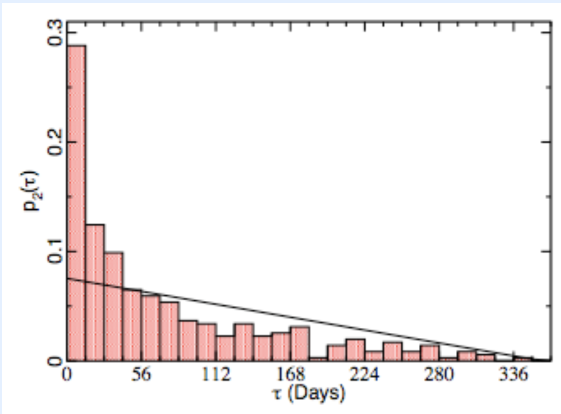


Your neighbors are at risk as well!

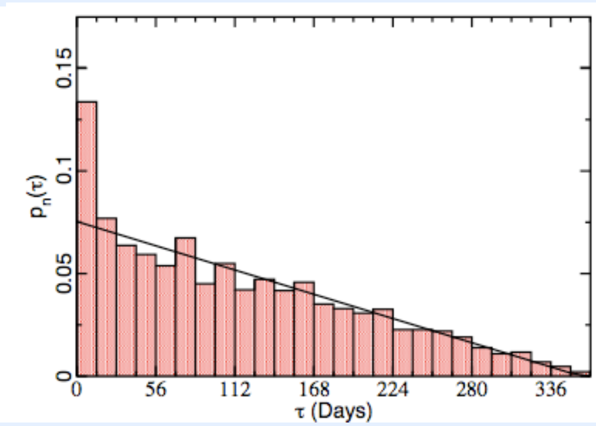


Bowers, K.J., and Johnson, S.D. (2005).
European Journal of Criminology, **2**(1), 67-92.

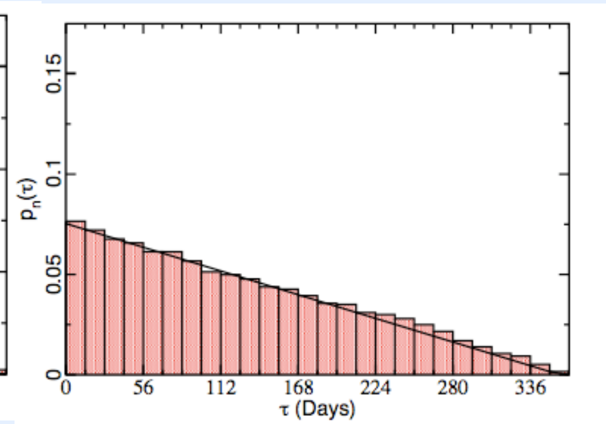
Residential burglary, Long Beach



0 meters



100 meters



4000 meters

Repeat and Near-repeat burglaries within one year

If random, repeat burglary probability within D days is

$$p_2(t) = \frac{2(D-t)}{D(D+1)}$$

Repeat burglary probability is higher at short distances

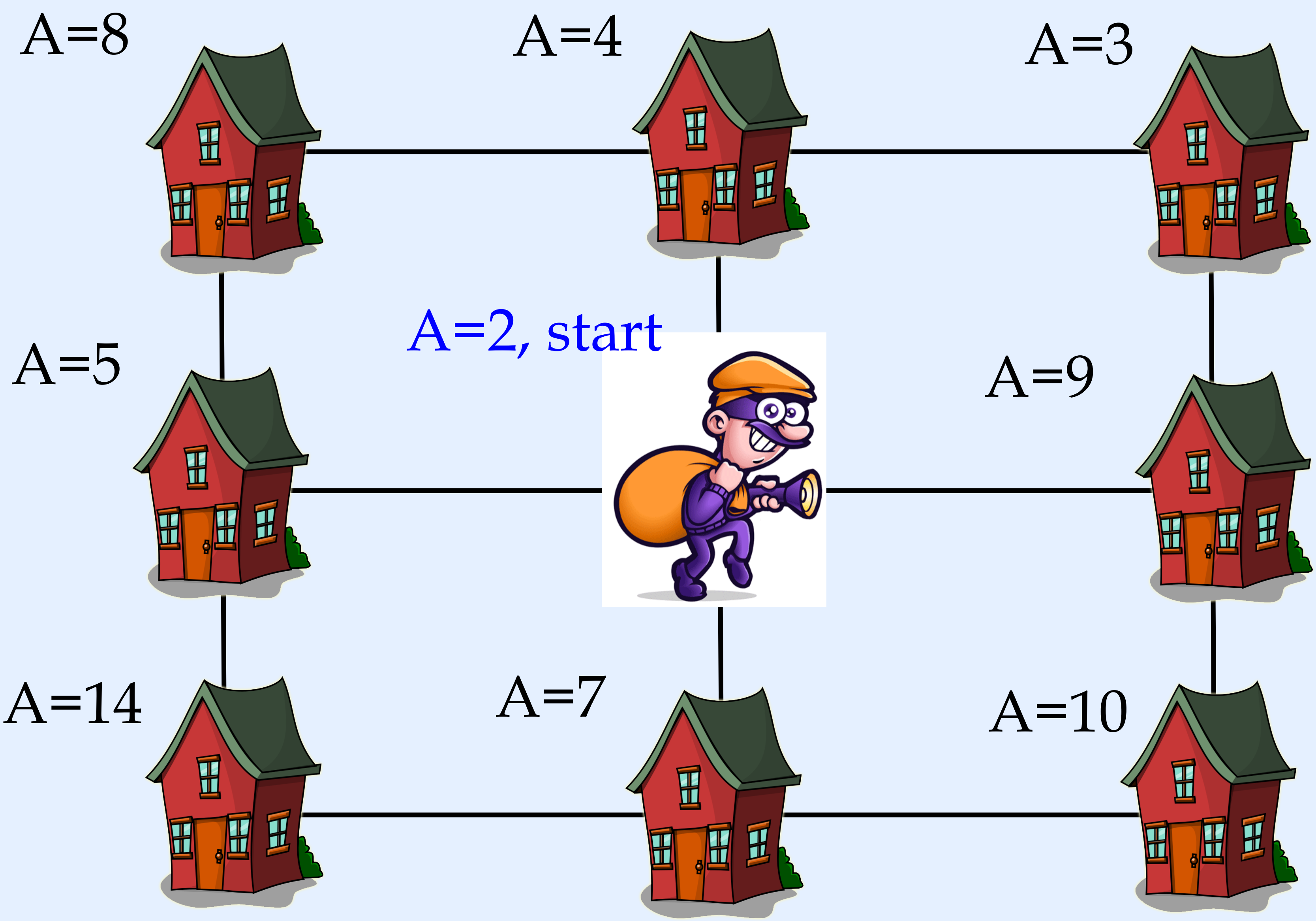
Ingredients

1. Residential burglary, crime of opportunity
2. Victimized once, easier to be victimized again
3. Introduce dynamically changing "environmental" attractiveness A(x,t)

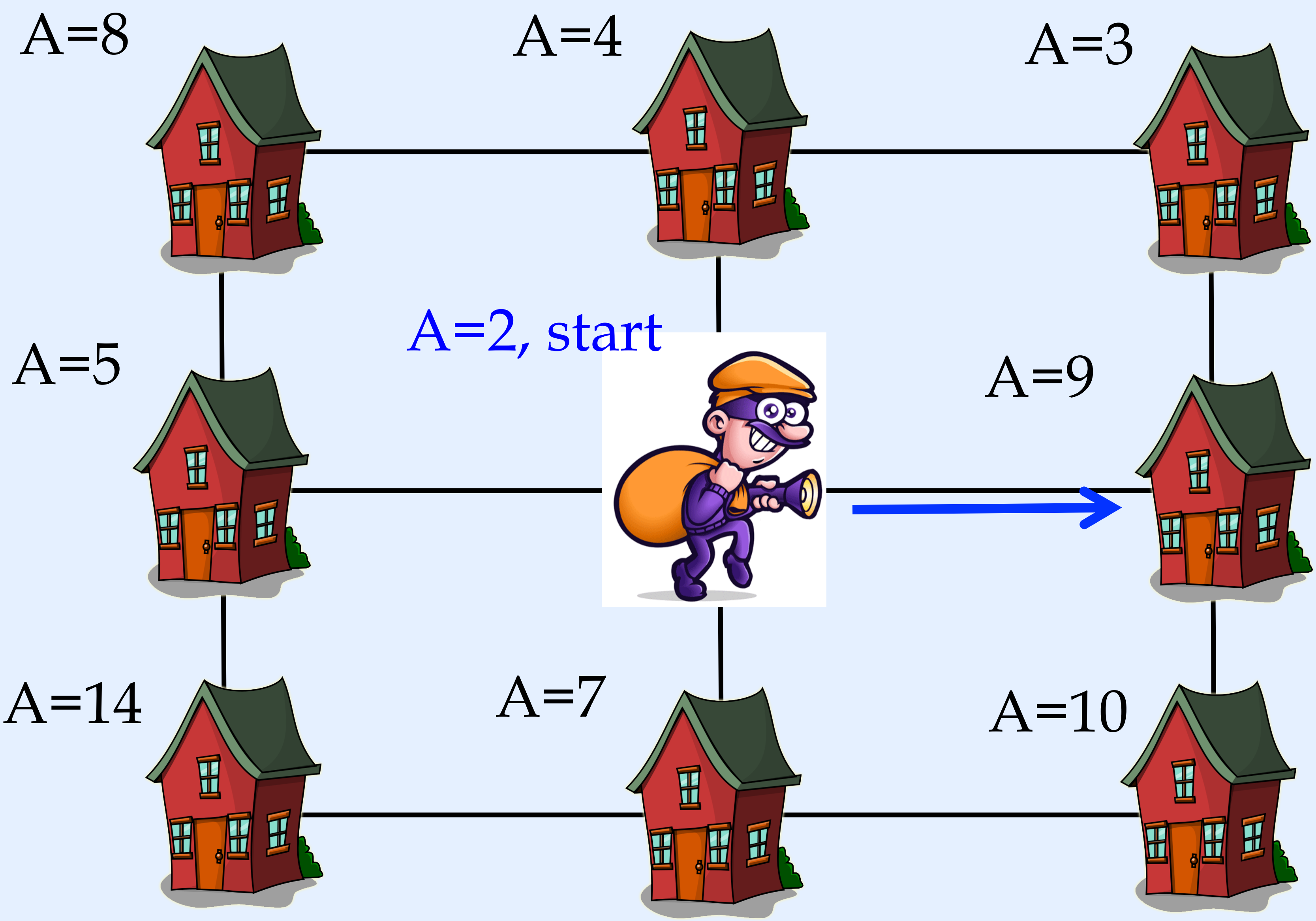
Sites are made "better" by criminal activity
Criminals are biased towards "better" sites

Feedback loops criminals/environment

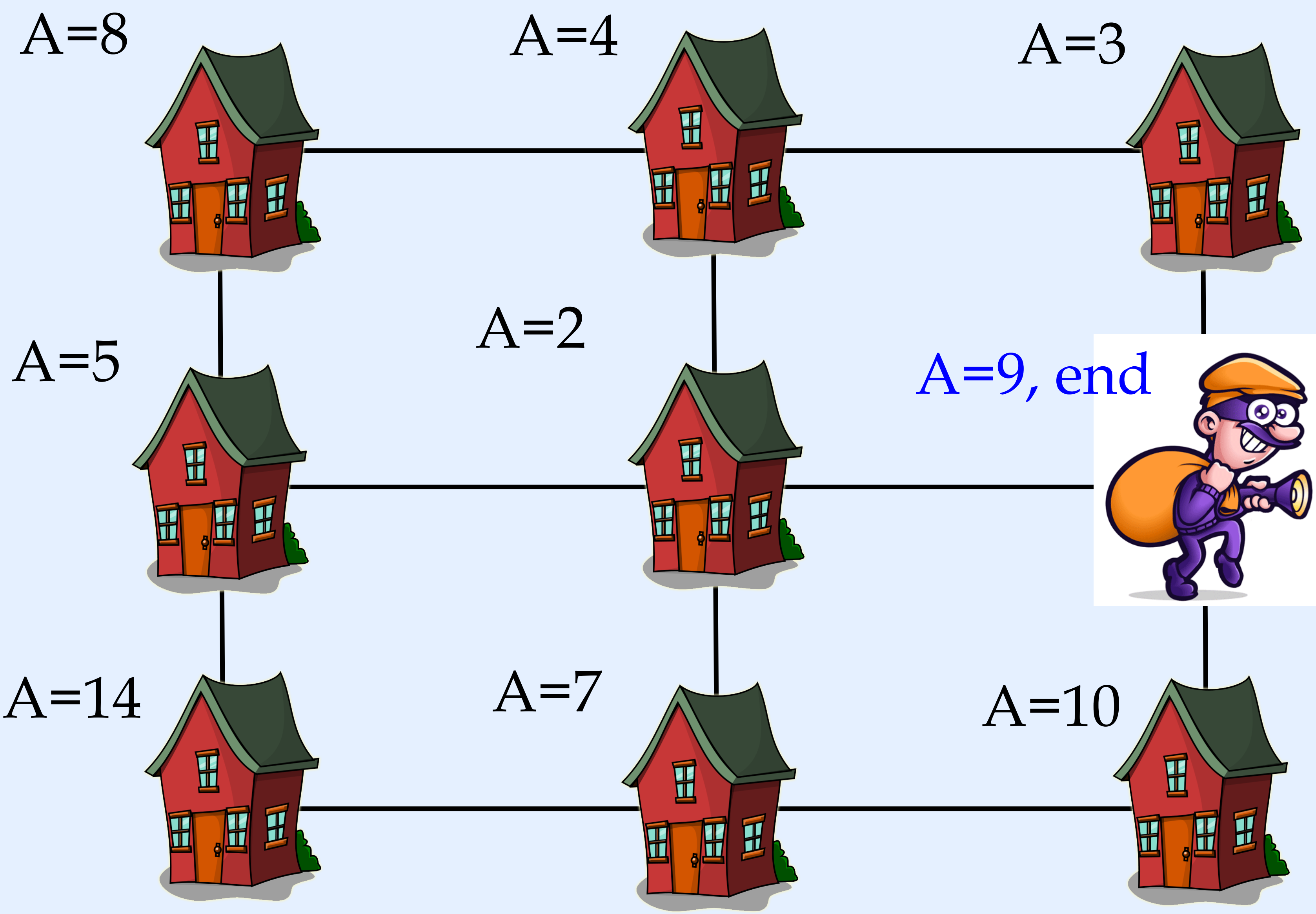
Agent based model, a cartoon



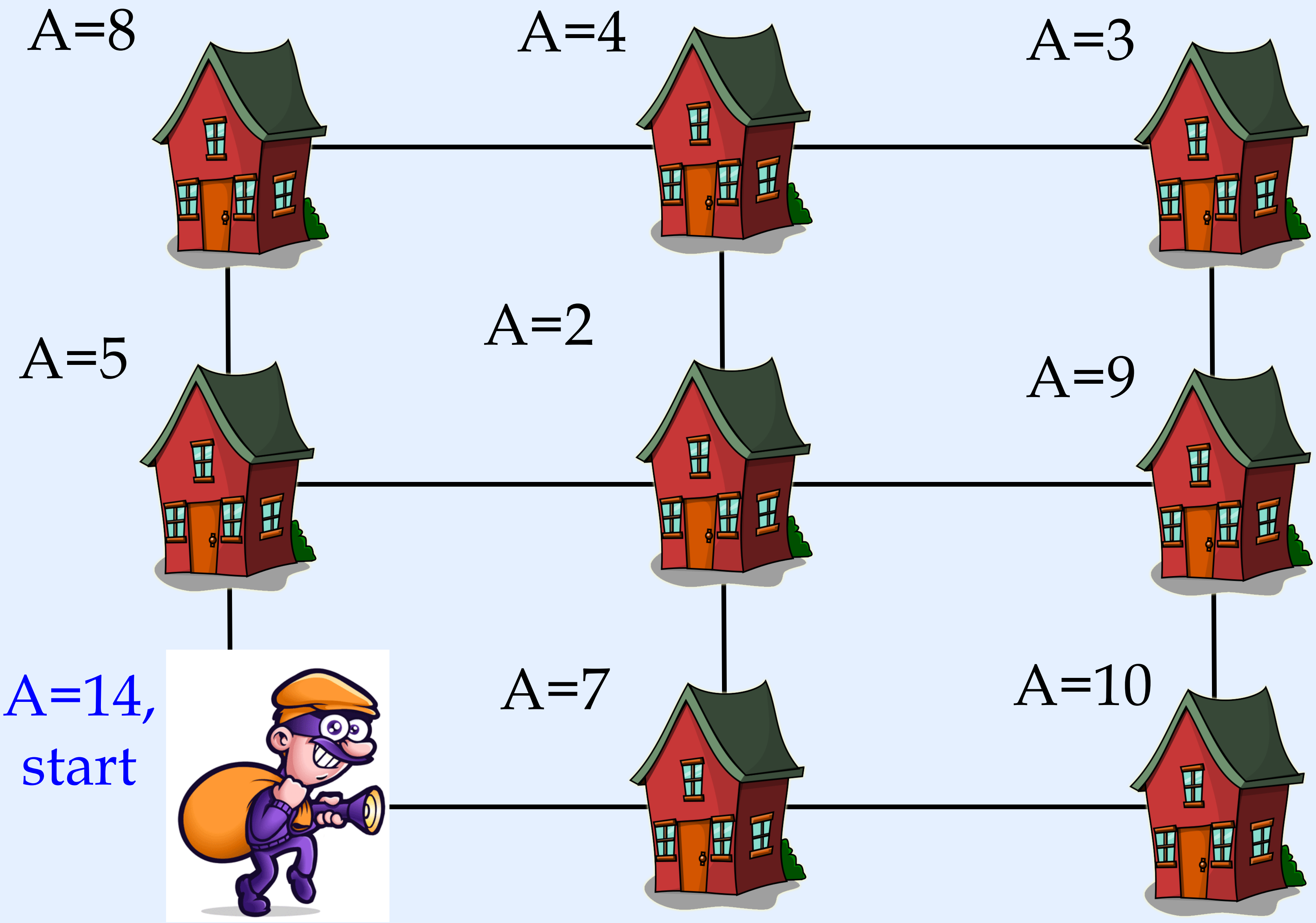
i) Site not very attractive, move



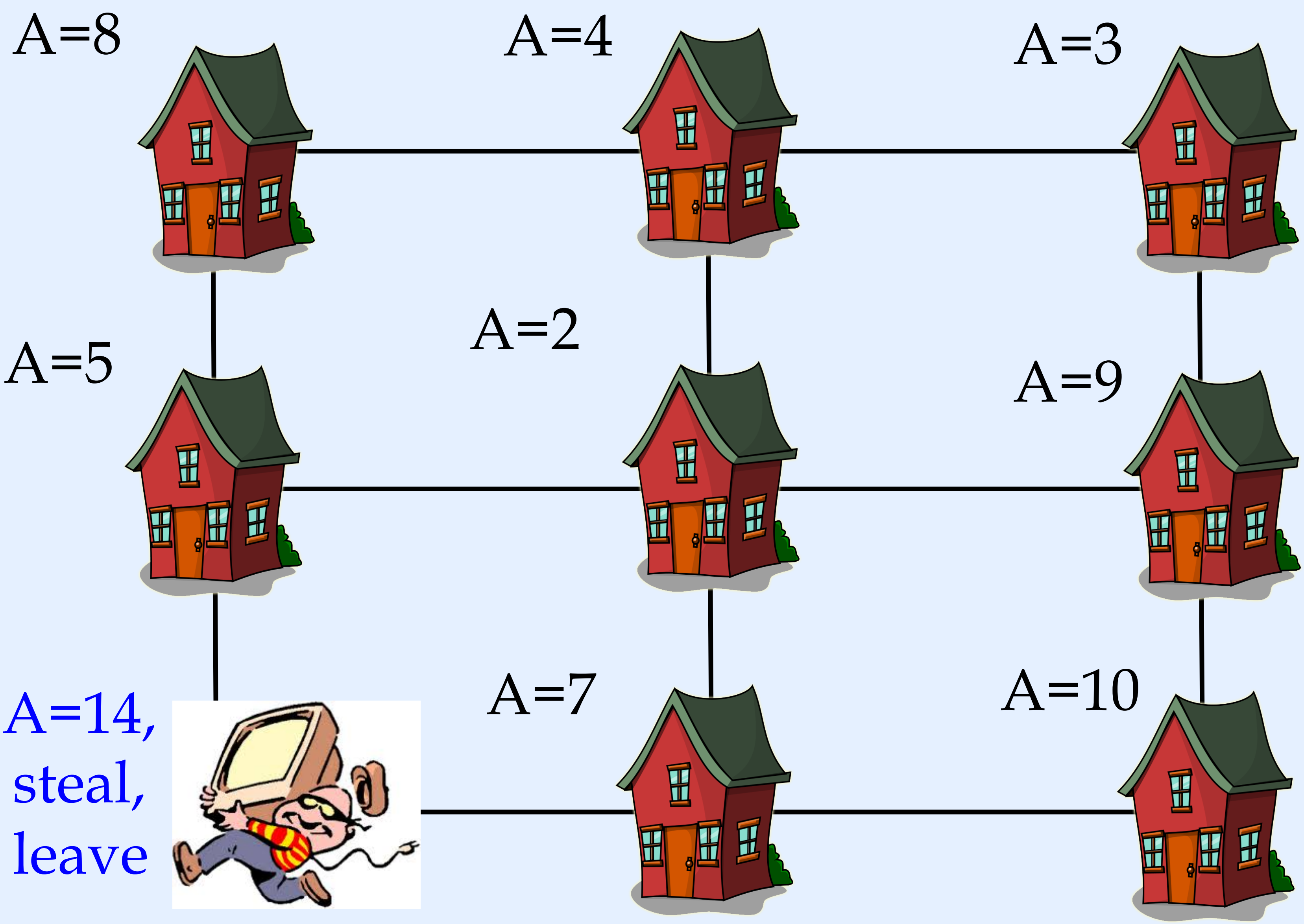
i) Site not very attractive, move



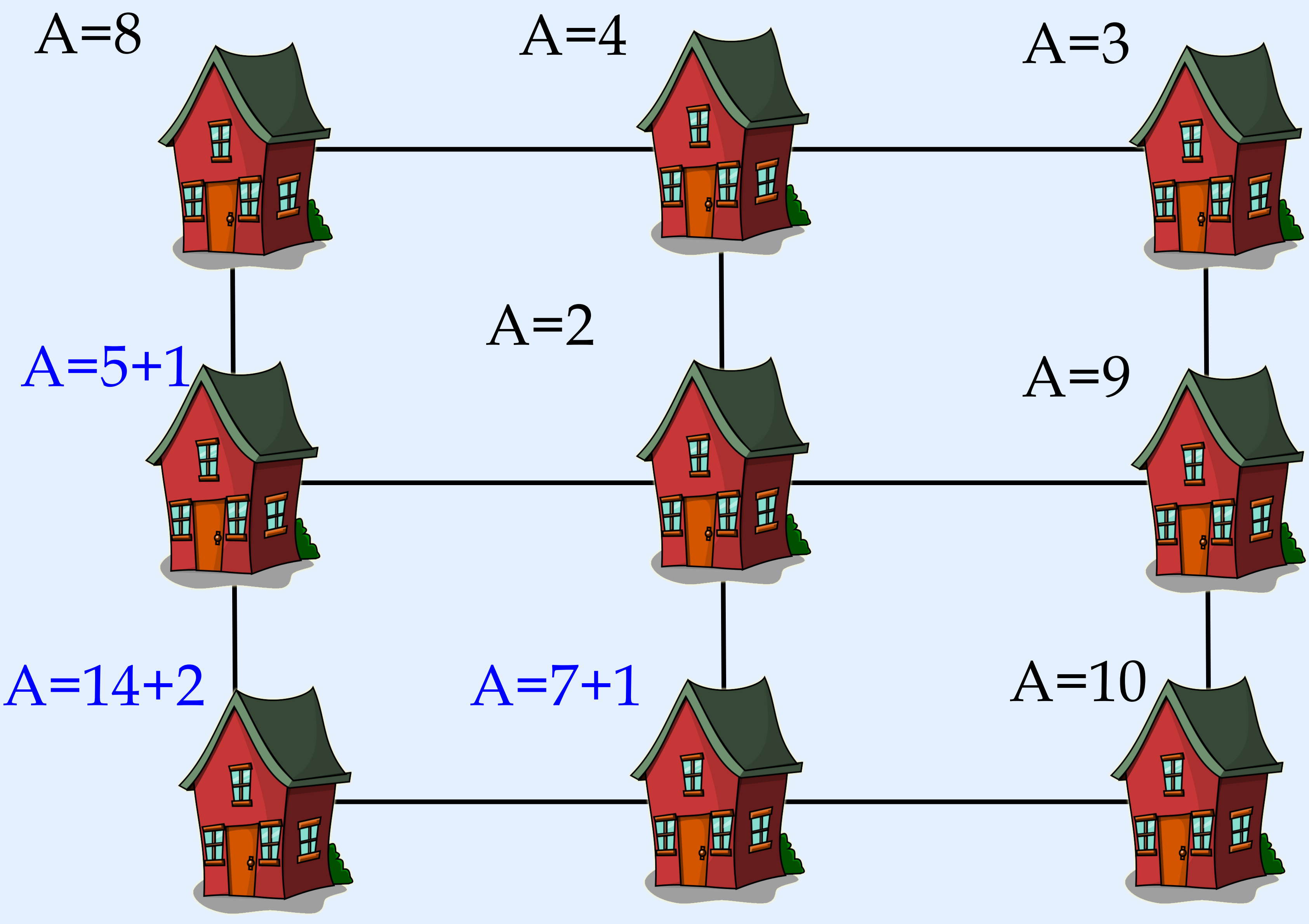
ii) Attractive site, steal



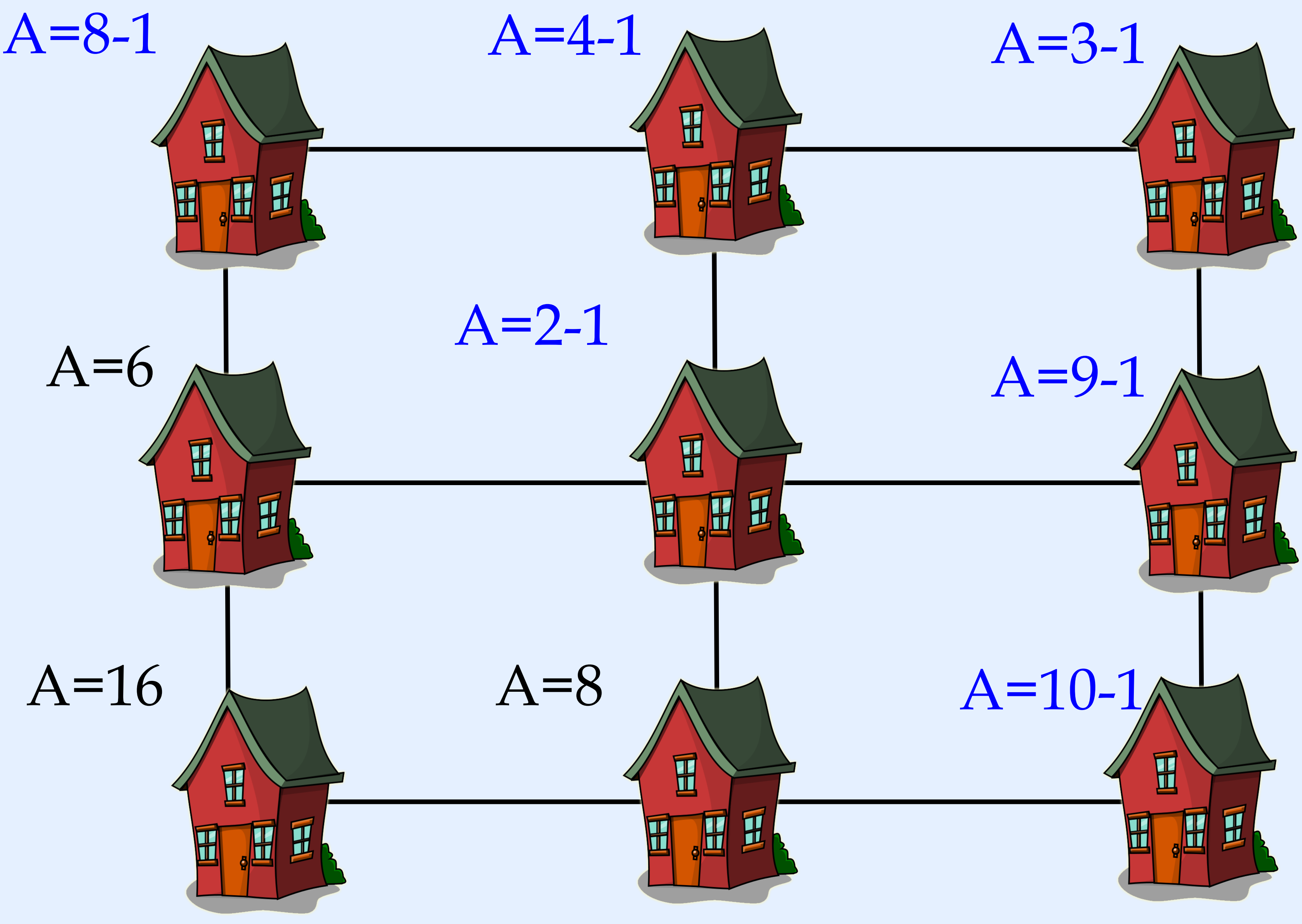
ii) Attractive site, steal



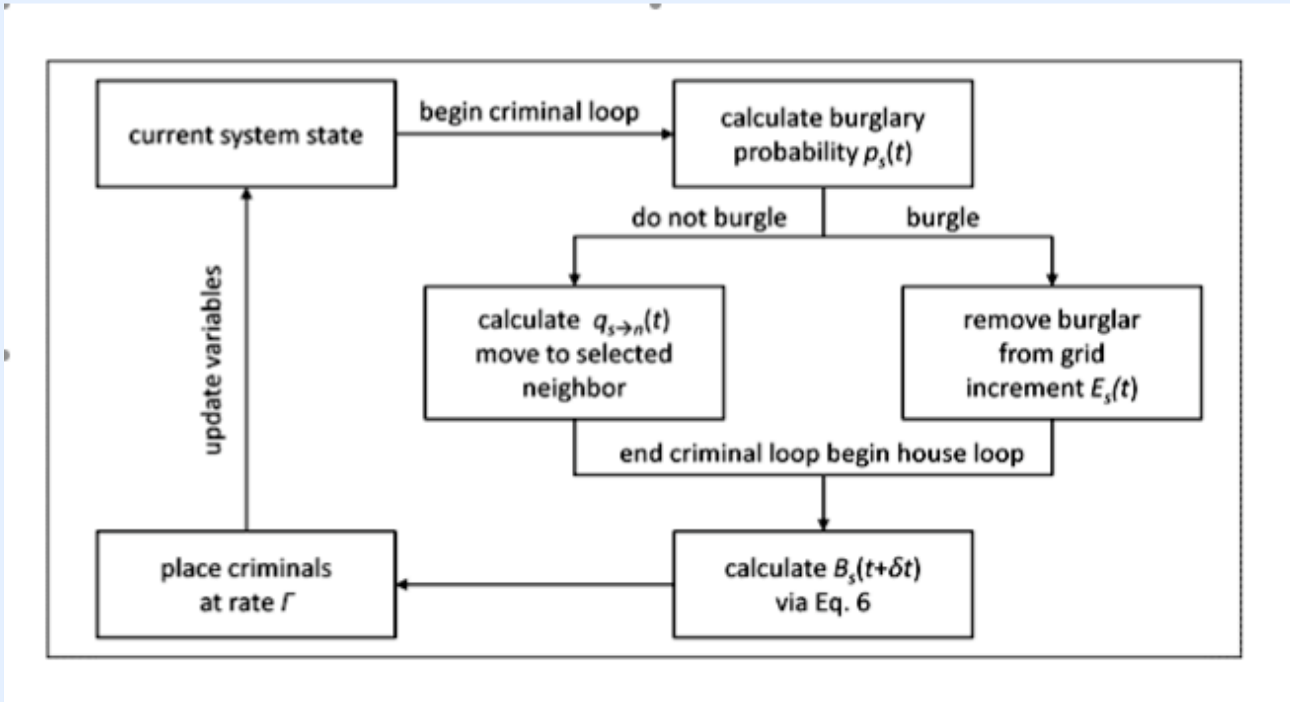
A increases close to theft site



A decreases away from theft site

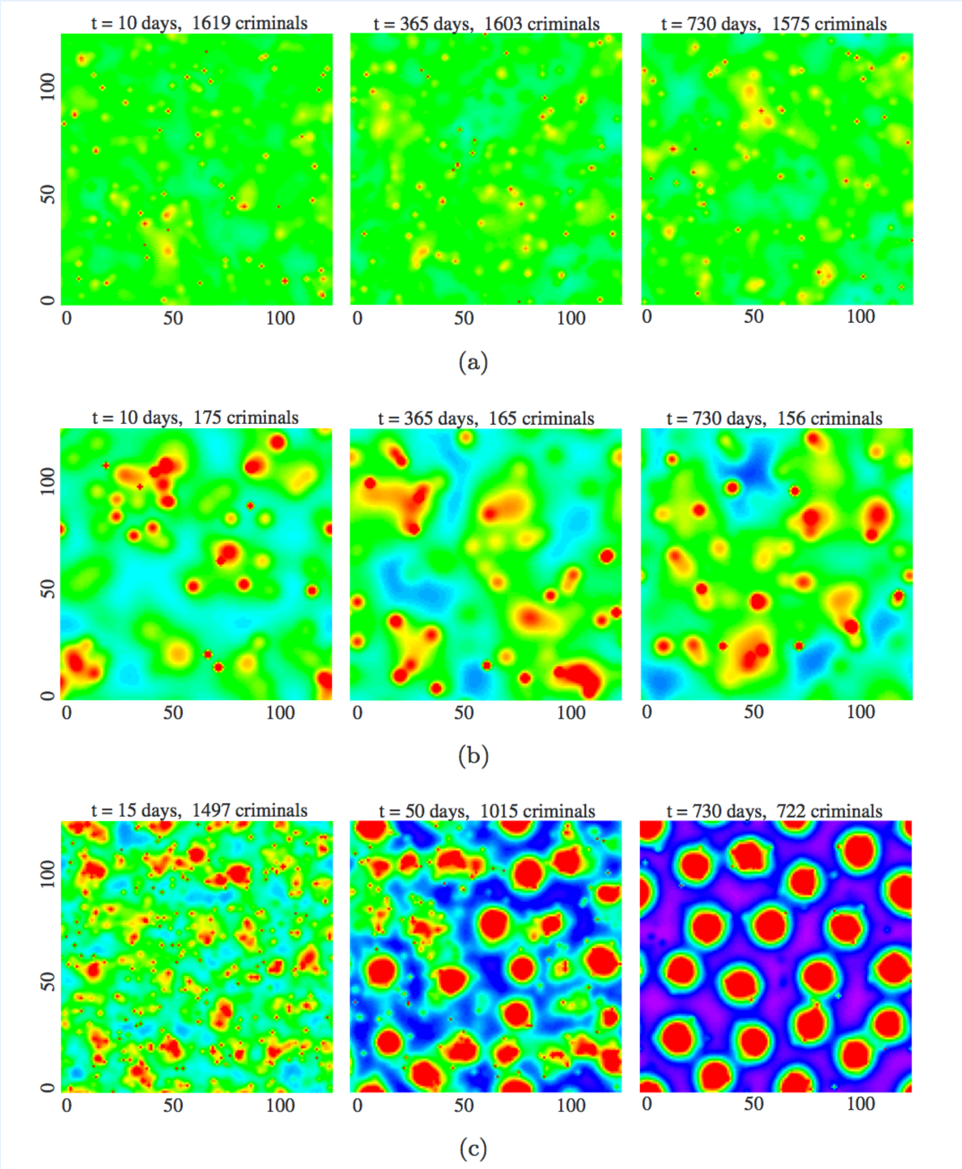


Scheme, parameters



Parameter Name	Meaning
ℓ	Grid spacing
δt	Time step
ω	Dynamic attractiveness decay rate
η	Measures neighborhood effects (ranging from 0 to 1)
θ	Increase in attractiveness due to one burglary event
A_s^0	Intrinsic attractiveness of site s
Γ	Rate of burglar generation at each site

Simulations



Many criminals,
low increase in A
No hotspots

Few criminals,
large increase in A
Transient hotspots

Many criminals,
large increase in A
Stationary hotspots

Continuum model

$$\frac{\partial A}{\partial t} = \eta \nabla^2 A - (A - A_0) + \rho A$$

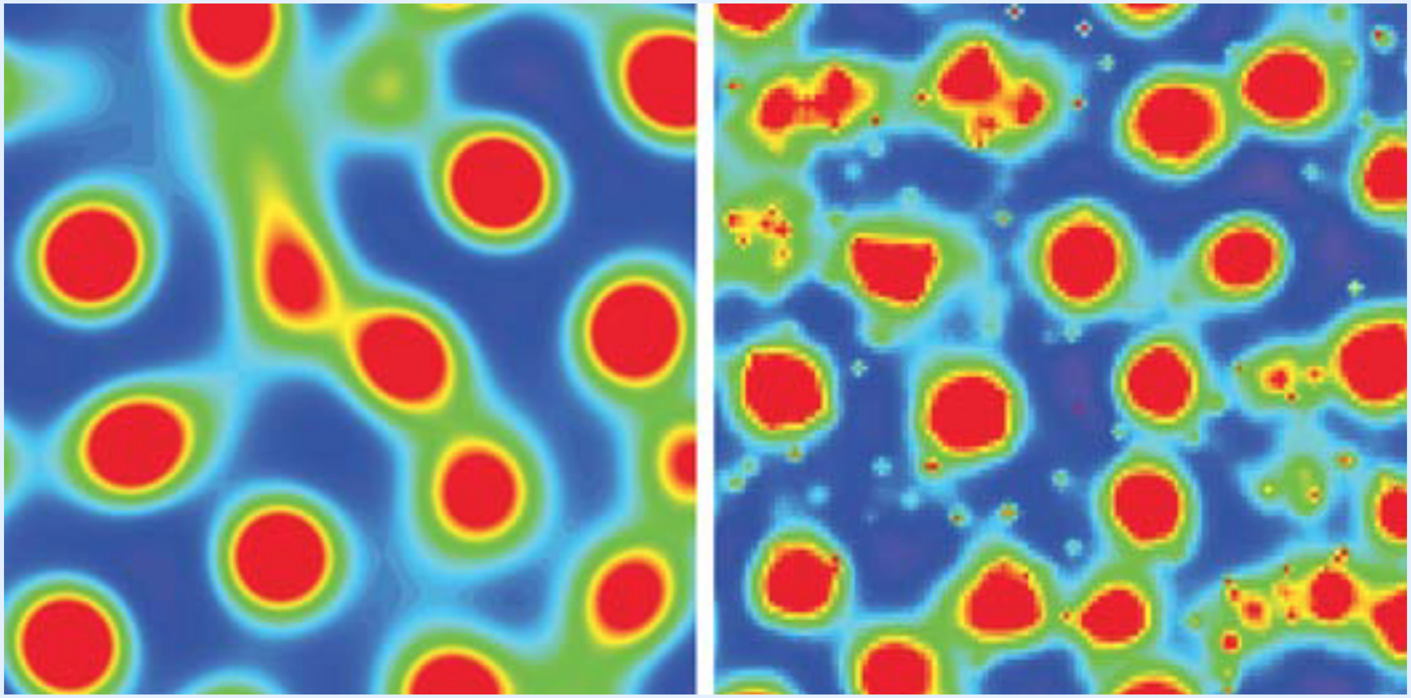
$$\frac{\partial \rho}{\partial t} = \vec{\nabla} \cdot \left[\vec{\nabla} \rho - \frac{2\rho}{A} \vec{\nabla} A \right] - \rho A + \Gamma$$

Attractiveness diffuses through environment
decay, replenished by criminal acts

Criminals depleted through reactions with system
spontaneously generated

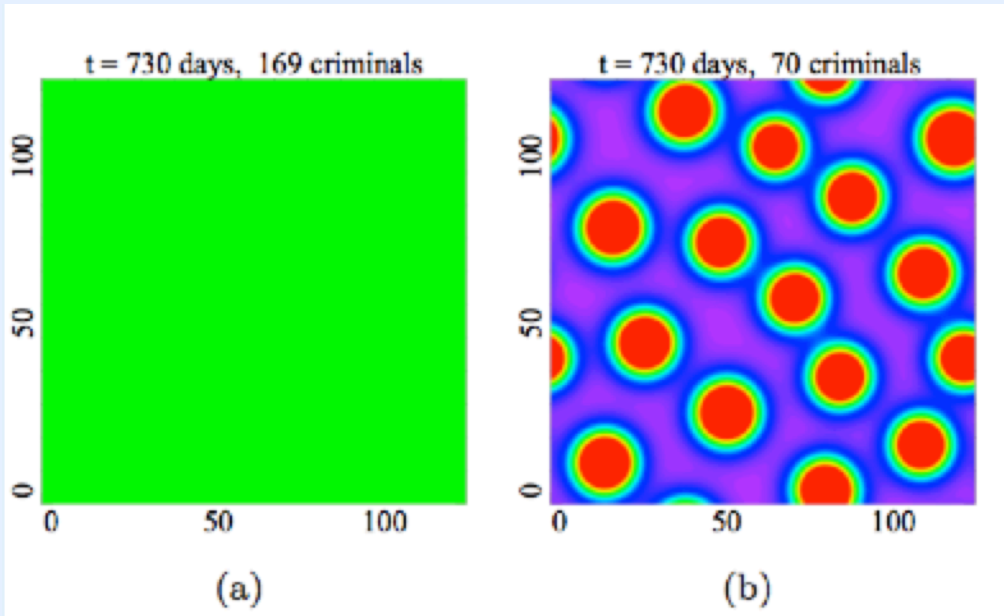
Diffusion and advective motion up gradients of A

Continuum vs. discrete



Steady state

Same parameters
as discrete (a), (b)



Same parameters
as discrete (c)

Many criminals,
large increase in A

Stationary hotspots: enhanced risk
of repeated crime is high enough to diffuse and be
sustained locally without binding distant crimes

Linear Stability Analysis

$$\frac{\partial A}{\partial t} = \eta \nabla^2 A - (A - A_0) + \rho A$$

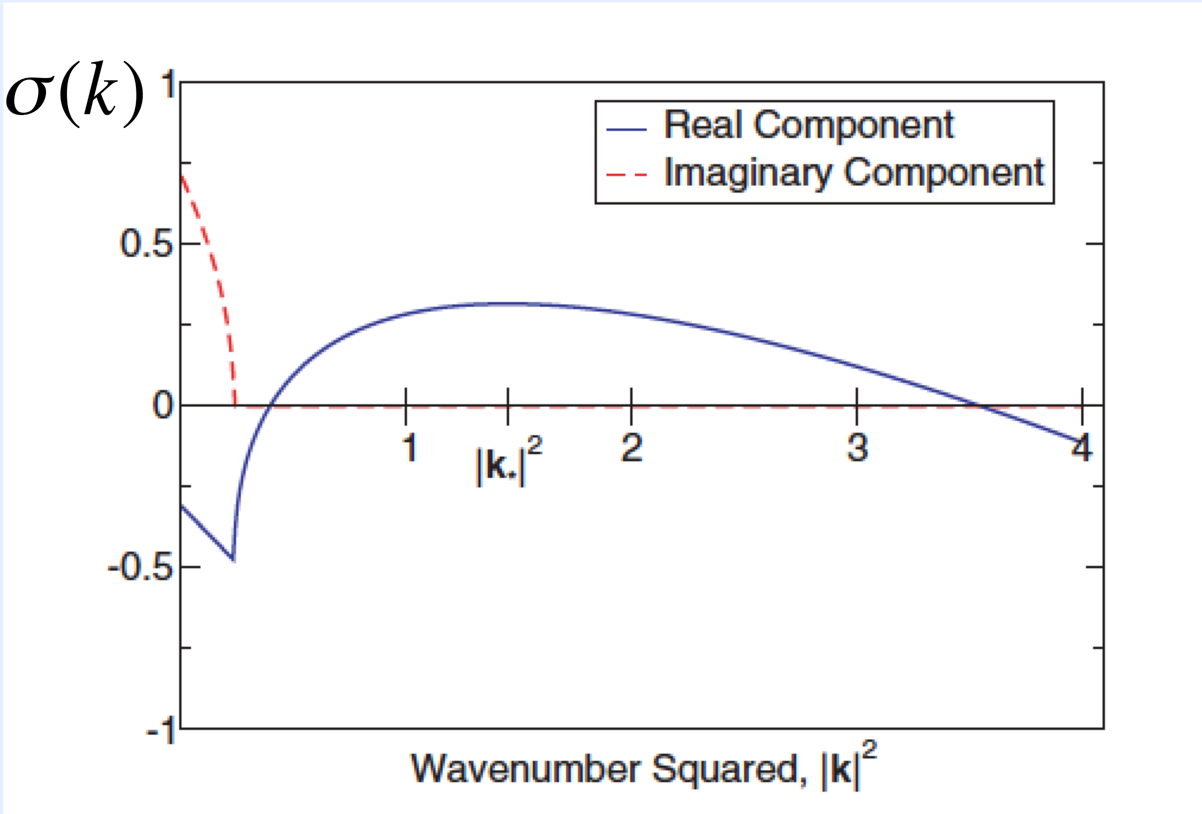
$$\frac{\partial \rho}{\partial t} = \vec{\nabla} \cdot \left[\vec{\nabla} \rho - \frac{2\rho}{A} \vec{\nabla} A \right] - \rho A + \Gamma$$

$$A^* = A_0 + \Gamma$$

$$\rho^* = \frac{\Gamma}{\Gamma + A_0}$$

$$\boxed{\begin{aligned} A &= A^* + \delta_A e^{\sigma t} e^{i\vec{k}\vec{x}} \\ \rho &= \rho^* + \delta_\rho e^{\sigma t} e^{i\vec{k}\vec{x}} \end{aligned}}$$

Linear Stability Analysis

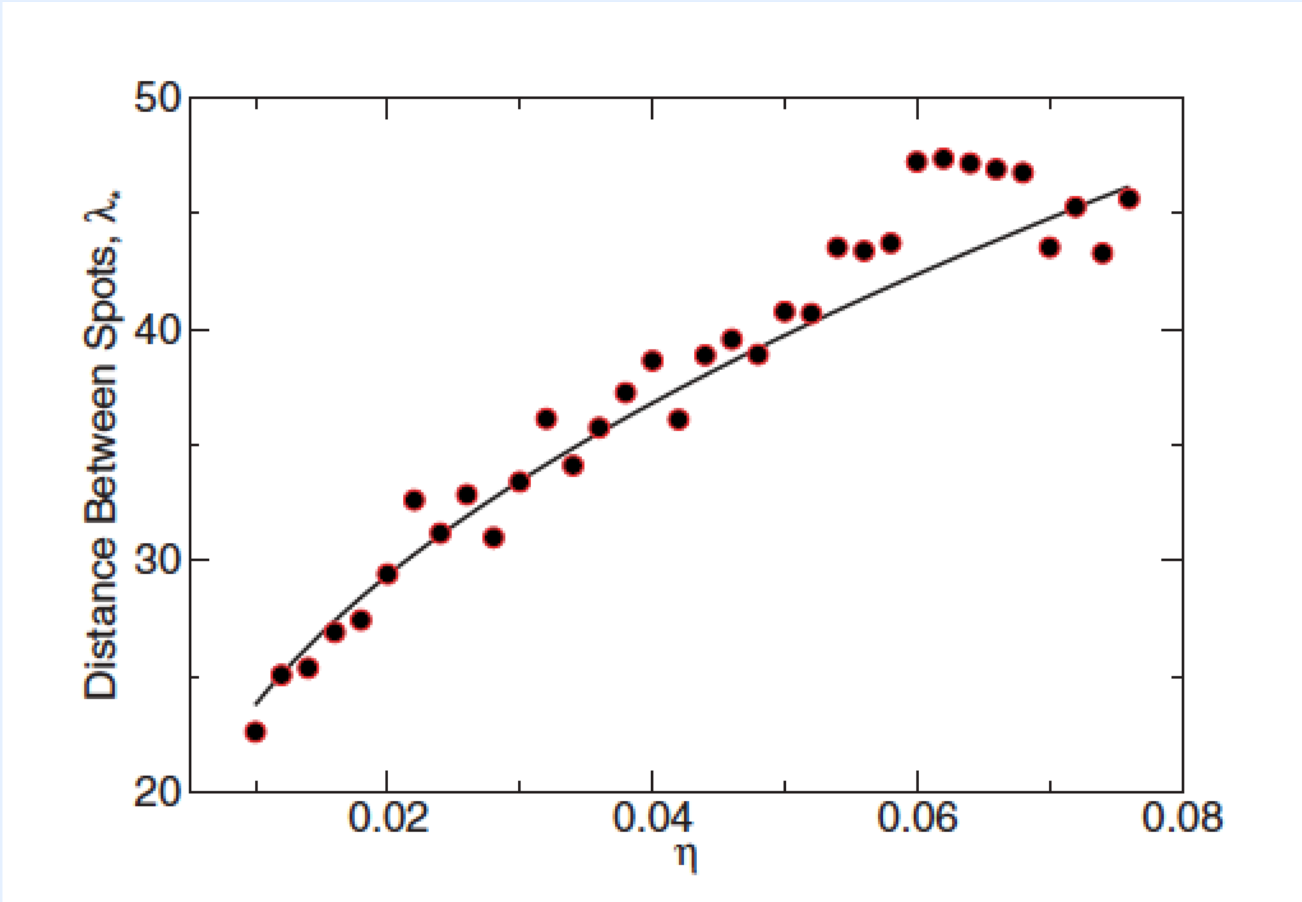


σ - imaginary

σ – real

Typical wavenumber k* -- Typical distance λ = 2π/k*

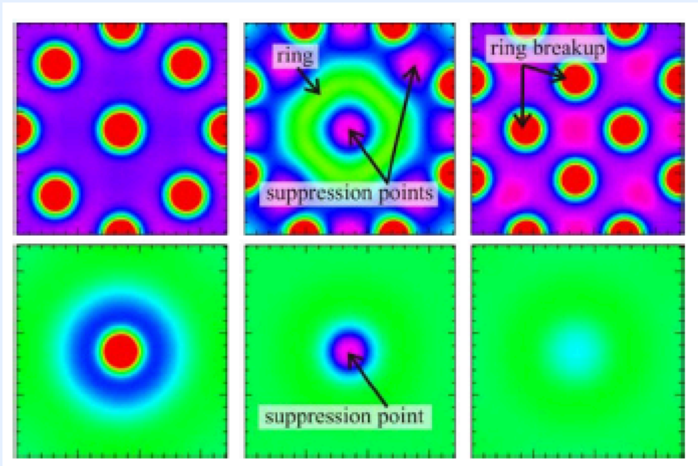
Hotspot separation



λ = $2\pi/k^*$ -- fitted vs. simulated data

Larger diffusivity → larger typical hotspot separation

Extensions



Suppression of hotspots

Policing

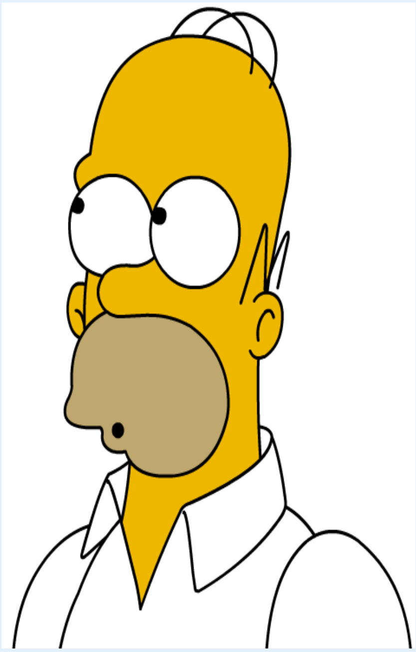
Spatial heterogeneity

Application to geographical data

Self exciting point processes

2. An informant and a criminal

The apathetic



The villain

The informant

The paladin

To commit a crime or not.
To collaborate or not.

Paladins: will not commit crime and will always report

Villains: will always commit crimes and never report

Apathetics: will not commit crimes but will not report

Informants: will commit crimes and will report

Assume constant population

$$P + V + A + I = N$$

strategies may change in time

The crime

Begin round - each player has value 1.

1. Choose a victimizer (V or I)
2. Choose a victim (any player)
3. A "crime" occurs with transfer of "wealth" $\delta<1$

Victim $\longrightarrow$ Victimizer

$1-\delta$ $\qquad$ $1+\delta$

4. Reporting?

If the victim is a villain or an apathetic no reporting end of round - start over

If the victim is a paladin or an informant reporting beginning of "investigation" - move to investigation

The investigation

Only if victim is paladin or informant.

1. "Witness" pool of m_P, m_I, m_A, m_V individuals
2. How many of them will corroborate victim's story?

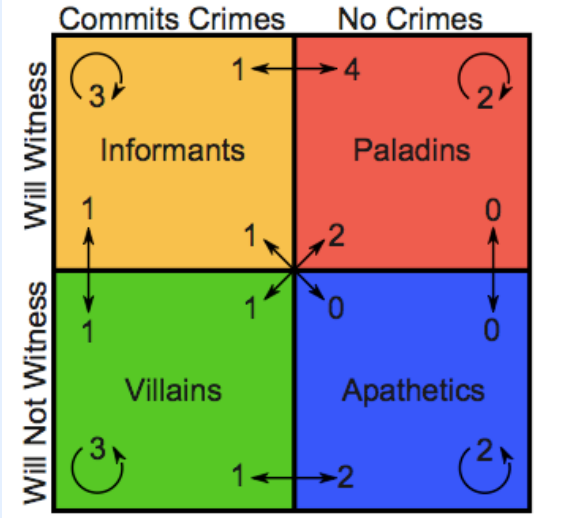
$$\omega = \frac{m_P + m_I}{m_P + m_I + m_A + m_V}$$

3. Victimizer is convicted with probability ω. The δ "loot" is returned to the victim. The victimizer is punished by θ.

4. Victimizer is free with probability $1-\omega$. The δ "loot" is kept by the victimizer. The victim gets extra loss ε due to retaliation.

Changing strategy

Find "loser" between victim and victimizer
(player with lower payoff)



Change his/her strategy by comparing
final payoffs and imitating

Imitate victimizer - copy total strategy
Loser becomes an informant or a villain

OR

Imitate victim - become non criminal
copy only reporting strategy
Loser becomes a paladin or an apathetic

Bias removed, "empathy"

Agent based vs. continuum equations

$$\dot{P} = (I+V)\left[(P+I)^2 \frac{1}{2-\theta} + I(A+V)\frac{1-\delta-\epsilon}{2-\epsilon} - P(A+V)\frac{1+\delta}{2-\epsilon} \right],$$

$$\dot{A} = (I+V)\left[V\frac{1-\delta}{2} - A\frac{1+\delta}{2} \right],$$

$$\dot{I} = I\left[(A+V)\frac{1+\delta}{2} + P(A+V)\frac{1+\delta}{2-\epsilon} - (P+I)^2\frac{1}{2-\theta} - I(A+V)\frac{1-\delta-\epsilon}{2-\epsilon} - V(A+V) \right],$$

$$\dot{V} = V\left[(P+I)(A+V)\frac{1+\delta}{2-\epsilon} + (A-I)\frac{1+\delta}{2} - (P+I)^2\frac{1}{2-\theta} - (I+V)\frac{1-\delta}{2} \right],$$

At t=0 I_0, P_0, V_0, A_0

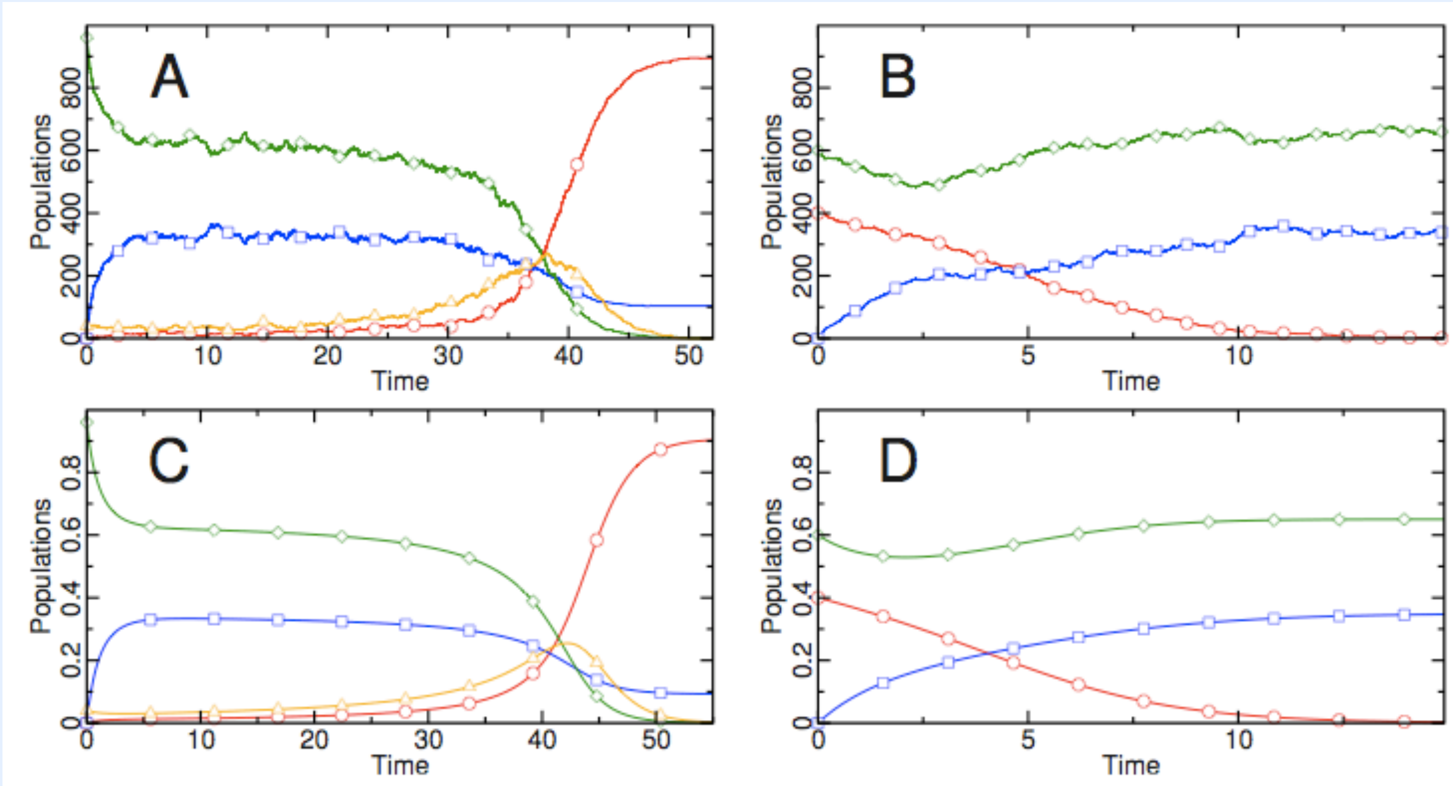
Study dynamics
Compare agent based (stochastic) and continuum descriptions
Find equilibria.
Stability?

Dynamics

I_0=40, A_0=P_0=0, V_0=960 I_0=A_0=0, P_0=400,V_0=600

Stochastic

Deterministic



θ = 0.6,
ε = 0.2, δ = 0.3
N=1000

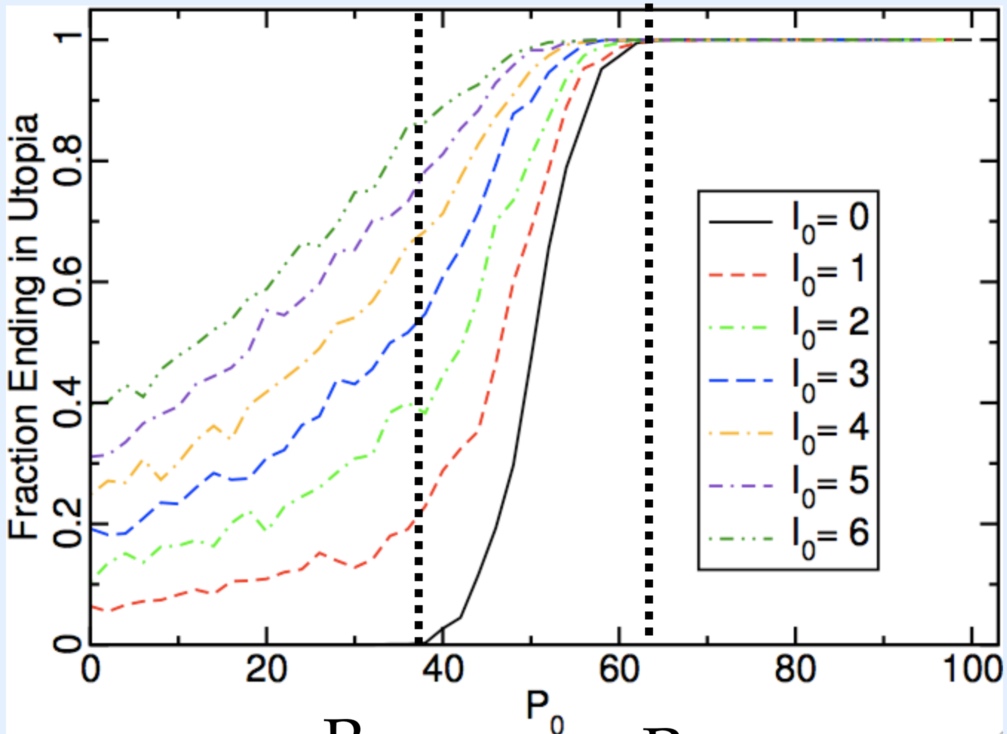
Paladins "win"
UTOPIA

Villains "win"
DYSTOPIA

Upon closer inspection

N=100

A_0=0, V_0 = N - P_0 - $I_{0.}$



P_{T1} P_{T2}

Achieving utopia is strongly dependent upon I_0.

For $P_0 < P_{T2}$, adding one informant leads to 5-10% increases in the probability of reaching utopia

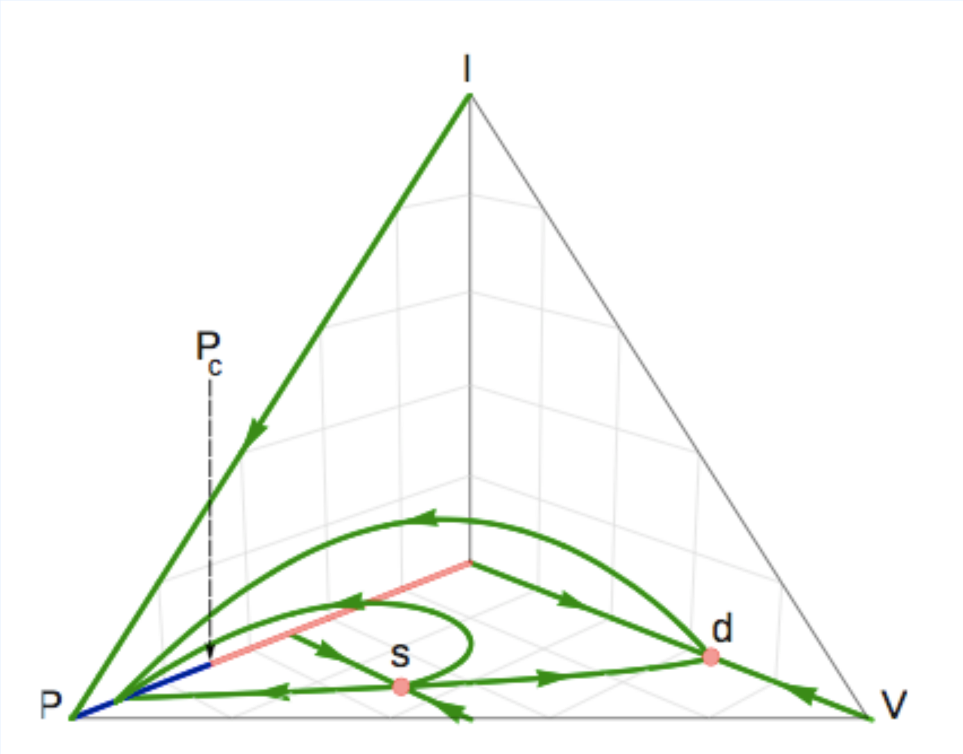
Linear Stability Analysis

From the deterministic case:

Dystopia is unstable to the addition of informants

A saddle point emerges

Utopia stable for P > Pc



Hence, if $I_0 > 0$

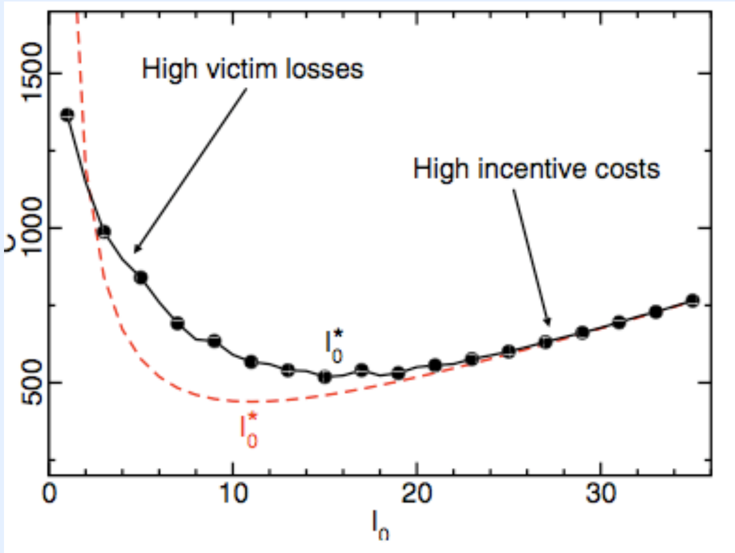
We always end up in utopia
according to the deterministic equations

Costs?

To achieve utopia we need informants
Let's recruit them from the villain pool.

This comes at a price.
at t=0 we convert I_0 villains

Cost = cost to convert informants + losses to society due to crime



Stochastic vs. deterministic

OPTIMAL CONVERSION

What do real people do?

Here is what UCI college kids do

			Session Variables			Strategies in last 5 rds.				
Session	Initialization	Available Strategies	Number of Subjects	Parameter Profile	Number of Periods	P	A	I	V	End State
1	90%V - 10%I	All	22	A	25	77%	1%	14%	8%	Utopia
2	90%V - 10%I	No A	24	A	25	66%	0%	21%	13%	Utopia
3	90%V - 10%P	No I	18	A	32	28%	17%	0%	56%	Dystopia
4	90%V - 10%I	All	18	B	30	64%	8%	17%	11%	Utopia
5	90%V - 10%P	No I	24	B	27	12%	16%	0%	73%	Dystopia

16 experiments, 10 treatments, ~ 400 students

Learning rounds, inertia, ‘parallelized’

Outcomes

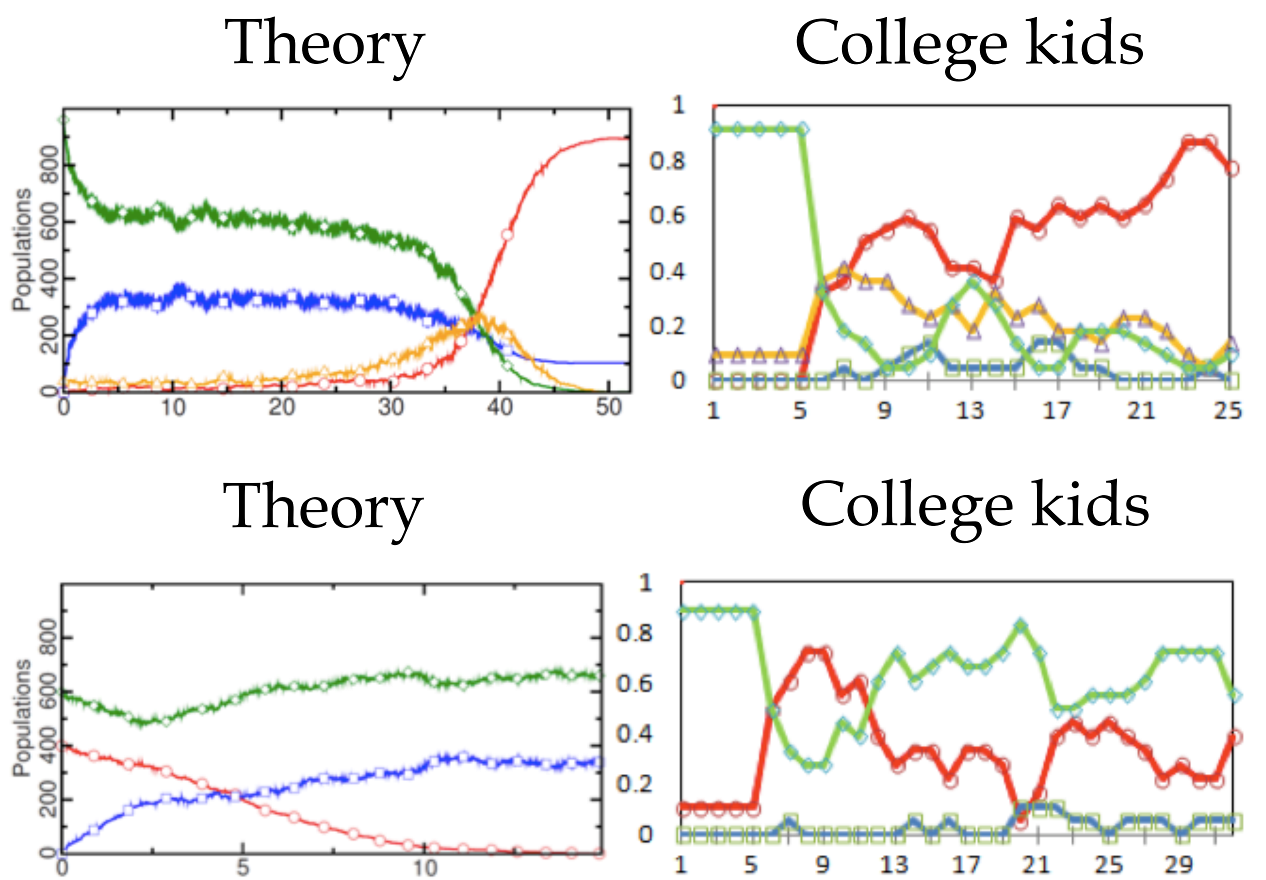
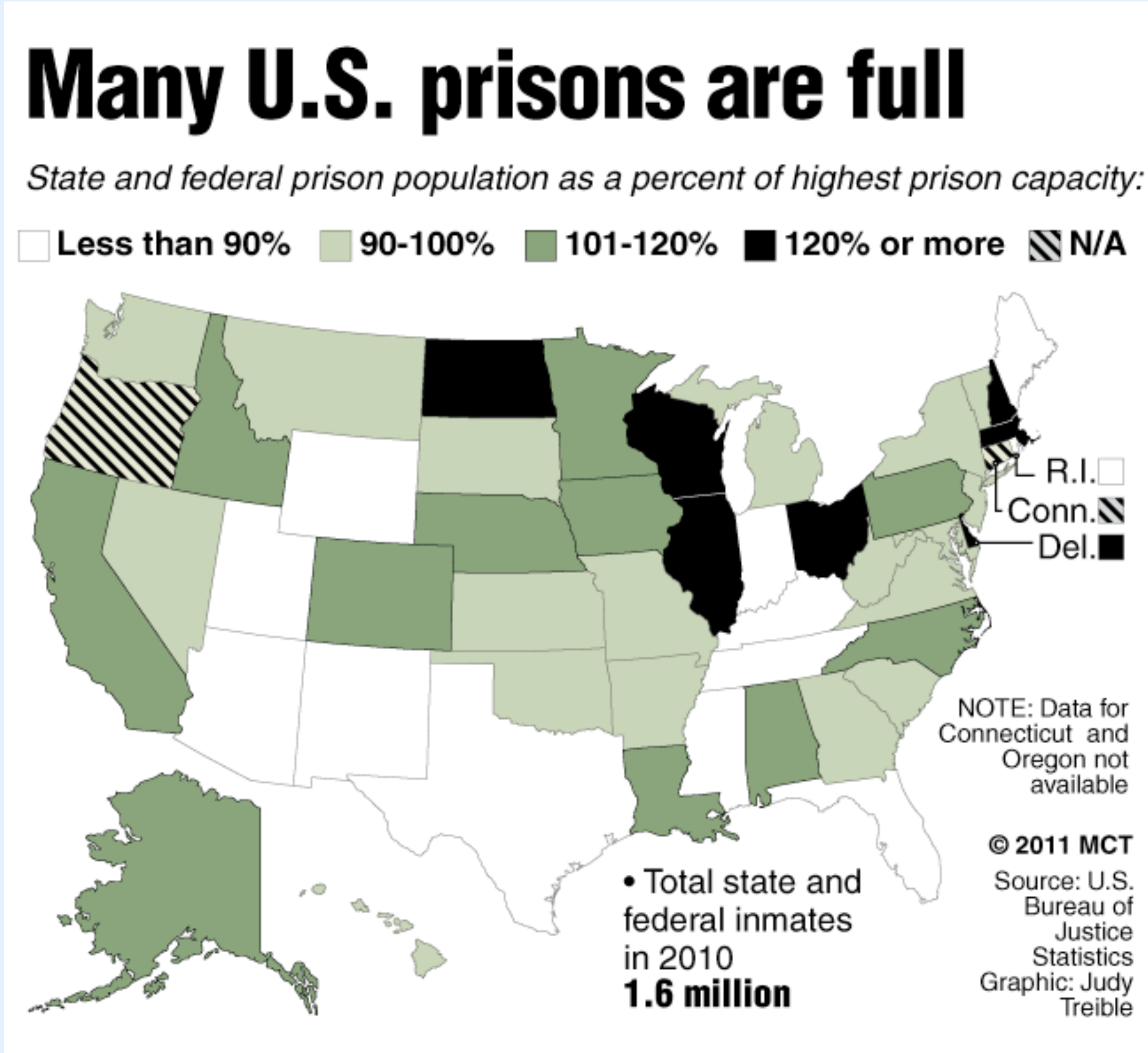


Fig 2. Evolution of Strategies. A and C depict the evolution of strategies from simulations using the imitation dynamic from (*27*) with with N=1000. B and D depict the evolution of strategies from our experiment Sessions 1 and 3. A and B allow all strategies; C and D do not allow the Informant strategy. In all figures, Paladins are red, Apathetics are blue, Informants are orange, and Villains are green.

3. Rehabilitation and Recidivism

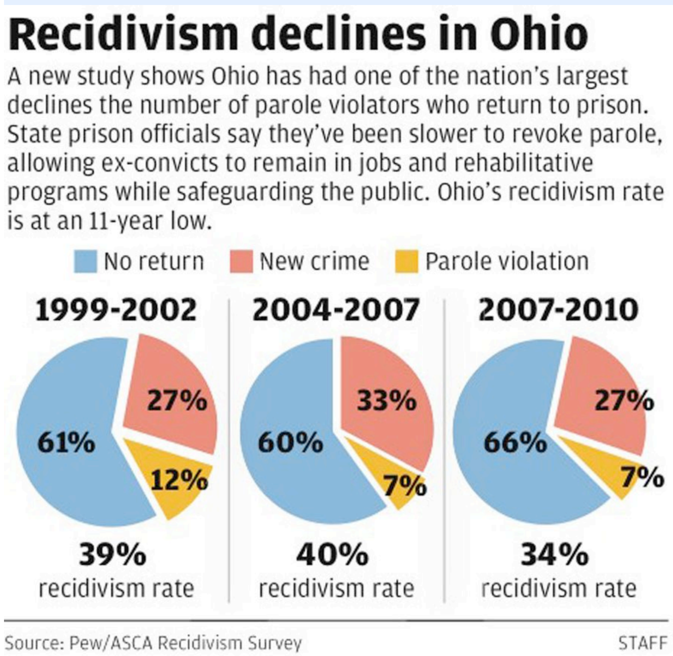


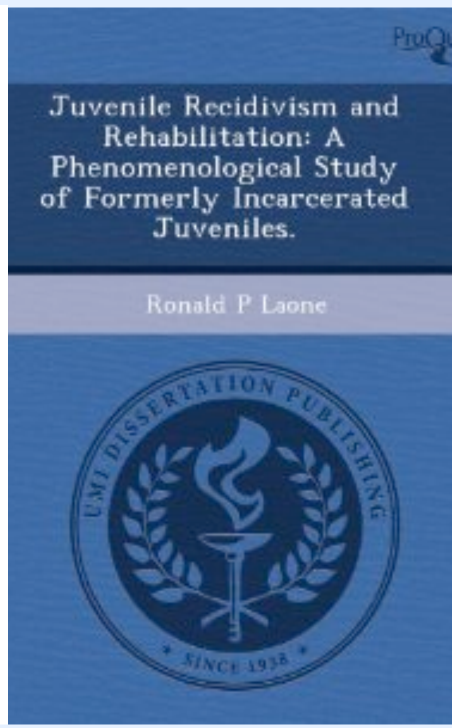
The carrot and the stick

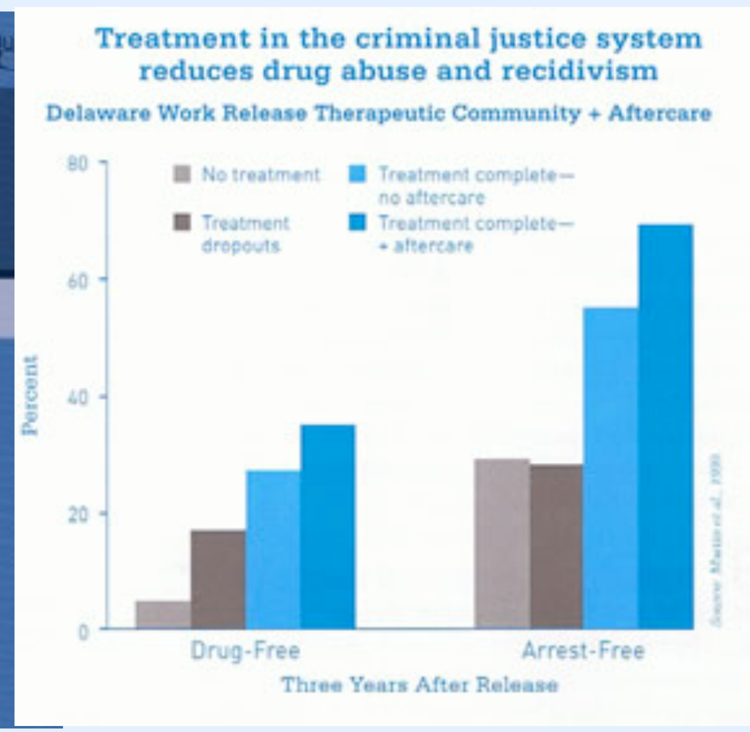
Report: Recidivism Rate Down as Texas Focuses on Treatment

Thanks partly to greatly expanded rehabilitation and treatment programs, Texas sent 11 percent fewer ex-convicts back to prison in recent years a significant drop in recidivism that is being replicated across the country, according to a new study.

BY MCCLATCHY NEWS | SEPTEMBER 25, 2012







The criminal life

Players are exposed to criminal activities
and choose to commit crimes or not based on:

1. Past criminal history
2. Surrounding environment
3. If recidivists: enrollment in rehabilitation programs

Build and keep track of players criminal history

$N_{0,} N_{1,}\ N_{2,}\ \ldots N_{k,} \ldots$

The criminal life

For each convicted crime: PUNISH AND REHABILITATE

Players may permanently reform: PALADINS

If the number of crimes reaches a threshold criminals are irreducible:

UNREFORMABLES

Paladins and Unreformables

$$
\begin{tikzcd}
 & P & P & P & P \\
N_0 \arrow[r] & N_1 \arrow[r] \arrow[u] & \dots \arrow[r] \arrow[u] & N_k \arrow[r] \arrow[u] & N_{R-1} \arrow[u] \arrow[d] \\
 & & & & U
\end{tikzcd}
$$

At the end of our irreversible game players
in one of two sinks:

Paladins P
Unreformables U

Define natural ``order parameter” P/U

Decisions – Committing crimes?

For each individual i

k_u unpunished crimes k_p punished crimes

k total = $k_u + k_p$

Personal history - p_i

Societal input - s_i

Attenuation due to rehabilitation – a_i

$$p_{Crime} = \left(\frac{p_i + s_i}{2} \right) a_i$$

Mathematical choices

$$p_i = \frac{p_0 + k_u}{k_u + p_0 + \theta k_p}$$

θ punishment
unpunished crimes - k_u
punished crimes – k_p

$$s_i = \frac{\sum_{k \neq 0} N_k + U}{N}$$

Crime generates more crime

$$a_i = (1 - h\ e^{-t/\tau})$$

h incentives
τ time duration

$$p_{Crime} = \left(\frac{p_i + s_i}{2}\right) a_i$$

After crime:
punish with probability $\alpha = 1/4$

After the crime

If no crime committed : reform with probability

$$p_{reform} = \frac{\alpha P}{N}$$

If a crime committed

if not arrested/punished
don't reform

$$p_{reform} = 0$$

if arrested/punished
reform with probability

$$p_{reform} = \frac{1}{2}\left(\frac{\alpha P}{N} + \frac{\theta k_p}{k_u + p_0 + \theta k_p}\right)$$

Variables/Parameters

P	paladins
U	unreformables (who have have been punished R times)
N_0	neutral citizens that have committed no crimes
N_k	citizens that have committed $k = k_u + k_p$ crimes
k_u	number of unpunished crimes
k_p	number of punished crimes
h	parameter quantifying resources
τ	duration of intervention
θ	severity of punishment
p_0	punishment amplitude parameter
α	arrest and conviction probability
R	maximum number of punished crimes

Irreversible system, very sensitive to initial conditions

Runs

400 individuals, 25 replicas

τ=2 p_0=0.1 α= ¼ R=3

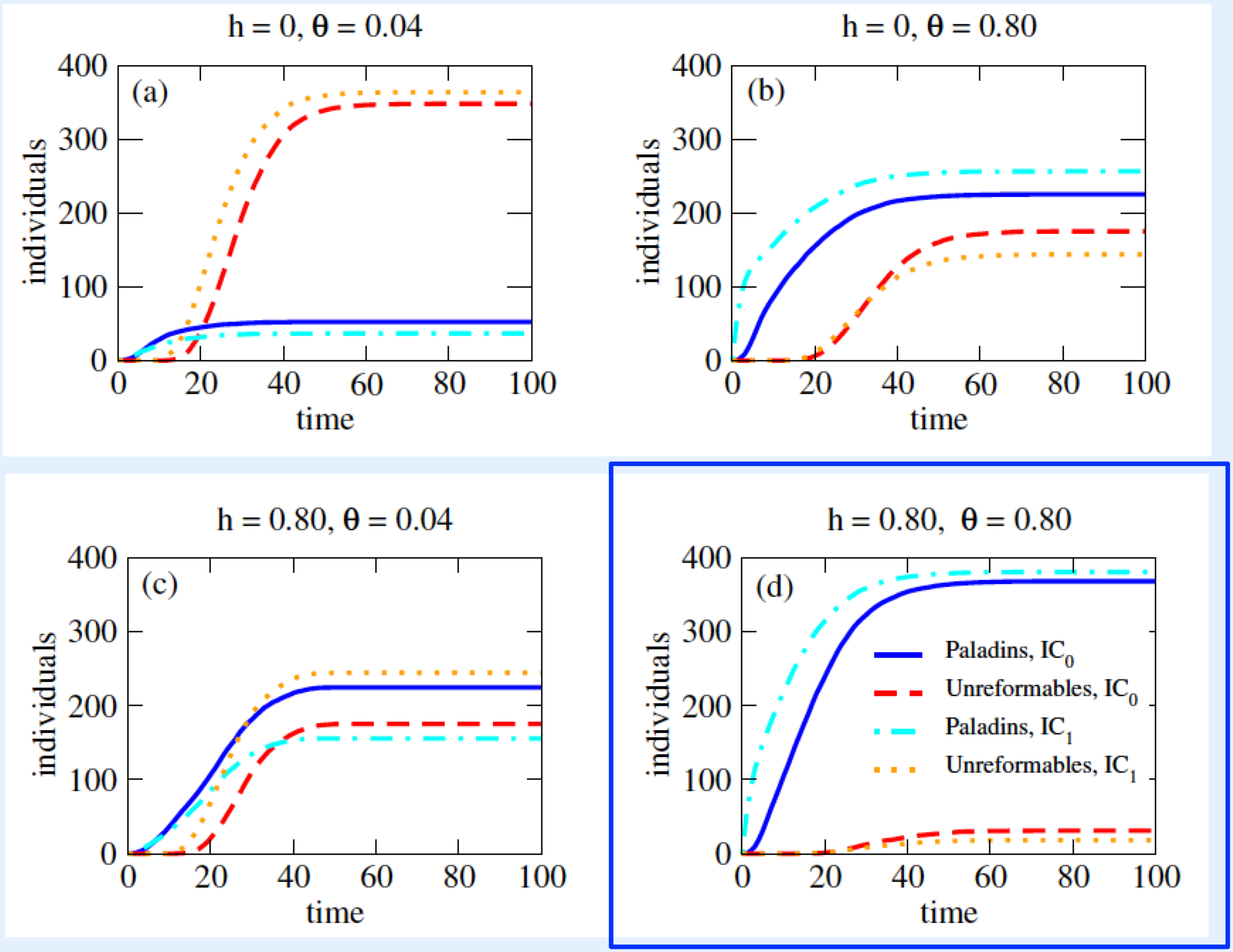
vary h, θ

IC0 : 400 in N_0 at time t=0

IC1: 200 in N_0, 200 in N_1 at time t=0

Parallel updates

Results



h = 0, θ = 0.04
(a)
h = 0, θ = 0.80
(b)
h = 0.80, θ = 0.04
(c)
h = 0.80, θ = 0.80
(d)
individuals
time
Paladins, IC_0
Unreformables, IC_0
Paladins, IC_1
Unreformables, IC_1

Total resources are finite!

Carrot, stick or both?

The carrot:
Rehabilitation resources $h\tau$

The stick:
Punishment resources θ

Increase one,
Decrease the other

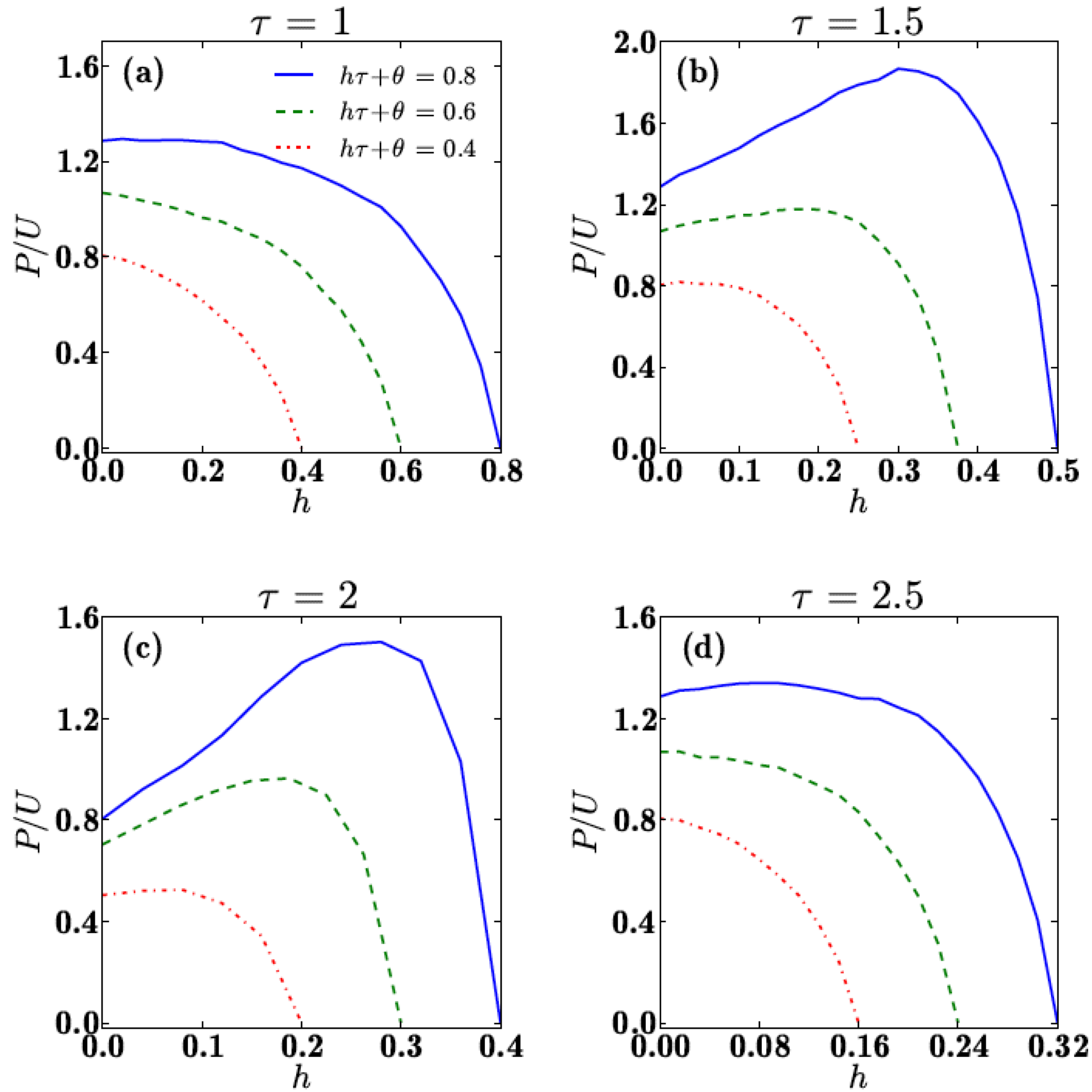
$$h\tau + \theta = const$$

P/U with finite resources

$h\tau + \theta = 0.8$

$h\tau + \theta = 0.6$

$h\tau + \theta = 0.4$



P/U with finite resources

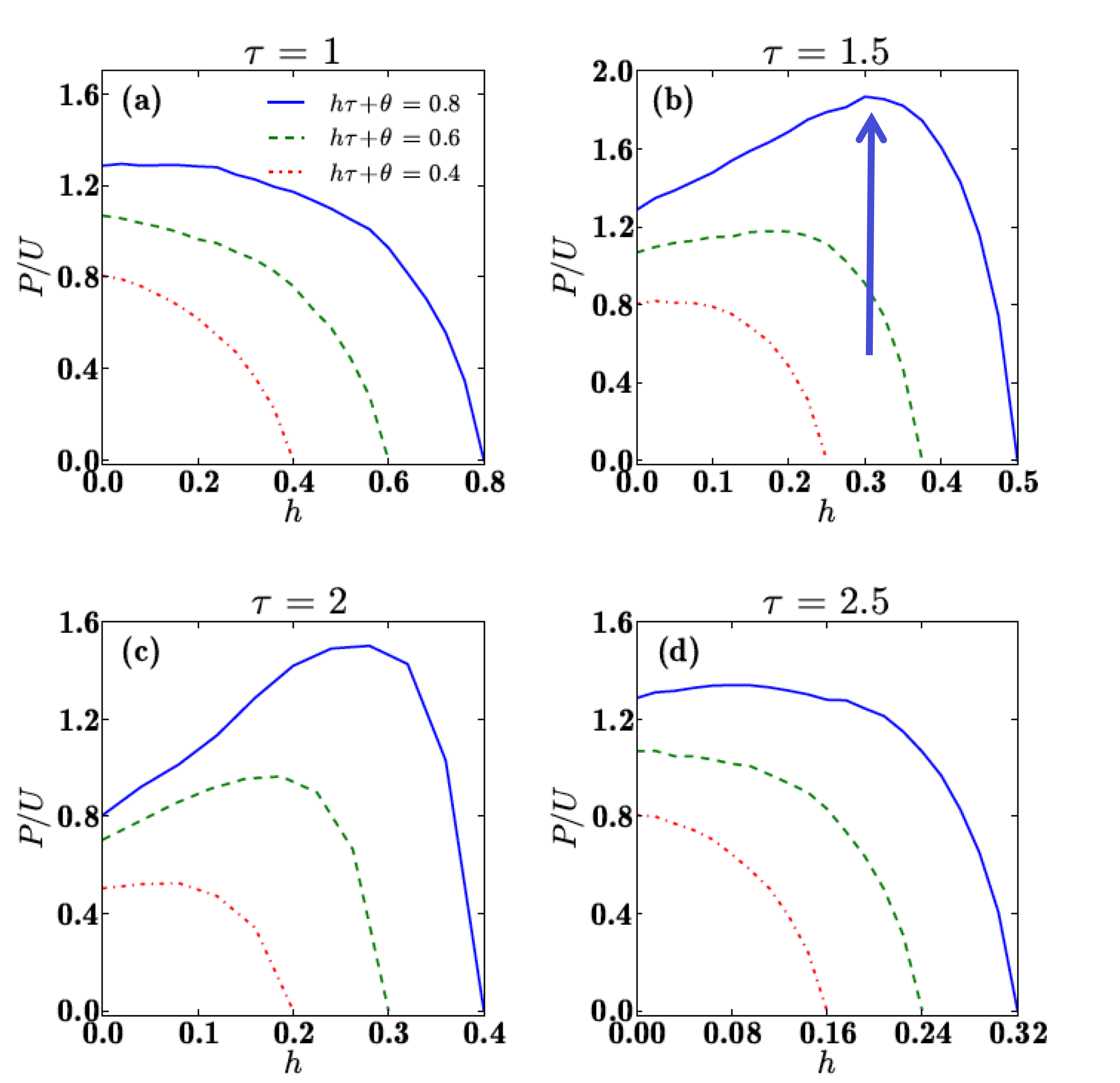
$$h\tau + \theta = 0.8$$

Optimal parameter set

$$h = 0.30$$

$$\tau = 1.5$$

$$\theta = 0.35$$



Best strategy

Enough stick

Enough carrots
For enough time

Too lenient or too harsh punishments are not
as effective as judicious balancing of the two

ODEs

Challenges:

Memory effects
Open ended
Parallel vs. Sequential updates

Keep track of indeces k_u, k_p

$N_k \rightarrow N_{ku,kp}$

$p_{crime} \rightarrow c_{\ ku,kp}$

$P_{reform} \rightarrow r_{\ ku,kp}$

Agent based vs. continuum equations

$$
\begin{aligned}
\dot{N}_{0,0} &= -\left[c_{0,0} + (1-c_{0,0})\frac{\alpha P}{N}\right] N_{0,0}, \\
\dot{N}_{0,k_p} &= -\left[c_{0,k_p} + (1-c_{0,k_p})\frac{\alpha P}{N}\right] N_{0,k_p} + \alpha c_{0,k_p-1}(1-r_{0,k_p})N_{0,k_p-1}, \\
& \qquad \text{for } k_p = 1, \cdots, R-1 \\
\dot{N}_{k_u,0} &= -\left[c_{k_u,0} + (1-c_{k_u,0})\frac{\alpha P}{N}\right] N_{k_u,0} + c_{k_u-1,0}(1-\alpha)N_{k_u-1,0}, \\
& \qquad \text{for } k_u \geq 1 \\
\dot{N}_{k_u,k_p} &= -\left[c_{k_u,k_p} + (1-c_{k_u,k_p})\frac{\alpha P}{N}\right] N_{k_u,k_p} + c_{k_u-1,k_p}(1-\alpha)N_{k_u-1,k_p} \\
& \quad +\alpha c_{k_u,k_p-1}(1-r_{k_u,k_p})N_{k_u,k_p-1}, \\
& \qquad \text{for } k_u \geq 1 \text{ and } 1 \leq k_p \leq R-1
\end{aligned}
$$

ODEs

$$\dot{P} = \sum_{k_u=0}^{\infty}\sum_{k_p=0}^{R-1}\left[(1-c_{k_u,k_p})\frac{\alpha P}{N}\right]N_{k_u,k_p} + \alpha\sum_{k_u=0}^{\infty}\sum_{k_p=0}^{R-2} c_{k_u,k_p} r_{k_u,k_p+1} N_{k_u,k_p},$$

$$\dot{U} = \alpha\sum_{k_u=0}^{\infty} c_{k_u,R-1} N_{k_u,R-1},$$

$$c_{k_u,k_p}(t) = \frac{1}{2}\left[\frac{p_0+k_u}{p_0+k_u+\theta k_p} + \frac{\sum_{\{k_u,k_p\neq 0,0\}} N_{k_u,k_p}}{N}\right]\left(1-he^{-(t-t_{\text{last}})/\tau'}\right)$$

$$r_{k_u,k_p}(t) = \frac{1}{2}\left[\frac{h\alpha P}{N} + \frac{\theta k_p}{\theta k_p + k_u + p_0}\right].$$

$$t - t_{last} = t / k_p$$

Conservation of population

$$\sum_{k_u=0}^{\infty} \sum_{k_p=0}^{R-1} \dot{N}_{k_u,k_p} + \dot{P} + \dot{U} = 0,$$

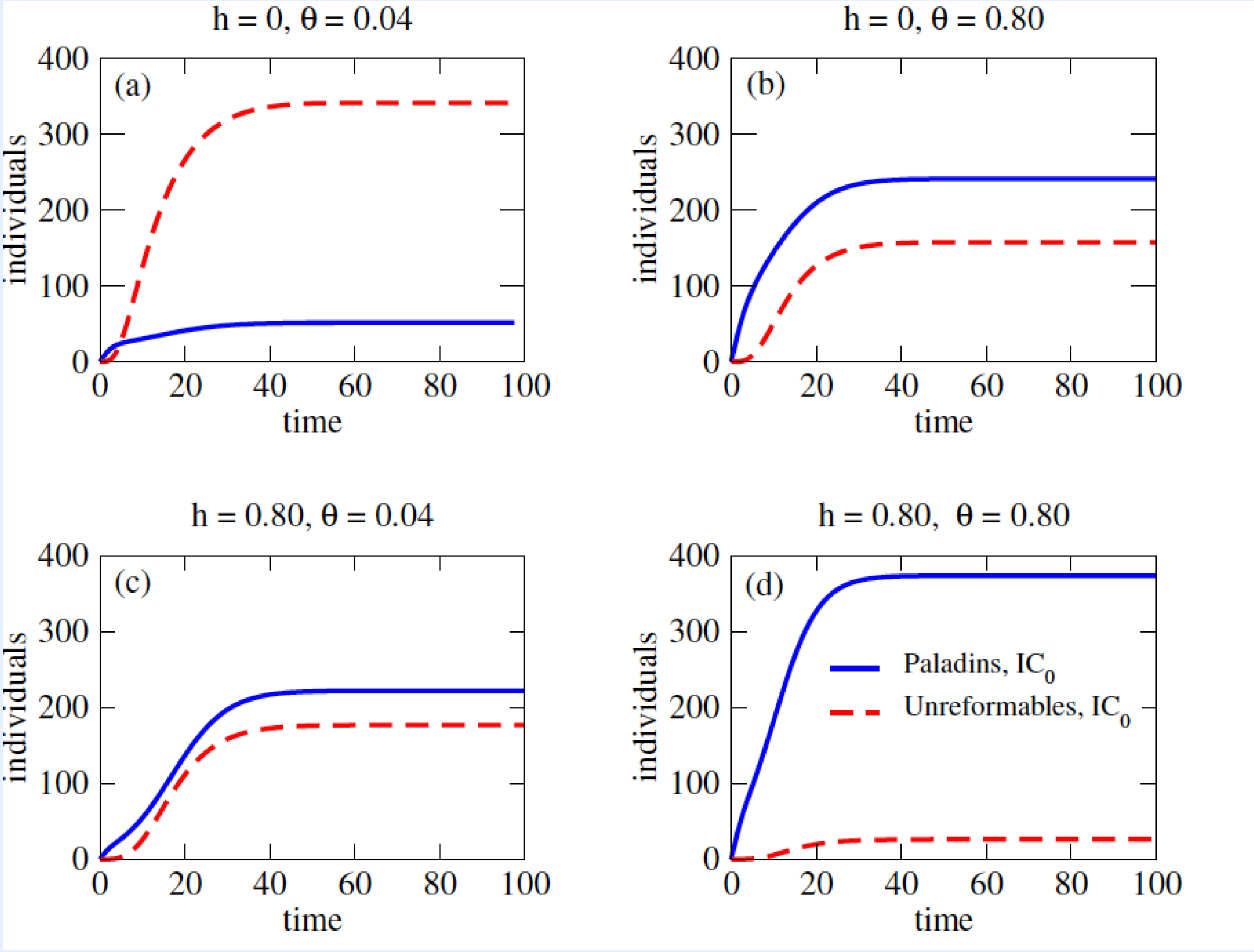
Truncate

$$\dot{N}_{\text{uncatch}} = (1-\alpha) \sum_{k_p=0}^{R-1} c_{k_u^*,k_p} N_{k_u^*,k_p}.$$

Relevant parameters

h	resources for rehabilitation
τ	time for rehabilitation
ϑ	punishment

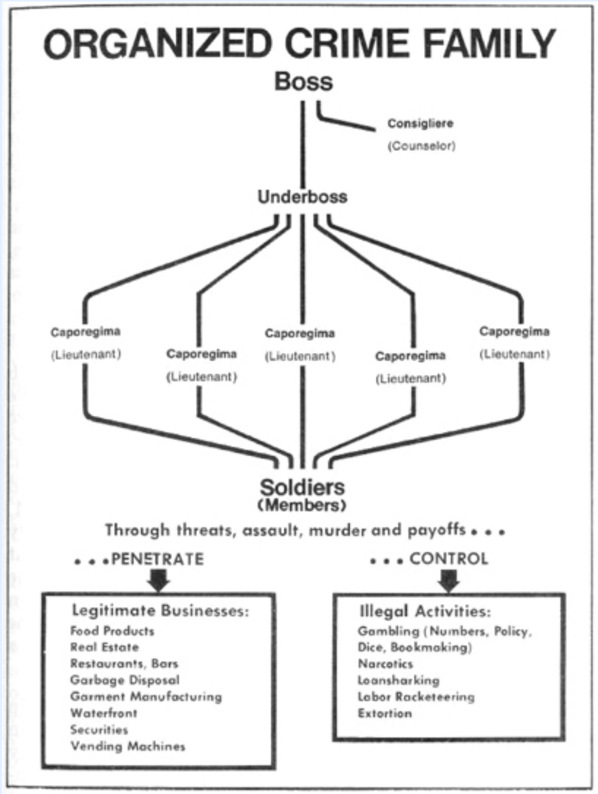
A qualitative match, IC0

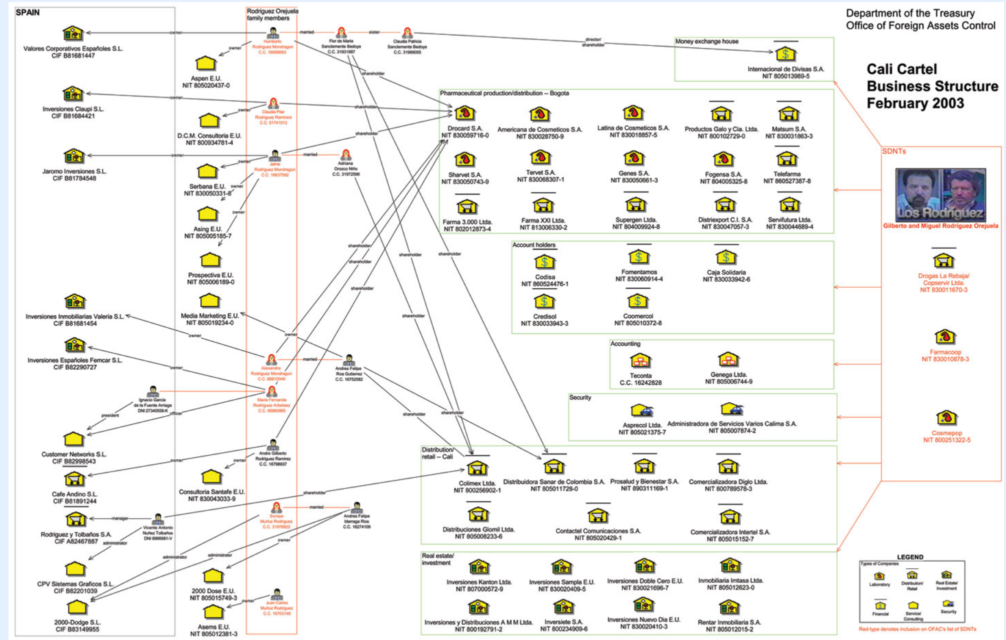


4. Criminal Networks

Organized crime: complex hierarchical structures

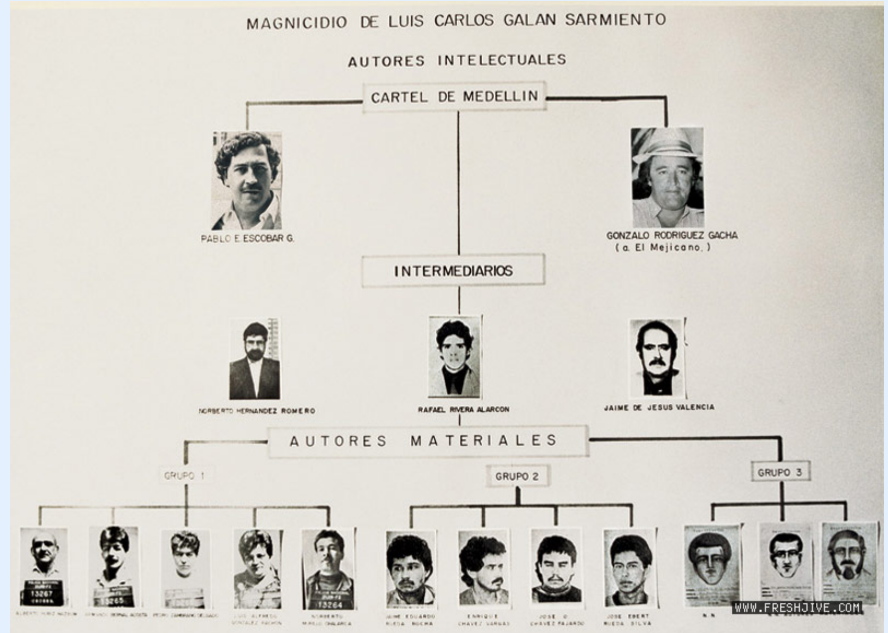
Pablo Escobar (Medellin) El Chapo (Sinaloa)
The Godfather (Cosa Nostra)

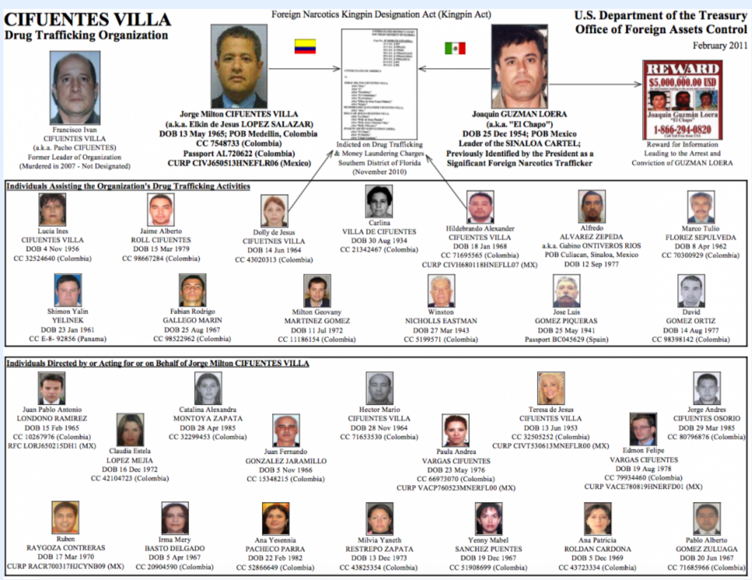




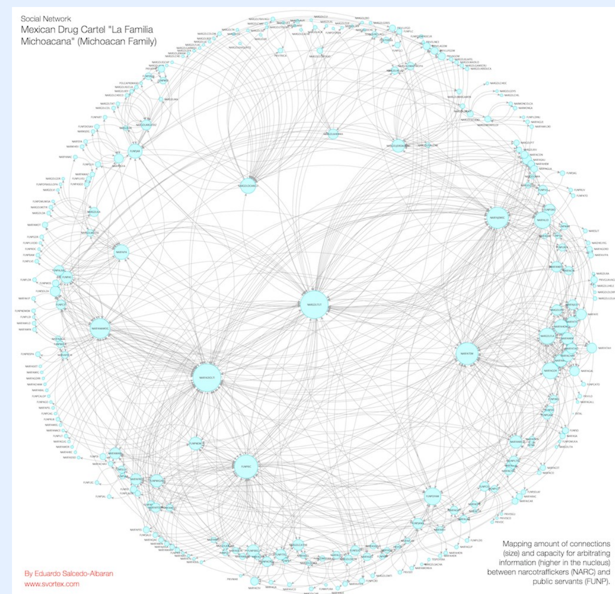
Cali, Sinaloa Mexico

Medellin, Colombia



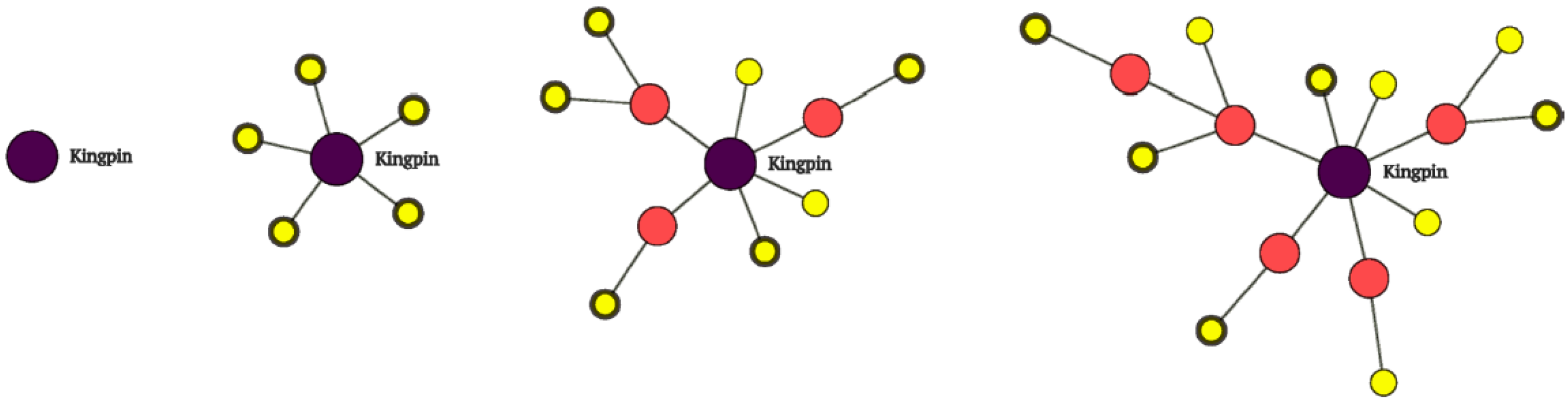


Cifuentes Villa,
Colombia



La familia Michoacana,
Mexico

How to stop the growth of a criminal network?



growth and containment

Network growth

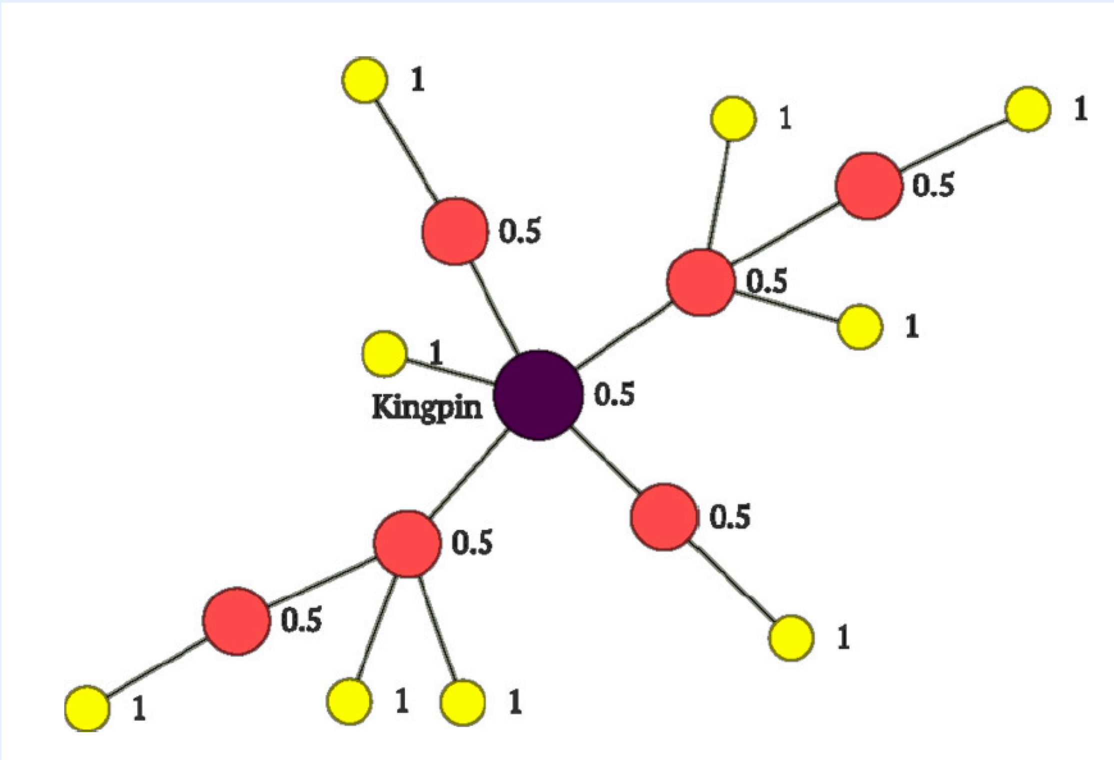
- Start with "kingpin"
- Grow hierarchically -- k recruits at each time step
- Preferential attachment at "street criminals"

Probability of attachment for each node w(j,t)
Distance from "street" σ(j,t)

$$w(j,t) = \frac{1}{\sigma(j,t) + a}$$

If node j on street σ(j,t)=0, w(j,t) = 1/a
If node j very far from street σ(j,t) >> a, w(j,t) ~ 0

Network growth



$$w(j,t) = \frac{1}{\sigma(j,t) + a}$$

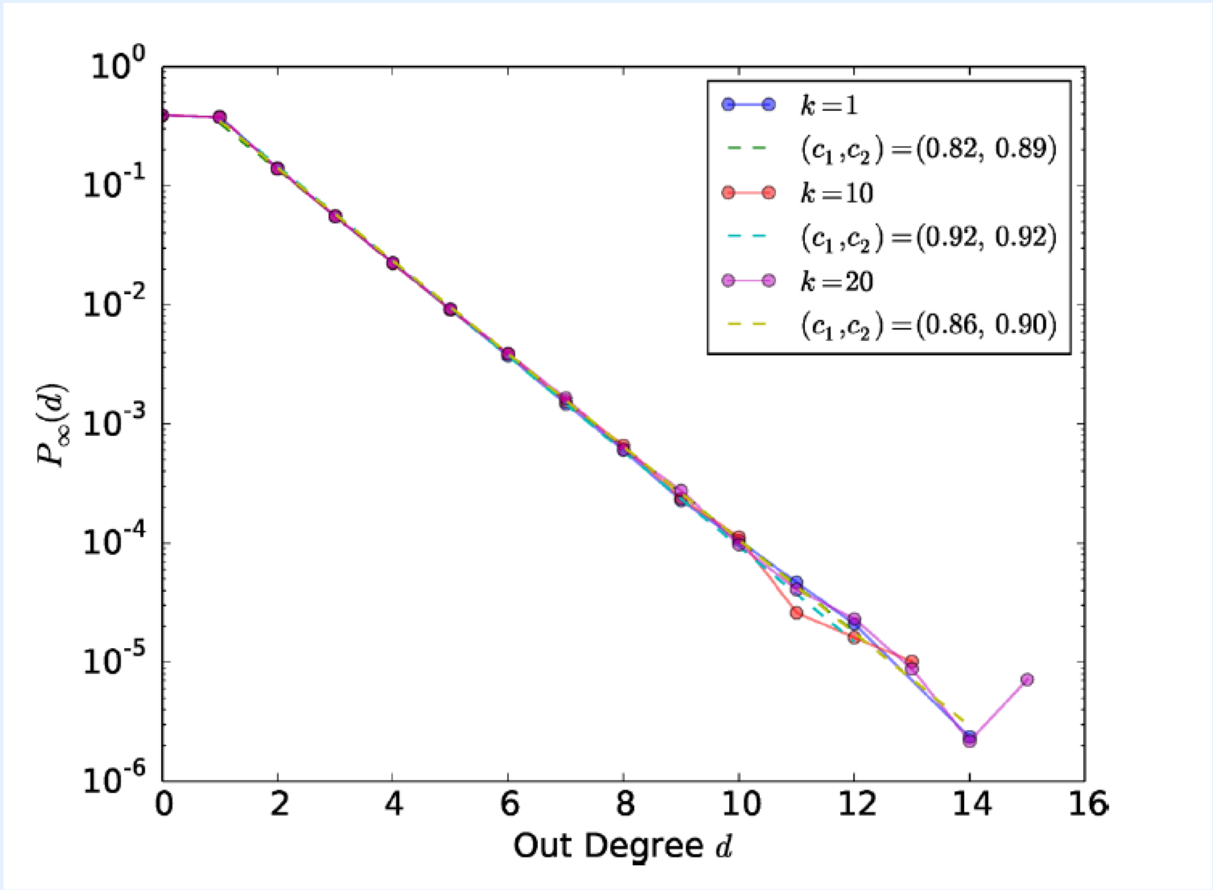
k=5, a=1, t=3
Street criminals
yellow

kingpin σ=1, w=1/2
street criminals σ=0, w=1

Node distribution

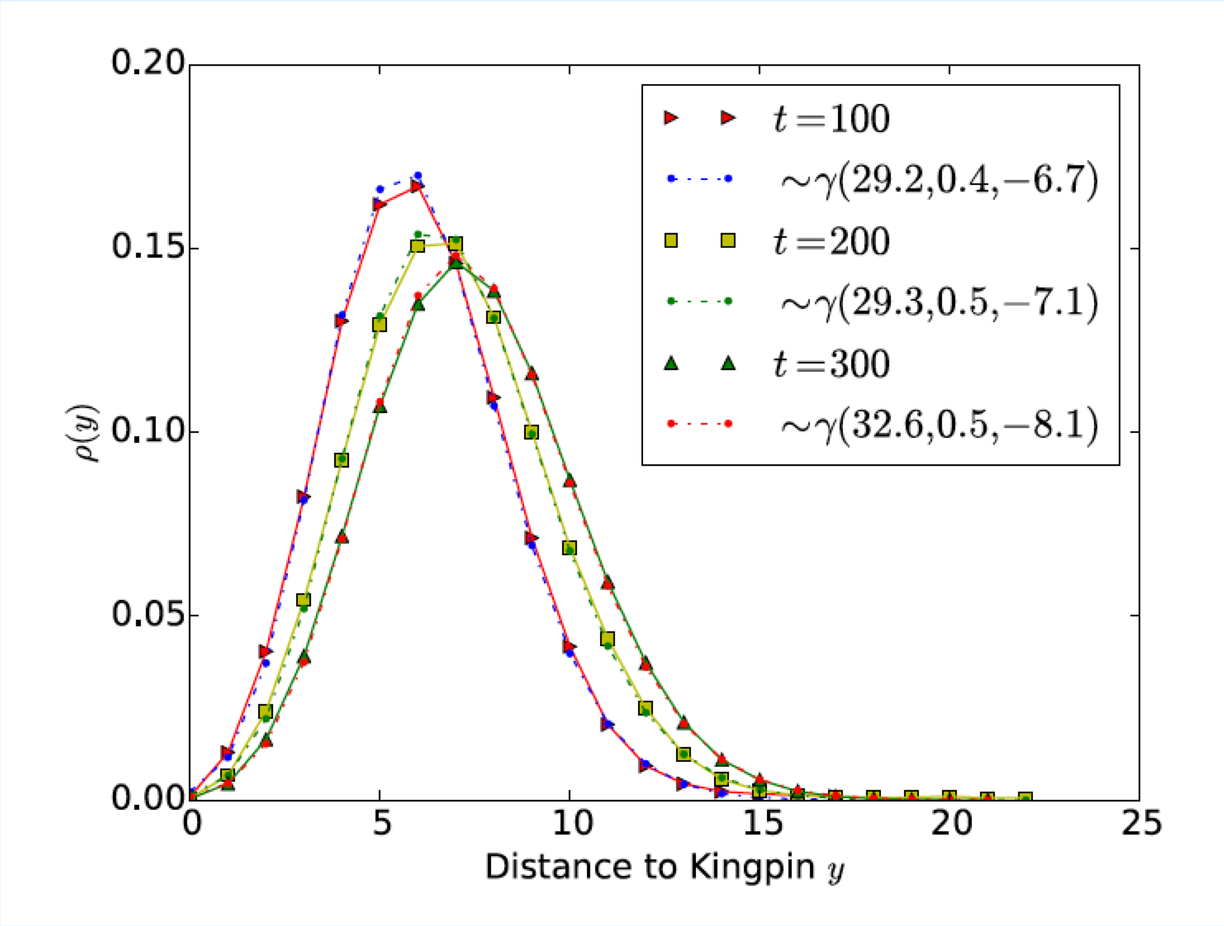
P(d,t) = probability any node has d underlings at time t

For t>>1, P(d,t) ~ c_1 exp[-c_2t]



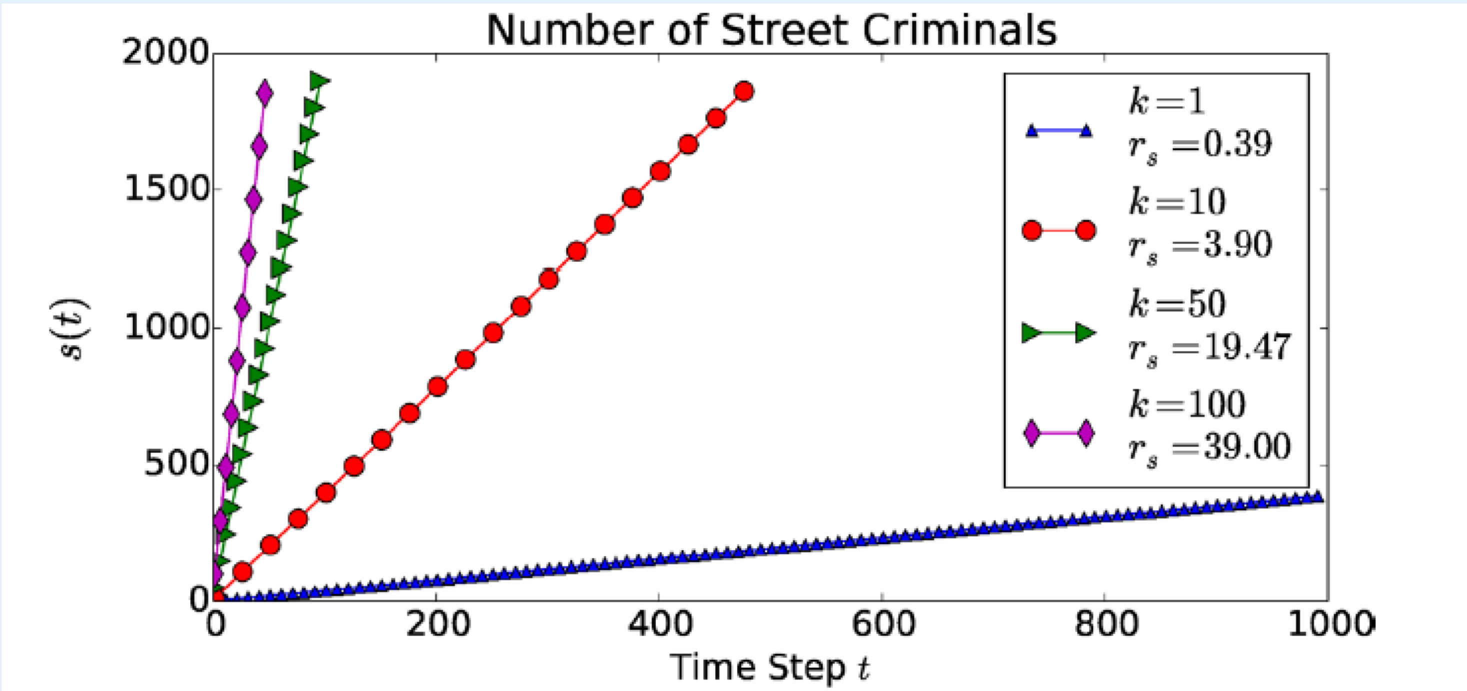
Distance from kingpin

On averages increases with time and follows a shifted Γ probability density



Number of street criminals s(t)

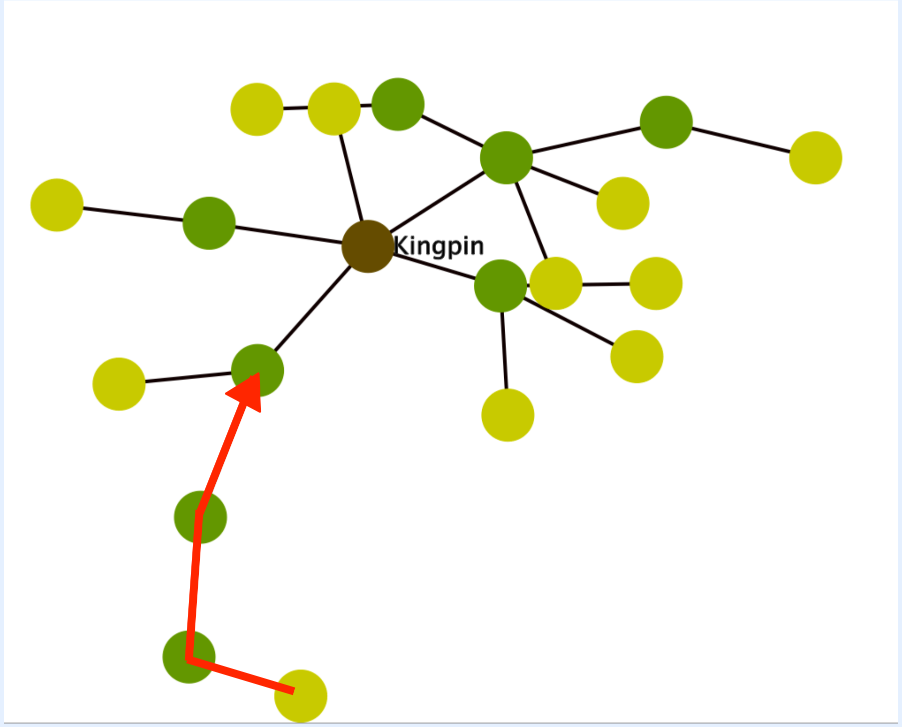
increases linearly with time, and with k



$$s(t+1) \simeq s(t) + \frac{\sum_{j \in \mathcal{C}(t)} w(j;t) - s(t)}{\sum_{j \in \mathcal{C}(t)} w(j;t)} k.$$

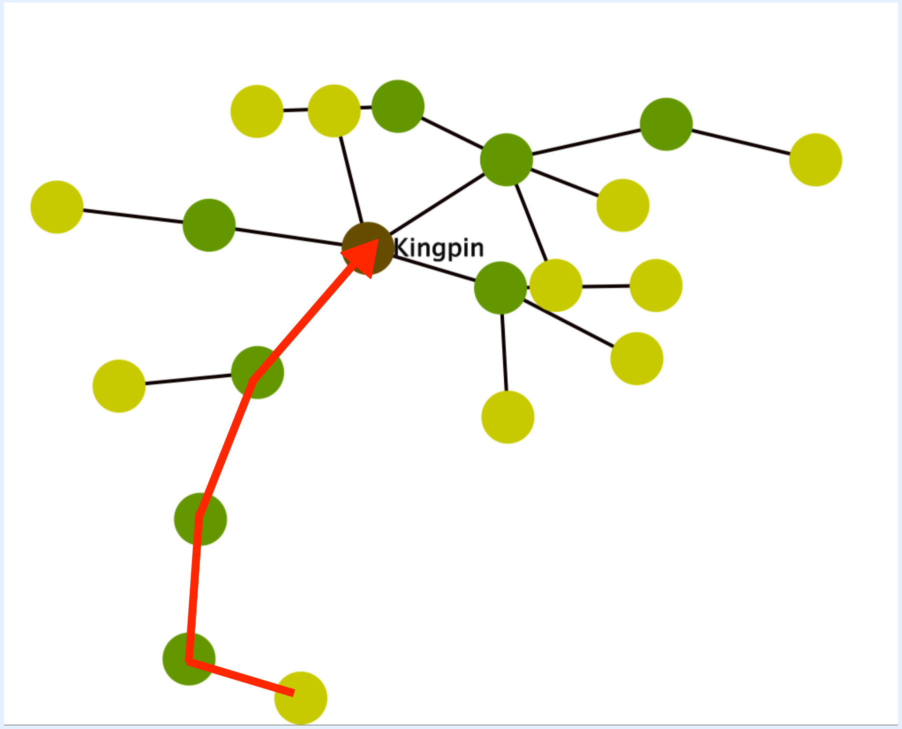
Network containment

- Police always start ``investigating" street criminals
  - Move up via self-avoiding random walks
    - Dark network – structure unknown
      - Network is still growing



Three strategies

1. Stop when kingpin is reached
2. Stop if criminal of distance q from street is reached
3. Stop after p steps



Three strategies

1. Stop when kingpin is reached
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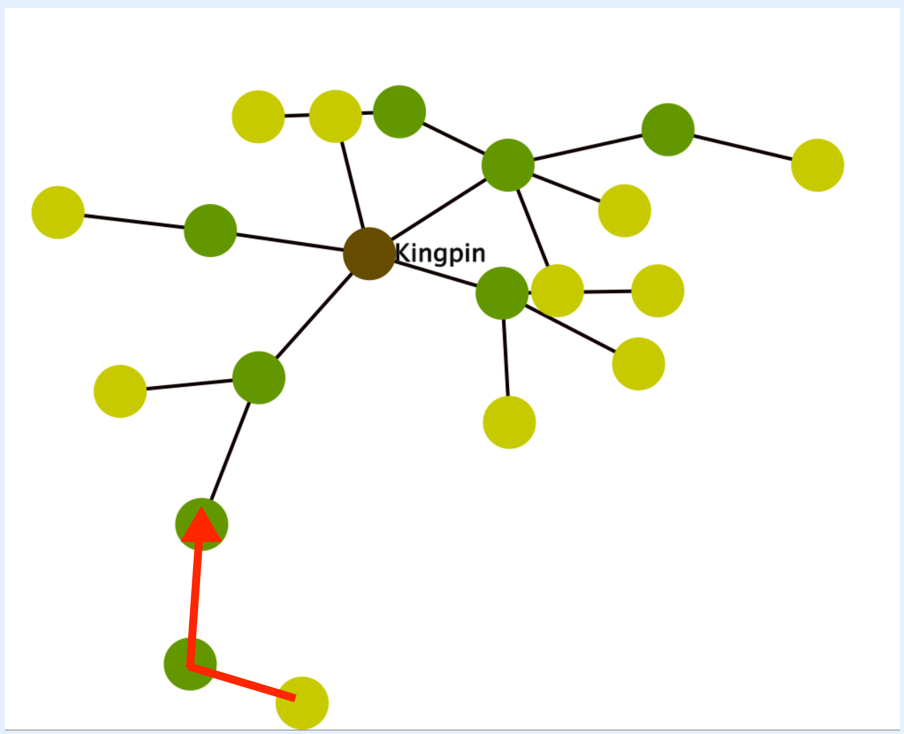
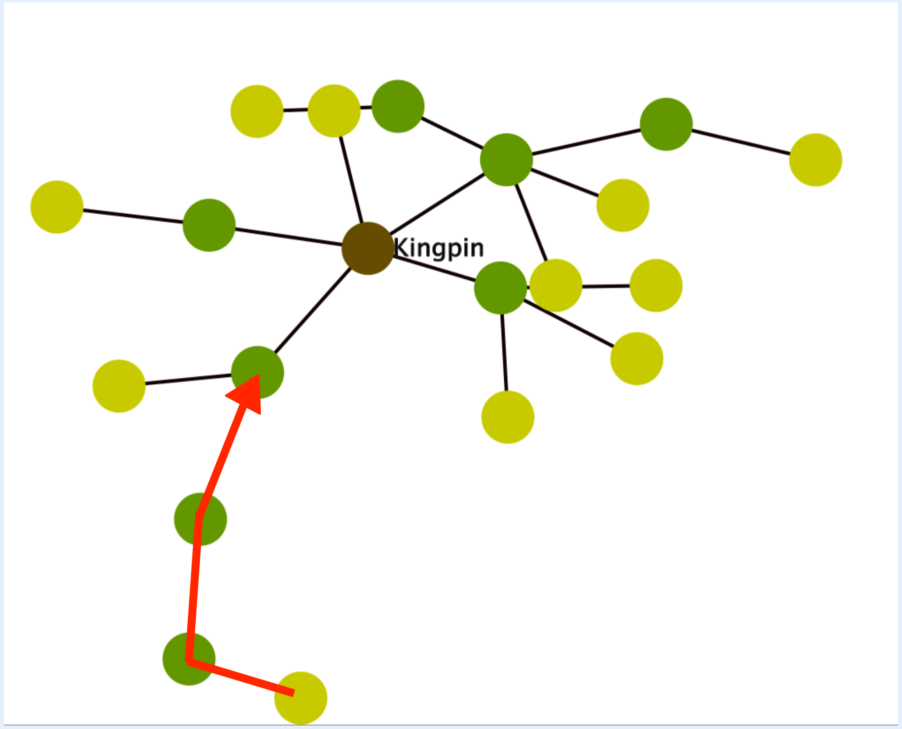


q=2

Three strategies

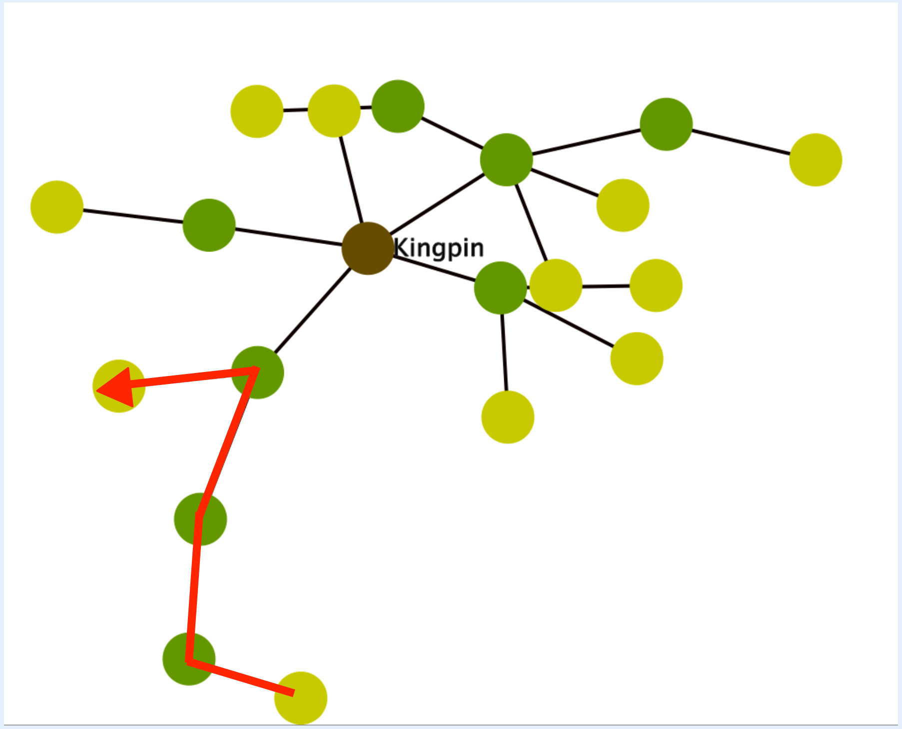
1. Stop when kingpin is reached
2. Stop if criminal of distance q from street is reached
3. Stop after p steps



p=3

Reaching dead-ends

If number of investigations until stopping is too large, higher risk of dead-ends



Eradication probability

2. Stop if criminal of distance q from street is reached
grow network until n=2000 people

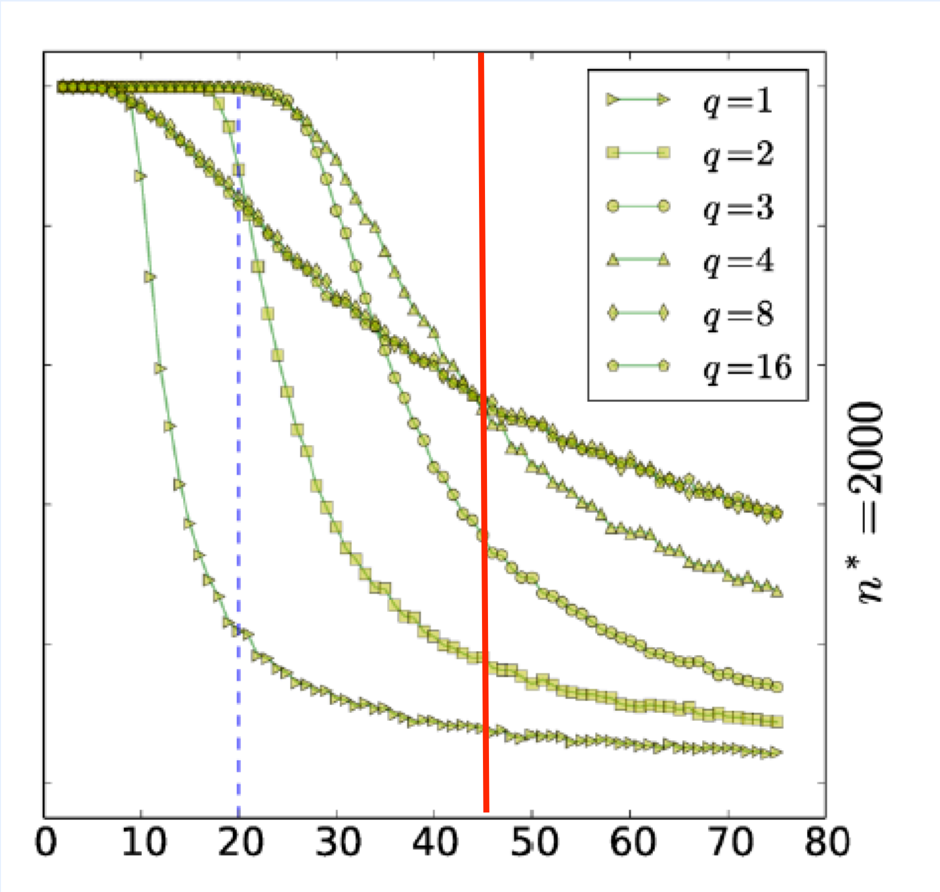
Eradication decreases for k

For small k~40, q intermediate is optimal

For large k>40, q large is optimal

P=1

P=0



k, network growth

Lessons?

If network is growing fast,
Aim to catch more senior criminals
more risky, more rewards

If network is growing slowly,
Aim to catch lower level criminals
less risky, reach kingpin slowly but surely

Summary and conclusions

Simplified models, mathematically tractable

Basic concepts from sociology, criminology,
anthropology analyzed through math tools

Some validation through data, experiments

Predictions, discussion, ideas,
more questions to be asked

Integration with data?

Thank you

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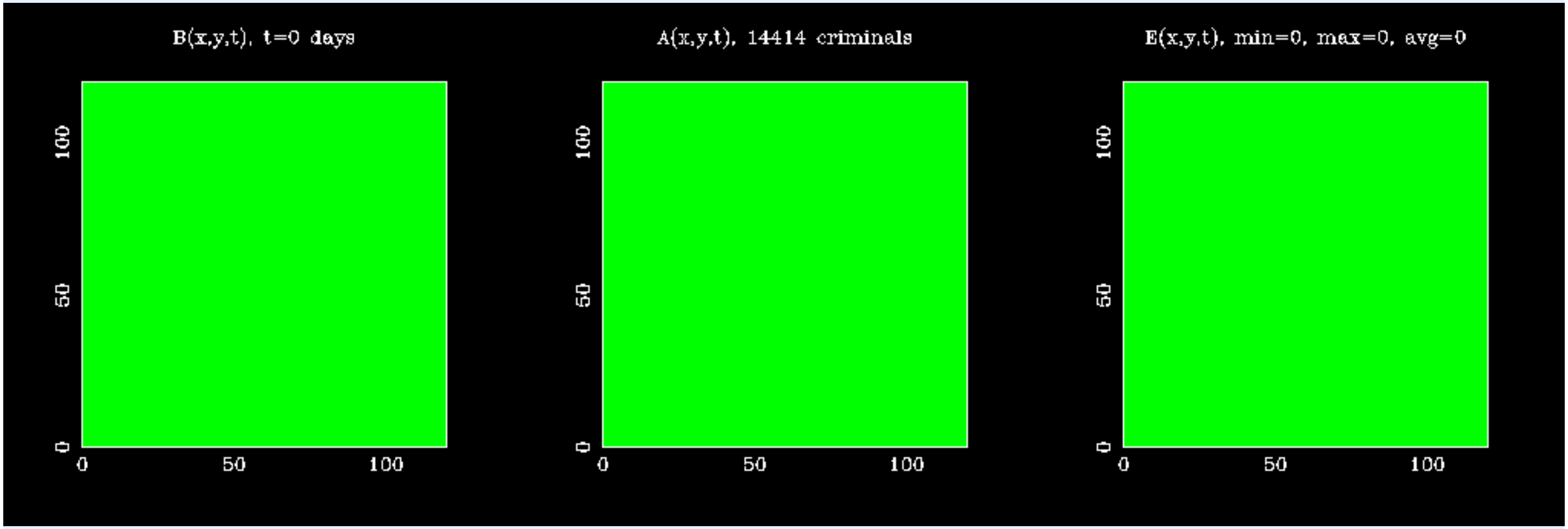
Bijan Berenji (UCLA)
P. Jeff Brantingham (UCLA)
George Tita (UCI)
Charlie Marsak (UCLA)
Milind Tambe (USC)

Discrete simulation



Attractiveness vs. Crimes

Best Response Analysis

Strategies are not chosen by imitation
but depend on payoff maximization

Once in a game, the first player will choose to
victimize only if his/her expected payoff is higher than 1.

Once victimized, the victim will choose whether
to report or not in order to improve his/her payoff odds.

Informants do not necessarily drive the system to utopia
(not always a best response)

But it is a best response for low enough punishment to the
informant