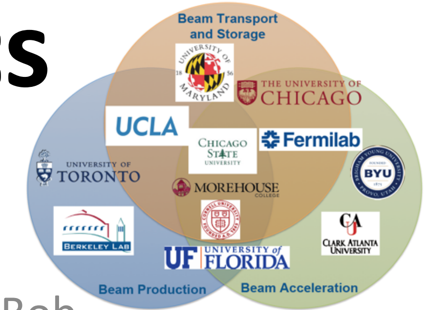
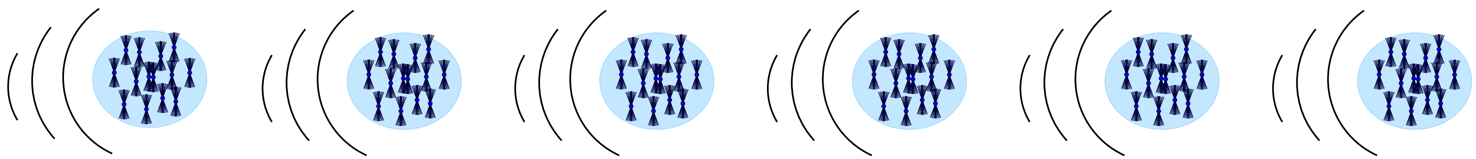
Mathematical questions about particle beam dynamics

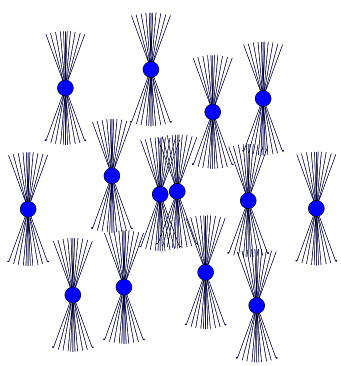


James P. Sethna, Center for Bright Beams
Archishman Raju, Sayan Chowdhury
Cornell University

(thanks to Jared Maxson, David Rubin, Jamie Rosenzweig, Bob Meller, Richard Rand, Georg Hoffstaetter…)

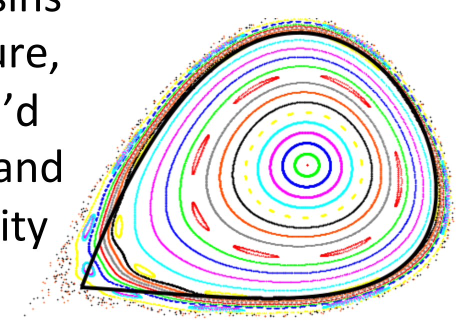
Bunch trains. Linear stability theory; Integro-differential delay eqns; Pseudospectra?





Particle interactions in a bunch: large-N, continuum vs. discrete

Single particle dynamics: chaos, resonances, basins Dynamic aperture, noise vs. Arnol'd diffusion, KAM and near-integrability



Translator. Amateur both in mathematics and in accelerator beam dynamics. (Statistical mechanics, dynamical systems, condensed matter physics…)

Archishman Raju, Choudhury, David Rubin, JPS

Chaos, KAM, and all that

Single particle dynamics

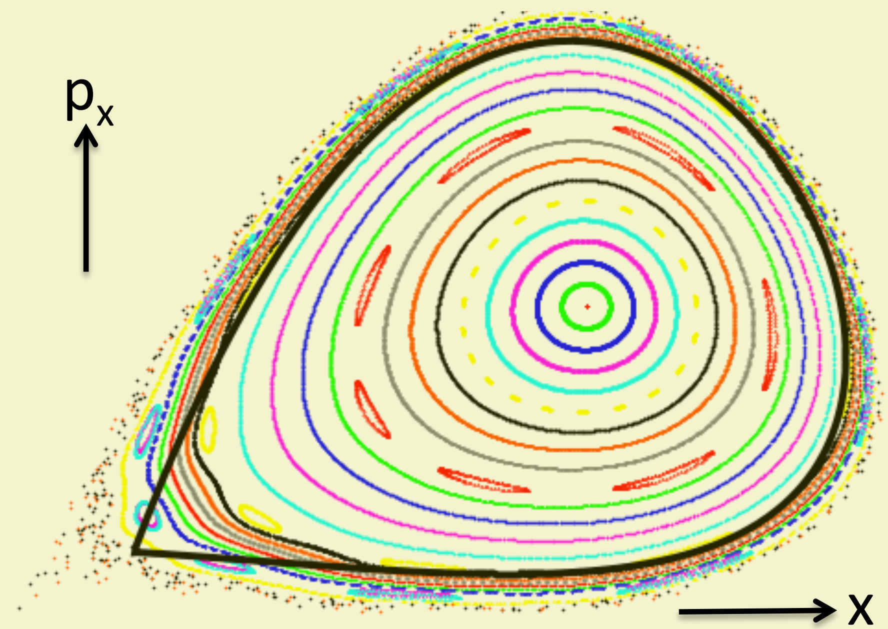
Time dependent Hamiltonian $\mathcal{H}(t)$

Liouville's theorem:

- Volume-preserving 6D Poincare map $\{x, p_x, y, p_y, z, p_z\}$
- *Emittance* ~ phase-space volume of bunch distribution

- KAM tori for irrational winding numbers (ratios of 'betatron tunes')
- Chaotic regions at rational 'resonances'
- Dynamic aperture = stability boundary.

Ignore interactions between particles

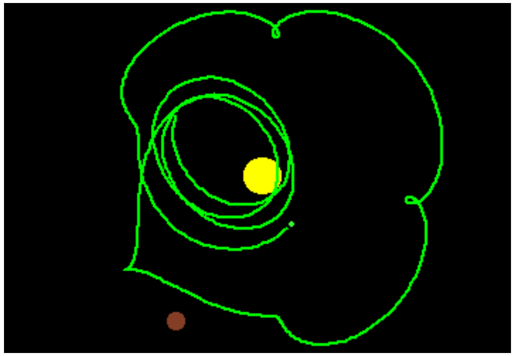


$\{x, p_x\}$ area preserving map. Black curve is dynamic aperture for 2nd BCH Floquet $\mathcal{H}_{eff}$. $\mathcal{H}_{eff}$ ignores chaos.

(Archishman Raju, Sayan Choudhury)

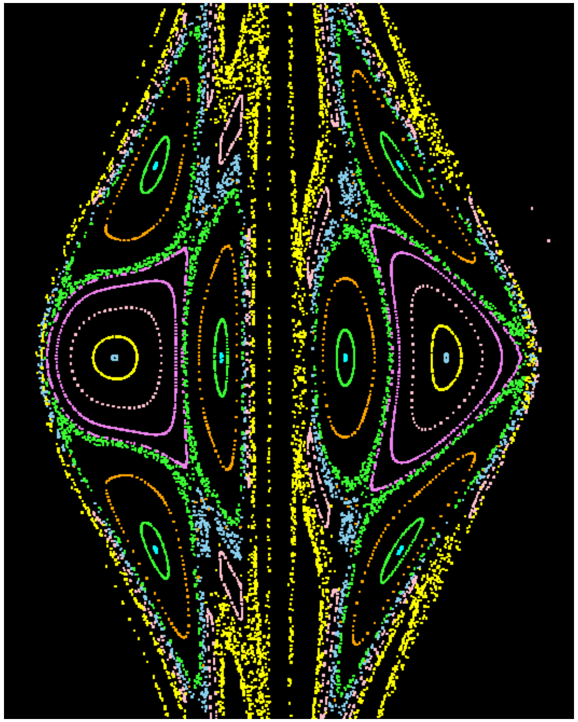
Analogy with planetary motion

Single particle dynamics



Why has the Earth not left
the Solar System?
The three-body problem.

Billions of years = Billions of orbits



- Solved by Kolmogorov, Arnol'd and Moser (KAM) for nearly integrable systems (small masses/nonlinearities)
- Proved, for sufficiently irrational winding numbers, that tori were preserved
- Solved 'small denominator' problem with superconvergent Newton's method plus smoothing
- Tori do not partition phase space for high dimensions: Arnol'd diffusion expected to connect all chaotic regions
- Dynamical aperture is probably the *complement of an open dense set of rational chaotic regions*

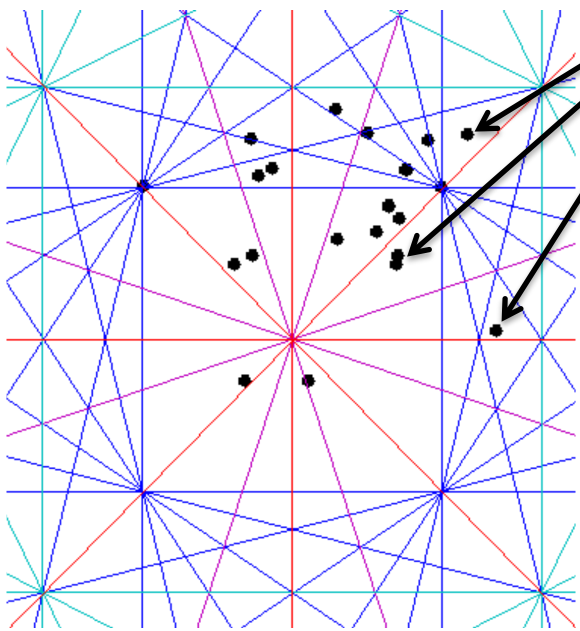
Can we mathematically justify accelerator physicist's simpler picture? Can we formalize ignoring the chaos?

Archishman Raju, Choudhury, David Rubin, JPS

Avoiding chaos

Single particle dynamics

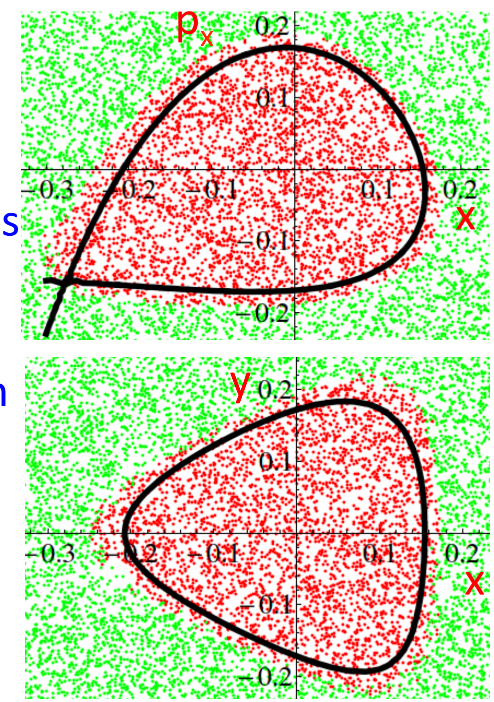
1) Avoiding rational winding numbers (Tune plot)



2) Tuning sextupoles to cancel low order resonances, or (IOTA) generate 'integrable accelerator'

3) Dynamic aperture for higher dimensions is quite compact. Particle loss, emittance growth mostly when resonances 'overlap'.

4D map, single effective H

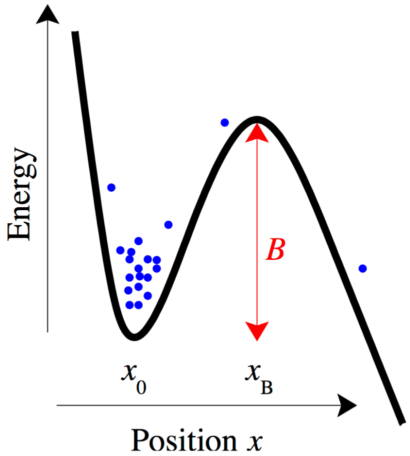


Can we tune nonlinear map to cancel all resonances (perhaps in a region)? C^∞ integrability? Fourier, Lie operator normal forms, BCH?

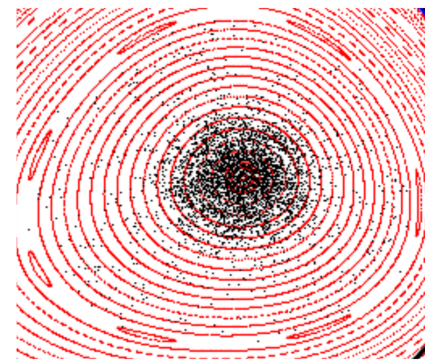
Archishman Raju, Choudhury, David Rubin, JPS

Adding noise: statistical mechanics

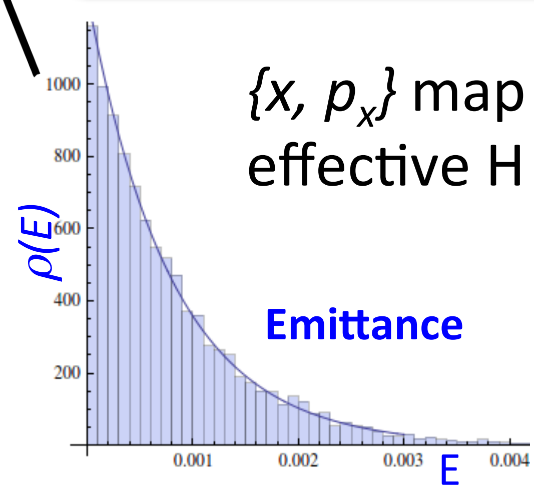
Single particle dynamics

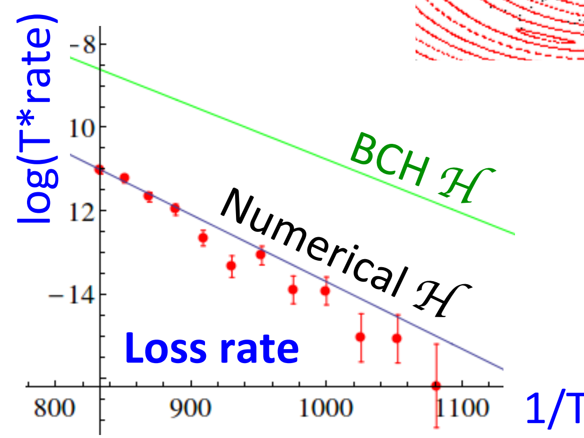


Chemical reactions	Accelerator bunch
Thermal noise	X-ray photon noise
Mean-square vibrations	Emittance
Reaction rate	Loss rate



$\{x, p_x\}$ map effective H





Higher dimensions (Rubin)
'Integrable' approximation
Separate temperatures for x, y, z
(correct for integrable H plus noise,
one temperature for each
conserved integral)

How does the distribution evolve in the limit of weak nonlinearity (Arnol'd diffusion) and weak noise? Asymptotics for emittances and loss rates.

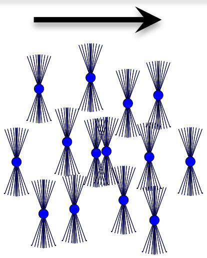
Jared Maxson, Bob Meller
(not me)

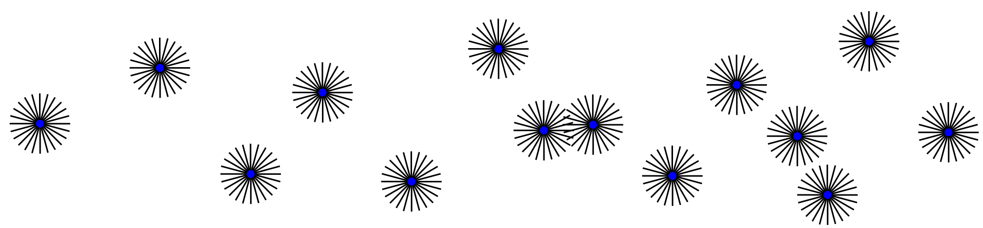
One bunch: Continuum interactions

Particle interactions inside a bunch

Up to and around $N \sim 10^{10}$ particles in a bunch.
Continuum limit (spacing << Debye length << packet size)?
Integro-differential PDEs, continuing algorithmic challenges

Lab frame
Lorentz
contraction

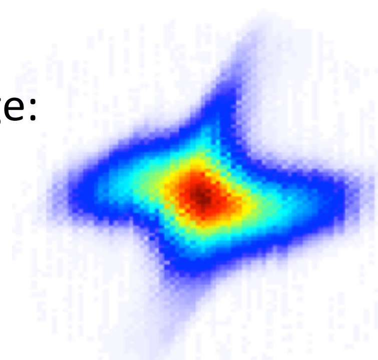




Bunch frame

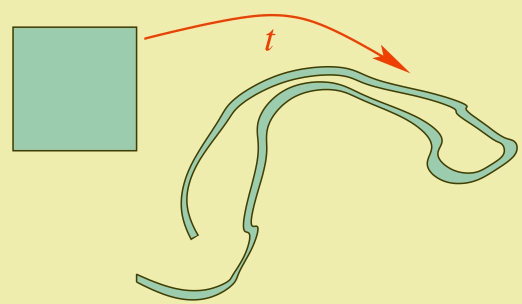
Coulomb interactions suppressed by relativity

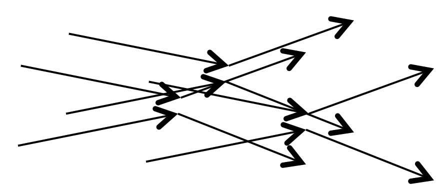
Space charge:
'average'
Coulomb
repulsion



6N-D emittance conserved
6D emittance retrievable,
if bunch stays elliptical

Practical emittance = $\Delta x\ \Delta p_x\ \Delta y\ \Delta p_y$ grows under space charge, other bunch-warping effects





- Intra-beam scattering (IBS, diffusion)
- Intra-bunch instabilities (head-tail, microwave, …)

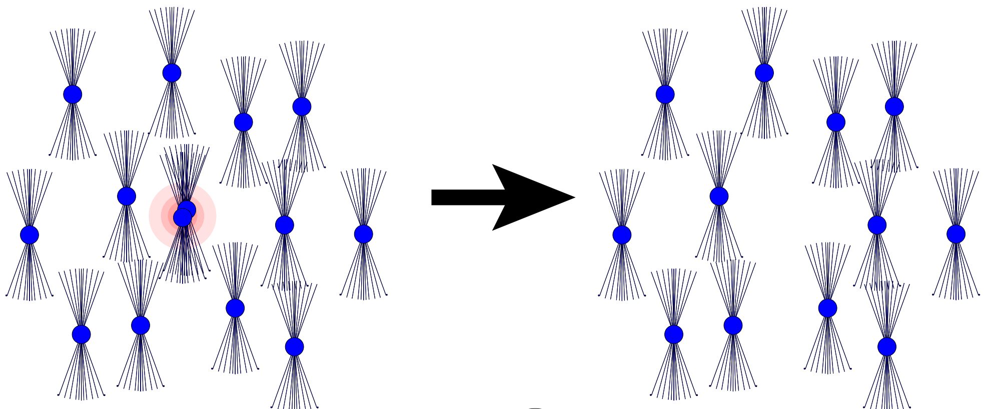
Jared Maxson, Jamie Rosenzweig
(not me)

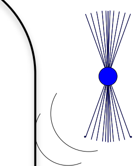
One bunch: Discreteness effects

Particle interactions inside a bunch

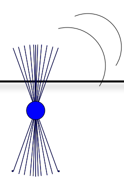
Binary collisions cause particle loss, increase emittance

Large angle scattering (vs. IBS)
'Touschek effect': particle loss

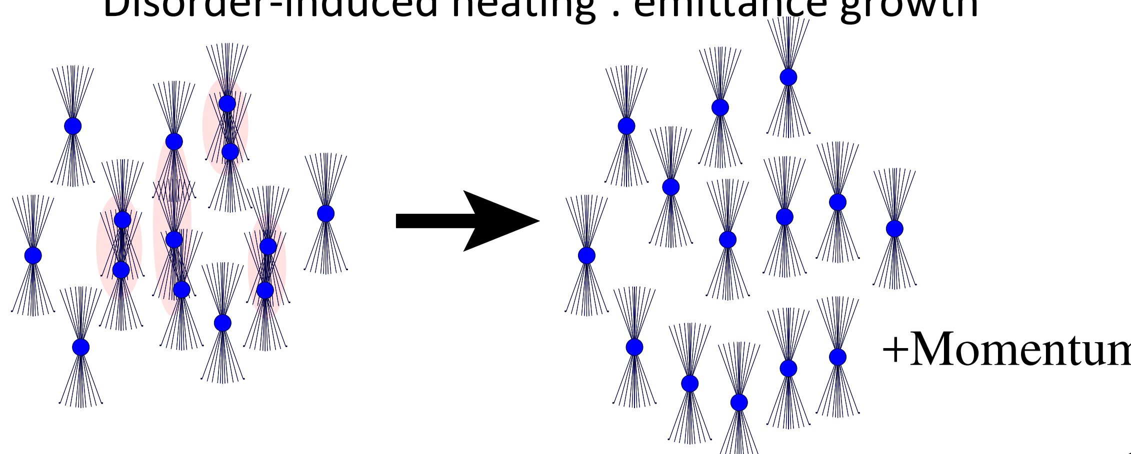




Can one prove convergence to large-N limit [continuum + binary collisions]? Homogenization? Analogy to random walk [CLT + extreme value statistics]



'Disorder-induced heating': emittance growth

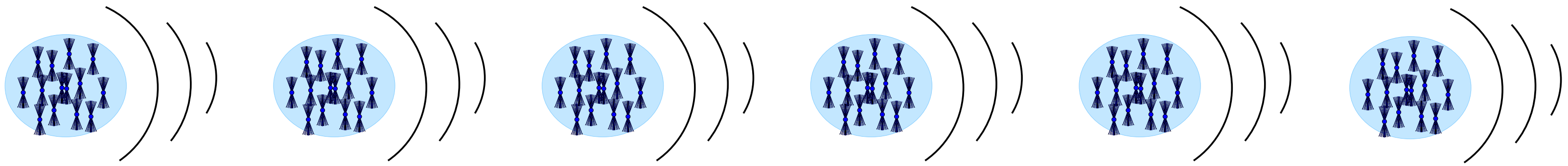


Also microbunching (longitudinal space charge instability)

Richard Rand, Bob Meller
(not me)

Linear stability theory

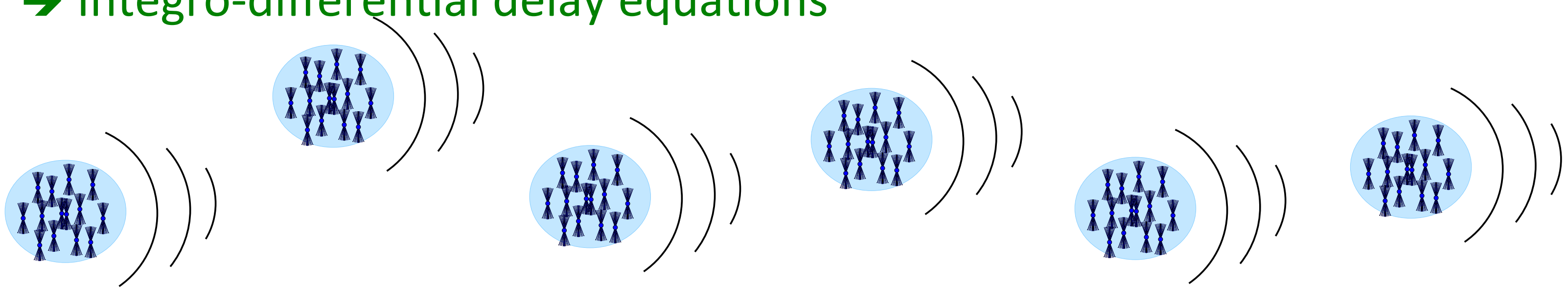
Bunch train dynamics



- Bunches lose energy via X-ray radiation as they bend (especially electrons in rings).
- Bunches gain energy in resonant cavities.

Linear multi-bunch stability theory, bunch ⬄ environment. (Nonlinear stability too…)

- Radiation from one bunch interacts with other bunches
  - ➔ Wake field instabilities
- Walls, 'Higher-order' resonances in resonant cavities excited by bunches
  - ➔ Beam breakup instability, resistive wall instability
- 'Plasma' scattering from free electrons and residual gas ions
  - ➔ Fast ion instability, …
- Time-delayed interaction via radiation, resonances, residual gas
  - ➔ Integro-differential delay equations



Nick Trefethen, Hoffstaetter
(not me)

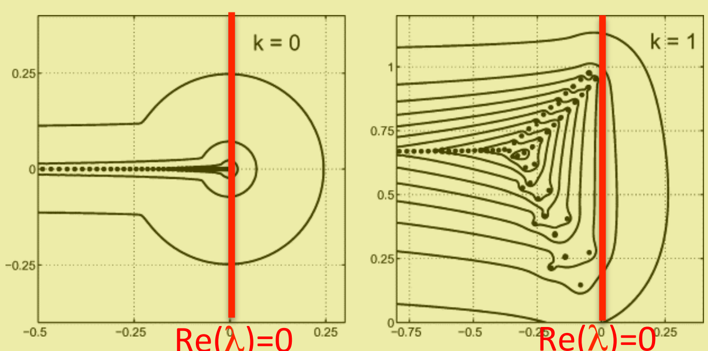
Pseudospectra

Bunch train dynamics

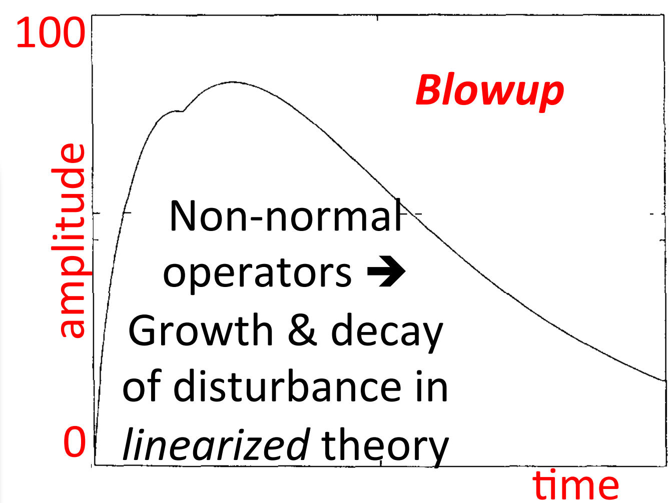
Are accelerator beam instabilities like pipe turbulence? (2-stream instability for protons)

Flow of water in a pipe is turbulent for (inertia/viscous) Re ≥ 4000

Pipe flow is linearly stable, all Re



Dots = eigenvalues
lines = pseudospectra

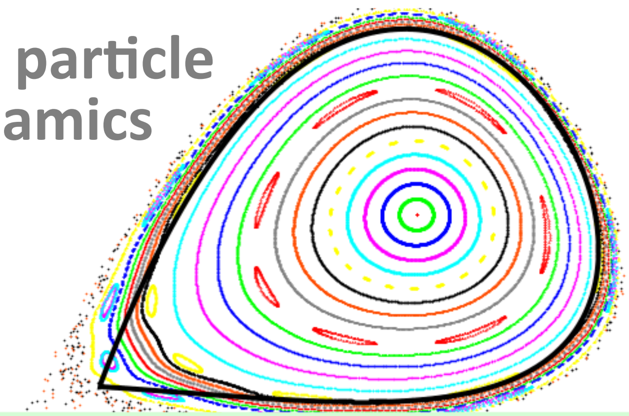


Pseudospectra = Spectra of *nearby* linearized dynamics. (*Structural susceptibility*)

Stable for infinite bunch train on linac? Pseudospectra to test if practical stability = linearized stability?

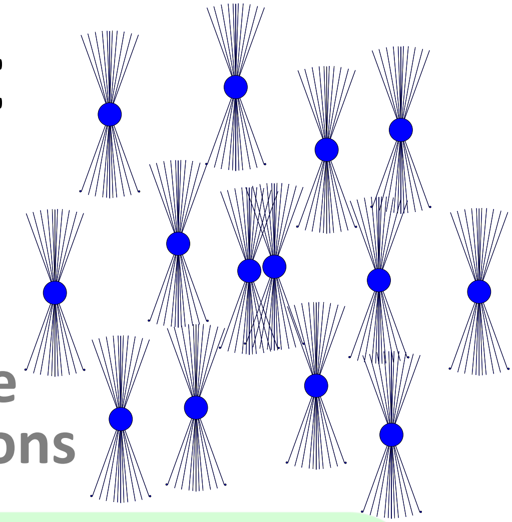
Mathematical questions about particle beam dynamics

Single particle dynamics



Does nonlinear map tuned to cancel all resonances converge to effective integrable system, justifying the simple dynamic aperture & 3 temperature models? Adding noise and Arnol'd diffusion, can we derive asymptotics for emittance and loss rate?

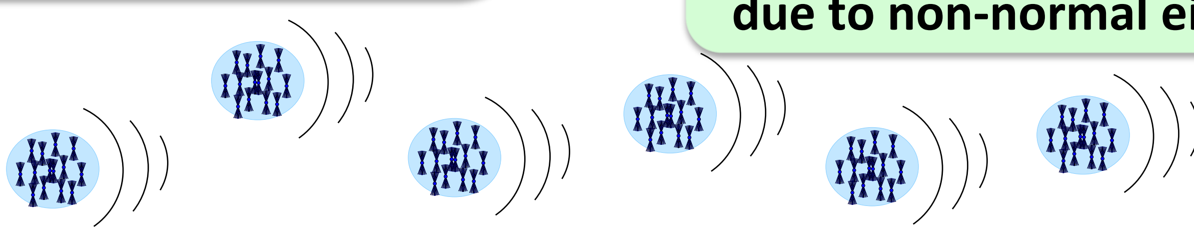
Particle interactions



Can one derive large-N limit of interacting Coulomb particles, effective for both continuum and particle-level effects?

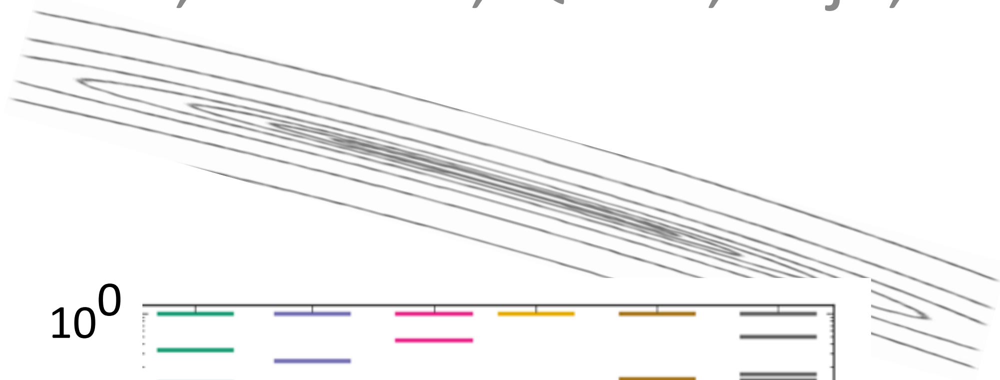
Is linear stability theory safe? Pseudospectral tests for blowup due to non-normal eigenvectors

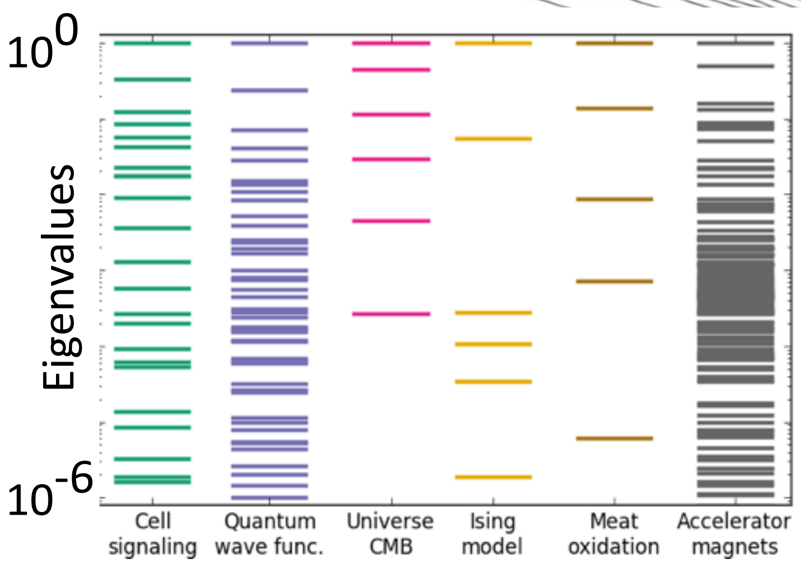
Bunch interactions



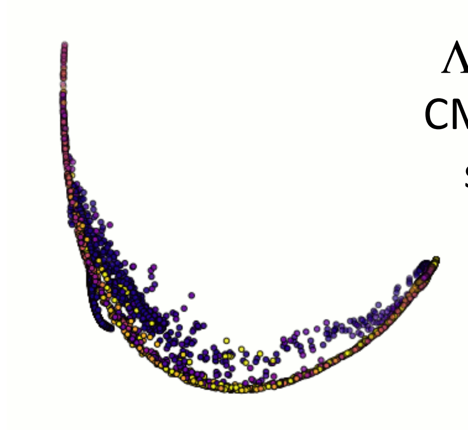
Sloppy models and multiparameter emittance optimization

Rubin, Bazarov, Quinn, Raju, Clement, Bergan, He, Bartnik





Multiparameter systems sensitive to only a few stiff eigenvector 'knobs', other directions sloppy



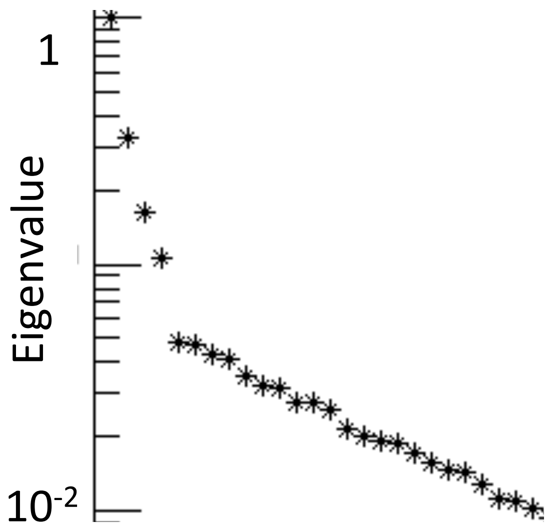
ΛCDM model CMB spectra of space of all universes

2D Ising model

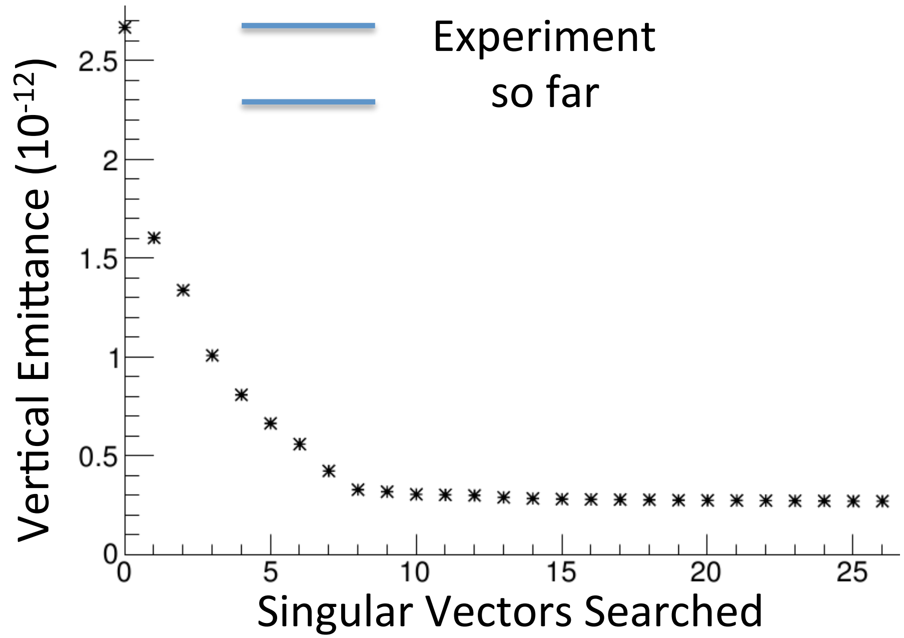
Model manifold in behavior space is a hyperribbon: thin directions from sloppy parameter combinations.
Information geometry, interpolation theory, & renormalization group

Sloppy models and multiparameter emittance optimization

Rubin, Bazarov, Quinn, Raju, Clement, Bergan, He, Bartnik



CESR storage ring calculated emittance depends mostly on only a few combinations of 85 corrector magnets



Searching stiff directions of energy (after regular tuning protocol) leads to rapid emittance decrease in simulation, not yet in experiment