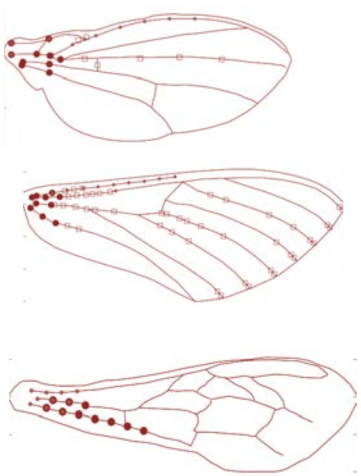
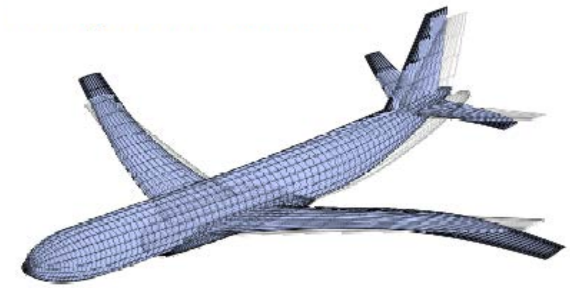
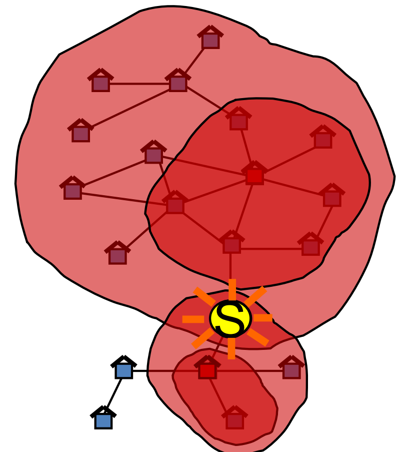
Analytical and Empirical Tools for Nonlinear Network Observability in Autonomous Systems

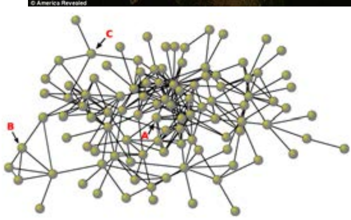









Prof. Kristi A. Morgansen



William E. Boeing Department of Aeronautics & Astronautics
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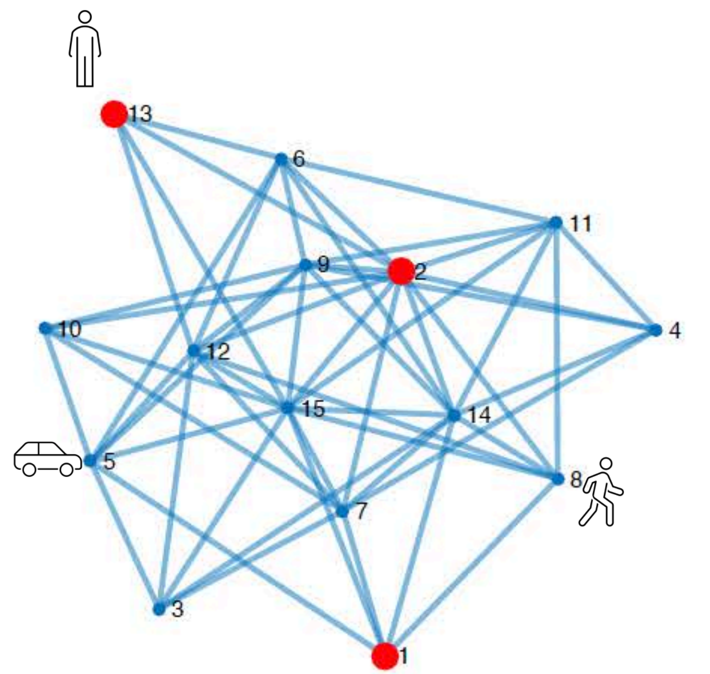
Nonlinear Dynamics and Control Lab

- Autonomous Vehicles
- Bioinspired Systems
- Shape Actuated Dynamics
- Integrated Sensing and Actuation

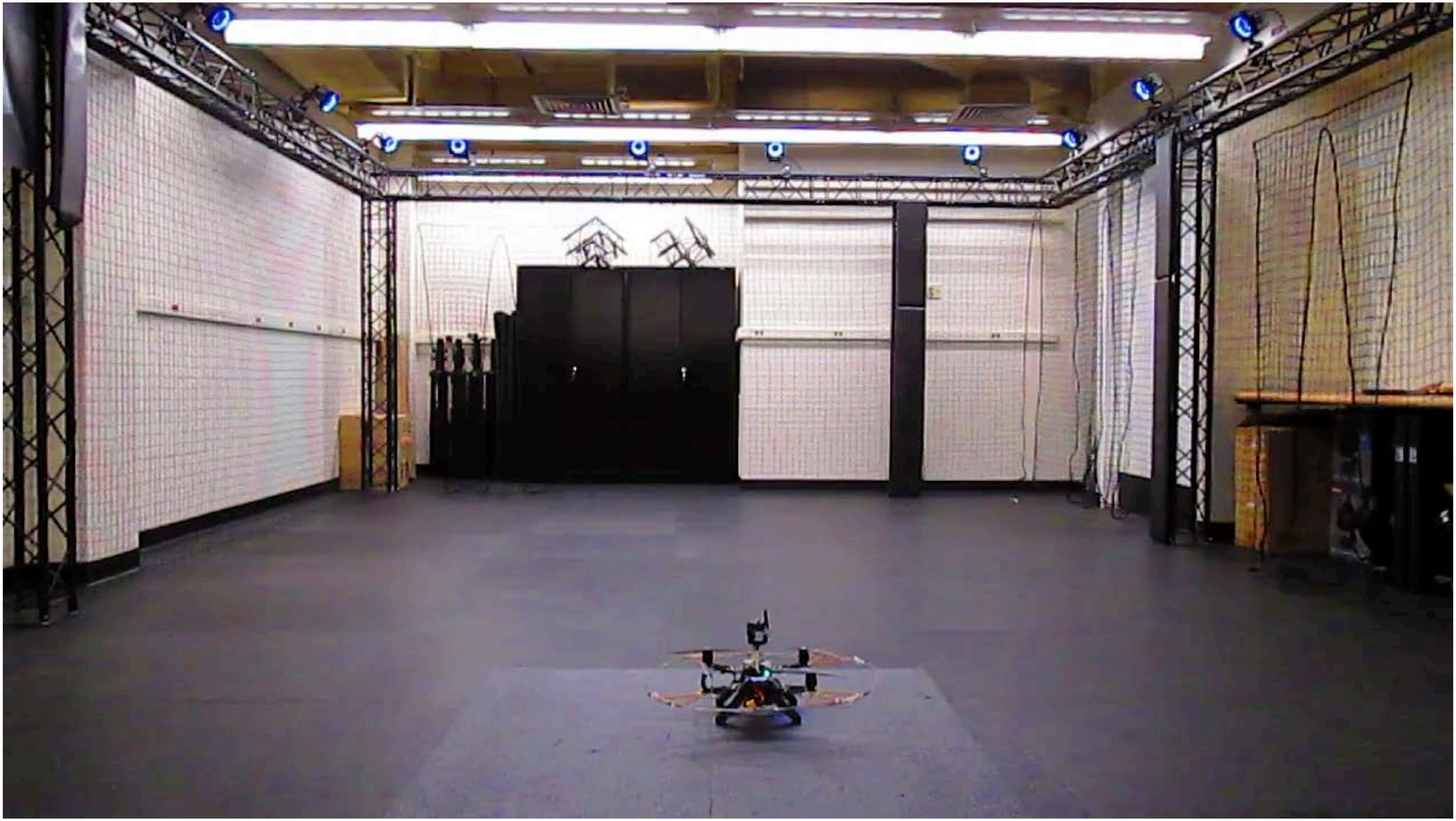


Key contributors for the work presented today

Nathan Powel, Atiye Alaeddini, Trevor Avant, Brian Hinson

Collaboration with Thomas Kunz,
Crystal Schaaf, Alan Strahler, BU

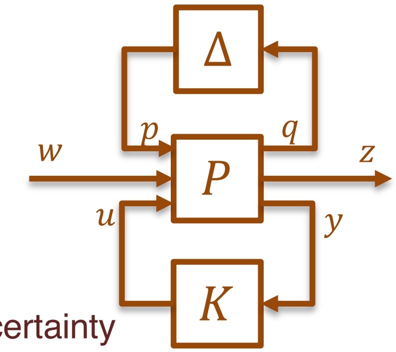
Remote Sensing

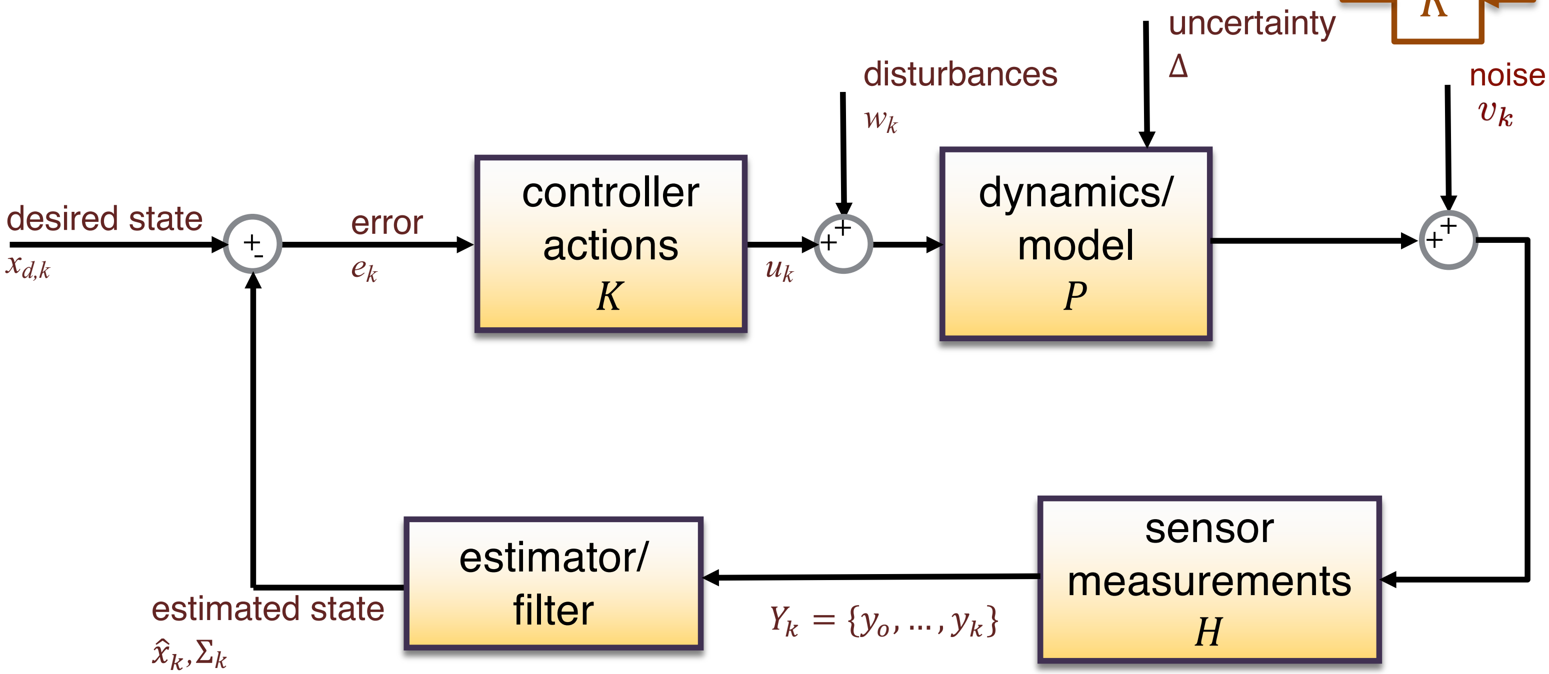


Dynamics, Control, Sensing, Robustness

Robustness determination

- Control design as function of Δ
- Sensing/Estimation design as function of Σ_k





$\hat{x}_k, \Sigma_k \leftarrow \boxed{\hat{G} = \{\hat{P}, \hat{H}, K\}} \leftarrow Y_k$

$Q_k = E[w_k w_k']$

$R_k = E[v_k v_k']$

Error covariance is bounded by observability

$$\mathcal{P} = \Sigma^{-1} \preccurlyeq F \preccurlyeq \underline{\sigma}(R^{-1}) f(W_o)$$

Agility and localization in biological systems

Agility and sensing in biological systems far exceed engineering capabilities.

Owls use head movements to maximize auditory signals to locate a single mouse. [photo: wikipedia]

Surging/casting behaviour in plume tracking of insects with chemosensing to localize mate or food. [photo: wikipedia]

Halteres in flies to provide gyroscopic information. [photo:wikipedia]

Movement is a necessary and essential element of effective biological sensing (active sensing)

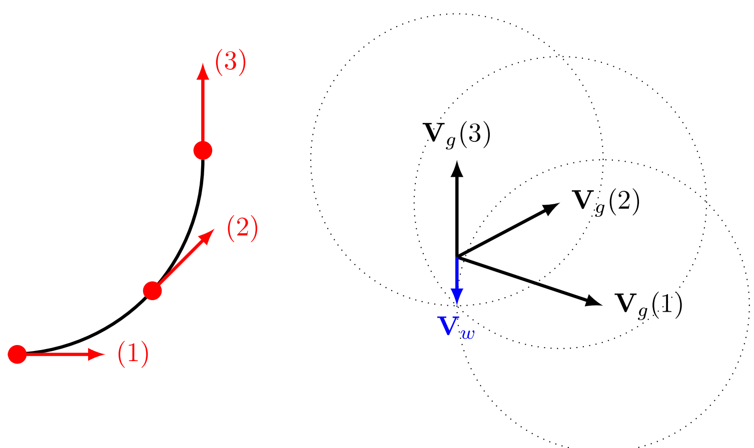
Active sensing in engineered systems: Wind-finding

Aerosonde

Aerosonde sensors

- GPS - inertial position
- Differential GPS - ground speed
- Pitot probe - airspeed

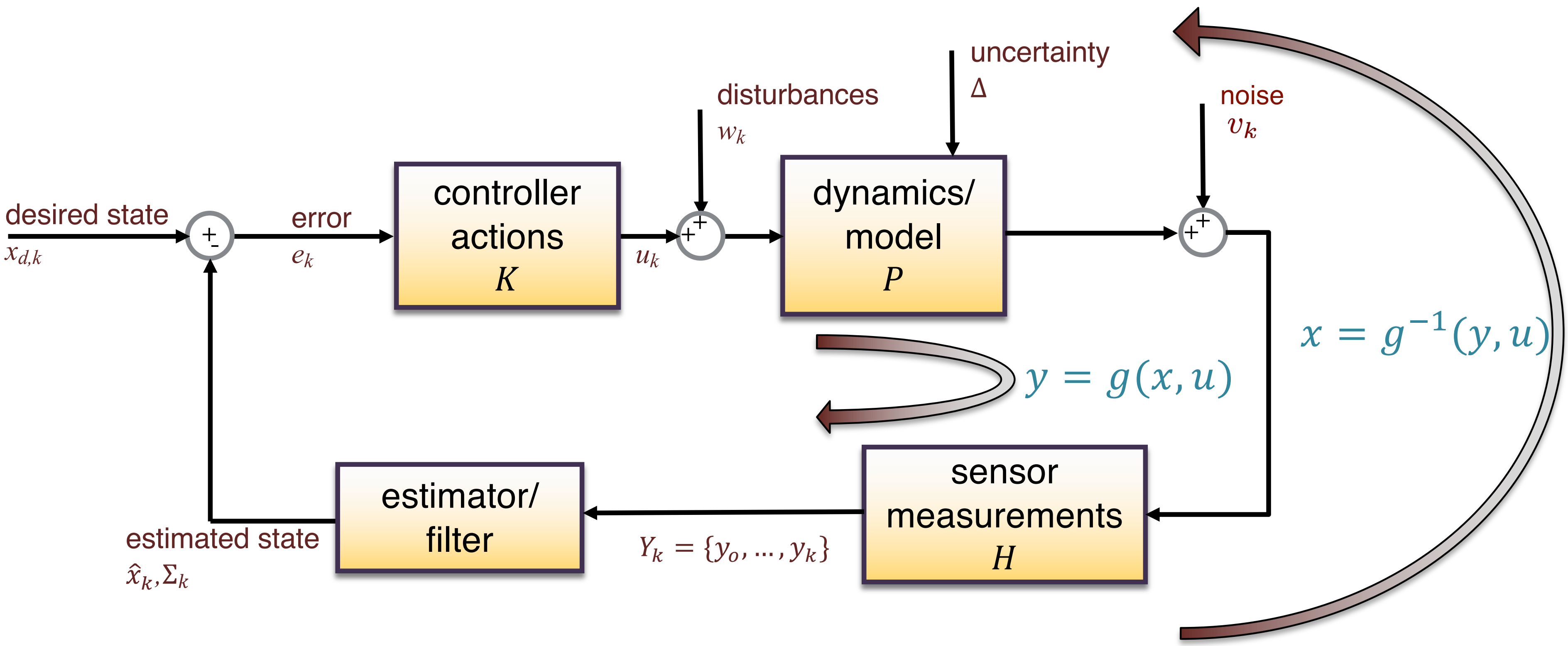
- No heading measurement → direct estimation of wind speed and direction not possible
- S-turns used to sample wind at different directions, thus making wind "observable"



Planned Path   Wind Vector Triangulation

Observability

Observability (informal): Given a set of measurements over a finite time interval, can quantities of interest be reconstructed?



Observability

Linear systems

$$\begin{aligned} \dot{x} &= Ax + Bu, \;\; x \in \mathcal{R}^n \\ y &= Cx, \;\; y \in \mathcal{R}^p \end{aligned}$$

$$\begin{bmatrix} y \\ \\ \\ \end{bmatrix} = \begin{bmatrix} C \\ \\ \\ \end{bmatrix} x + \begin{bmatrix} 0 \\ \\ \\ \end{bmatrix}$$

Known and differentiable measurements

To reconstruct all states from measurement, this matrix must be full rank.

Assuming differentiable and known controls, subtract from LHS.

Observability in linear systems does not depend on the control inputs

Observability

Nonlinear systems

Control affine system

$$\dot{x}(t) = f_0(x) + \sum_{i=1}^{m} f_i(x)u_i(t) = f(x,u,t),\ x \in \mathcal{R}^n$$

$$y(t) = h(x),\ h(x) \in \mathcal{R}$$

Output derivatives with nonzero control

$$\begin{bmatrix} y \\ y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(n)} \end{bmatrix} = \begin{bmatrix} h(x) \\ \frac{\partial h}{\partial x}\dot{x} \\ \frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\dot{x}\right)\dot{x} \\ \vdots \\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(\cdots)\right)\dot{x} \end{bmatrix} = \begin{bmatrix} L_f^0 h(x) \\ L_f^1 h(x) \\ L_f^2 h(x) \\ \vdots \\ L_f^n h(x) \end{bmatrix} = g(x,u) \neq \begin{bmatrix} L_{f_0}^0 h(x) \\ L_{f_0}^1 h(x) \\ L_{f_0}^2 h(x) \\ \vdots \\ L_{f_0}^n h(x) \end{bmatrix}$$

Known and differentiable measurements

To reconstruct all states from measurement, this relation must be invertible.

Relation between outputs and controls may be highly coupled with the control terms.

Observability

Nonlinear observability

To determine an inverse relation between output functions and states, employ a local approximation via the inverse function theorem:

Theorem (Observability Rank Condition for Analytic Systems)
The nonlinear system is locally observable at x_0 iff $d\mathbf{g} = \frac{\partial}{\partial \mathbf{x}}\mathbf{g}$ has rank n at x_0, and the first n-1 derivatives determine local observability.

What are characteristics of admissible controls that ensure or maximize full state observability?

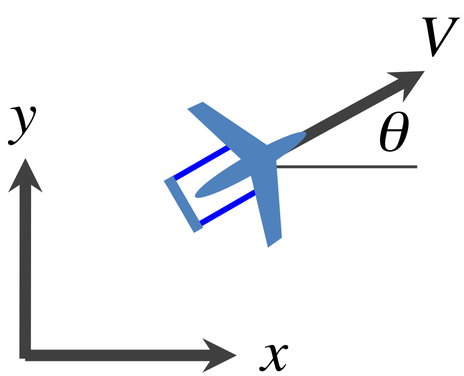
What are characteristics of sensor placement that ensure or maximize full state observability?

Wind-finding

Aerosonde

S-turn maneuvers can be derived using nonlinear observability theory

Model Aerosonde as constant speed unicycle in uniform flowfield



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{W}_x \\ \dot{W}_y \end{bmatrix} = \begin{bmatrix} W_x + V\cos(\theta) \\ W_y + V\sin(\theta) \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \dot{\theta}$$

$$h(x) = \begin{bmatrix} x \\ y \end{bmatrix}$$

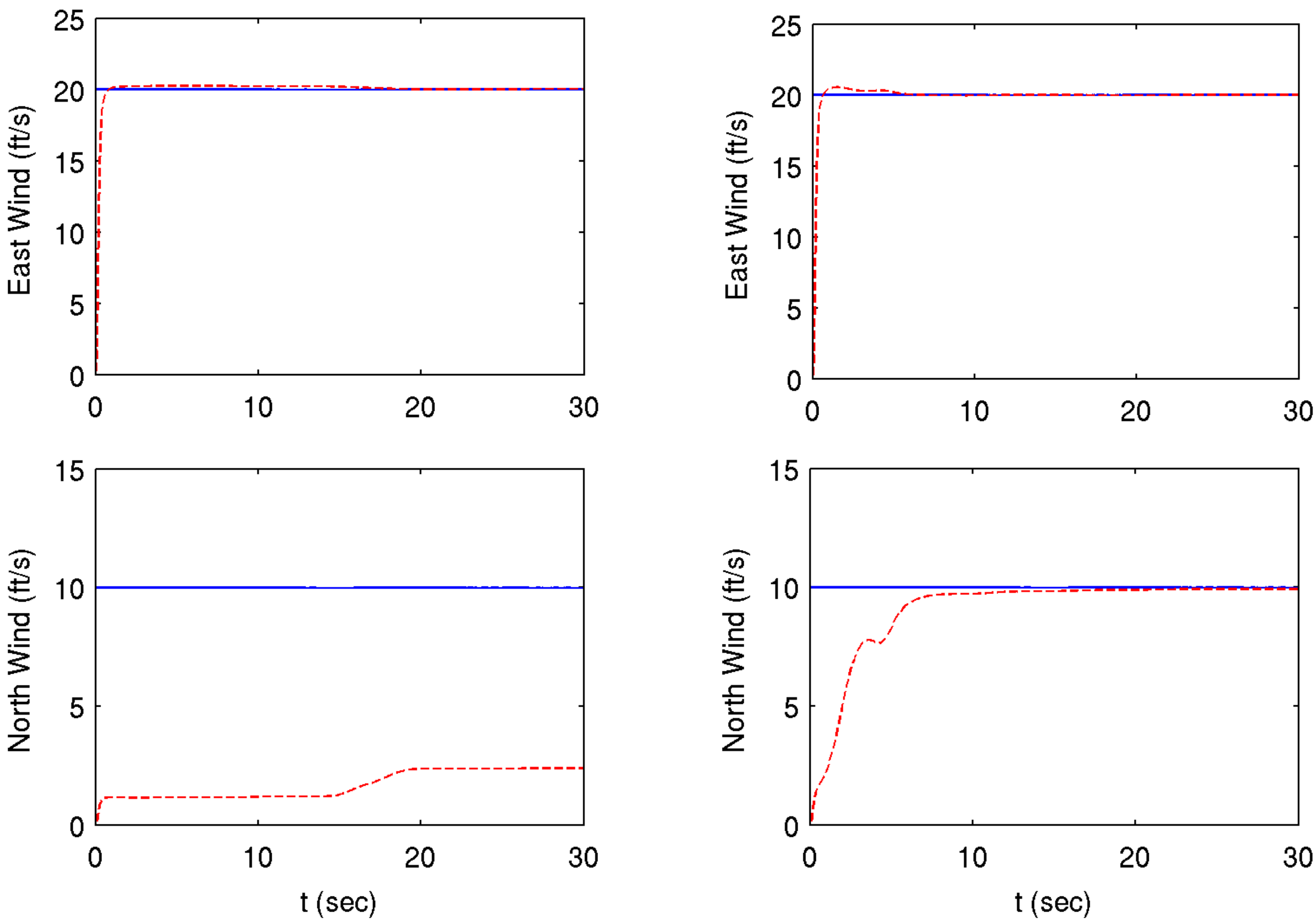
- System is unobservable with constant control inputs
- System is globally observable with cyclic inputs in the heading control, which yields s-turns like those used on the Aerosonde!

Wind-finding

Aerosonde – estimation results

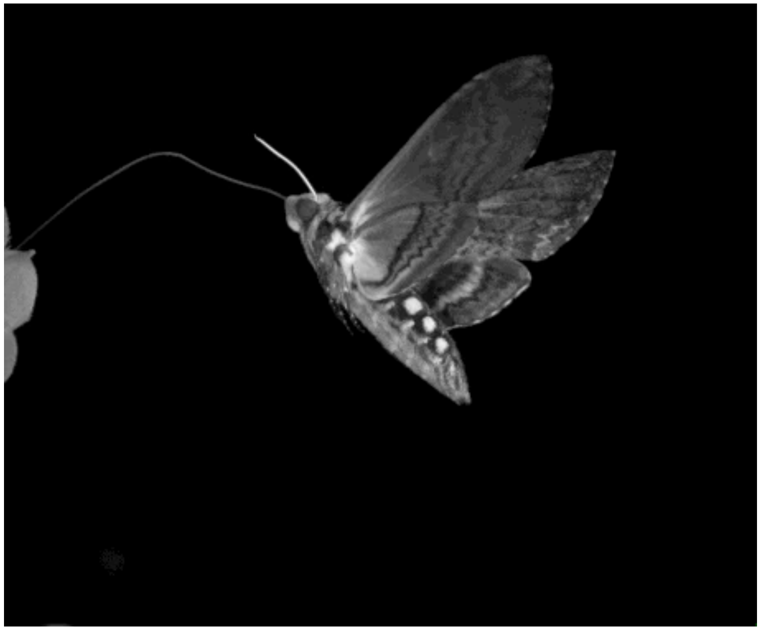
Constant Inputs

Cyclic Inputs

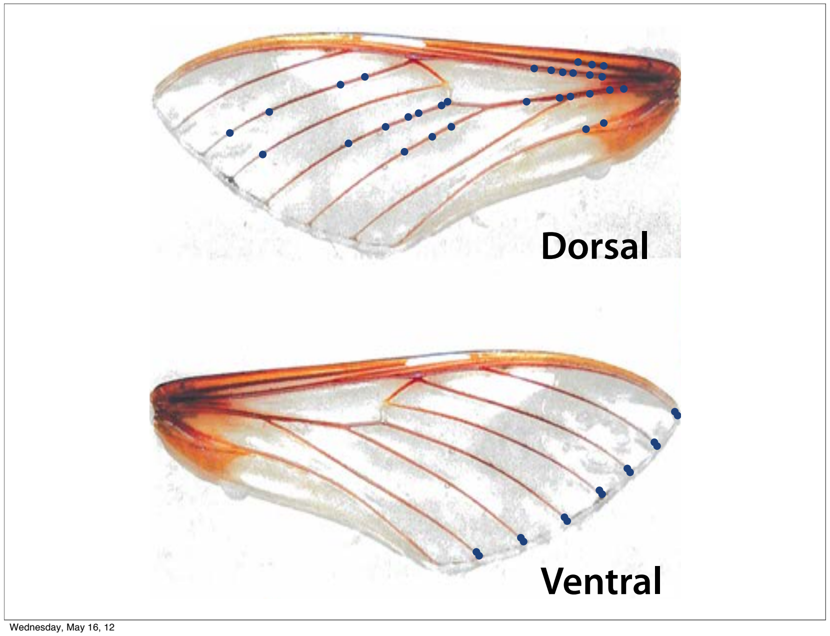


Gyroscopic sensing in insect wings

- Similar to fly halteres and antennae in moths, the wings may serve as gyroscopic sensors
- Wings are covered in hundreds of campaniform sensilla
- Can strain measurements encode body rotation rates?



Hawkmoth hovering [Daniel lab]



Locations of campaniform sensilla on a hawkmoth forewing [Brad Dickerson}

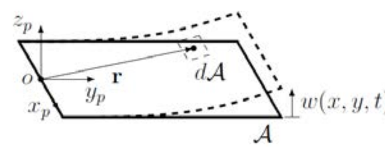
Hinson et. al., "Sensing from control: Airframe deformation for simultaneous actuation and state estimation," *Conference on Decision and Control*, 2013.
Hinson & Morgansen, "Gyroscopic sensing in the wings of the hawkmoth," *Bioinspir. and Biomim.*, 10(5), 2015.

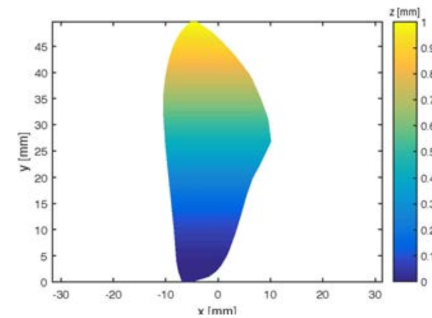
Reduced-order modeling

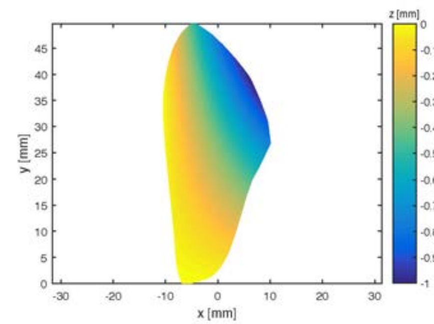
Mode shapes

Rotation rates

$$\mathbf{x} = \begin{bmatrix} \eta^T & \dot{\eta}^T & P & Q & R \end{bmatrix}^T$$







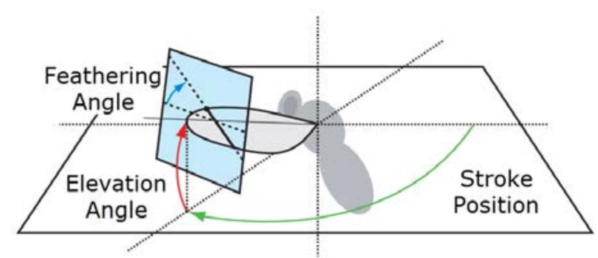
Stiffness matrix

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\eta} \\ K(\omega_0)\eta + (2\mathbf{M}_1 x_r + \mathbf{M}_3)PR - \mathbf{M}_2 QR \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -\mathbf{M}_2 & -\mathbf{M}_1 x_r - \mathbf{M}_3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{u}$$

Mass matrix columns

$$y = H\eta$$

Full rank

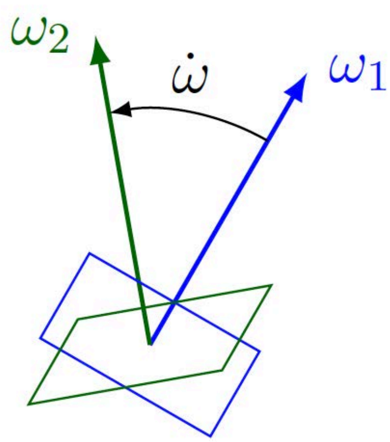


Observability

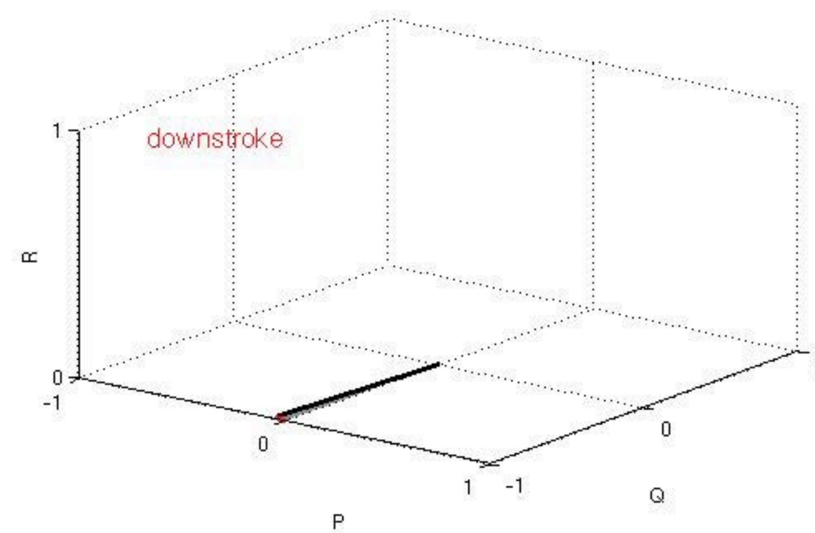
Wing flapping motion required for body rate observability

$$|dg| = \left| \frac{\partial}{\partial x} \begin{bmatrix} h \\ L_{f_0} h \\ L^2_{f_0} h \\ L^3_{f_0} h \\ L^4_{f_0} h \end{bmatrix} \right| = 0$$

$$|dg| = \left| \frac{\partial}{\partial x} \begin{bmatrix} h \\ L_{f_0} h \\ L^2_{f_0} h \\ L^3_{f_0} h \\ L_{f_i} L^2_{f_0} h \end{bmatrix} \right| \neq 0$$



Direction of Coriolis force sensitivity



Wing angular velocity vector

Challenges

Analyticity of models

Complexity of models

Observability via linearization about trajectory

Linearize a nonlinear system about $(\mathbf{x}^0(t),\mathbf{u}^0(t))$ to get an LTV system:

$$\begin{aligned}\dot{\tilde{\mathbf{x}}} &= A(\mathbf{x^0(t)},\mathbf{u^0(t)})\tilde{\mathbf{x}}\\ \tilde{\mathbf{y}} &= C(\mathbf{x^0(t)})\tilde{\mathbf{x}}\end{aligned}$$

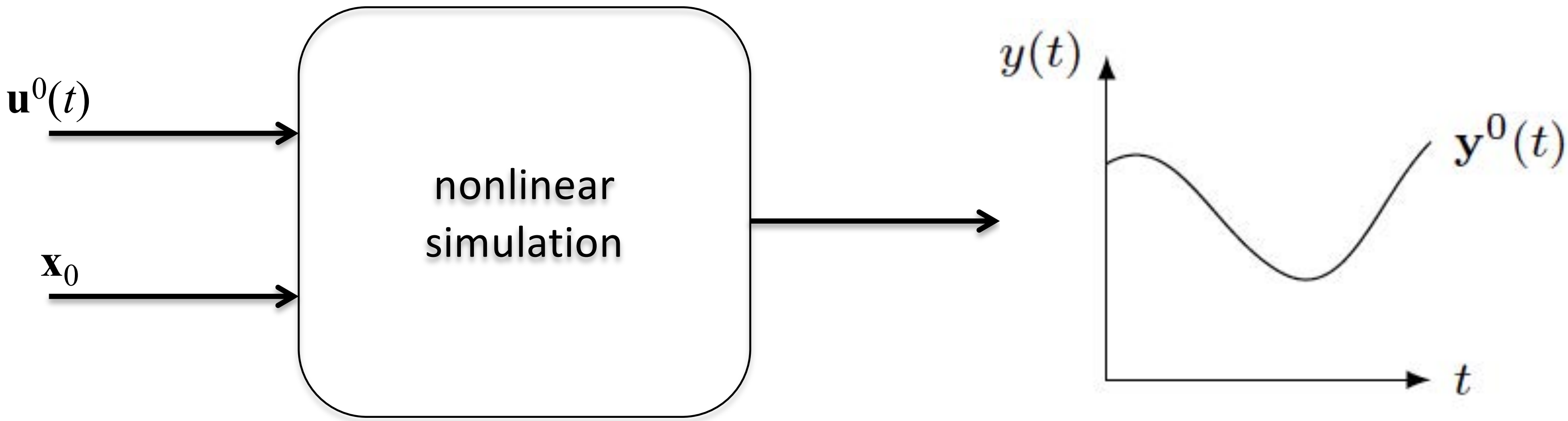
Compute output energy:

$$\begin{aligned}\|\tilde{\mathbf{y}}(t)\| &= \tilde{\mathbf{x}}_0^T \int_{t_0}^{t_1} \Phi^T(\tau,t_0)C^T(\mathbf{x^0}(\tau))\mathbf{C}(\mathbf{x^0}(\tau))\mathbf{\Phi}(\tau,\mathbf{t_0})\mathrm{d}\tau\tilde{\mathbf{x}}_\mathbf{0}\\ &= \tilde{\mathbf{x}}_0^T W(t_1,t_0)\tilde{\mathbf{x}}_0\end{aligned}$$

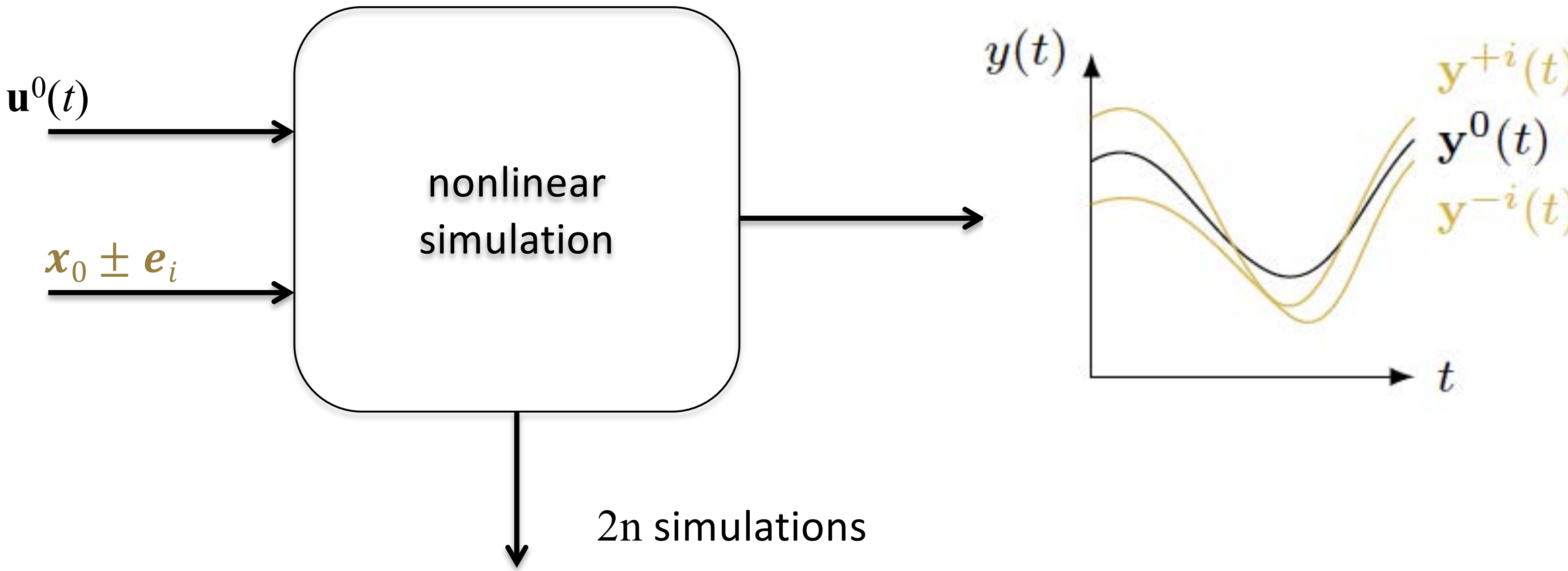
Theorem (LTV Observability)

An LTV system is observable, and the NL system is locally observable about $(\mathbf{x}^0(t),\mathbf{u}^0(t))$, iff $W(t_1,t_0)$ has rank n.

Empirical observability Gramian



Empirical observability Gramian



$$\tilde{W} = \frac{1}{4\epsilon^2} \int_{t_0}^{t_f} \begin{bmatrix} \Delta \mathbf{y}_1^{\mathbf{T}}(\mathbf{t}) \\ \vdots \\ \Delta \mathbf{y}_{\mathbf{n}}^{\mathbf{T}}(\mathbf{t}) \end{bmatrix} \begin{bmatrix} \Delta \mathbf{y}_1(\mathbf{t}) & \cdots & \Delta \mathbf{y}_{\mathbf{n}}(\mathbf{t}) \end{bmatrix} \mathrm{d}t$$

Limit case

Theorem

If there exists u such that

$$\operatorname{rank}\left(\lim_{\epsilon\to 0} W_0^\epsilon(\tau, x_0, u)\right) = n$$

for some $\tau > 0$, then the system is weakly observable at x_0.

- Analogue to rank condition of linear Gramian
- Rigorously connects empirical Gramian to nonlinear observability
- Weak observability at x_0 implies that $y(t)$ is sufficient to distinguish x_0 from its neighbors (if observed for long enough)
  - Weaker than local weak observability from Lie derivative tools

Powel & Morgansen, "Empirical observability Gramian rank condition for weak observability of nonlinear systems with control," *2015 IEEE Conf. Decision and Control.*

Finite epsilon case

Theorem

If there exists some u such that

$$\underline{\sigma}(W_0^\epsilon) > \sup_{t\in[9,\tau]} \left(\frac{\sqrt{n}\epsilon^2\tau}{3} \left\| \frac{\partial y}{\partial x_0} \right\|_2 \Gamma + \frac{n\epsilon^4\tau}{36}\Gamma^2 \right)$$

where

$$\Gamma(t, x_0, u) = \max_i \sup_{\eta\in I_i^\epsilon} \left\| D^3 y(\eta)(e_i, e_i, e_i) \right\|$$

and

$$I_i^\epsilon = [x_0 - \epsilon e_i, x_0 + \epsilon e_i]$$

for some $\tau > 0$, then the system is weakly observable at x_0.

- Much more useful; limit form of Gramian not generally evaluable
- Basically: If the empirical Gramian is “sufficiently” positive definite, the system is weakly observable

Caveat: Generally not possible to find Γ analytically; must be done numerically

Fisher information bound

Theorem

Given a system with measurement noise:

$$\tilde{y} = h(x) + v, \;\; v \sim N(0, R)$$

we have

$$F(t) \preccurlyeq \underline{\sigma}(R^{-1}) \frac{d}{dt} \lim_{\epsilon \to 0} W_0^{\epsilon}(t, x_0, u)$$

- Gramian can also bound the Fisher information matrix
- Fisher information matrix bounds estimator covariance (Cramer-Rao)
- Therefore, the empirical observability Gramian is connected to ideal estimator performance

Error covariance is bounded by observability

$$\mathcal{P} = \Sigma^{-1} \preccurlyeq F \preccurlyeq \underline{\sigma}(R^{-1}) f(W_o)$$

Sensor Selection – Problem framework

Given a nonlinear control affine system

$$\dot{\mathbf{x}} = \mathbf{f}_0(\mathbf{x}) + \sum_{i=1}^{m} \mathbf{f}_i(\mathbf{x})\mathbf{u}_i$$

System dynamics

$$\mathbf{y}_i = \mathbf{h}_i(\mathbf{x}, \mathbf{s}_i), \quad \mathbf{s}_i \in \mathcal{S}, \quad \mathbf{h}_i \in \mathcal{H}, \quad \mathbf{i} = \mathbf{1}, \cdots, \mathbf{p},$$

Measurements

Sensor switches

Choose s_i and h_i to optimize the observability Gramian

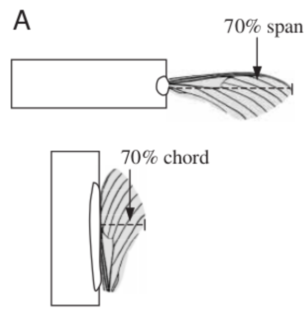
$$\begin{aligned} \min_{\mathbf{s},\mathbf{h}} \quad & J[W(\mathbf{s},\mathbf{h})] \\ \text{subject to} \quad & \mathbf{s}_i \in \mathcal{S} \\ & \mathbf{h}_i \in \mathcal{H} \end{aligned}$$

Metric on observability (analytical/empirical)

Constraints

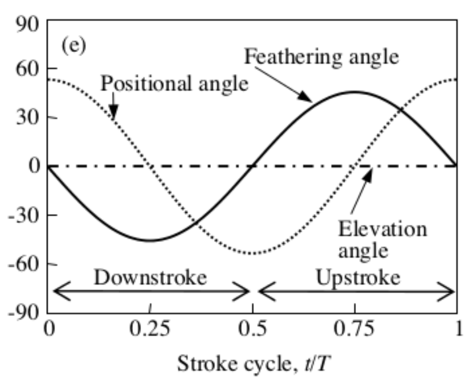
Reduced-order modeling

Wing properties



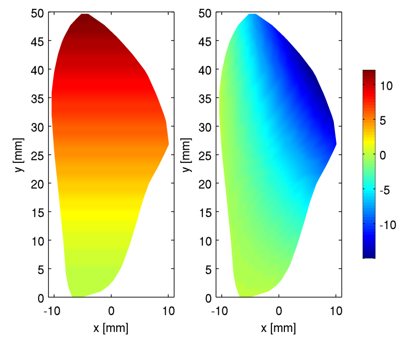
Material properties estimation [Combes & Daniel]

Stroke kinematics



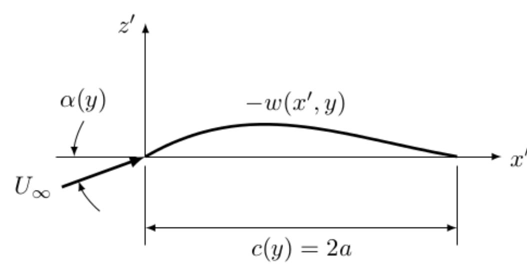
Fit of measured wing kinematics [Nakata & Liu]

Finite element model



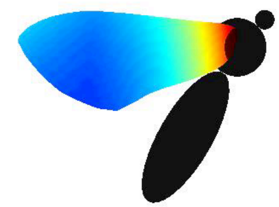
Wing mode shapes

Aerodynamics



Quasi-steady aerodynamics using local deformed wing shape

Simulated flapping



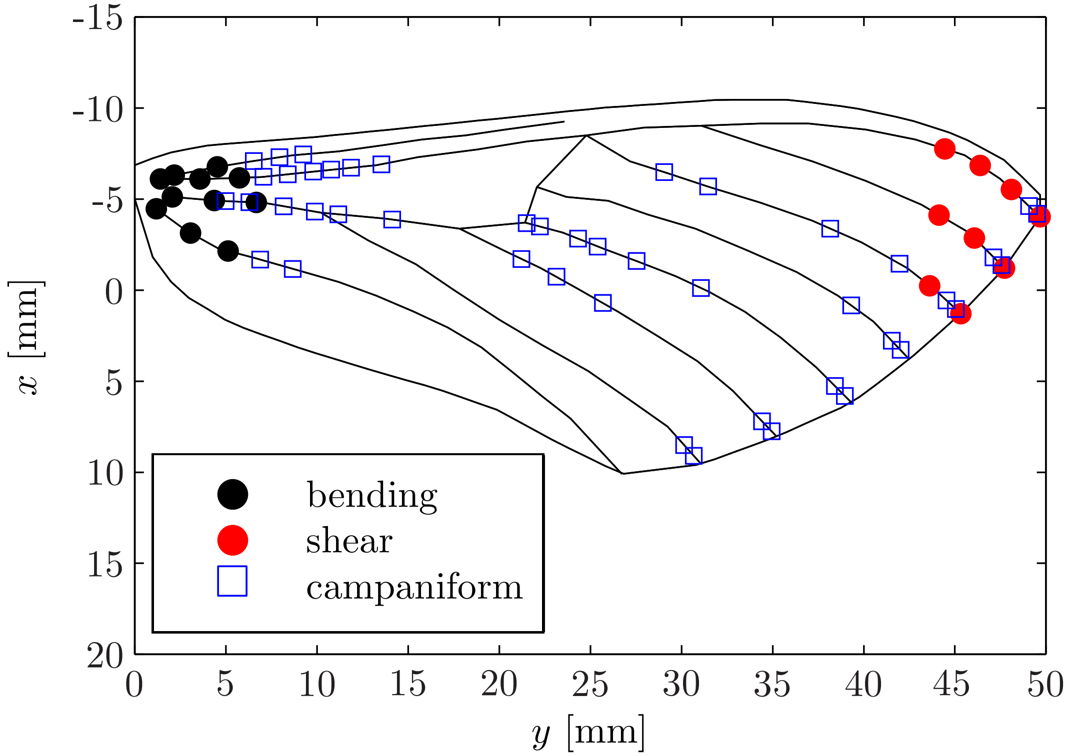
Side view

Front view

Coupled fluid-structure model

Sensor placement results

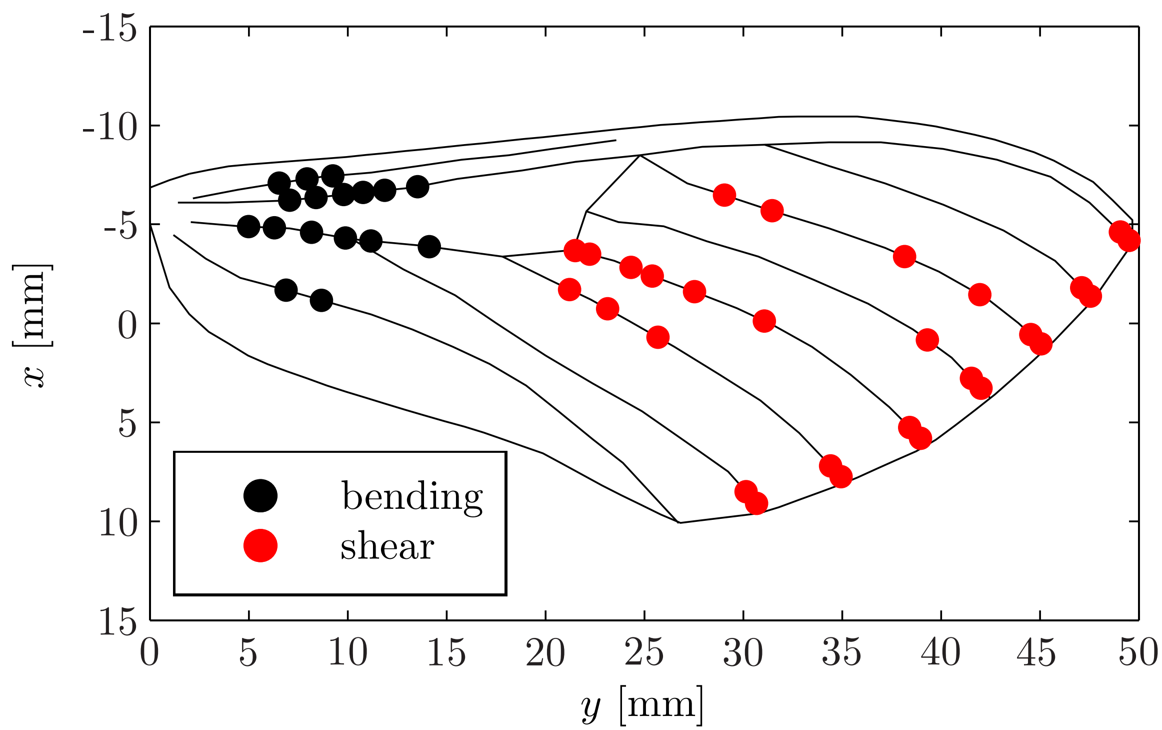
- Wing flapping motion required for body rate observability
- Shear strain due to wing twisting provides the most information about body rotation rates
- Optimal sensor set forms two clusters



comparison of sensor placement with campaniform sensilla locations

Sensor placement results

- Wing flapping motion required for body rate observability
- Shear strain due to wing twisting provides the most information about body rotation rates
- Optimal sensor set forms two clusters
- Optimal sensor type hypothesizes heterogeneity distribution



sensor type selection for known campaniform sensilla locations

Optimal sensor placement

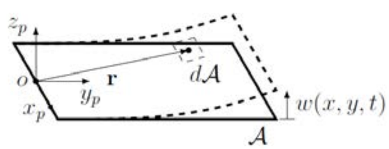
Cantilever and FEA shell models

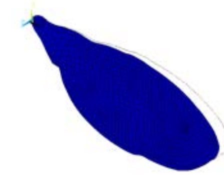
Hawkmoth
Cantilever

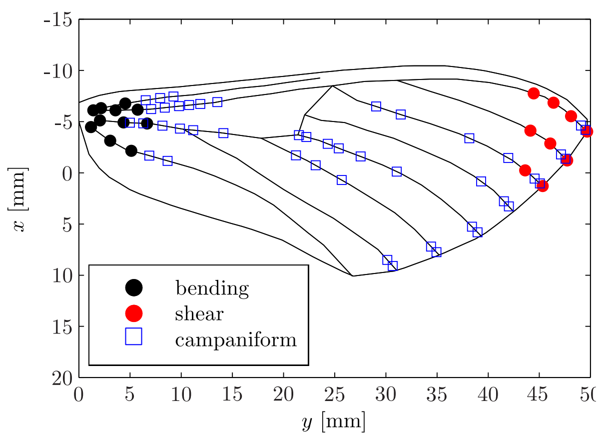
Hawkmoth
FEA mode 1

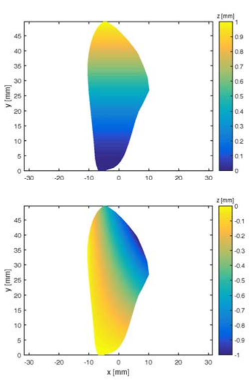
Honeybee
FEA mode 1

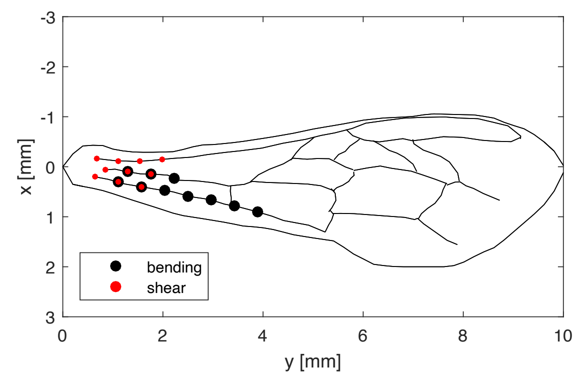
Drosophila
FEA mode 1

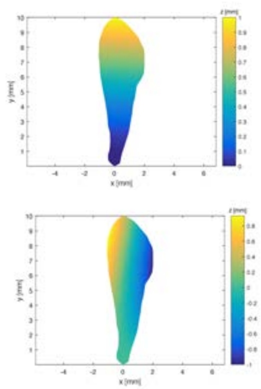


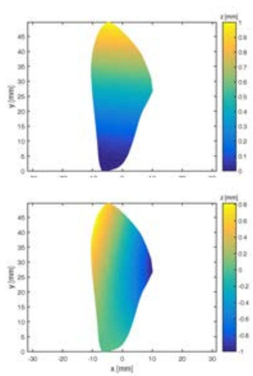


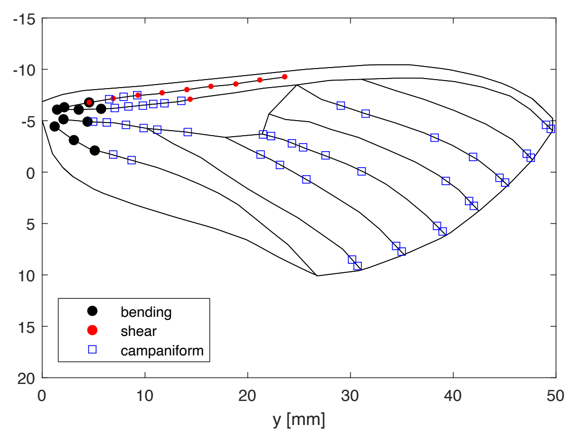


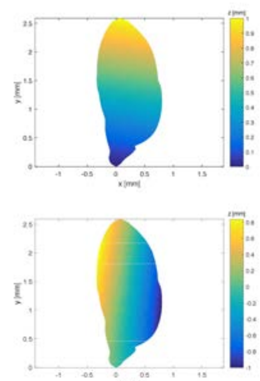


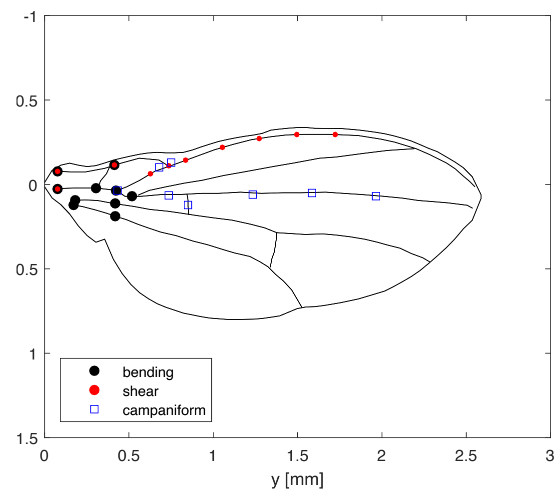












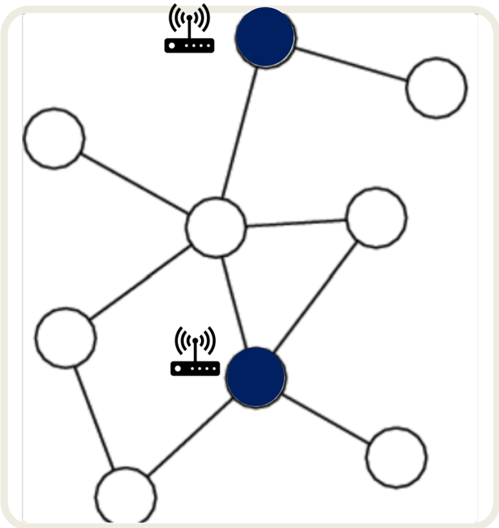
Network Observability

Consider a system of N agents with dynamics and measurement

$$\dot{x}_i = f_i(x_i, u_i)$$

$$u_i = \sum_{i=1}^{N} k_{ij}(x_i, x_j)$$

$$y^i = x_i, \;\; i = 1, \ldots, N$$



Define a binary variable $\zeta \in \mathbb{R}^N$
where

- $\zeta_i = 1$ if there is a sensor on node i
- $\zeta_i = 0$ otherwise

The measurement model then has the form

$$y = \begin{bmatrix} \zeta_1 y^1 & \zeta_2 y^2 & \cdots & \zeta_N y^N \end{bmatrix}$$

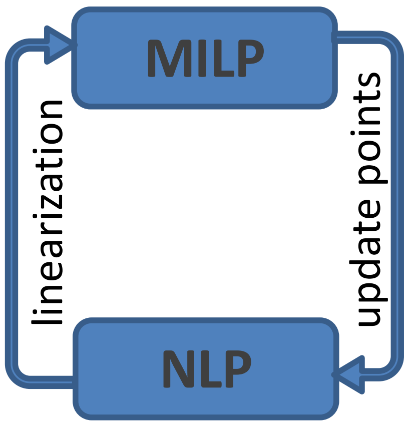
Optimization Algorithm

Outer Approximation (Duran and Grossman, 1986)

Motivation: avoid solving huge number of NLPs

Exploit MILP/NLP solvers:

- Decompose integer and nonlinear parts



This algorithm gives a lower bound on the optimal value.

MILP Optimization Problem

If $F(\zeta)$ refers to the cost function

$$\nabla_j F(\zeta) = -\text{trace}\left[\left(\sum_{i=1}^{N} \zeta_i W_{0,i}\right)^{-1} W_{0,j}\right]$$

Then a relaxation of the optimization is

$$\min_{\zeta_1,\ldots,\zeta_N} \quad \alpha$$

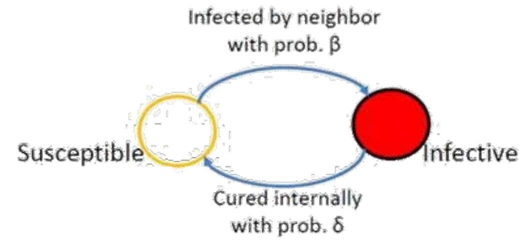
$$\text{Subject to} \quad \begin{aligned} &\nabla F(\zeta - \bar{\zeta}) + F(\bar{\zeta}) \leq \alpha \\ &\sum_{i=1}^{N} \zeta_i \leq r \\ &\zeta_i \in \{0,1\} \end{aligned}$$

Virus Spreading Model (SIS)

Each node is either infected or susceptible.

infection rate: β_i

curing rate: δ_i



The state of each node i is described by a binary random variable $X_i(t) \in \{H, I\}$, i.e. at time t node i has 2 states:

1. Infected with probability $\Pr[X_i(t) = I]$, and
2. Healthy with probability $\Pr[X_i(t) = H]$.

Evolution of the states is described by a Markov Process, two possible state transitions:

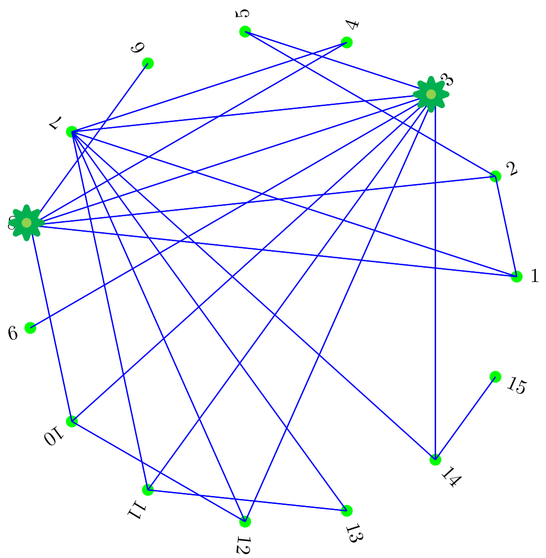
1. $\Pr[X_i(t+\Delta t) = I | X_i(t) = H] = \sum_{j \in N_i} A[i,j]\beta_i X_j(t)\Delta t + o(\Delta t)$
2. $\Pr[X_i(t+\Delta t) = H | X_i(t) = I] = \delta_i \Delta t + o(\Delta t)$

Denoting $x_i(t) = \Pr[X_i(t) = I]$, the Markov differential equation in matrix form is:

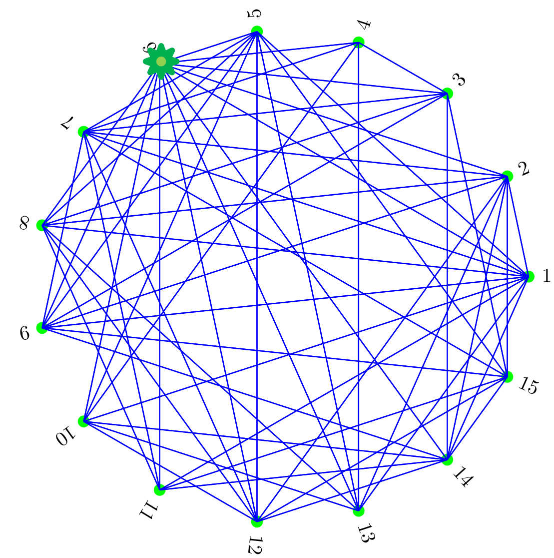
$$\dot{x}(t) = (BA - D)x(t) - \left(\sum_{j=1}^{N} e_j e_j^T x(t) e_j^T\right) BAx(t)$$

Sparse or Dense Network Node Sensor Selection

Sparse Structure



Dense Structure



Ordinary Node
Observing Node

Alaeddini & Morgansen, "Optimal disease outbreak detection in a community using network observability," *2016 American Control Conference.*

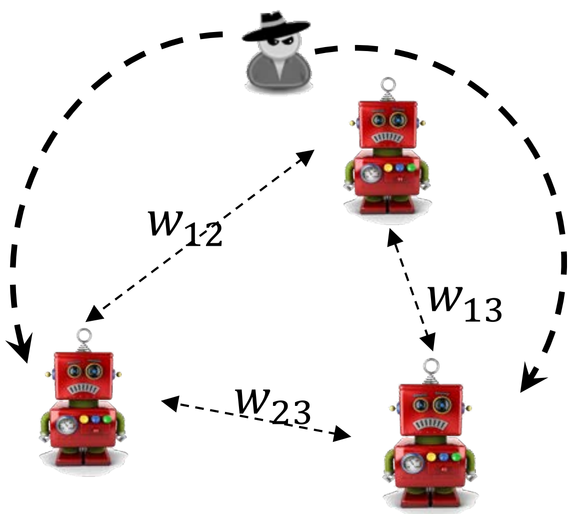
Privacy in Networked Systems

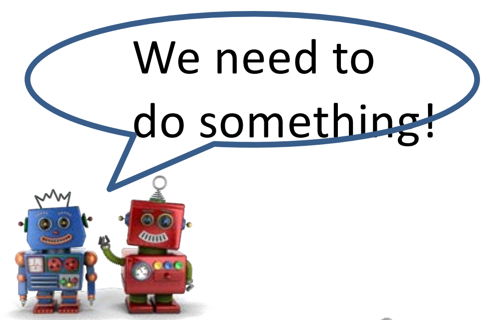
- ✓ When agents exchange sensitive data, our concern is ensuring that privacy is kept.
- ✓ A minimum observable topology of interaction will be proposed, which translates to minimization of the data being exposed to a foreign agent.

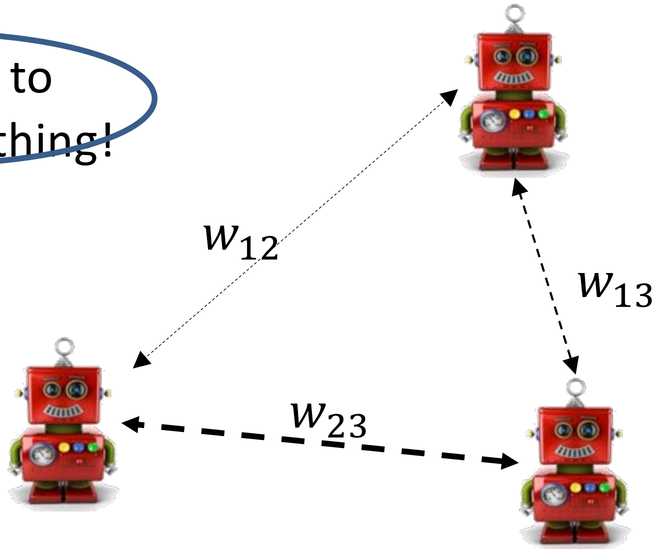
Network Security

- Communication between agents in a network is represented by a graph
- Each agent is denoted as a node, and edges represent communication links
- Node i is connected to node j with an edge with weight w_{ij}.
- A hacker attacks the network by connecting to a node in network.
- The objective is minimize the information that the hacker can attain.

$$\min_{w} \operatorname{trace} W_O$$







Mathematical Modeling

Dynamics of the network

$$\dot{x} = A(w)x; \quad \mathrm{A(w)} = \mathrm{A}_0 + \sum_{l=1}^{M} A_l w[l]$$

$$y = C_t x; \qquad C_t = e_k, \;\; k \in \{1, 2, \cdots, N\}$$

Loss function (trace of the empirical observability Gramian)

$$\text{trace}\, W_O \quad = \frac{1}{4\epsilon^2} \int_0^{t_f} \sum_{i=1}^{N} \left(y^{+i}(t) - y^{-i}(t) \right)^2 dt$$

$$= \sum_t \underbrace{\int_t^{t+\Delta} \left[e^{2A(w)\tau} \right]_{k,k} d\tau}_{f_t(w)}$$

Optimization Problem:

$$\min_w f_t(w) + \gamma \|w\|^2$$

Subject to $w > 0$

Log T Bound on Regret

The regret of an action sequence $\{w_t\}$:

$$\mathcal{R}_T = \sum_{t=1}^{T} (f_t(w_t) - f_t(w^*))$$

Lemma 1 The matrix $A(w)$ is negative definite for all $w > 0$.

Lemma 3 The cost function has gradient values upper bounded by a $G > 0$.

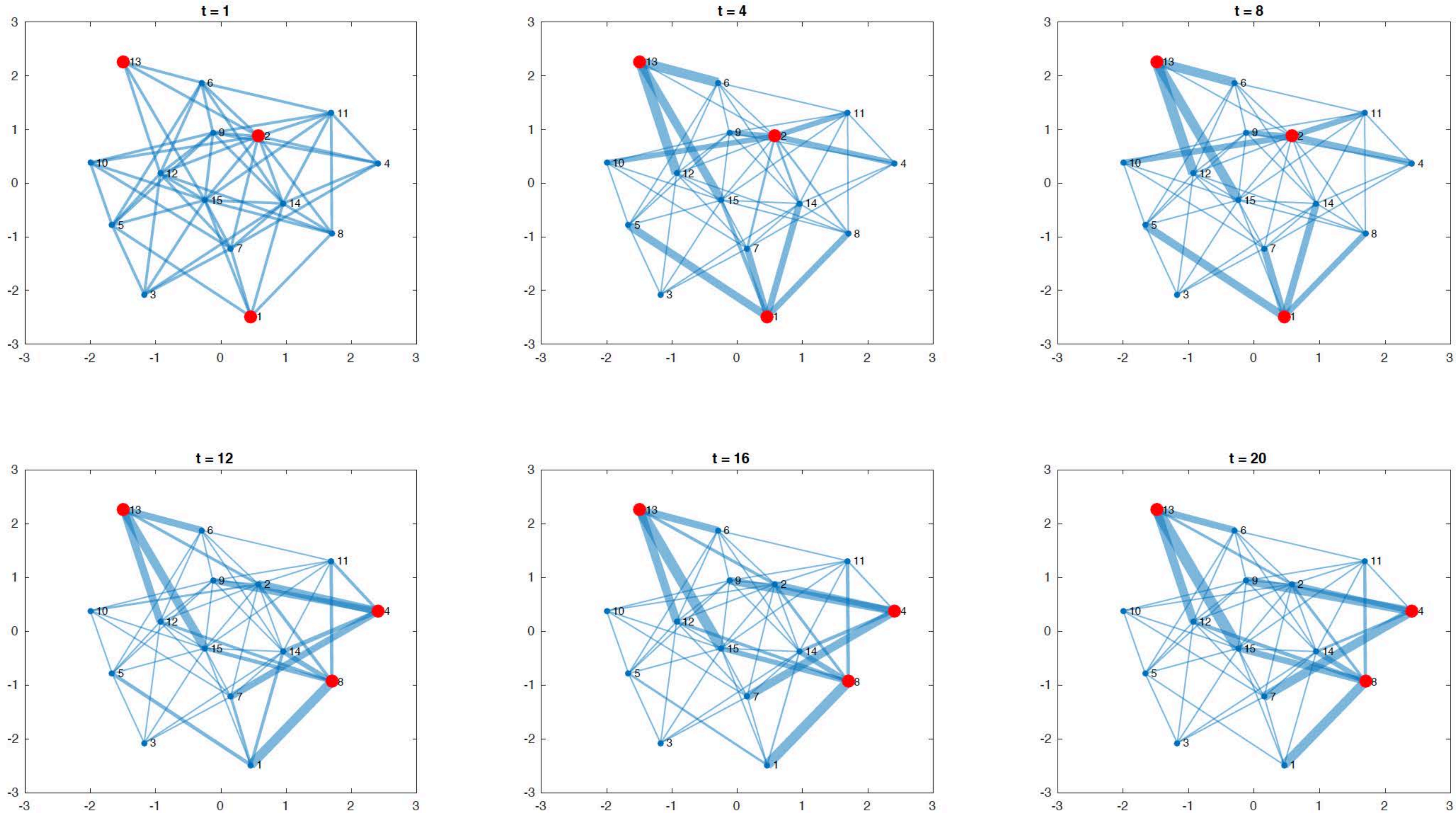
Lemma 2 The function $f_t(w)$ is convex.

Thus, the privacy problem can be accomplished by *Online Newton Step,* presented by Hazan *et al*., 2007.

Theorem If step size is chosen at $\theta_k = \frac{1}{hk}$ then Online Newton Step algorithm has $\mathcal{R}_T \leq O(\log T)$ regret bound, which is sub-linear.

Alaeddini, Mesbahi & Morgansen, "Adaptive communication networks with privacy guarantees," *2017 American Control Conference.*

Results

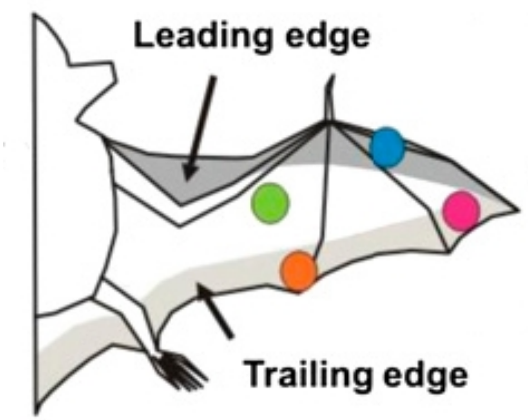


Reweighting of a network with multiple foreign/adversarial nodes.

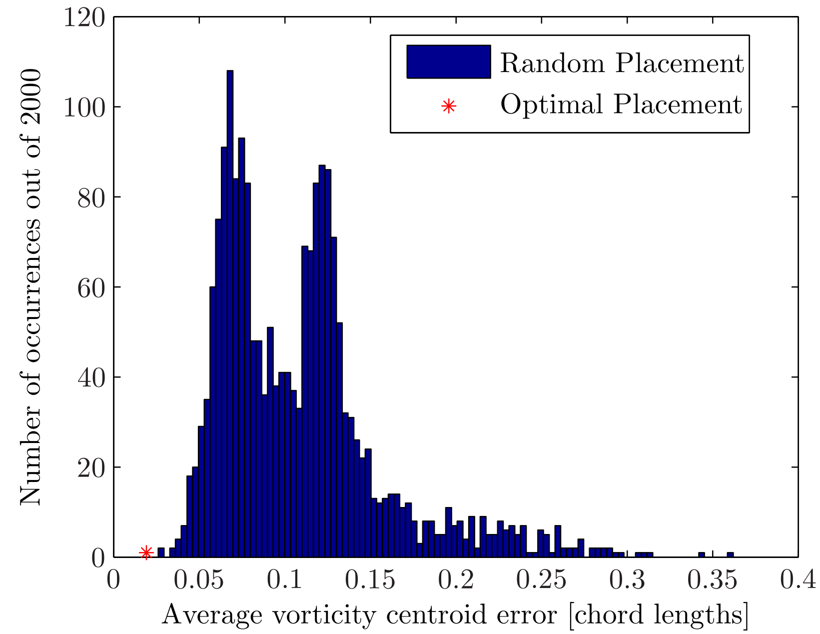
Optimal sensor locations for vortex sensing

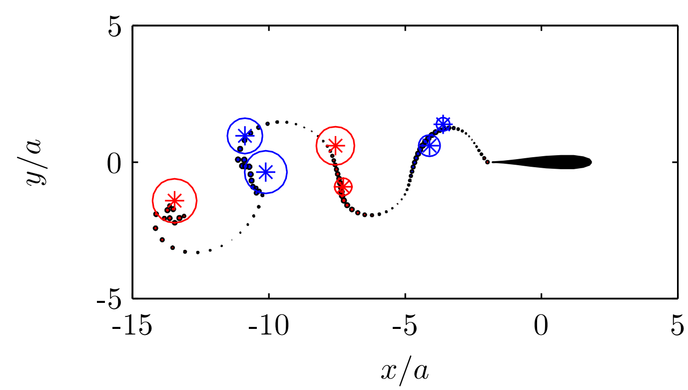
Sensors placed in pairs on upper and lower surface near trailing edge

Compares well with bat study



Sensory hairs on bat wings [Sterbing-D'Angelo, 2011]

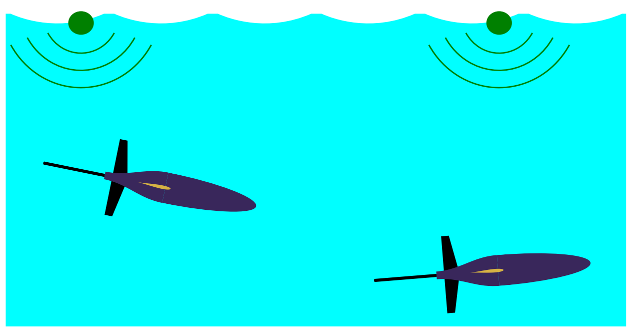




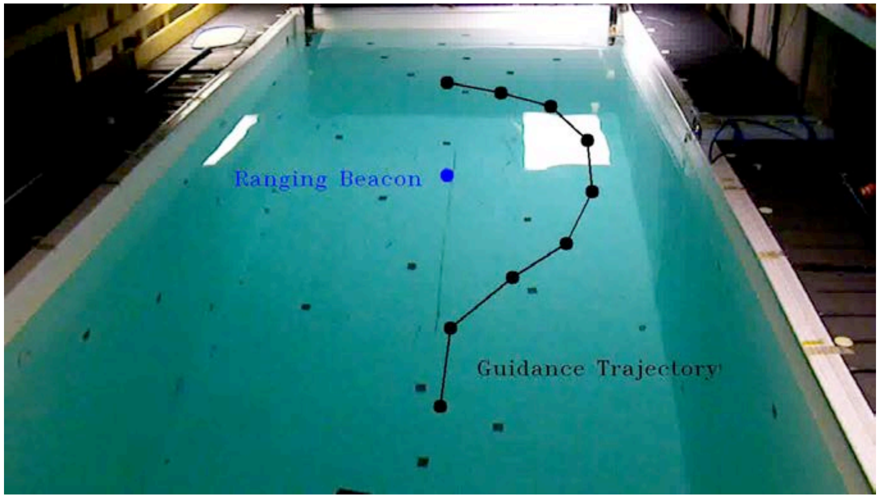
Estimation results

Range-only and bearing-only navigation

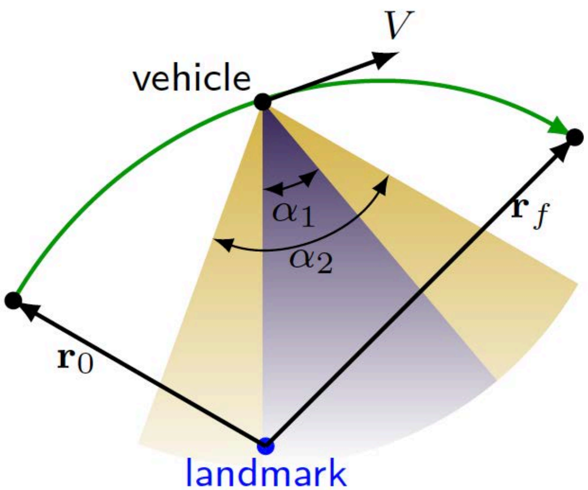
Vehicle localization with limited sensors



Acoustic localization



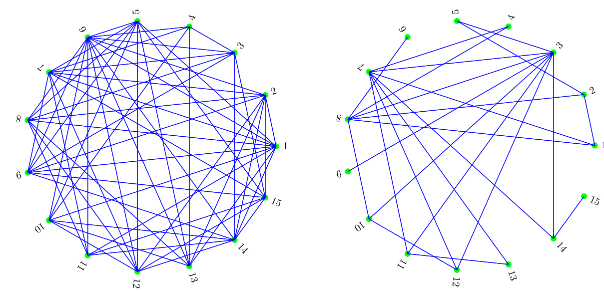
Target tracking



Ongoing work

Motion camouflage

Network observability from sparse sensing (epidemics)



Stochastic systems

$$dx = f_0(x,t)dt + \sum_{i=1}^{m} f_i(x)u_i dt + \gamma dw$$

$$y = h(x,t)$$

Network security

Acknowledgements

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http://www.aa.washington.edu/research/ndcl