

# On Resilient Control for Secure Connected Vehicles

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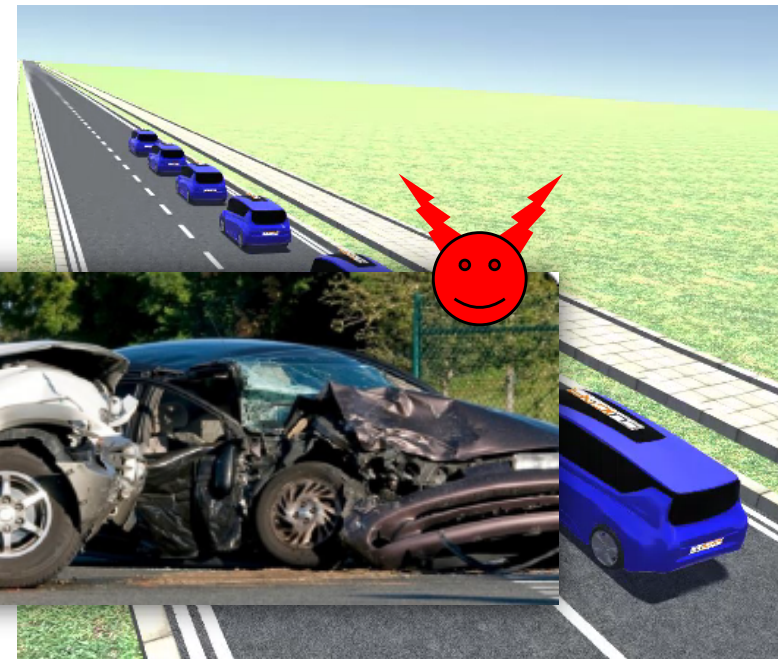
Connected vehicle image from:  
<https://www.geotab.com/blog/connected-vehicle-technology/>

## Connected Autonomous Vehicles<sup>1,2,3,4</sup> :

- Expected to improve:
  - Traffic throughput.
  - Safety.
  - Fuel economy.
- V2V and V2I communications through Dedicated Short-Range Communication (DSRC).

## Famous application is Cooperative Adaptive Cruise Control (CACC):

- Relies on V2V.
- Employs on-board sensors.
- Autonomous vehicles



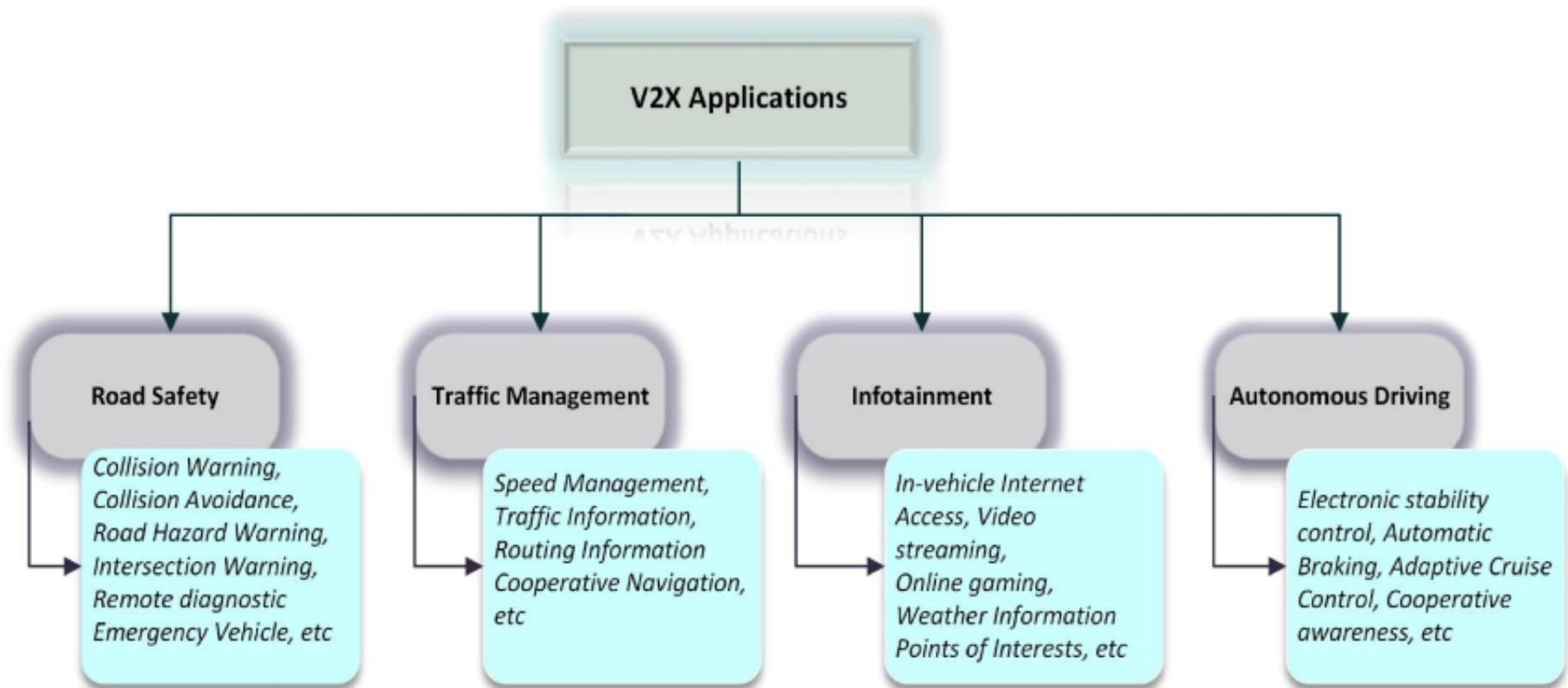
## Vulnerable to Cyber-Attacks

<sup>1</sup>Smart city challenge. <https://www.transportation.gov/sites/dot.gov/files/docs/Smart%20City%20Challenge%20Lessons%20Learned.pdf>.

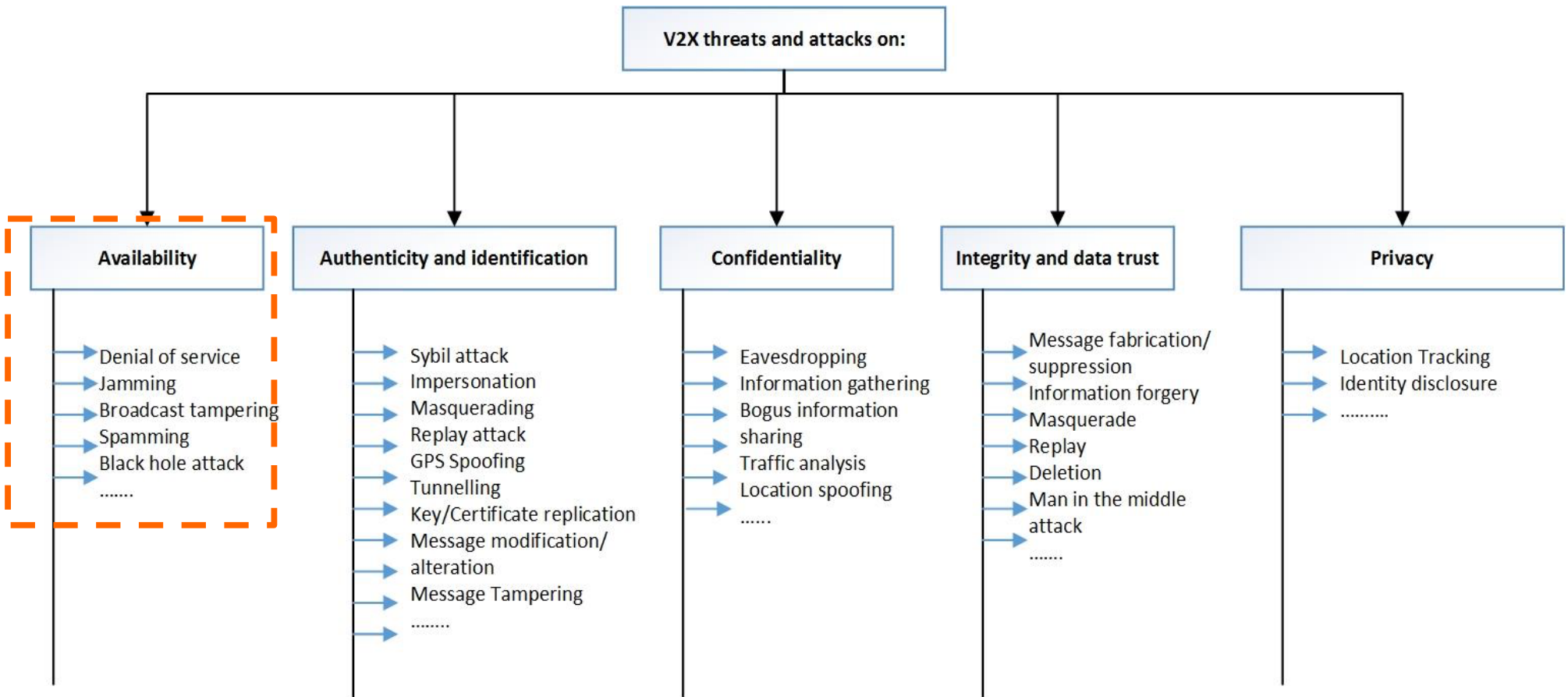
<sup>2</sup>J. K. Hedrick, M. Tomizuka, and P. Varaiya. Control issues in automated highway systems. IEEE Control Systems, 14(6):21–32, 1994.

<sup>3</sup>J. N. Lu, N. Cheng, N. Zhang, X. Shen, and J. W. Mark. Connected vehicles: Solutions and challenges. IEEE Internet of Things Journal, 1(4):289–299, 2014.

<sup>4</sup>B. Van Arem, C. J. G. Van Driel, and R. Visser. The impact of cooperative adaptive cruise control on traffic-flow characteristics. IEEE Trans on ITS, 7(4):429–436, 2006.



## V2X Security: Types of Attacks on V2X



Security threats have become more sophisticated, and cars already on the road require updated security mechanisms that address new risks.

## Existing solutions

Bit commitment and signature → Availability ( e.g. DoS)

Digital certification and zero knowledge → Identification and authenticity ( e.g. Man in the middle, GPS spoofing replay attack)

Trusted hardware → Identification and authenticity

Group management system → Integrity and data trust (e.g. Message tampering)

Encryption of data and the corresponding positioning and vehicle identification → Confidentiality and privacy (e.g. Traffic analysis)

- 
- Security must not come at the expense of performance.
  - In traffic safety scenarios, security verification must be performed in real time.
  - Queueing of V2V messages is not an option.
  - Finally, security systems must be certified. Certification of the complete solution assures safety.

## V2X Network Cyber Attack on Communication

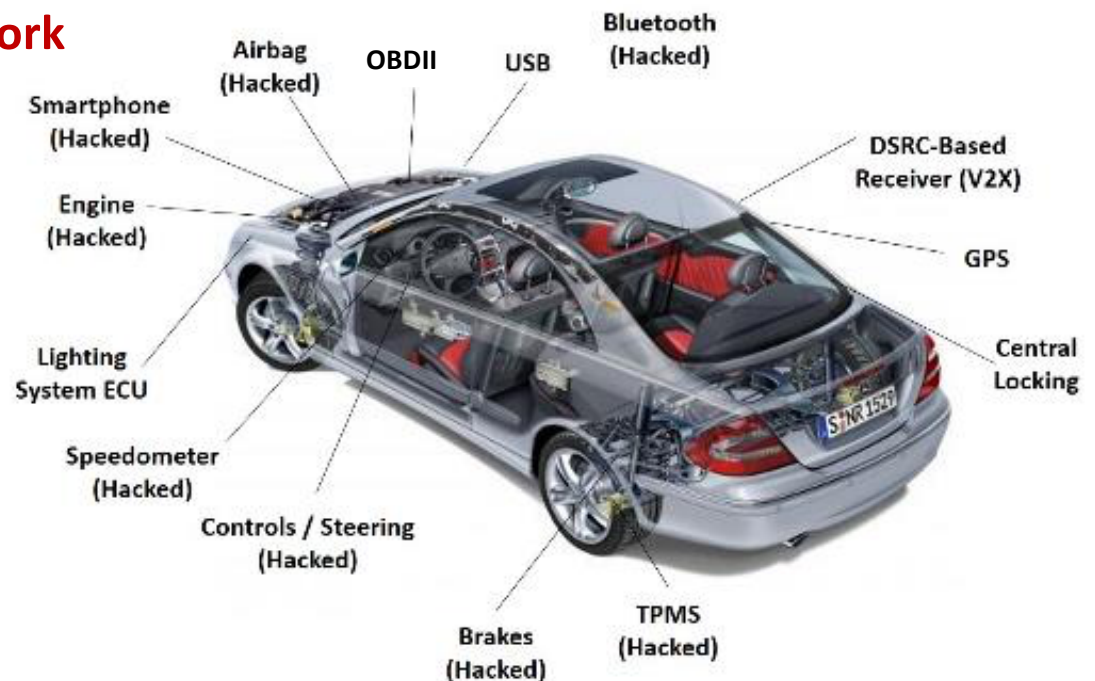
Network flooding, affecting the availability of the network

Changing the content and integrity of the package

## V2X Cyber Attack on In-Vehicle Network

Hacking the equipment, sensors and actuators

Affecting CAN bus readings, messages and high level controls



## Denial of Service

$$\begin{bmatrix} \dot{d}_i(t) \\ \dot{v}_i(t) \\ \dot{a}_i(t) \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ \frac{k_p}{h} & -\left(k_p + \frac{k_d}{h}\right) & \left(k_p + \frac{1}{h}\right) \end{bmatrix} \begin{bmatrix} d_i(t) \\ v_i(t) \\ a_i(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \frac{k_d}{h} \end{bmatrix} v_{i-1}(t) + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{h} \end{bmatrix} a_{i-1}(t - \tau)$$

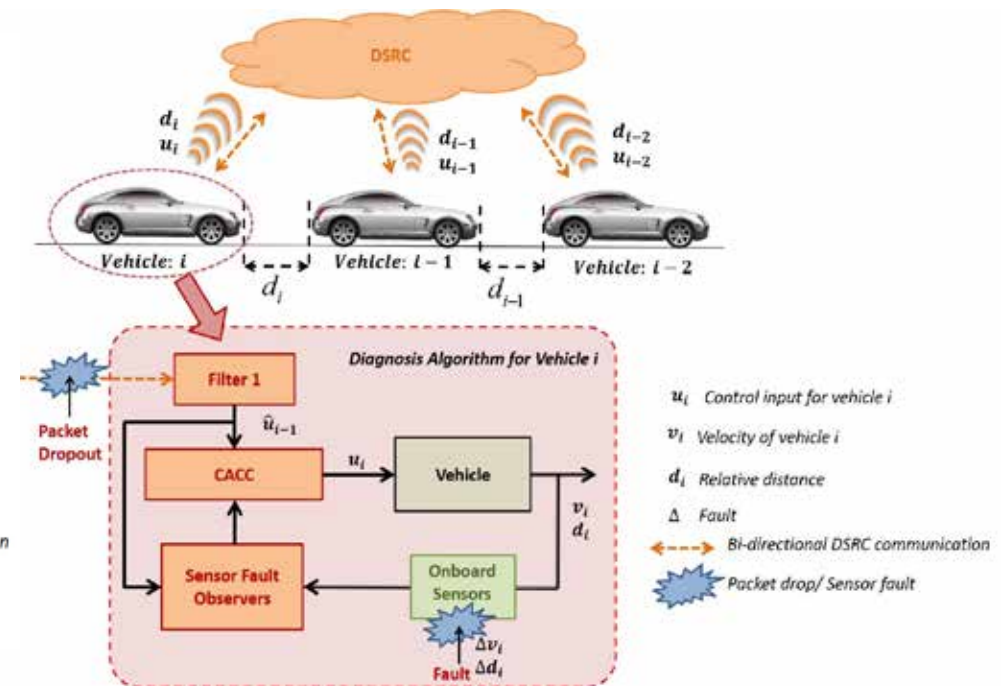
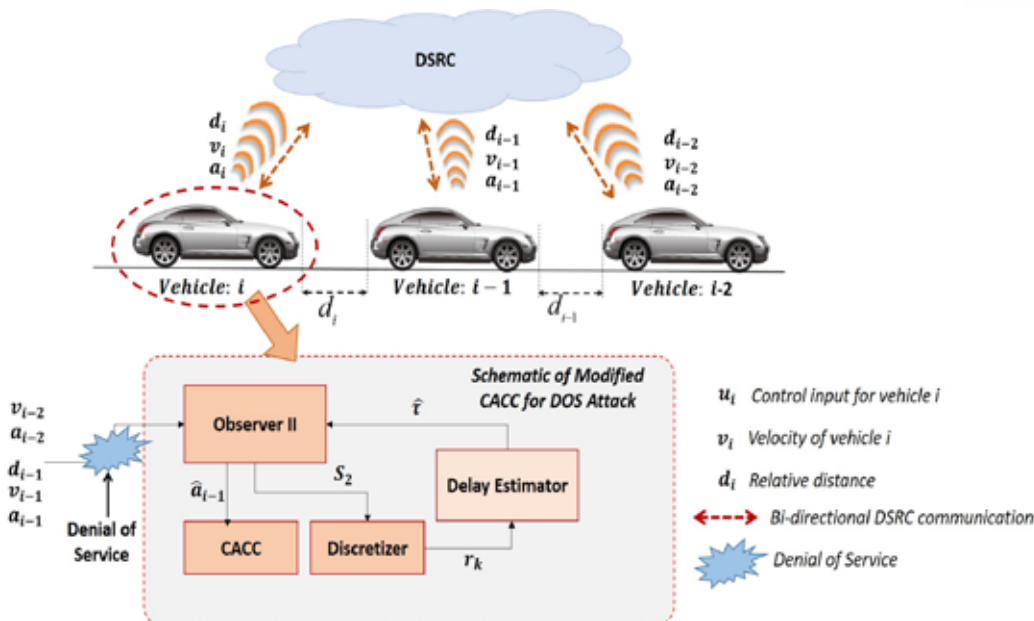
## Packet drop out

$$\chi(k) \in \{0, 1\}$$

$$\begin{cases} p(\chi(k) = 0) = \lambda \\ p(\chi(k) = 1) = 1 - \lambda \end{cases}$$

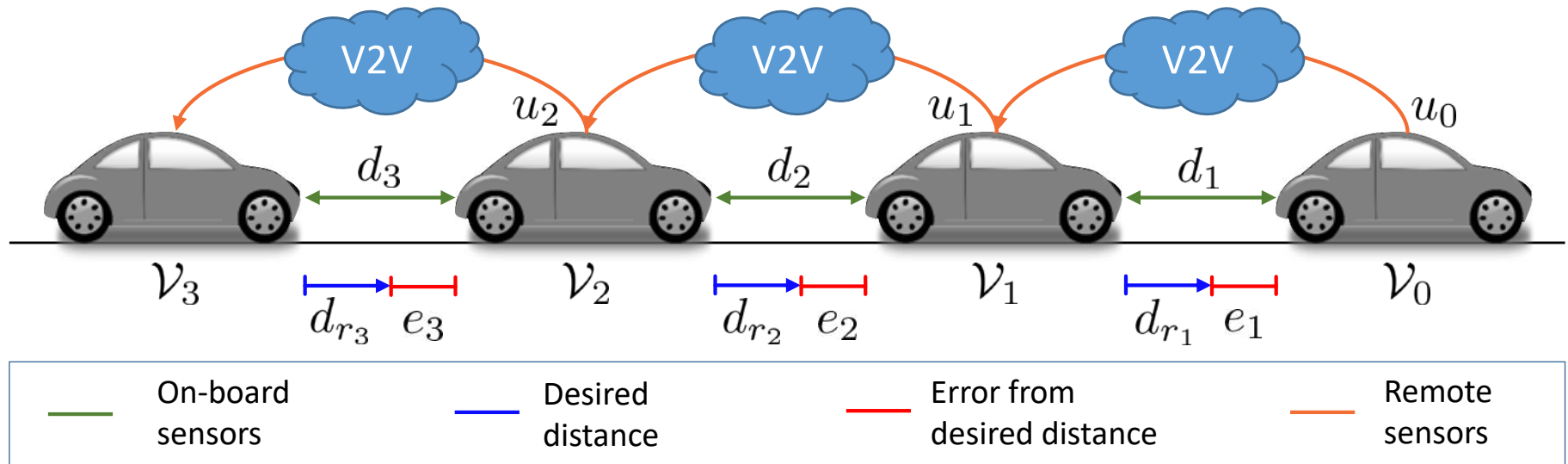
$$k \times T_s \leq t < (k+1) \times T_s$$

$$\dot{u}_i = -\frac{1}{h} u_i + \frac{1}{h} (k_p e_i + k_d \dot{e}_i) + \frac{1}{h} \chi(k) \times u_{i-1}$$



- Attacker provides several requests to access the network → communication network will be busy for real requests

# Vehicle Platooning: Cooperative Adaptive Cruise Control (CACC)



## Adaptive Cruise Control (ACC)

- Reference:  $d_{r_i} = r + h v_i$
- Error:  $e_i = d_i - d_r$   
 $e_i(t) \rightarrow 0$
- String stability (shock waves attenuation)

## Cooperative Adaptive Cruise Control (CACC)



- Shared information
- Closer inter-vehicle distance

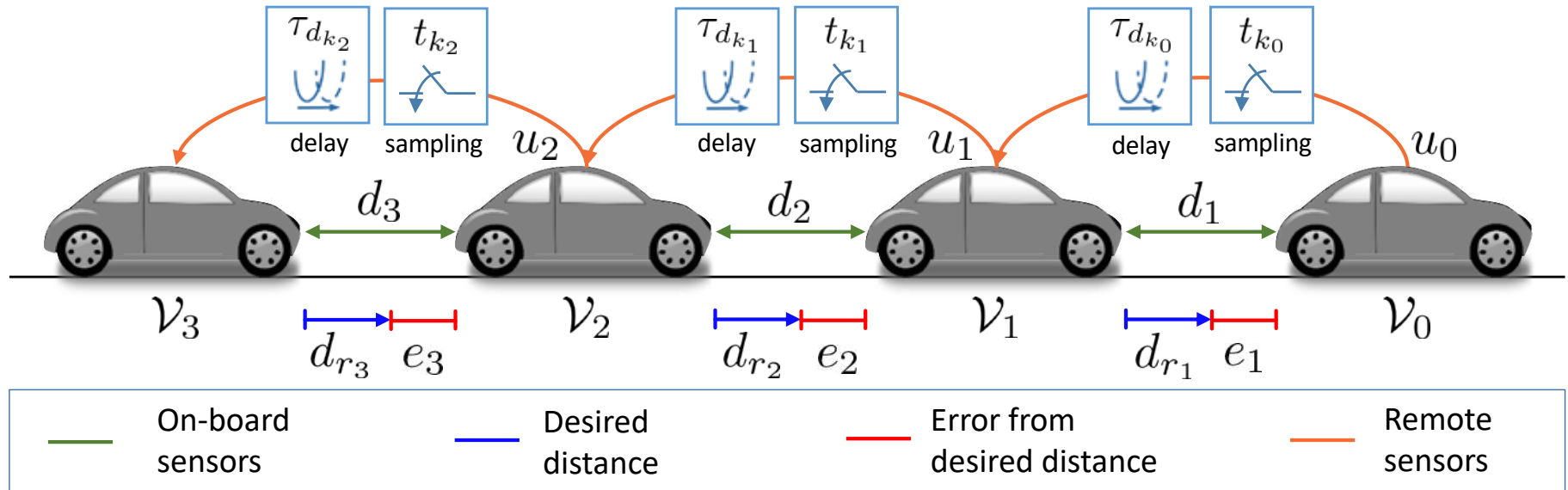
## Vehicle and Controller Dynamics<sup>1</sup>:

$$\mathcal{V}_i : \begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} v_{i-1} - v_i - h a_i \\ a_i \\ -\frac{1}{\tau_d} a_i + \frac{1}{\tau_d} u_i \end{pmatrix}$$

$$\dot{u}_i = -\frac{1}{h} u_i + \frac{1}{h} (k_p e_i + k_d \dot{e}_i + u_{i-1})$$

<sup>1</sup>J. Ploeg, B. T. Scheepers, E. Van Nunen, N. Van de Wouw, and H. Nijmeijer, "Design and experimental evaluation of cooperative adaptive cruise control," in 14th International IEEE Conference on Intelligent Transportation Systems (ITSC), pp. 260–265, IEEE, 2011.

# Vehicle Platooning with Sporadic Measurements



The presence of the network leads to network imperfections such as<sup>1</sup>:

## Question:

How do we deal with these phenomena in our model?

<sup>1</sup>W.P.M.H. Heemels, A. R. Teel, N. Van de Wouw, and D. Nešić. Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance. IEEE Transactions on Automatic Control, 55(8):1781–1796, 2010.

# Vehicle Model with Networked Sporadic Measurements

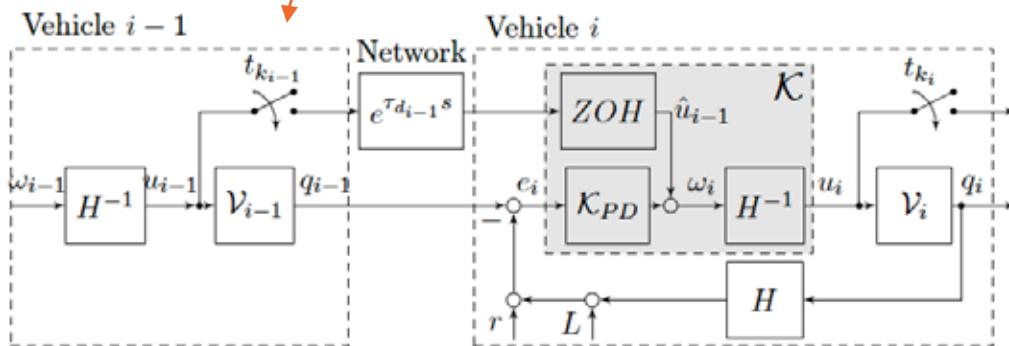
Controller Dynamics:

$$\dot{u}_i = -\frac{1}{h}u_i + \frac{1}{h}(k_p e_i + k_d \dot{e}_i + u_{i-1})$$

on-board sensors

V2V

sporadic measurements



## Assumptions

maximum allowable transmission interval

1. Variable transmission intervals:

$$\delta_T \leq t_{k_{i-1}+1} - t_{k_{i-1}} < T_{mati}$$

2. Variable network delays

$$0 \leq \tau_{d_{k_i}} \leq T_{mad} \leq T_{mati}$$

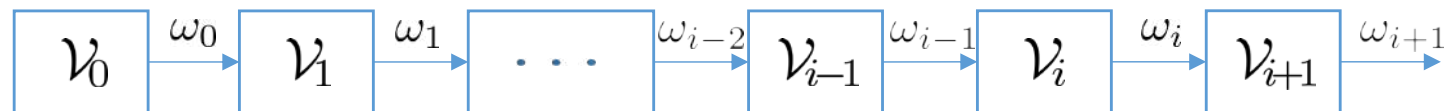
maximum allowable delay

Ensures transmitted output is received by controller before next sample is sent

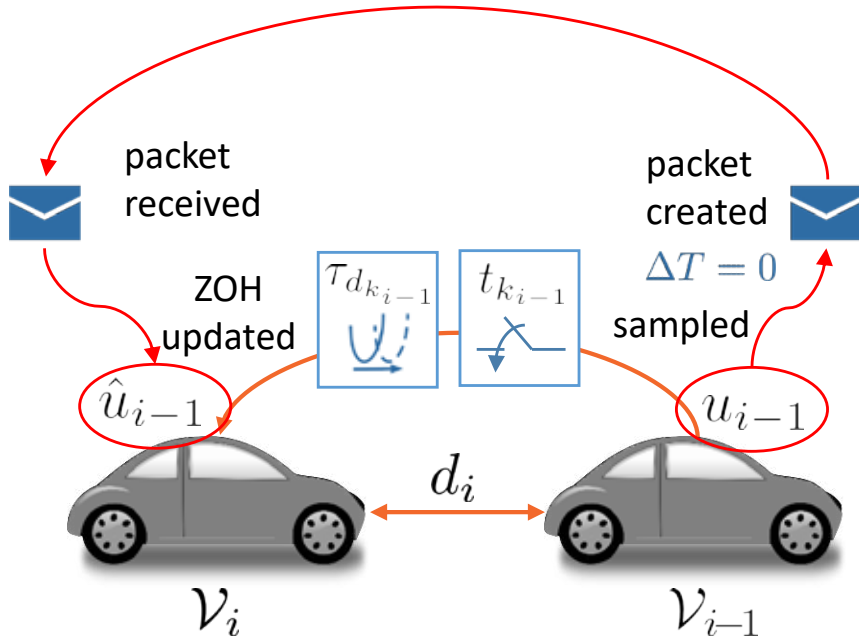
The state of the error dynamics is modified such that the whole platoon is considered as a cascade of dynamical systems.

Benefits in achieving **by design**

- String stability
- Performance



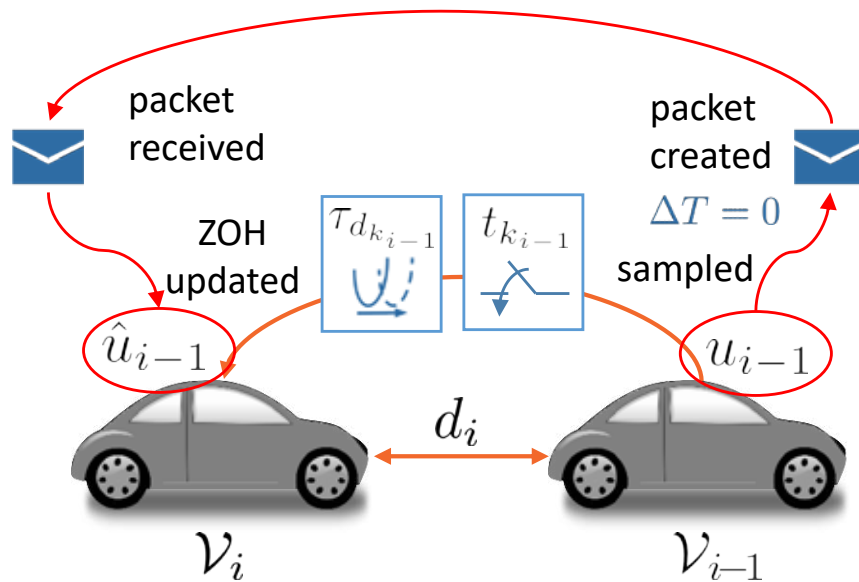
# Vehicle Model with Networked Sporadic Measurements



The controller relies on  $u_{i-1}$  is sampled and sent throughout the vehicle network

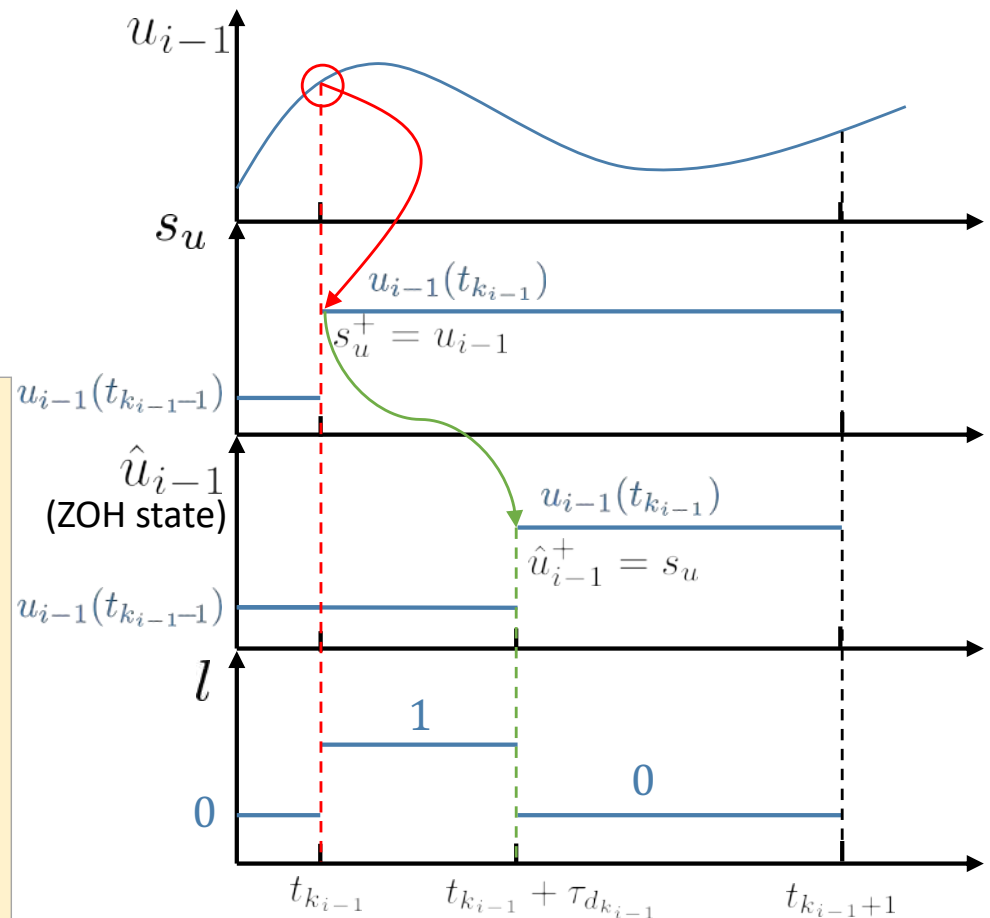
Each vehicle can be represented as linear system with jumps

$$\mathcal{V}_i \left\{ \begin{array}{l} \dot{e}_i(t) = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{k_p}{\tau_d} & -\frac{k_d}{\tau_d} & -\frac{1}{\tau_d} \end{bmatrix}}_{A_e} e_i(t) + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_d} \end{bmatrix} u_{i-1}(t) + \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{\tau_d} \end{bmatrix} \hat{u}_{i-1}(t) \\ \dot{\hat{u}}_{i-1}(t) = -\frac{1}{h} u_{i-1}(t) + \frac{1}{h} \omega_{i-1}(t) \\ \hat{u}_{i-1}(t) = 0 \\ \omega_{i-1}(t) = k_p e_{i-1}(t) + k_d \dot{e}_{i-1}(t) + u_{i-2}(t) \\ \begin{cases} e_i(t^+) = e_i(t) \\ u_{i-1}(t^+) = u_{i-1}(t) \\ \hat{u}_{i-1}(t^+) = u_{i-1}(t) \end{cases} \\ \omega_i(t) = k_p e_i(t) + k_d \dot{e}_i(t) + u_{i-1}(t) \end{array} \right. \quad \begin{array}{l} \forall t \neq t_{k_{i-1}} \quad \forall t_{k_{i-1}} \in \bigcup_{n \in \mathbb{N}} H_n \\ \forall t = t_{k_{i-1}} \quad \wedge t_{k_{i-1}} \notin \bigcup_{n \in \mathbb{N}} H_n \end{array}$$

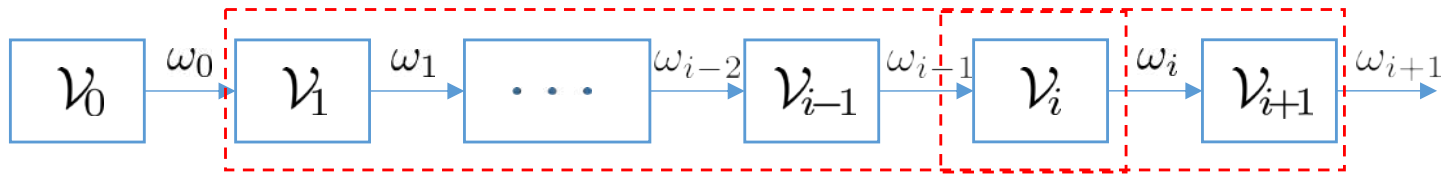


## Modelling Characteristics:

- Platooning as cascade of dynamical systems
- We follow the Hybrid Systems framework proposed by R. Goebel, R. Sanfelice, A. Teel<sup>1</sup>
- State space of the model enlarged with the auxiliary variables subject to reset:
  - $\hat{u}_{i-1}, s_u, l$
  - $\tau_{i-1}$  for time-based network trigger



## String stability



$$\frac{\|\omega_0\|_{\mathcal{L}_2}}{\|\omega_{i-1}\|_{\mathcal{L}_2}} \leq 1 \quad \Longrightarrow \quad \mathcal{L}_2 \text{ stability of } \mathcal{V}_i \quad \frac{\|\omega_i\|_{\mathcal{L}_2}}{\|\omega_{i-1}\|_{\mathcal{L}_2}} \leq 1$$

**Definition.** The vehicle platooning is said to be string stable if the systems  $\mathcal{V}_i$  in the platoon are  $\mathcal{L}_2$ -stable from the input  $\omega_{i-1} \in \mathcal{L}_2$  to the output  $\omega_i \in \mathcal{L}_2$  with an  $\mathcal{L}_2$ -gain less than or equal to one.

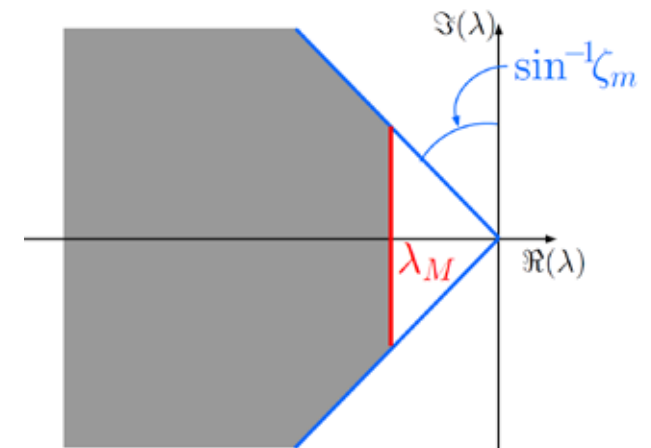
## Individual vehicle stability ( $\lim_{t \rightarrow \infty} e_i(t) = 0$ )

“Networked-free” error dynamics with performance requirements

$$\mathbb{P}(\lambda_M, \zeta_m) := \{A \in \mathbb{R}^{n \times n} \mid \Lambda_{\max}(A) = \lambda_M, \zeta_{\min}(A) \geq \zeta_m\}$$

$$\lambda_M < 0, \zeta_m \in (0, 1]$$

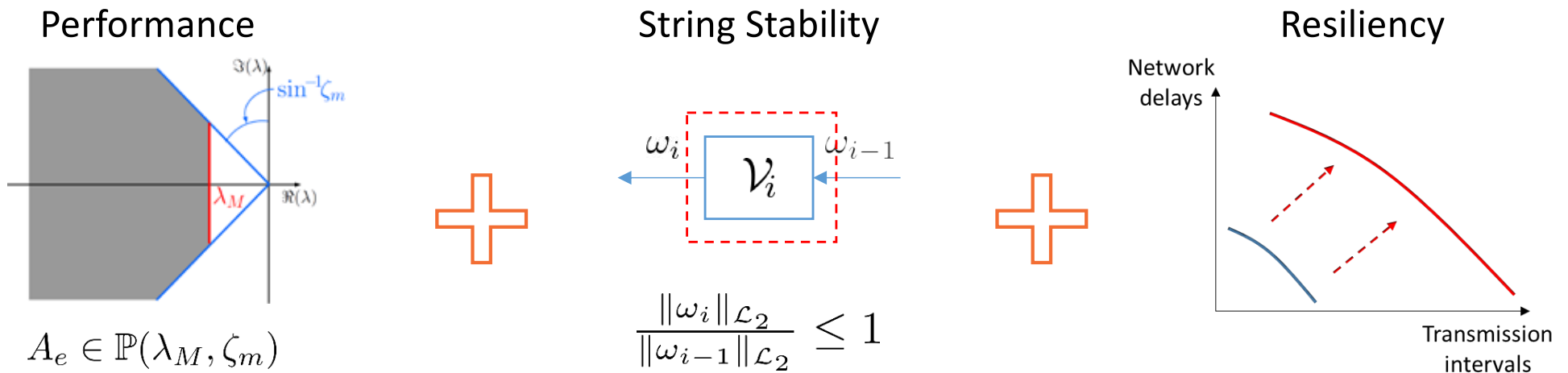
$$A_e = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{k_p}{\tau_d} & -\frac{k_d}{\tau_d} & -\frac{1}{\tau_d} \end{bmatrix} \in \mathbb{P}(\lambda_M, \zeta_m)$$

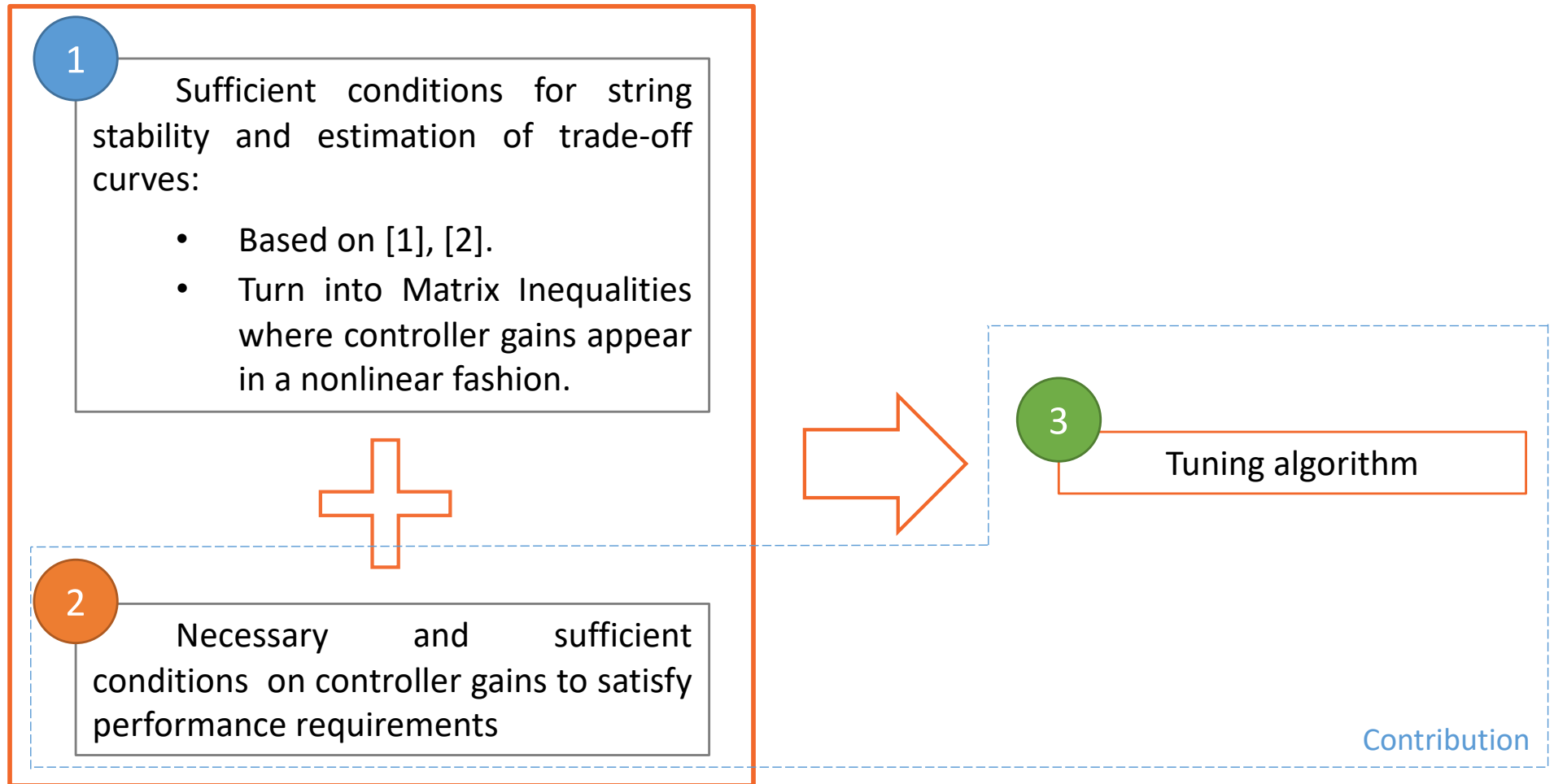


**Problem.** Given the platooning parameters  $h$  and  $\tau_d$ , and the performance requirements  $\mathbb{P}$ , design gains  $k_p$  and  $k_d$  for the hybrid controller such that the vehicle platooning satisfies the following properties with the largest achievable value of  $T_{mati}$  and  $T_{mad}$ :

(P1) Individual vehicle stability with performance  $\mathbb{P}$ , i.e.,  $A_e \in \mathbb{P}$ .

(P2) String stability.





[1] W. M. H. Heemels, A. R. Teel, N. Van de Wouw, and D. Nesić, "Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance," *IEEE Transactions on Automatic Control*, vol. 55, no. 8, pp. 1781–1796, 2010.

[2] V. S. Dolk, J. Ploeg, and W. M. H. Heemels, "Event-triggered control for string-stable vehicle platooning," *IEEE Transactions on Intelligent Transportation Systems*, vol. 18, no. 12, pp. 3486–3500, 2017.

1

**Assumption 1.** There exist constants  $\gamma, \epsilon \in \mathbb{R}_{\geq 0}, \mu \in \mathbb{R}_{> 0}$  and  $P = P^\top > 0$  such that

$$\begin{bmatrix} \text{He}(PA_{11}) + A_{21}^\top A_{21} + \mu C^\top C & PA_{12} + \mu C^\top D & A_{21}^\top A_{23} + PA_{13} \\ \bullet & \mu D^\top D - \gamma^2 & 0 \\ \bullet & \bullet & A_{23}^\top A_{23} - \mu(1 + \epsilon) \end{bmatrix} < 0$$

Nonlinear in controller gains

$\mathcal{L}_2$  gain from network imperfection to error dynamics

$$\gamma \downarrow \Rightarrow T_{mati} \uparrow, T_{mad} \uparrow$$

**Assumption 2.** There exists a pair  $(T_{mati}, T_{mad})$  such that

$$\begin{aligned} \gamma_1 \phi_1(\tau_{i-1}) &\geq \gamma_0 \phi_0(\tau_{i-1}), \quad \forall \tau_{i-1} \in [0, T_{mad}] \\ \gamma_0 \phi_0(\tau_{i-1}) &\geq \lambda^2 \gamma_1 \phi_1(0), \quad \forall \tau_{i-1} \in [0, T_{mati}] \end{aligned}$$

For the computation of trade-off curves

with  $T_{mati} \geq T_{mad} \geq 0, \lambda \in (0, 1)$ , constants  $\gamma_0 := \gamma$  and  $\gamma_1 := \frac{\gamma}{\lambda}$ , and where  $\phi_{l_{i-1}} : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}, l_{i-1} \in \{0, 1\}$  is the unique maximal solution to

$$\dot{\phi}_{l_{i-1}} = -\gamma_{l_{i-1}}(\phi_{l_{i-1}}^2 + 1)$$

with initial conditions  $\phi_1(0) \geq \phi_0(0) \geq \lambda^2 \phi_1(0), \phi_0(T_{mati}) \geq 0$ .

**Theorem.** Let Assumption 1 with  $\epsilon = 0$  and Assumption 2 hold. Then, hybrid system  $\mathcal{V}_i$  is  $\mathcal{L}_2$ -stable from the input  $\omega_{i-1}$  to the output  $\omega_i$  with an  $\mathcal{L}_2$ -gain less than or equal to one.

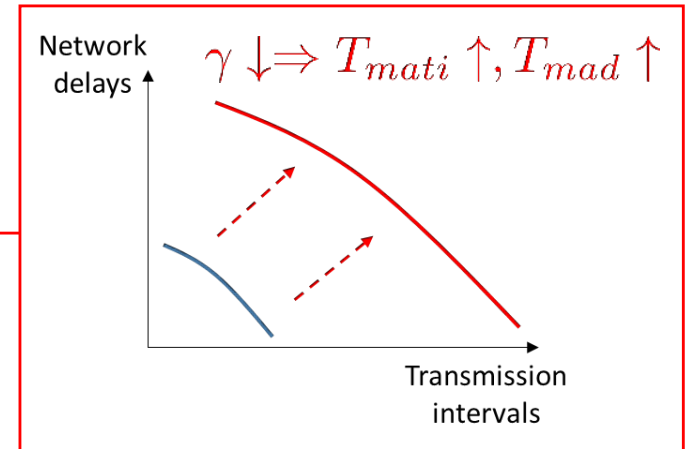
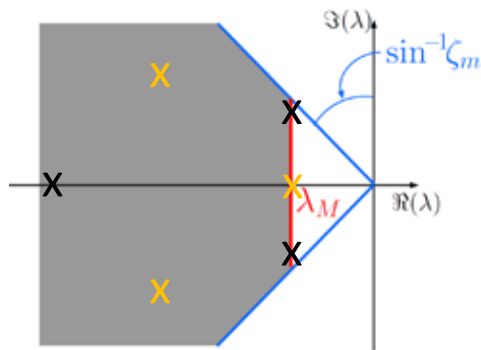
Based on:

- [1] W. M. H. Heemels, A. R. Teel, N. Van de Wouw, and D. Nesić, "Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance," IEEE Transactions on Automatic Control, vol. 55, no. 8, pp. 1781–1796, 2010.
- [2] V. S. Dolk, J. Ploeg, and W. M. H. Heemels, "Event-triggered control for string-stable vehicle platooning," IEEE Transactions on Intelligent Transportation Systems, vol. 18, no. 12, pp. 3486–3500, 2017.

Suff. Cond. for string stability and estimation of trade-off curves are based on the solution of the matrix inequality :

$$\begin{bmatrix} \text{He}(PA_{11}) + A_{21}^\top A_{21} + \mu C^\top C & PA_{12} + \mu C^\top D & A_{21}^\top A_{23} + PA_{13} \\ \bullet & \mu D^\top D - \gamma^2 & 0 \\ \bullet & \bullet & A_{23}^\top A_{23} - \mu(1 + \epsilon) \end{bmatrix} < 0$$

Nonlinear in controller parameters.  
Proposed solution:



**Condition (C1).** The real eigenvalue is equal to  $\lambda_M$ .



$$k_d = f_{C1}(k_p) \\ k_p \in [\underline{k}_{pC1}, \bar{k}_{pC1}]$$

**Condition (C2).** Single couple of complex eigenvalues with real part equal to  $\lambda_M$ .



$$k_d = f_{C2}(k_p) \\ k_p \in (\underline{k}_{pC2}, \bar{k}_{pC2}]$$

2

**Proposition 1** (N.S.C. for C1). Let  $k_p, k_d \in \mathbb{R}$ ,  $\lambda_M \in \mathbb{R}_{<0}$ , and  $\zeta_m \in \mathbb{R}_{>0}$ . Then, C1 is satisfied if and only if the following conditions hold:

$$k_d = f_{C1}(k_p) := -\frac{1}{\lambda_M}k_p - \lambda_M^2\tau_d - \lambda_M$$

$$k_p \leq \frac{|\lambda_M|(\lambda_M\tau_d + 1)^2}{4\tau_d\zeta_m^2} := \bar{k}_{pC1}$$

$$k_p \geq 2\tau_d\lambda_M^3 + \lambda_M^2 := \underline{k}_{pC1}$$

$$\lambda_M > -\frac{1}{3\tau_d}$$

**Proposition 2** (N.S.C. for C2). Let  $k_p, k_d \in \mathbb{R}$ ,  $\lambda_M \in \mathbb{R}_{<0}$ , and  $\zeta_m \in (0, 1)$ . Then, C2 holds if and only if the following conditions hold:

$$k_d = f_{C2}(k_p) := -\frac{8\lambda_M^3\tau_d^2 + 8\lambda_M^2\tau_d + 2\lambda_M - \tau_d k_p}{2\lambda_M\tau_d + 1}$$

$$k_p \leq \frac{2\tau_d\lambda_M^3 + \lambda_M^2}{\zeta_m^2} := \bar{k}_{pC2}$$

$$k_p > 2\tau_d\lambda_M^3 + \lambda_M^2 := \underline{k}_{pC2}$$

$$\lambda_M > -\frac{1}{3\tau_d}$$

Number of control variables is reduced

Performances are constrained by powertrain time constant

3

Design goal

Resiliency Metric  $\gamma \downarrow \Rightarrow T_{mati} \uparrow, T_{mad} \uparrow$

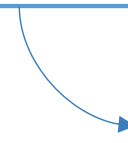
minimize  $\gamma, P, k_p, k_d, \mu$   
subject to

$\gamma$

Performance  $\mathbb{P}$ , Matrix Inequalities

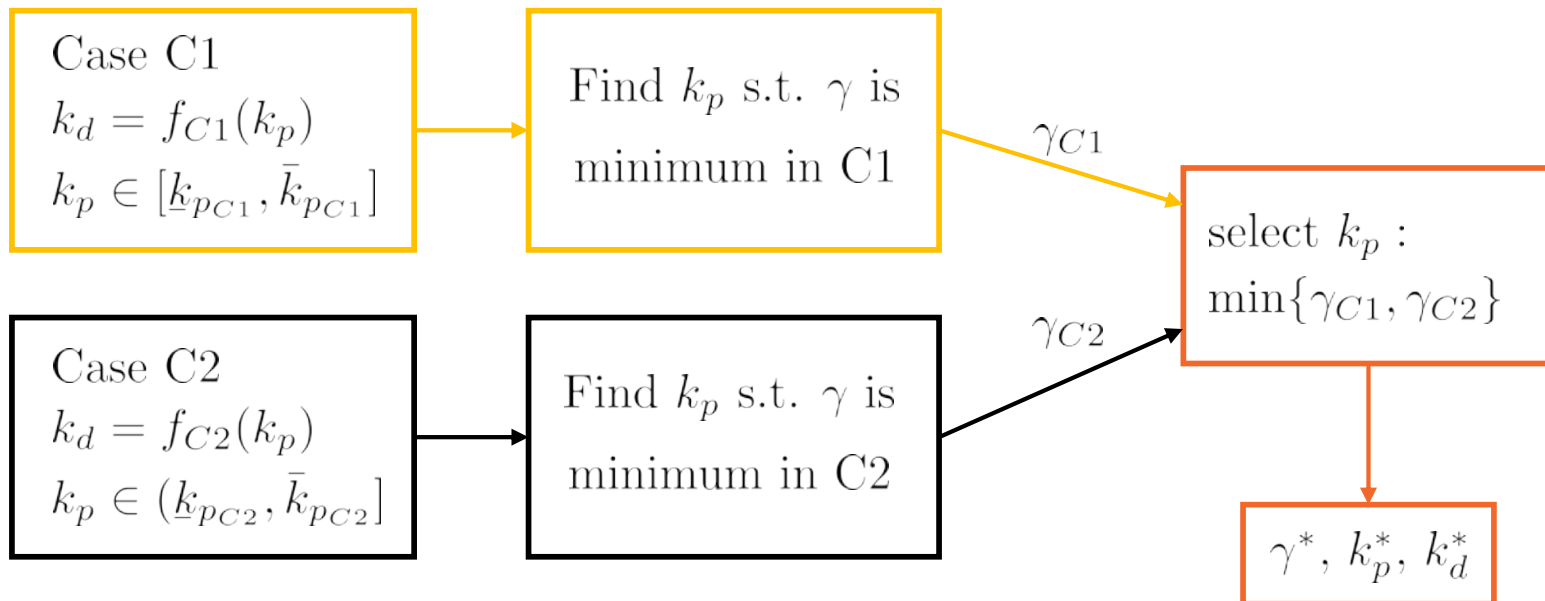


Performance Metric



Lyapunov-Based conditions

Design Algorithm

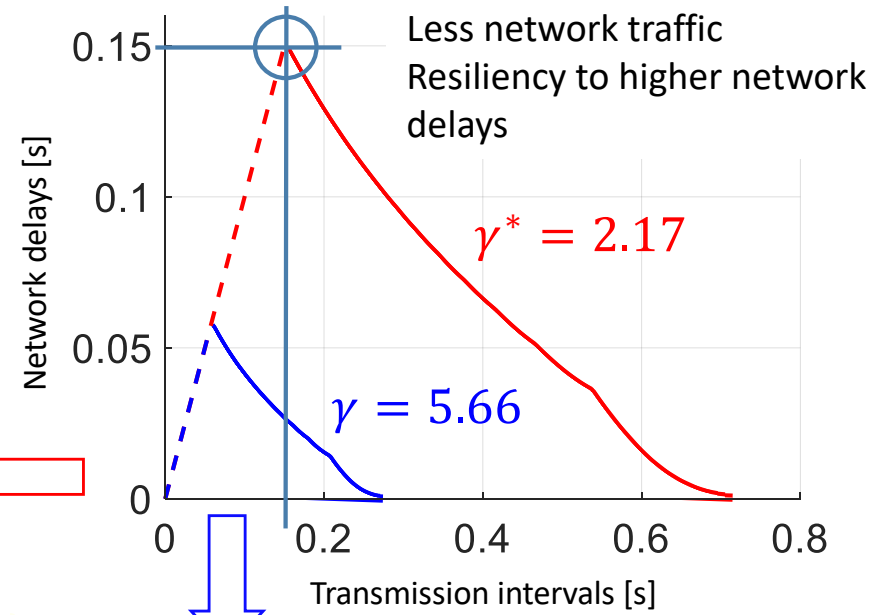
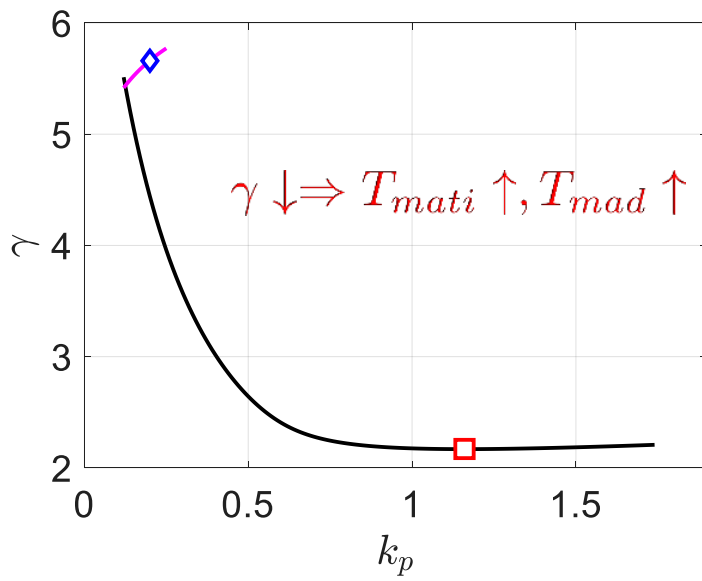


# Numerical Results

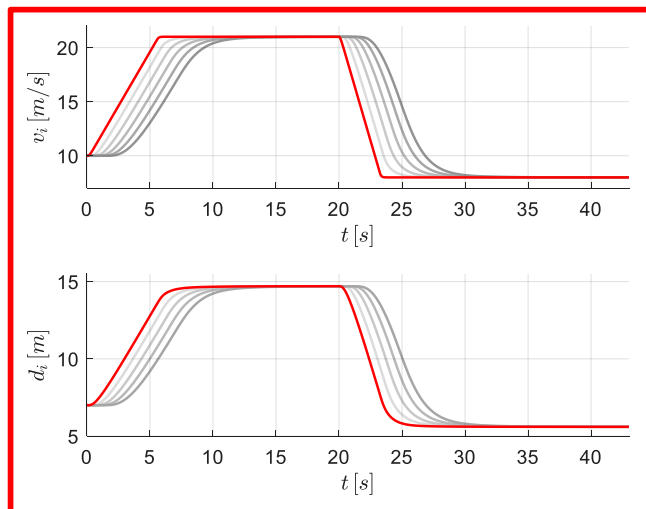


◇ Tuning as in Ploeg et al.

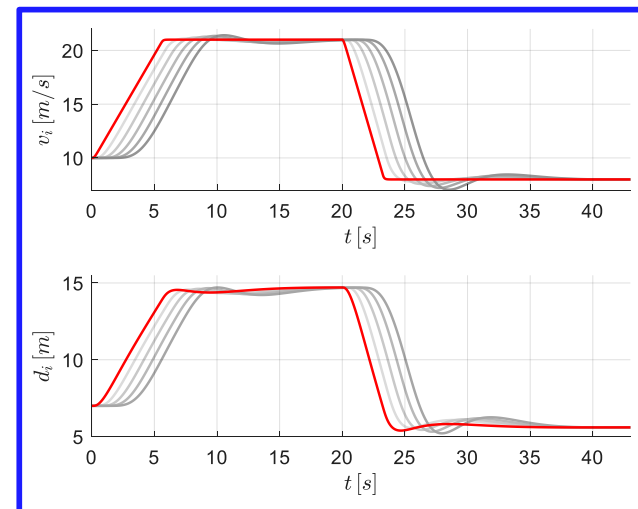
□ Our approach



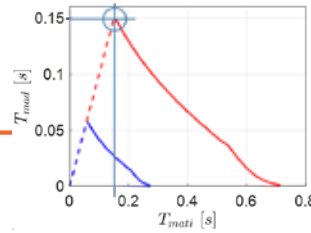
Our approach  
Network-resilient tuning



Tuning as in Ploeg et al.  
Non network-resilient tuning

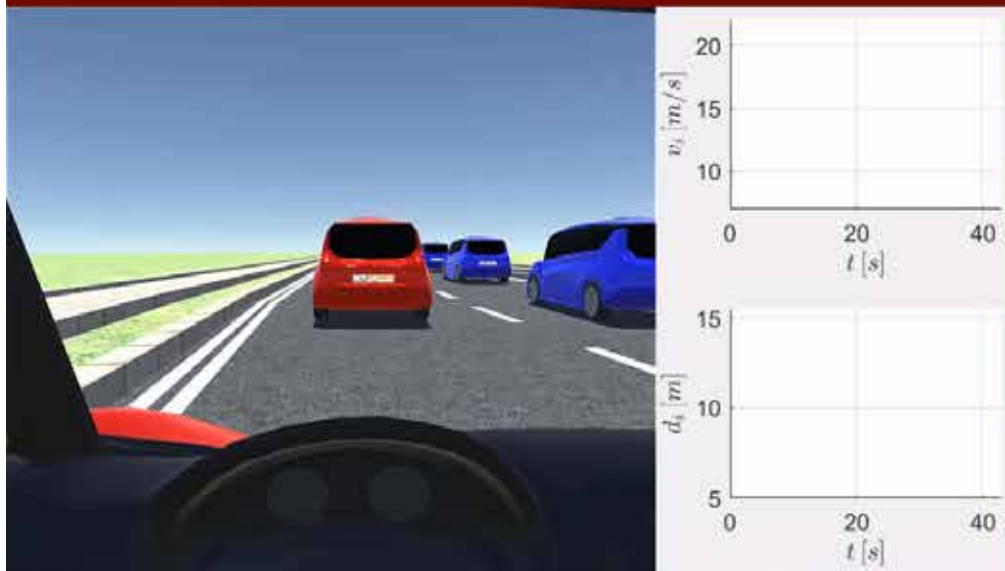
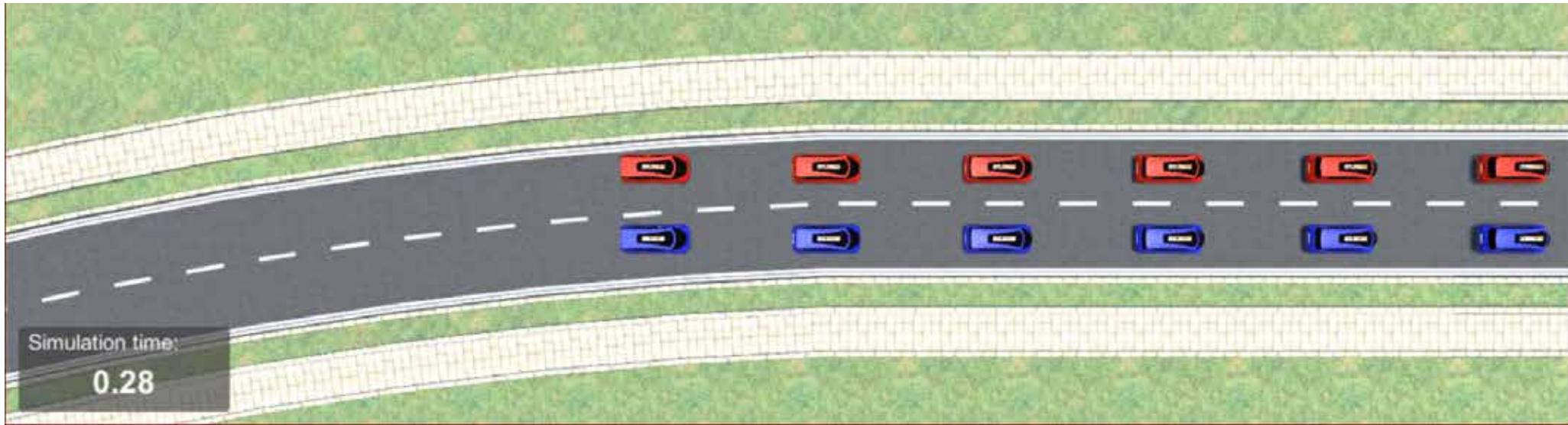


# Numerical Results

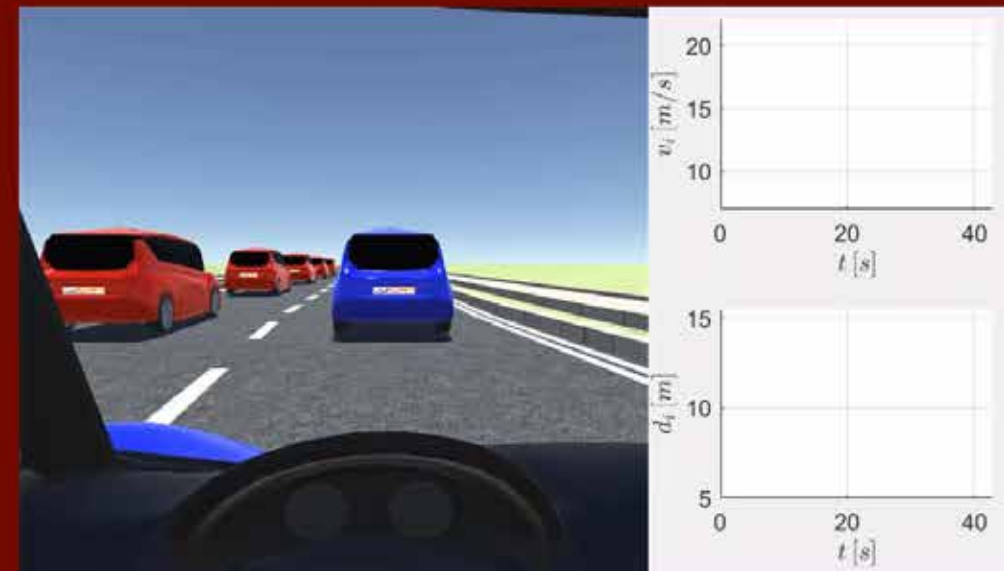


Left lane, in red, network-resilient tuning

Right lane, in blue, non network-resilient tuning



Inside last vehicle in the left lane  
network-resilient tuning



Inside last vehicle in the right lane  
non network-resilient tuning



## Resilient CACC Under Denial-of-Service Attacks

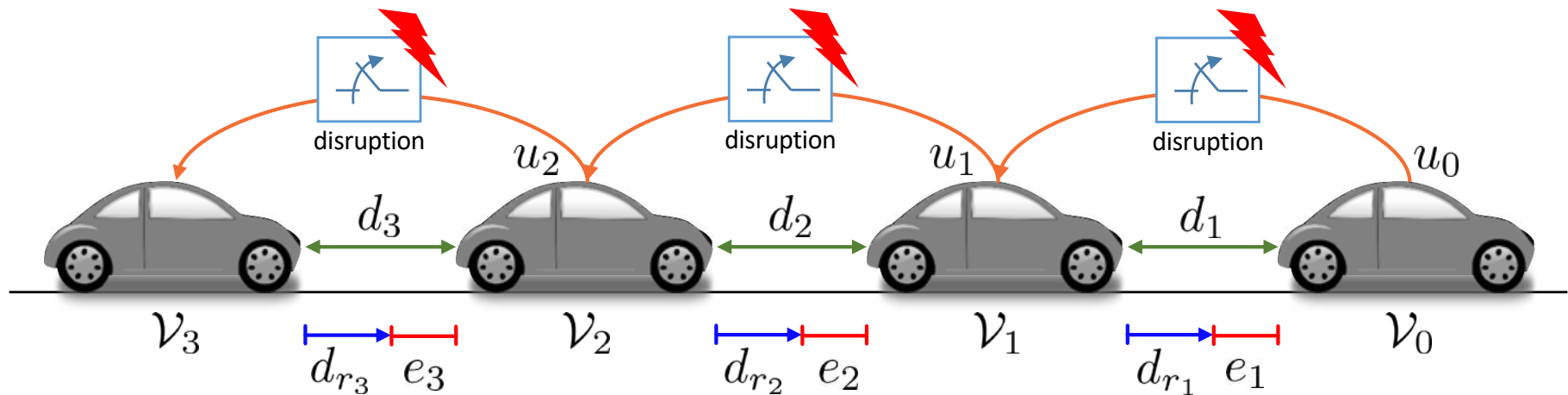
# Vehicle Platooning under Denial-of-Service (DoS) Attack



Radio jamming with unknown strategy<sup>1</sup>



Energy<sup>2</sup> and geography constrained<sup>1</sup>



The attacker generates DoS with the purpose of disrupting the network for the longest time possible as possible.

## CACC Pros:

Closer inter-vehicle distance:

- Traffic throughput
- Fuel economy

## CACC Cons:

String Stability influenced by:

- Network unreliability
- Cyber Attacks

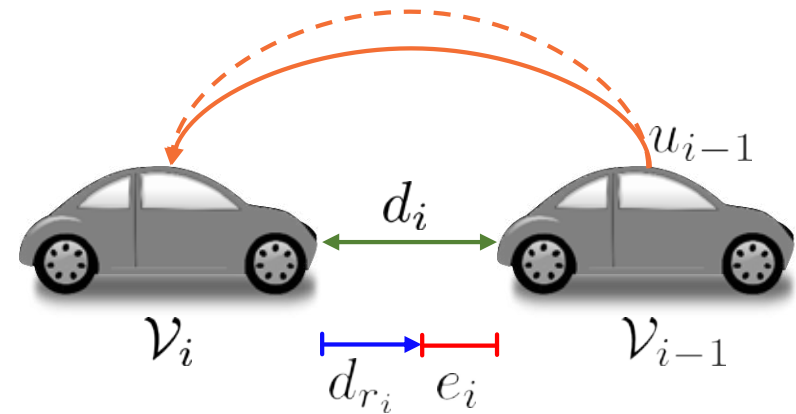
<sup>1</sup>M. Amoozadeh, A. Raghuramu, C.-N. Chuah, D. Ghosal, H. M. Zhang, J. Rowe, and K. Levitt. Security vulnerabilities of connected vehicle streams and their impact on cooperative driving. *IEEE Communications Magazine*, 53(6):126–132, 2015.

<sup>2</sup>V. S. Dolk, P. Tesi, C. De Persis, and W.P.M.H. Heemels. Event-triggered control systems under denial-of-service attacks. *IEEE Transactions on Control of Network Systems*, 4(1):93–105, 2017.

Adaptive Cruise Control (ACC)  
no communication

Estimation-based CACC<sup>1</sup>  
action is estimated by on-board sensors

Switching Strategy CACC/ACC<sup>2</sup>  
control strategy ACC and CACC switched  
over time based on communication status



Inter-vehicle time gaps without CACC  
**are larger than**  
inter-vehicle time gaps with CACC



Trade off between safety and  
performance is needed

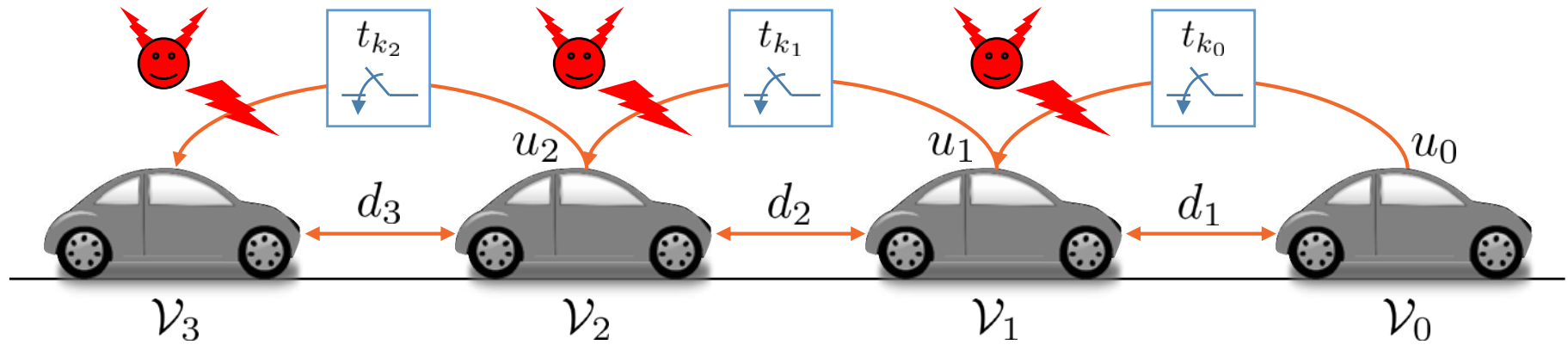
## Research questions:

1. What network unreliability and DoS attacks can we tolerate without the need to fall back to safe strategies?
2. How to design a CACC that maximize the resilience to network unreliability and DoS attack?

<sup>1</sup>J. Ploeg, E. Semsar-Kazerooni, G. Lijster, N. van de Wouw, and H. Nijmeijer. Graceful degradation of cooperative adaptive cruise control. IEEE Transactions on Intelligent Transportation Systems, 2014

<sup>2</sup>Y. A. Harfouch, S. Yuan, and S. Baldi. An adaptive switched control approach to heterogeneous platooning with intervehicle communication losses. IEEE Transactions on Control of Network Systems, 2017.

# Vehicle Platooning with DoS attacks



Controller Dynamics:

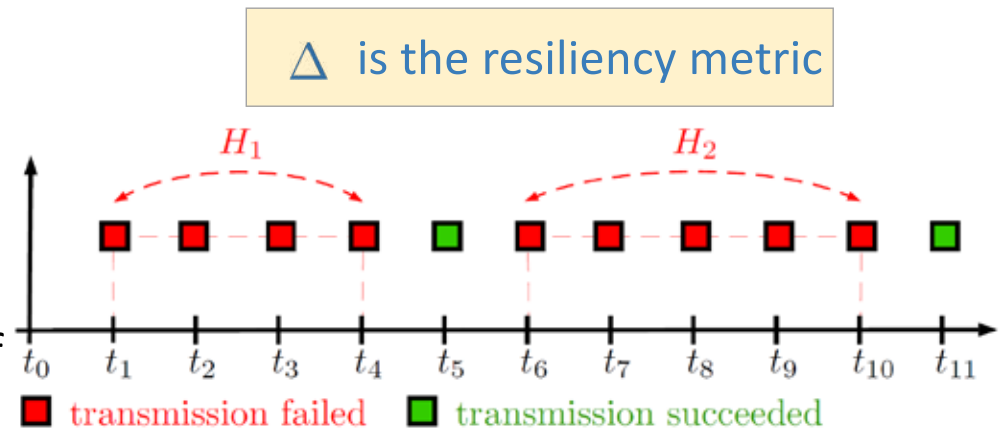
$$\dot{u}_i = -\frac{1}{h}u_i + \frac{1}{h}(k_p e_i + k_d \dot{e}_i + u_{i-1})$$

on-board sensors
V2V

Available in sporadic fashion: data shared with period  $T_s$

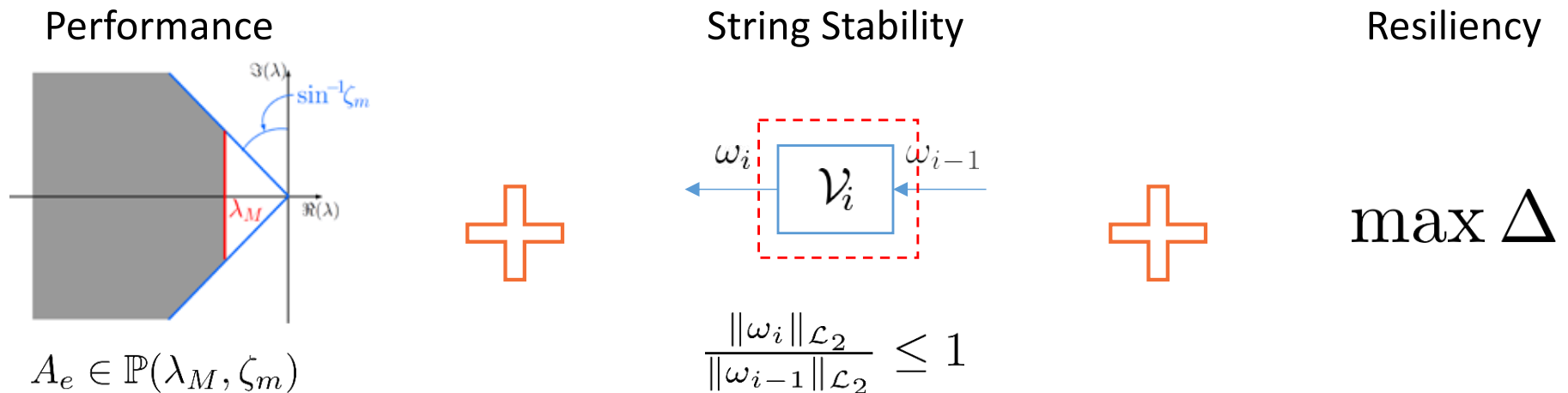
DoS attacks as a sequence of intervals  $\{H_n\}_{n \in \mathbb{N}}$  of limited length bounded by  $\Delta \in \mathbb{N}_0$

$\Delta \in \mathbb{N}_0$  is the maximum allowable number of successive packet dropouts (MANSD)



**Problem.** Given the platooning parameters  $h$  and  $\tau_d$ , and the performance requirements  $\mathbb{P}$ , design gains  $k_p$  and  $k_d$  for the hybrid controller such that the vehicle platooning satisfies the following properties with the largest achievable value of  $\Delta$ :

- (P1) Individual vehicle stability with performance  $\mathbb{P}$ , i.e.,  $A_e \in \mathbb{P}$ .
- (P2) String stability.



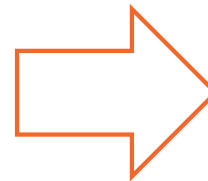
1

Necessary and Sufficient  
Conditions on controller gains to  
satisfy performance requirements



2

S.C. for string stability and  
estimation of  $\Delta$  :  
Matrix Inequalities where controller  
gains appear in a nonlinear fashion.



3

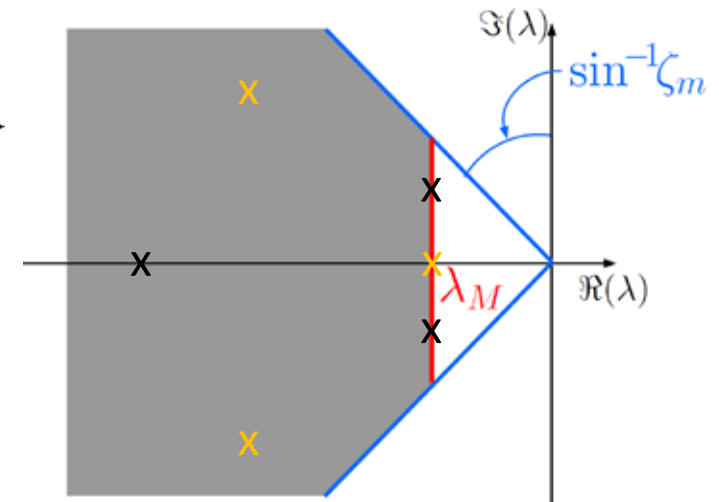
Tuning algorithm

1

To satisfy performance

$$\mathbb{P}(\lambda_M, \zeta_m) := \{A \in \mathbb{R}^{n \times n} \mid \Lambda_{\max}(A) = \lambda_M, \zeta_{\min}(A) \geq \zeta_m\}$$

$$A_e = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{k_p}{\tau_d} & -\frac{k_d}{\tau_d} & -\frac{1}{\tau_d} \end{bmatrix} \in \mathbb{P}(\lambda_M, \zeta_m)$$



**Condition (C1).** The real eigenvalue is equal to  $\lambda_M$  and, the other two eigenvalues have real part less than or equal to  $\lambda_M$  with damping ratio greater than  $\zeta_m$ .

**Condition (C2).** The spectrum of  $A_e$  is characterized by a single couple of complex eigenvalues with real part equal to  $\lambda_M$  and damping ratio greater than  $\zeta_m$ , and the other real eigenvalue is less than  $\lambda_M$ .

2

**Lemma.** Let  $P_1 \in \mathcal{S}_+^4$ ,  $p_2$ ,  $\delta$ ,  $\tau_d$ ,  $h$ , and  $T_s$  be given positive real number,  $\Delta \in \mathbb{N}_0$ , and  $k_p$ , and  $k_d$  be given real numbers. For each  $\tau_{i-1} \in [0, (\Delta + 1)T_s]$ , define  $\mathcal{M} : \tau_{i-1} \mapsto \mathcal{M}(\tau_{i-1})$ . Then,  $\text{rge}\mathcal{M} = \text{Co}\{\mathcal{M}(0), \mathcal{M}((\Delta + 1)T_s)\}$ . Therefore,  $\mathcal{M}(\tau_{i-1}) < 0, \forall \tau_{i-1} \in [0, (\Delta + 1)T_s]$  holds if and only if

$$\mathcal{M}(0) < 0, \quad \mathcal{M}((\Delta + 1)T_s) < 0$$

$$\mathcal{M}(\tau_{i-1}) = \begin{pmatrix} \text{He}(P_1 A_{xx}) + C_\omega^\top C_\omega & P_1 A_{x\eta} + C_\omega^\top + e^{-\delta\tau_{i-1}} p_2 A_{\eta x}^\top & P_1 A_{x\omega} \\ \bullet & -\delta p_2 e^{-\delta\tau_{i-1}} + 1 & -e^{-\delta\tau_{i-1}} p_2 / h \\ \bullet & \bullet & -1 \end{pmatrix} < 0, \forall \tau_{i-1} \in [0, (\Delta + 1)T_s]$$

$k_p, k_d, \delta$  fixed.

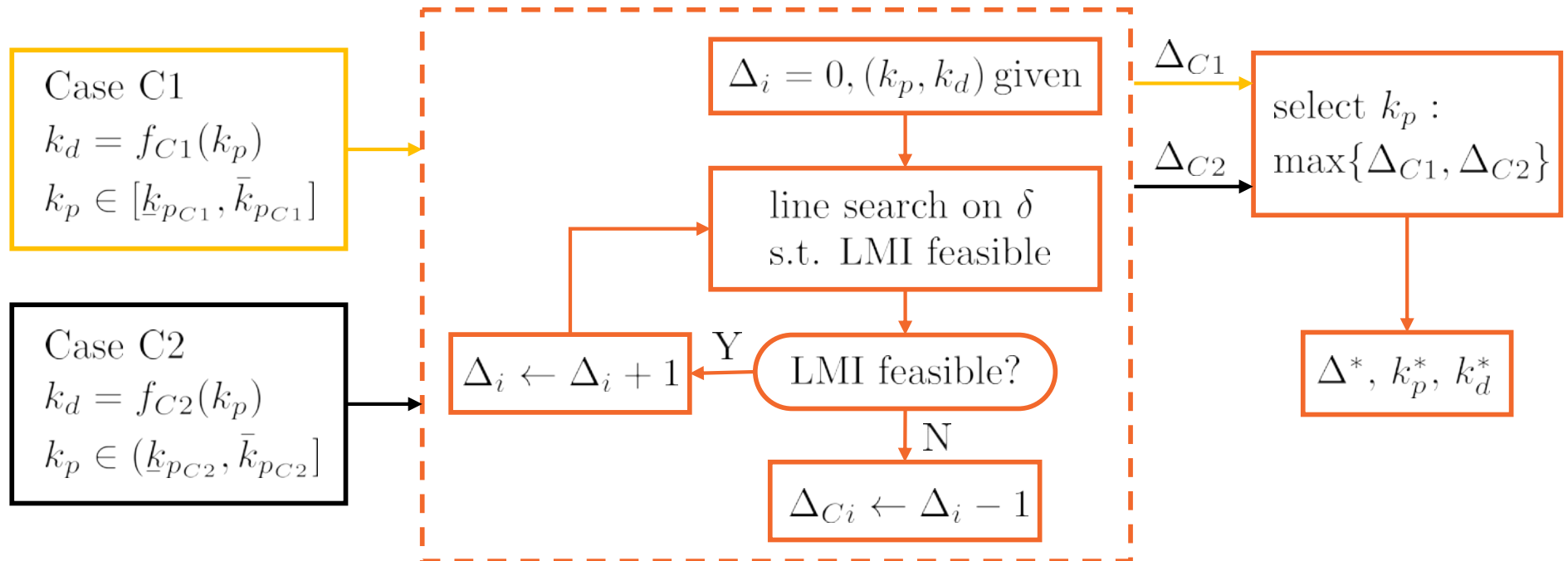
$k_d$  univocally determined by the choice of  $k_p$  via Conditions 1 and 2  
 $k_p$  gridding on an interval known a priori.

3

## Design goal

$$\begin{aligned} & \text{maximize} && \Delta \\ & P_1, p_2, \delta, k_p, k_d \\ & \text{subject to} && A_e \in \mathbb{P}, \mathcal{M}(0) < 0, \mathcal{M}((\Delta + 1)T_s) < 0 \end{aligned}$$

## Design Algorithm



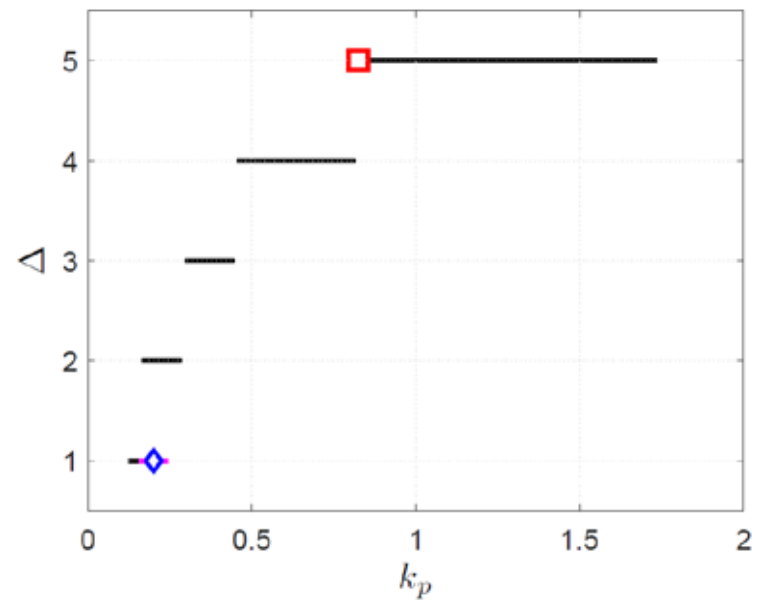
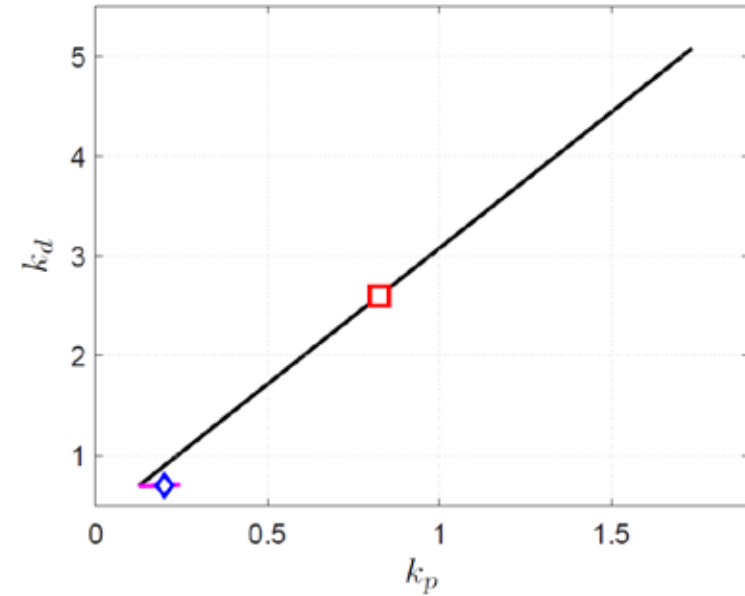
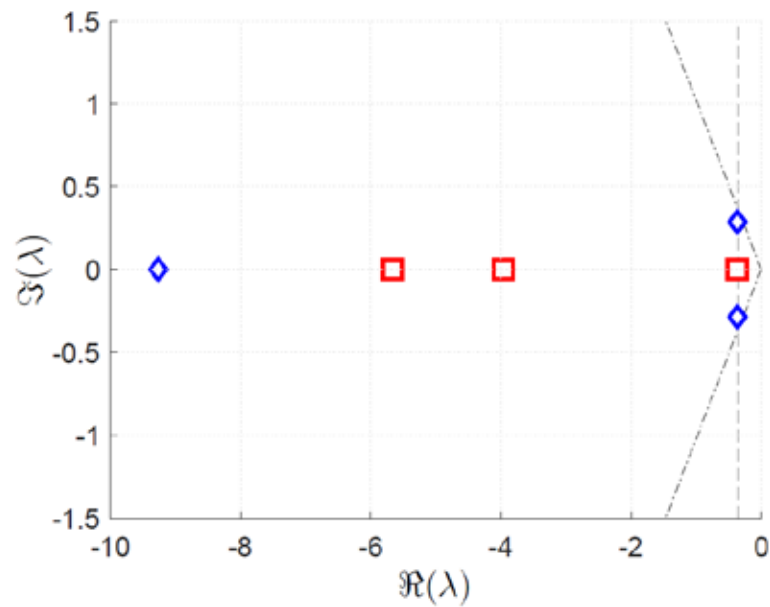


Tuning as in [Ploeg et al CITS '11]<sup>1</sup>

<sup>1</sup>J. Ploeg et al, "Design and experimental evaluation of cooperative adaptive cruise control," in IEEE ITSC, 2011.



Tuning as our approach

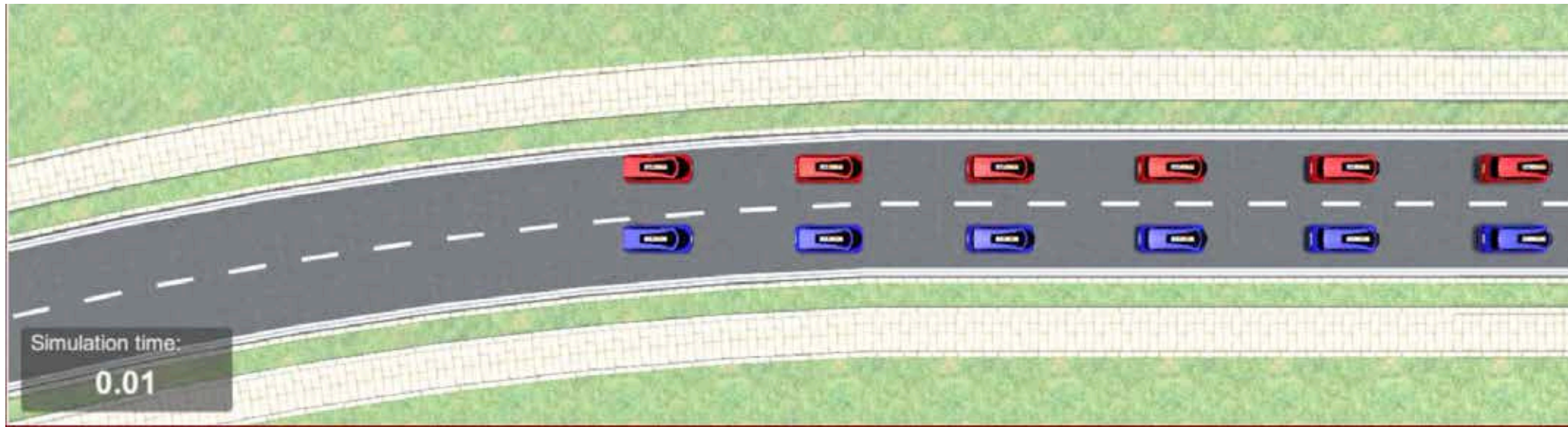


# Numerical Results – DoS

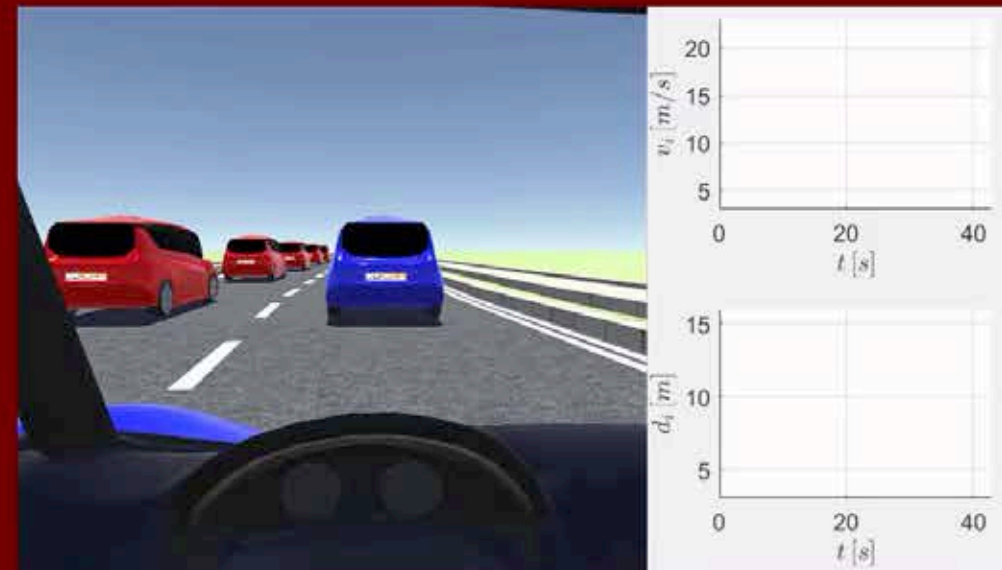


Left lane, in red, network-resilient tuning

Right lane, in blue, non network-resilient tuning



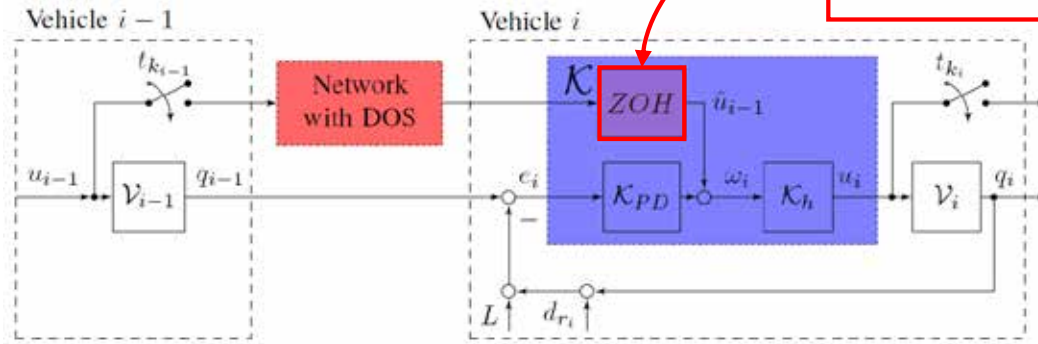
Inside last vehicle in the left lane  
network-resilient tuning



Inside last vehicle in the right lane  
non network-resilient tuning

## Zero-Order Hold

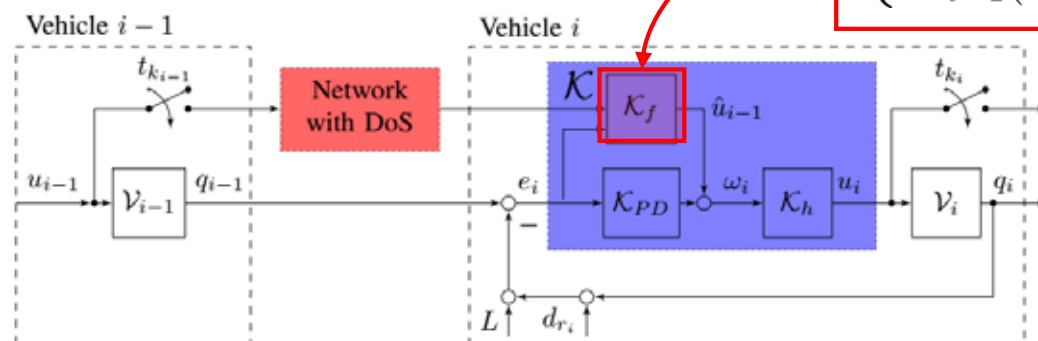
$$\begin{cases} \dot{\hat{u}}_{i-1}(t) = 0 & \forall t \neq t_{k_{i-1}} \vee \text{NoDoS} \\ \hat{u}_{i-1}(t^+) = u_{i-1}(t_{k_{i-1}}) & \forall t = t_{k_{i-1}} \wedge \text{DoS} \end{cases}$$

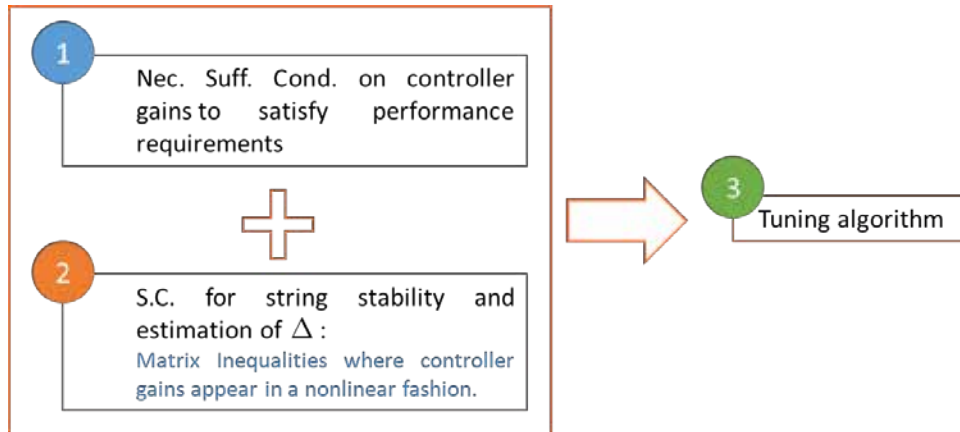


Can we enhance the ZOH by using continuous-time measurements in between communication updates?

## Intersample Dynamics

$$\begin{cases} \dot{\hat{u}}_{i-1}(t) = \gamma_p e_i + \gamma_d \dot{e}_i & \forall t \neq t_{k_{i-1}} \vee \text{NoDoS} \\ \hat{u}_{i-1}(t^+) = u_{i-1}(t_{k_{i-1}}) & \forall t = t_{k_{i-1}} \wedge \text{DoS} \end{cases}$$





Same design approach.

LMI conditions become design tool for intersample dynamics.

2

$$\mathcal{M}(\tau_{i-1}) = \begin{pmatrix} \text{He}(P_1 A_{xx}) + C_\omega^\top C_\omega & A_{x\eta} + C_\omega^\top + e^{-\delta\tau_{i-1}} \begin{bmatrix} \gamma_p p_2 & \gamma_d p_2 & 0 & p_2/h \end{bmatrix}^\top & P_1 A_{x\omega} \\ \vdots & \bullet & -p_2 e^{-\delta\tau_{i-1}}/h \\ \vdots & \bullet & -(1 + \epsilon) \end{pmatrix}$$

Change of variables

$$\gamma_p = \psi_p/p_2$$

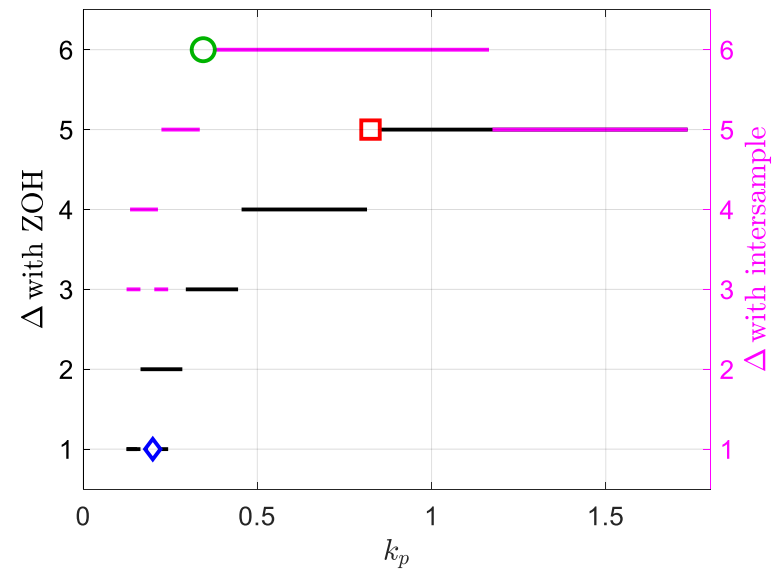
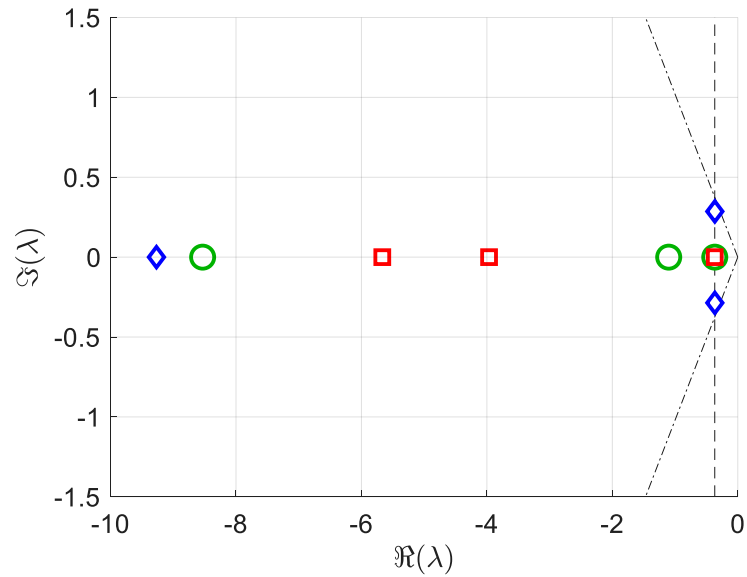
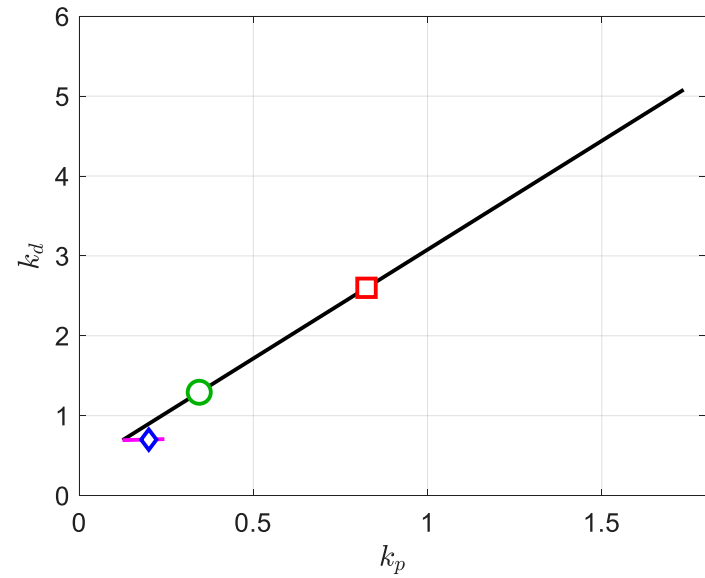
$$\gamma_d = \psi_d/p_2$$

$$\widehat{\mathcal{M}}(\tau_{i-1}) = \begin{pmatrix} \text{He}(P_1 A_{xx}) + C_\omega^\top C_\omega & A_{x\eta} + C_\omega^\top + e^{-\delta\tau_{i-1}} \begin{bmatrix} \psi_p & \psi_d & 0 & p_2/h \end{bmatrix}^\top & P_1 A_{x\omega} \\ \vdots & \bullet & -p_2 e^{-\delta\tau_{i-1}}/h \\ \vdots & \bullet & -(1 + \epsilon) \end{pmatrix}$$

# Numerical Results



- ◆ Tuning as in [Ploeg et al CITS '11] (with ZOH)
- Tuning as our approach with ZOH
- Tuning as our approach with Intersample



# Numerical Results – evaluation for different time gaps

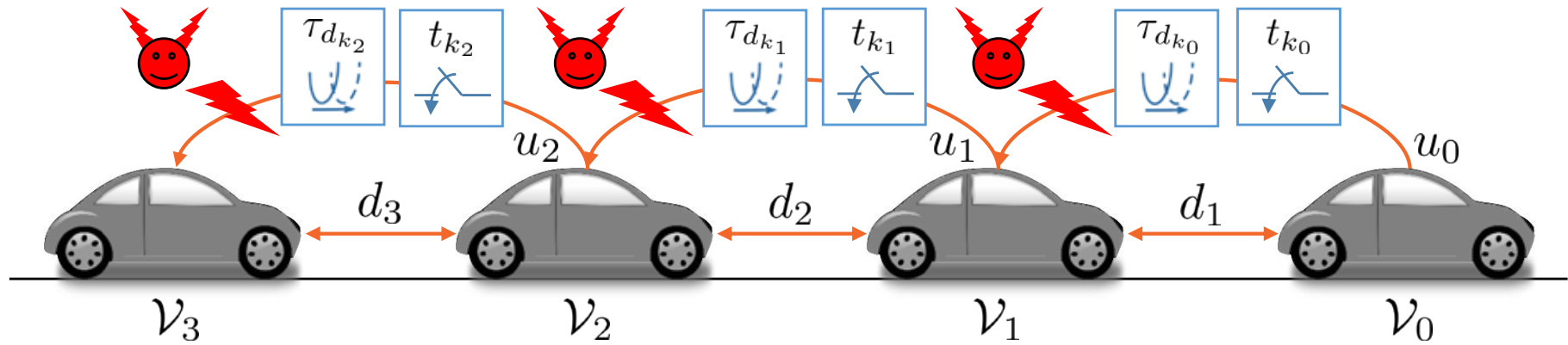


$h$ [s]	$\Delta$		$k_p$		$k_d$		$-\gamma_p$		$\gamma_d$	
	ZOH	INT	ZOH	INT	ZOH	INT	ZOH	INT	ZOH	INT
0.3	0	1	0.62	0.5	2.05	1.73	0	1.87	0	11.3
0.4	1	2	0.5	0.41	1.73	1.48	0	0.85	0	6.57
0.5	2	3	0.5	0.33	1.73	1.27	0	0.45	0	4.33
0.6	4	5	1.05	0.44	3.23	1.56	0	0.29	0	2.53
0.7	5	6	0.82	0.34	2.6	1.29	0	0.19	0	1.95
0.8	6	7	0.69	0.27	2.25	1.1	0	0.13	0	1.56
0.9	7	8	0.59	0.22	1.97	0.97	0	0.09	0	1.28
1	8	10	0.52	0.26	1.78	1.07	0	0.07	0	0.94
1.1	9	11	0.46	0.21	1.62	0.94	0	0.06	0	0.8
1.2	10	12	0.41	0.17	1.48	0.83	0	0.04	0	0.7

## Remarks:

- Increasing time gaps, resilience increases.
- CACC with intersample always guarantee higher resilience.
- Resilience gained by increasing derivative action.

# Adding Network Delays to the Resiliency Metric



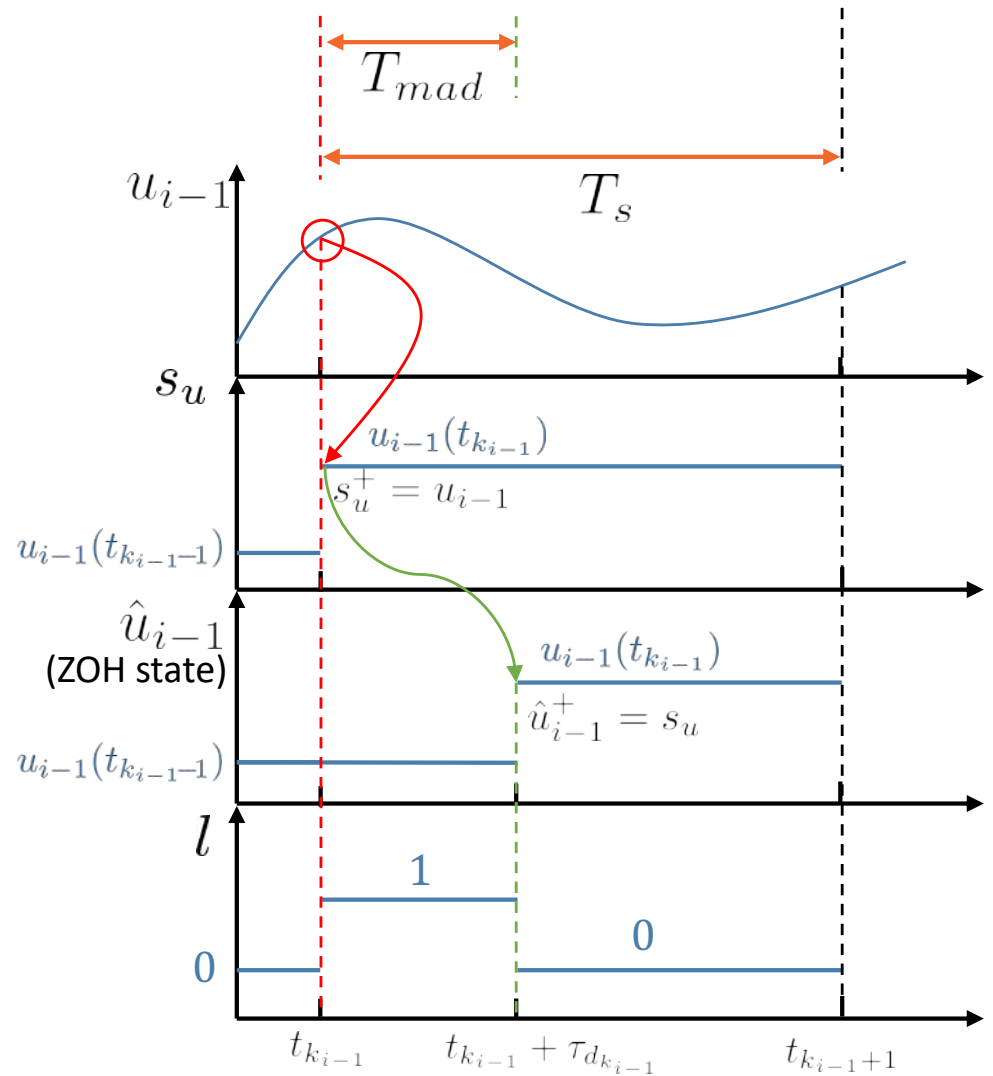
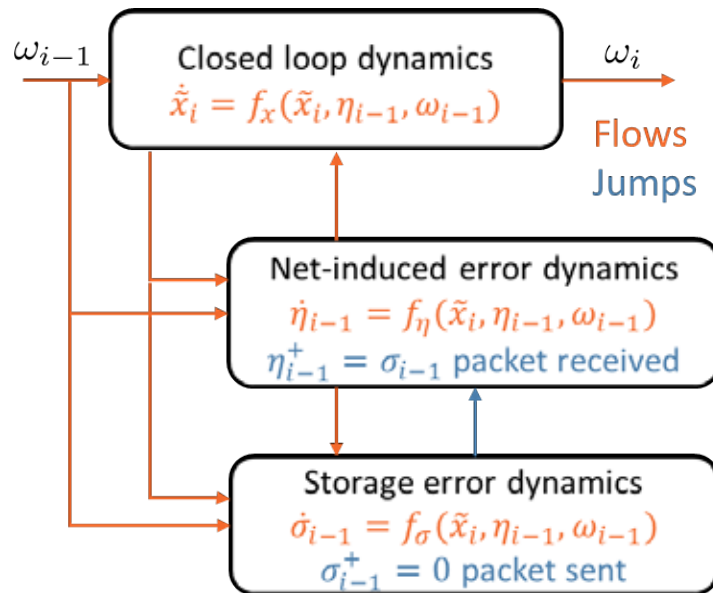
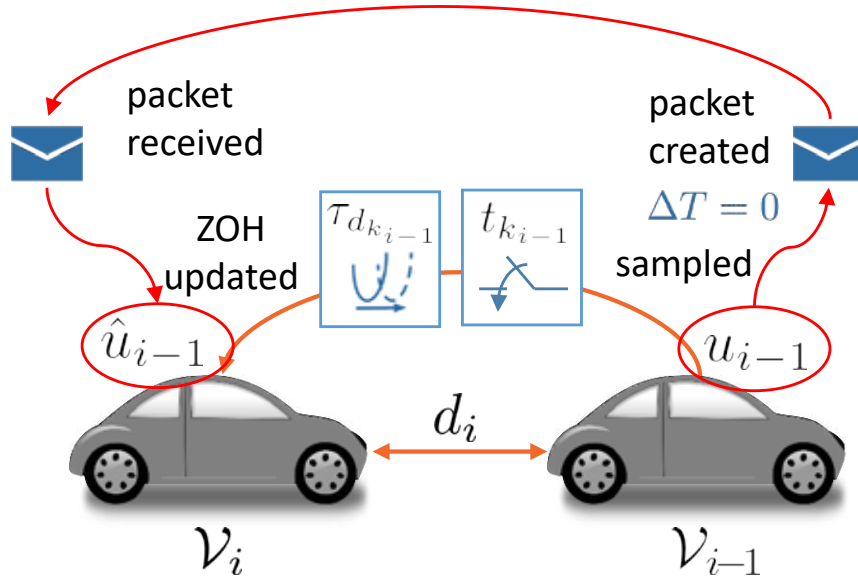
Assumption:

$$0 \leq \tau_{d_{k_i}} \leq T_{mad} \leq T_s$$

communication period
maximum allowable delay

- Packet disorder phenomena – forced dropouts for outdated information
- Packet dropouts and network delays captured in a simple and unified model
- Vehicle networks: communication frequency 10 to 20 Hz

# Modeling Network Delays



General results for NCS available at:

R. Merco, F. Ferrante, P. Pisu, "On DoS Resiliency Analysis of Networked Control Systems: Trade-off between Jamming Actions and Network Delays", IEEE Control Systems Letters, 2019

Sketch of the proof: based on Lyapunov theory for Hybrid Systems

$$V(x_i) = \underbrace{\tilde{x}_i^\top P_1 \tilde{x}_i}_{V_1(\tilde{x}_i)} + \underbrace{p_{2,l} \eta_{i-1}^2 e^{-\delta \tau_{i-1}}}_{V_2(\eta_{i-1}, \tau_{i-1}, l_{i-1})} + \underbrace{p_{3,l} \sigma_{i-1}^2 e^{-\delta \tau_{i-1}}}_{V_3(\sigma_{i-1}, \tau_{i-1}, l_{i-1})}$$

Decreases at jumps

$$p_{2,1} - e^{-\delta_2(1+\Delta)T_s} p_{2,0} \leq 0$$

$$p_{2,0} + p_{3,0} - p_{3,1} \leq 0$$

Decreases while flows

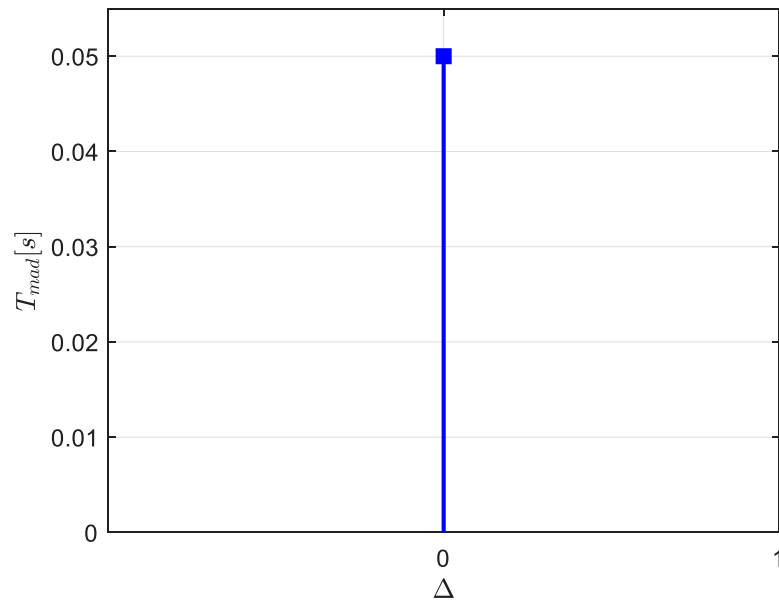
$$\mathcal{M}(\tau_{i-1}, l_{i-1}) < 0,$$

$$\forall \tau_{i-1} \in [0, (\Delta + 1)T_s], l_{i-1} \in \{0, 1\}$$

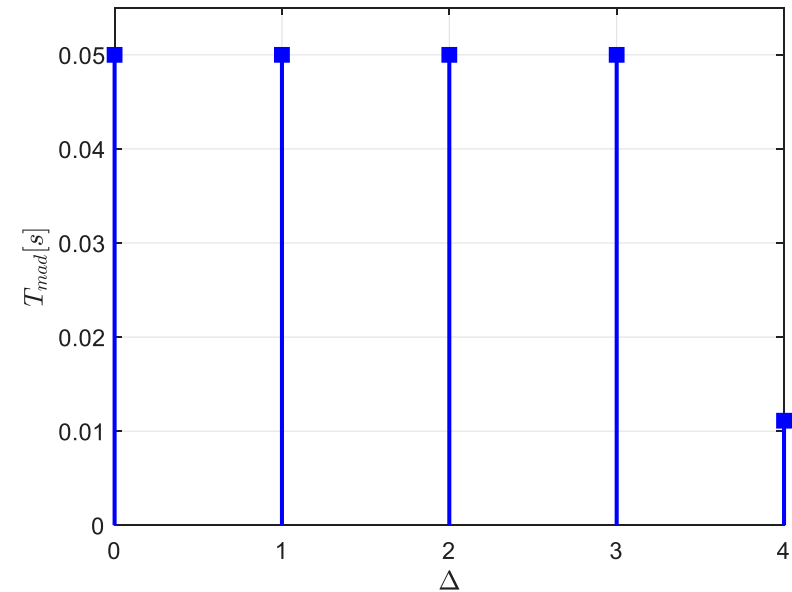
$$\mathcal{M}(\tau_{i-1}, l_{i-1}) = \begin{pmatrix} \text{He}(P_1 A_{xx}) + C_\omega^\top C_\omega & P_1 A_{x\eta} + C_\omega^\top + e^{-\delta \tau_{i-1}} p_{2,l_{i-1}} A_{\eta x}^\top & e^{-\delta \tau_{i-1}} p_{3,l_{i-1}} A_{\eta x}^\top & P_1 A_{xw} \\ \bullet & -\delta p_{2,l_{i-1}} e^{-\delta \tau_{i-1}} + 1 & 0 & -e^{-\delta \tau_{i-1}} p_{2,l_{i-1}} / h \\ \bullet & \bullet & -\delta e^{-\delta \tau_{i-1}} p_{3,l_{i-1}} & -e^{-\delta \tau_{i-1}} p_{3,l_{i-1}} / h \\ \bullet & \bullet & \bullet & -\theta^2 \end{pmatrix}$$

$\mathcal{L}_2$ -stability

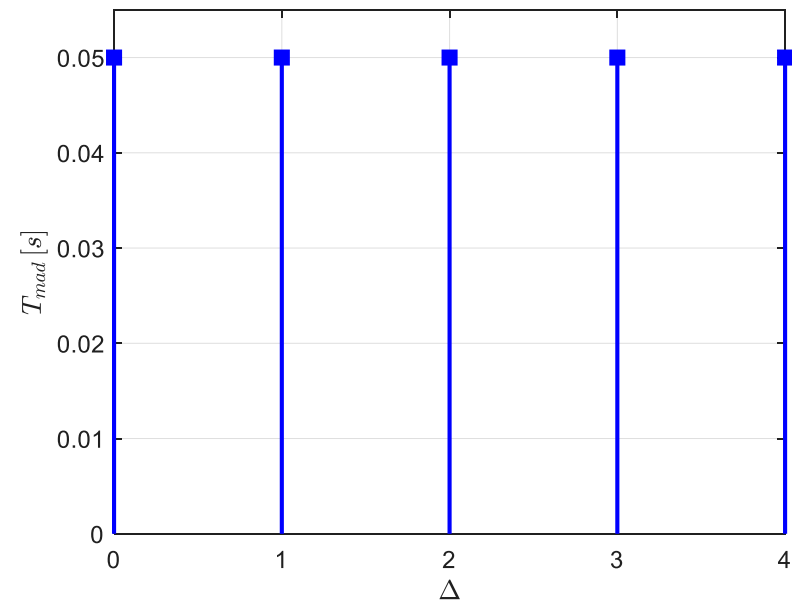
# Numerical Results



Tuning as [Ploeg et al CITS '11]

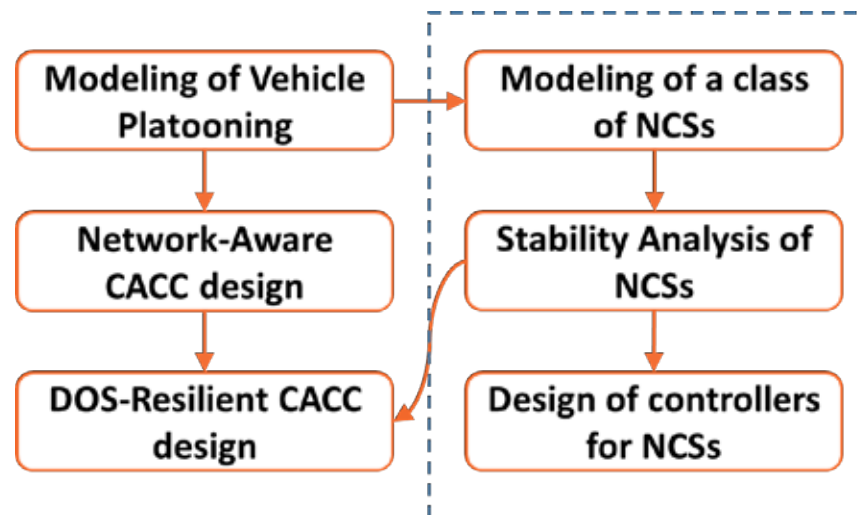


Tuning as our approach with ZOH

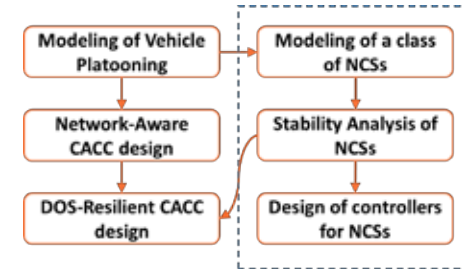
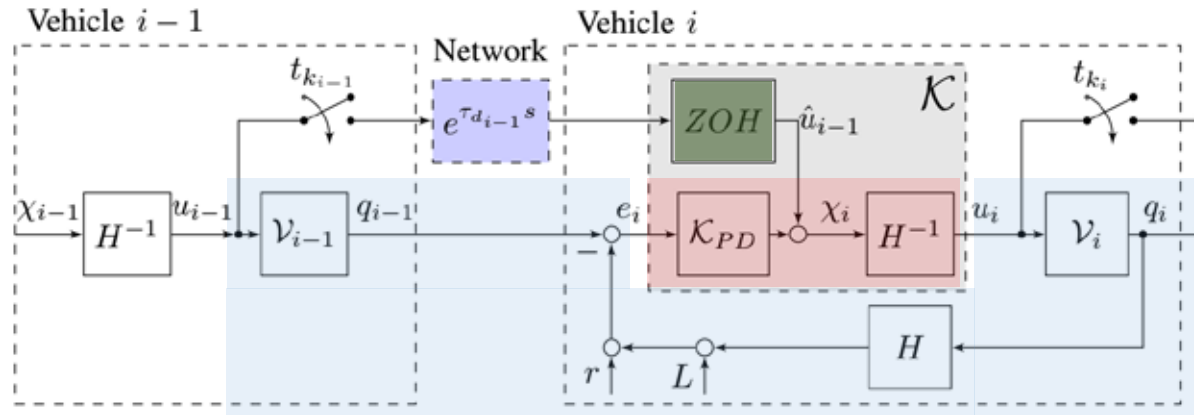


Tuning as our approach with Intersample

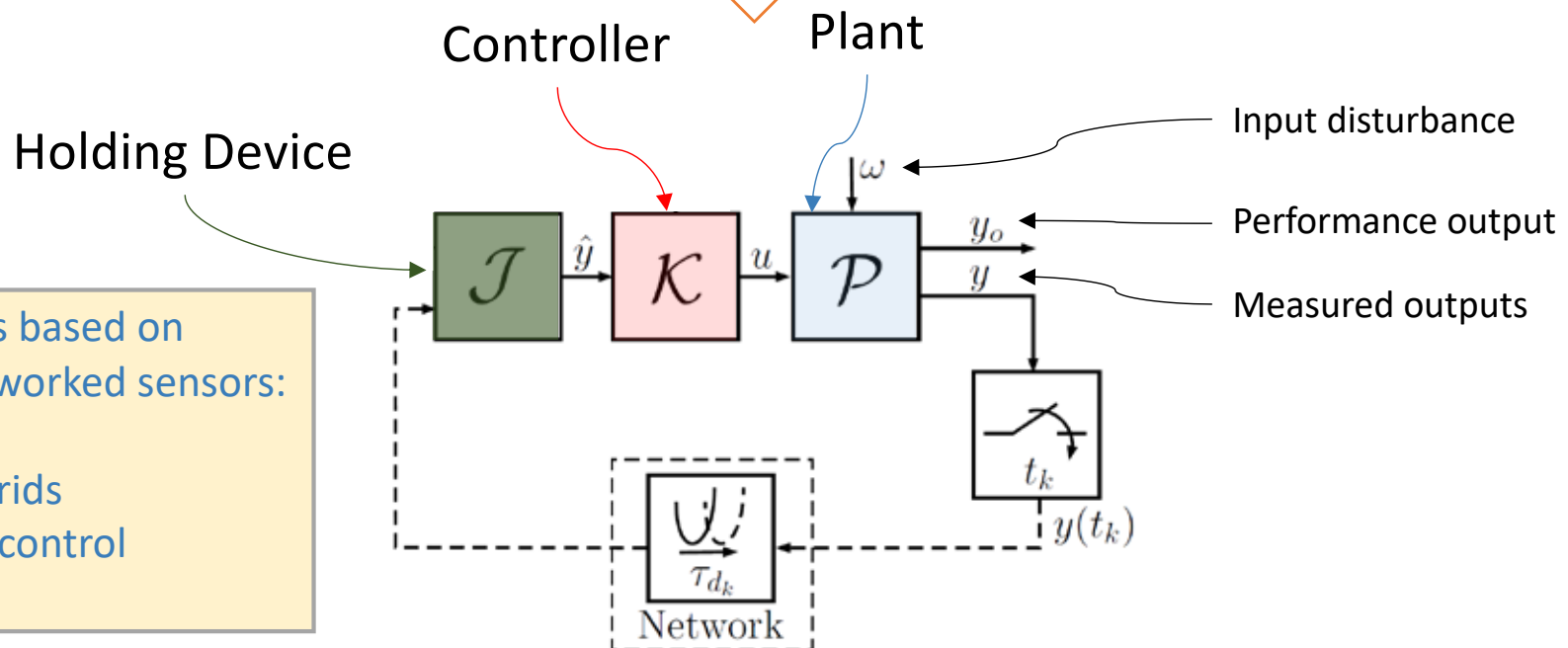
## From Vehicle Platooning to a Class of Networked Control Systems



## Vehicle Platooning model



## Generic Networked Control System



Applications based on remote networked sensors:

- Drones
- Smart Grids
- Process control
- ...

Majority of existing design approaches:

1. Does not consider dynamic output feedback controllers

$$\mathcal{K} \begin{cases} \dot{x}_c = A_c x_c + B_c \hat{y} \\ u = C_c x_c + D_c \hat{y} \end{cases}$$

More suitable to account  
for performance



For example:  
 $\mathcal{H}_\infty$  robust  
controllers

2. Are not suitable for the evaluation of trade-off curves as shown earlier  
When NCS are not modeled as hybrid systems, network delays and packet dropping are commonly not considered together

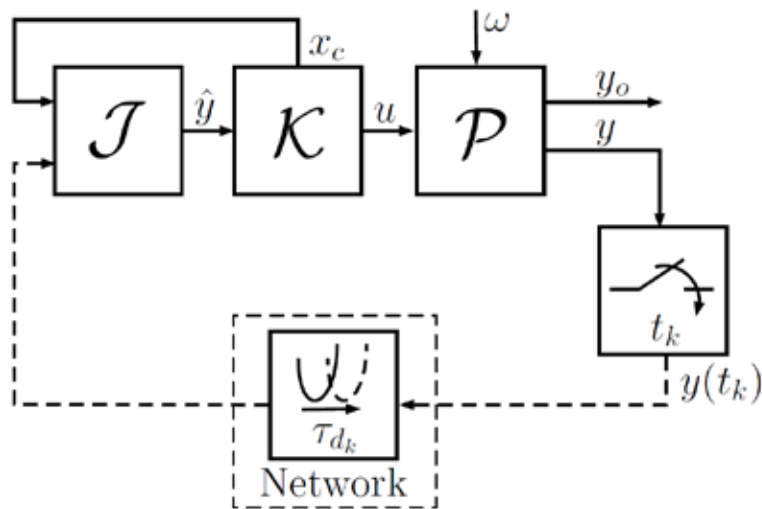
3. Use Zero-Order Hold (ZOH) as holding device

$$ZOH \begin{cases} \dot{\hat{y}}(t) = 0 & \forall t \neq t_k + \tau_{d_k} \\ \hat{y}(t^+) = y(t_k) & \forall t = t_k + \tau_{d_k} \end{cases}$$

Suitable for small transmission  
intervals, but potentially not for long  
period without network updates

4. Does not explicitly account for network unreliability metrics in the stability analysis

Network resilience does not appear directly in design conditions



$$\mathcal{P} \begin{cases} \dot{x}_p = A_p x_p + B_p u + W \omega \\ y = C_p x_p \\ y_o = C_o x_p \end{cases}$$

$$\mathcal{K} \begin{cases} \dot{x}_c = A_c x_c + B_c \hat{y} \\ u = C_c x_c + D_c \hat{y} \end{cases}$$

$$\mathcal{J} \begin{cases} \dot{\hat{y}}(t) = H \hat{y}(t) + E x_c(t) & \forall t \neq t_k + \tau_{dk} \\ \hat{y}(t^+) = y(t_k) & \forall t = t_k + \tau_{dk} \end{cases}$$

To address research gaps we solve:

## Problem #1 (Compute Trade-off Curves):

Assume controller and holding device given. Find trade-off curves between transmission intervals and network delays such that the NCS is input-output stable.

## Problem #2 (Controller Design):

Design controller and holding device such that the NCS is input-output stable with the largest transmission intervals.

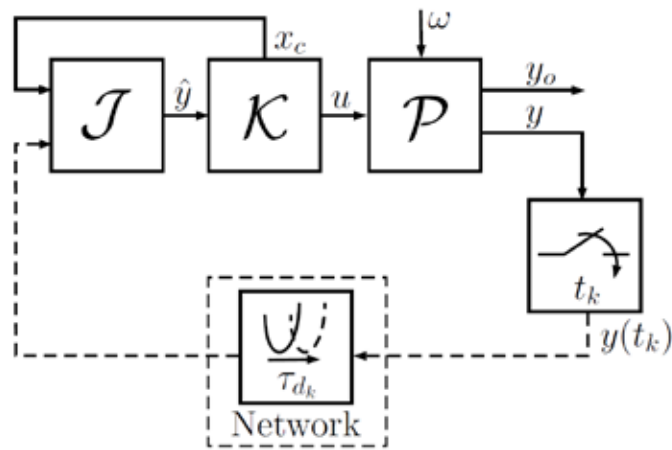
## Challenges for Problem #1 (Compute Tradeoff Curves) and Problem #2 (Controller Design):

1. Choice of clock-dependent Lyapunov function (related to Problem #1 and #2)
  - Lyapunov function for Hybrid Dynamical Systems must be properly designed:
    - Decreases during flows
    - Decreases as jumps
    - Conservatism of the trade-off curves
2. Linearization of matrix inequalities
  - Linearization of stability conditions (related to Problem #1 and #2)
  - Controller parameters linearization (related to Problem #2)

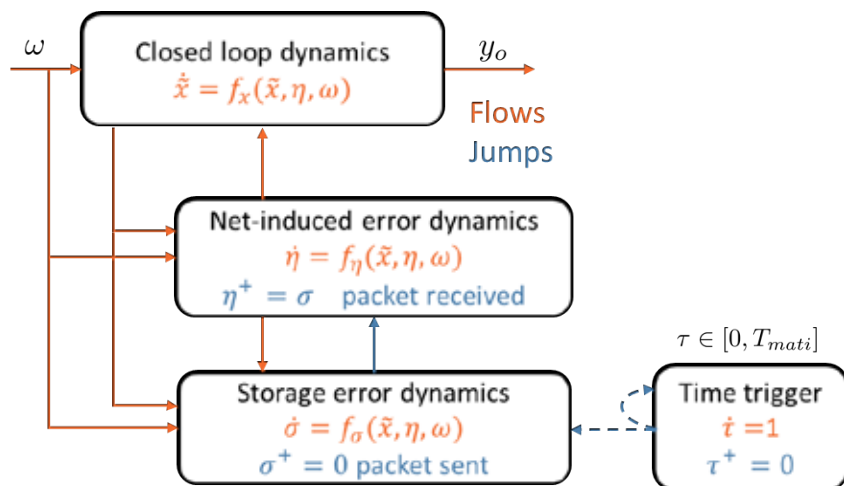
# Problem #1: Compute Trade-off Curves



## Networked Control System



## Hybrid dynamical system for stability analysis



### Challenge:

Find the Clock-Dependent Lyapunov function

$$V(x) = \underbrace{\tilde{x}^\top P_1 \tilde{x}}_{V_1(\tilde{x})} + \underbrace{e^{-\delta\tau} \eta^\top P_{2,l} \eta}_{V_2(\eta, \tau, l)} + \underbrace{e^{-\delta\tau} \sigma^\top P_{3,l} \sigma}_{V_3(\sigma, \tau, l)}$$

Decreases at jumps

$$P_{2,1} - e^{-\delta(T_{mati})} P_{2,0} \leq 0$$

$$P_{2,0} + P_{3,0} - P_{3,1} \leq 0$$

Decreases during flows

$$\mathcal{M}(\tau, l) < 0,$$

$$\forall \tau \in [0, T_{mati}], l \in \{0, 1\}$$

# Problem #1: Compute Trade-off Curves



## Clock-Dependent Lyapunov function

$$V(x) = \underbrace{\tilde{x}^\top P_1 \tilde{x}}_{V_1(\tilde{x})} + \underbrace{e^{-\delta\tau} \eta^\top P_{2,l} \eta}_{V_2(\eta, \tau, l)} + \underbrace{e^{-\delta\tau} \sigma^\top P_{3,l} \sigma}_{V_3(\sigma, \tau, l)}$$

Decreases at jumps

$$P_{2,1} - e^{-\delta(T_{mati})} P_{2,0} \leq 0$$
$$P_{2,0} + P_{3,0} - P_{3,1} \leq 0$$

Decreases during flows

$$\mathcal{M}(\tau, l) < 0,$$
$$\forall \tau \in [0, T_{mati}], l \in \{0, 1\}$$

### Challenges:

- Variables appear in a nonlinear fashion
- Infinite conditions to be satisfied

### Proposed solutions:

- Line search on  $\delta$
- Employ convexity of  $\mathcal{M}(\tau, l)$



$$\mathcal{M}(0, 1) < 0, \mathcal{M}(T_{mad}, 1) < 0,$$
$$\mathcal{M}(T_{mad}, 0) < 0, \mathcal{M}(T_{mati}, 0) < 0$$

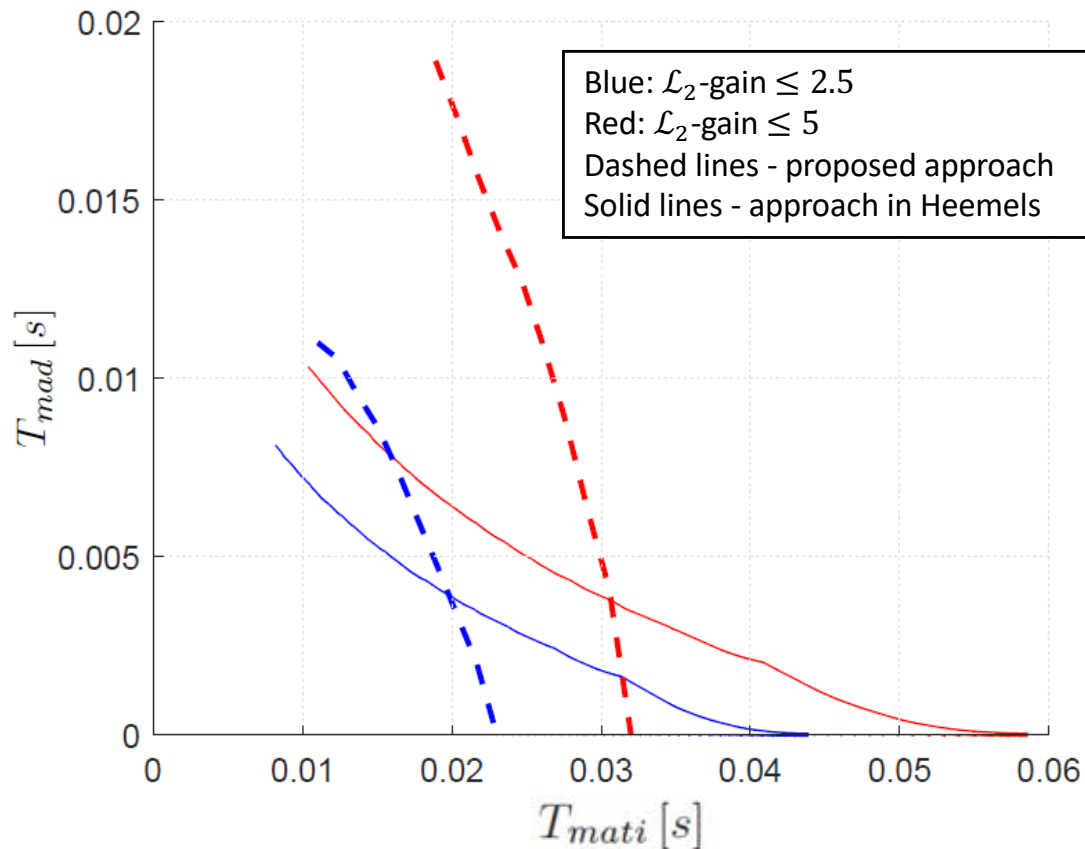
### Stability conditions expose trade-off curves parameters:

- Stability at a network operating point can be directly checked
- Trade-off curves can be obtained by an iterative algorithm by changing  $T_{mad}, T_{mati}$

# Problem #1: Compute Trade-off Curves – Example



Batch reactor: output feedback control system  
 $\mathcal{L}_2$  stability

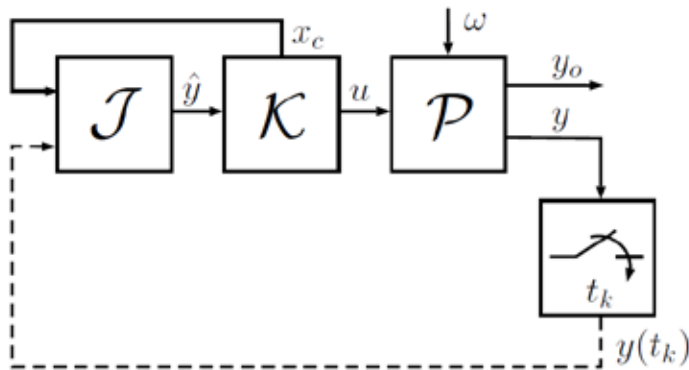


## For this control system:

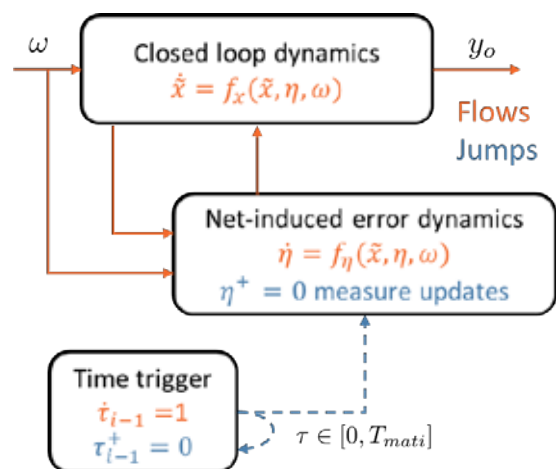
- Approach in Heemels et al. is less conservative for transmission intervals
- Proposed approach is less conservative for network delays
- Proposed approach accounts for intersample dynamics

# Problem #2: Dynamic Output Feedback Controller Design

## Networked Control System



## Hybrid dynamical system for stability analysis



$$\mathcal{H} \left\{ \begin{array}{l} \dot{x}_p = (A_p + B_p D_c C_p)x_p + B_p C_c x_c - B_p D_c \eta \\ \dot{x}_c = A_c x_c + B_c C_p x_p - B_c \eta \\ \dot{\eta} = (C_p A_p - H C_p)x_p + E x_c + H \eta \\ \dot{\tau} = -1 \quad \eta := y - \hat{y} \end{array} \right. \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} (x_p, x_c, \eta, \tau) \in \mathcal{C} \\ \\ \\ (x_p, x_c, \eta, \tau) \in \mathcal{D} \end{array}$$

$$\left. \begin{array}{l} x_p^+ = x_p \\ x_c^+ = x_c \\ \eta^+ = 0 \\ \tau^+ \in [T_1, T_2] \end{array} \right\} (x_p, x_c, \eta, \tau) \in \mathcal{D}$$

Approach in Scherer et al. introduced for  $\mathcal{H}_\infty$  control design, does not work in this case.

### Challenge:

Find the Clock-Dependent Lyapunov function such that stability conditions turn into linear matrix inequality.



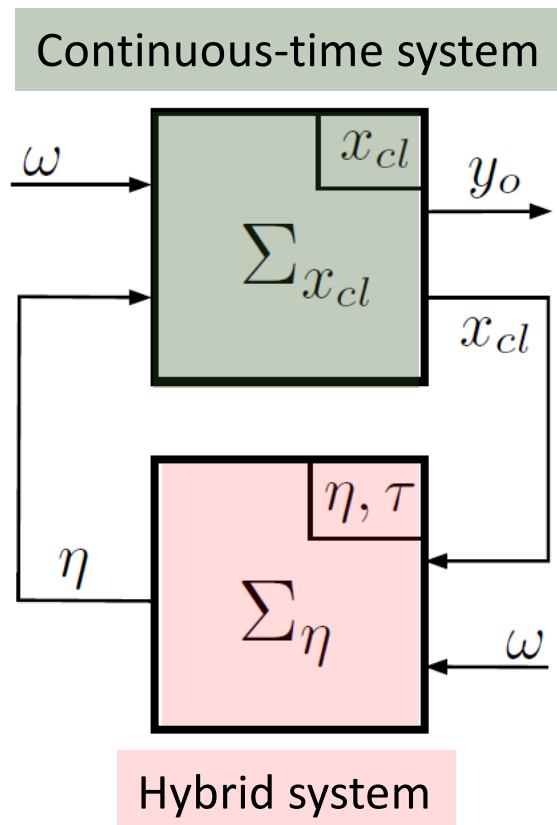
Parameters of controller and holding device can be computed by employing semidefinite programming tools.

# Problem #2: Dynamic Output Feedback Controller Design



## Proposed solutions:

Approach reminiscent of an “input-to-state stability small gain” philosophy.



$$\Sigma_{x_{cl}}: \begin{bmatrix} \dot{x}_p \\ \dot{x}_c \end{bmatrix} = \mathbb{A} \underbrace{\begin{bmatrix} x_p \\ x_c \end{bmatrix}}_{x_{cl}} + \mathbb{B}\eta$$

$$\Sigma_{\eta}: \begin{cases} \begin{bmatrix} \dot{\eta} \\ \dot{\tau} \end{bmatrix} = \begin{bmatrix} H\eta + \mathbb{J}\bar{x} \\ -1 \end{bmatrix} & \tau \in [0, T_2] \\ \begin{bmatrix} \eta^+ \\ \tau^+ \end{bmatrix} \in \begin{bmatrix} 0 \\ [T_1, T_2] \end{bmatrix} & \tau = 0 \end{cases}$$

$$\mathbb{A} := \left[ \begin{array}{c|c} A_p + B_p D_c C_p & B_p C_c \\ \hline B_c C_p & A_c \end{array} \right], \quad \mathbb{B} := - \left[ \begin{array}{c} B_p D_c \\ B_c \end{array} \right]$$

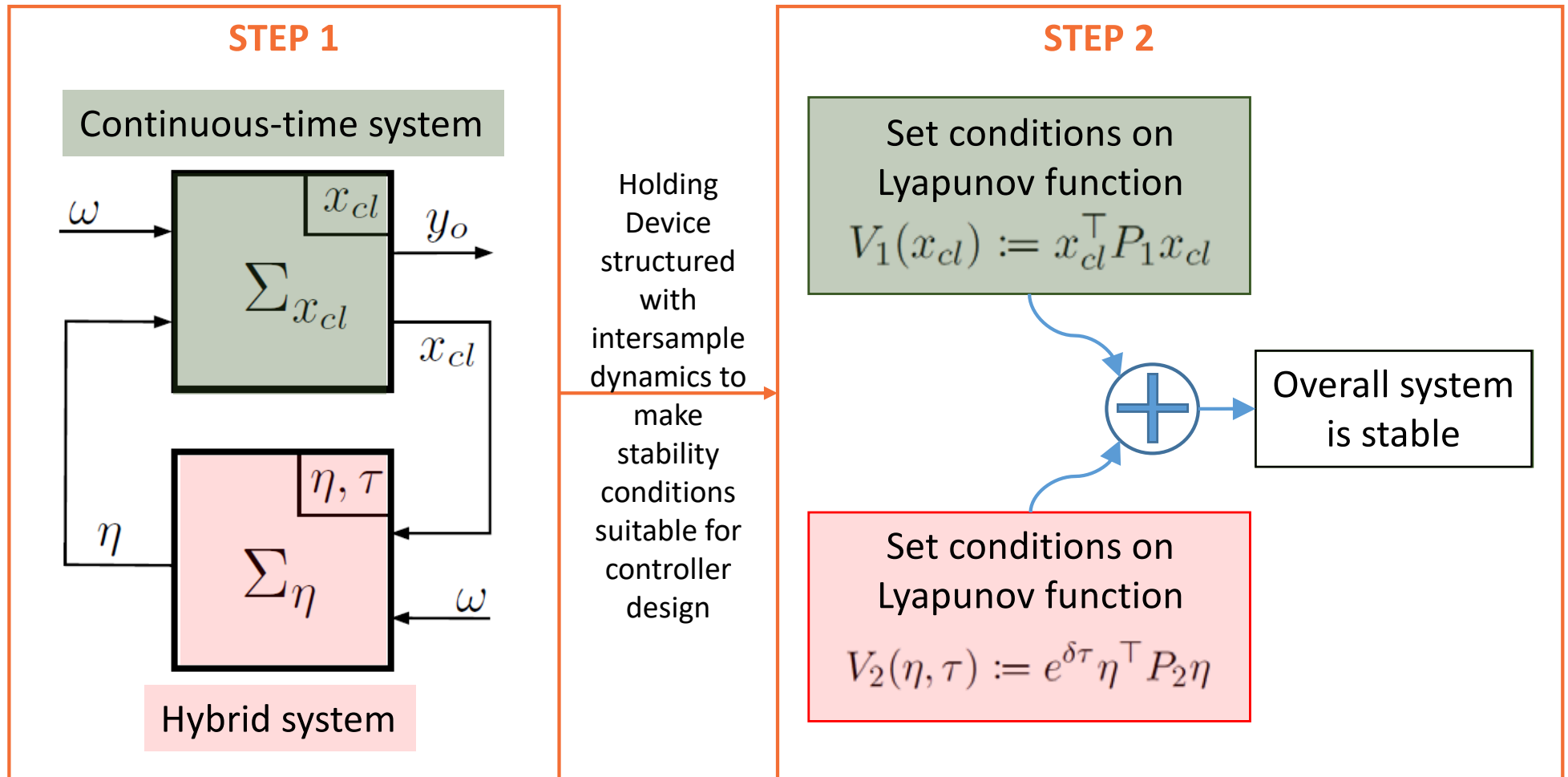
$$\mathbb{J} := \left[ \begin{array}{c|c} C_p A_p - H C_p & -E \end{array} \right]$$

# Problem #2: Dynamic Output Feedback Controller Design



## Proposed solutions:

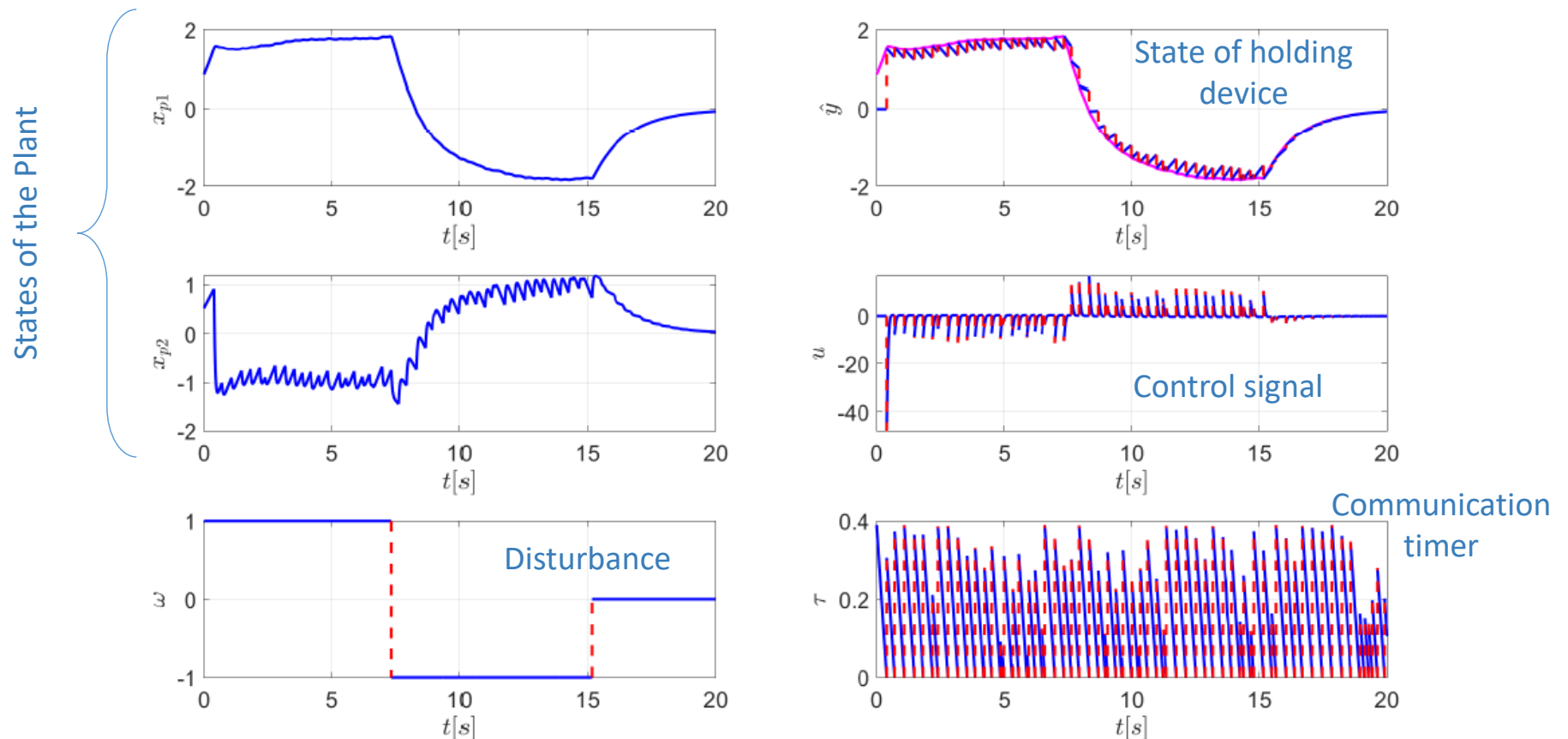
Approach reminiscent of an “input-to-state stability small gain” philosophy.



# Problem #2: Dynamic Output Feedback Controller Design – Example



$\mathcal{L}_2$  stability of a double integrator with  $\mathcal{L}_2$  gain less than or equal to 5.  
A value of  $T_{mati} = 0.39$  s is obtained by design.





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