# Instabilities in Homogeneous and Heterogeneous Traffic Flow

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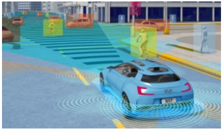

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## Support

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*Synergy: Control of vehicular traffic flow via low density autonomous vehicles*

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# Overview

# Overview



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# Need for Traffic Flow Models

- Why should we care about mathematical models for traffic flow? Traffic works fine. (Humans are actually surprisingly good at driving.)
- Because we (as a society) are about to fundamentally change the reality of traffic flow, by adding automation and connectivity.
- Connected and automated vehicles (CAVs) must be programmed to behave differently from humans (because humans make mistakes, at least once in a while).
- It will take many years until human drivers are removed from the roads, so we must understand mixed human–CAV traffic flow.
- And even if humans are taken out of the loop, achieving efficient flow is not a given ($\longrightarrow$ instability in existing ACC systems).

# Impact of Automation and Connectivity on Transportation

- Lots of attention on CAV impact on:
  - routing (user vs. system optima)
  - trip choices and travel demand (more trips okay)
  - vehicle miles taken (will increase)
  - urban sprawl (long trips okay)
  - safety (remove human error)
  - accessibility and comfort (relax or work during trip), etc.
- Less attention on the actual traffic flow dynamics.
- The near future of having only a few CAVs in the flow should receive particular attention.
- An important question that is not addressed here: (how) do human drivers change their driving behavior when surrounded by CAVs?

# Traffic Flow with a Few AVs

- Great news: Just a few AVs in the flow (about 4%) suffice to remove and prevent stop-and-go traffic waves.
- Caution: Having just few AVs in the flow will not fix urban congestion (because human drivers still are prone to physical bottlenecks).
- However: Removing traffic waves can result in significant reductions of fuel consumption, emissions, and accident risk (of all cars).
- In other words: Congestion is not removed, but many of the adverse consequences of congestion are mitigated.
- Caution again: Automation could also render flow less stable or less efficient than under pure human control:
  - unstable ACC systems;
  - humans frequently produce higher flow efficiency by not adhering to rules (speed limit) or guidelines (2-second rule). AVs that follow the rules or guidelines may reduce throughput (moving bottlenecks).

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# Overview



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## Observation: Traffic Waves on Long Road (video: [D. Helbing])

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## Experiment: Phantom Traffic Jam on Ring Road [Sugiyama et al.: New J. of Physics 2008]

## Wave-Dampening via a Single Autonomous Vehicle

- Traditional highway traffic controls (ramp metering, variable speed limits) do not have the resolution to dissipate traffic waves.
- Use autonomous vehicles: AVs will be on our roads anyways.
- Human-in-the-loop cyber-physical system: human drivers interact with AVs but with small amount of direct interaction.
- Ring road of $N$ vehicles with a single AV, as proxy for a long road with AV penetration rate $1/N$. Here: 4–5% AV penetration.

## Single AV with Local–Global Control Law

- Local: adjust velocity to safely follow lead vehicle.
- Global: attempt to drive at velocity equal to estimated average speed of traffic flow (without violating safety).

# Experimental Setup

## 360° Camera

## Elevated Camera

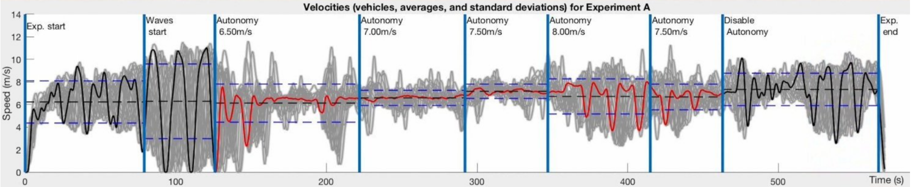
## OBD-II Sensor


## CATVehicle


## Research Team

# Experimental Results

| Time (s) | Interval | Velocity st. dev (m/s) | Fuel consumption (liters/100km) | Braking (events/vehicle/km) | Throughput (vehicles/hour) |
|---|---|---|---|---|---|
| 000 | Experiment start | 1.87 | 18.8 | 1.66 | 1809 |
| 079 | Waves start | 3.31 | 24.6 | 8.58 | 1827 |
| 126 | Autonomy 6.50m/s | 1.69 | 18.0 | 3.45 | 1780 |
| 222 | Autonomy 7.00m/s | 0.67 | 15.0 | 0.21 | 1915 |
| 292 | Autonomy 7.50m/s | 0.64 | 14.1 | 0.12 | 2085 |
| 347 | Autonomy 8.00m/s | 1.56 | 17.7 | 2.50 | 1952 |
| 415 | Autonomy [illegible].50m/s | 1.14 | 16.7 | 0.31 | 1938 |
| 463 | Disable Autonomy | 1.44 | 17.4 | 2.95 | 2133 |
| 567 | Experiment end | - | - | - | - |

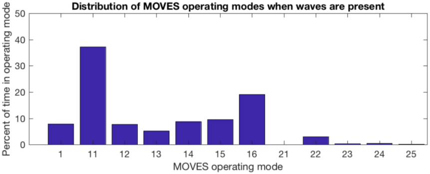


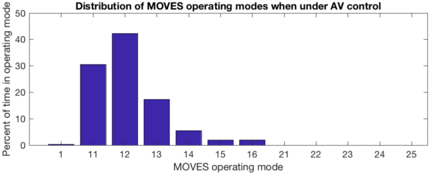

## Best AV-based controller vs. strongest waves

- economy: almost half the fuel consumption
- safety: $70\times$ less strong braking
- clean air: $5\times$ less velocity variation
- efficiency: throughput up by 14% (but: no physical bottlenecks here)

# Fleet Emissions

## MOVES (EPA, 2015) Modes (higher mode = more emissions)




## Environmental Impact of Control on Whole Fleet

| Metric/Pollutant | unit | waves | control | reduction |
|---|---|---|---|---|
| vehicle st. dev. | m/s | 3.31 | 0.64 | 81% |
| strong braking | 1/veh/km | 8.58 | 0.12 | 99% |
| fuel consumption | $\ell$/100km | 24.6 | 14.1 | 42% |
| carbon dioxide ($CO_2$) | g/mi | 1246 | 863.1 | 30.7% |
| carbon monoxide (CO) | g/mi | 2.430 | 1.481 | 39.1% |
| hydrocarbons (HC) | g/mi | 0.010 | 0.005 | 51.5% |
| nitrogen oxides ($NO_x$) | g/mi | 0.107 | 0.028 | 73.5% |

# Need for Traffic Flow Models

- CAV control needed at different scales: local (executing wave dampening), and global (distribute the control vehicles along the road network to maximize control impact, while satisfying constraints imposed by them acting as transportation vessels as well).
- Both tasks require models for the mixed flow.
  Dream: a single multi-scale model framework that can capture both aspects at once ($\longrightarrow$ more later).

## Objective of Modeling

- Find the simplest model that captures the phenomenon, by means of correct dominant underlying mechanisms (interpretability).
- For stop-and-go waves, that does not mean to reproduce the precise trajectories of the vehicles, nor to predict the position of the waves. Rather, the goal is to capture the reproducible characteristics of waves, such as growth rate, wave speed, or average velocity drop.

# Overview



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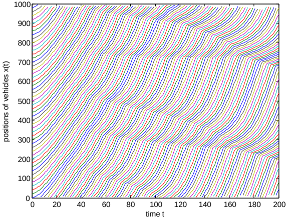
# Traffic Models Possessing Phantom Jams

## Microscopic Models

$$\ddot{x}_j = f(x_{j+1} - x_j, u_j, u_{j+1})$$

or

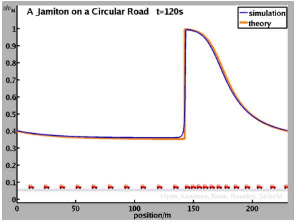
$$\dot{x}_j(t+\tau) = V(x_{j+1} - x_j)(t)$$




Describe individual vehicles.

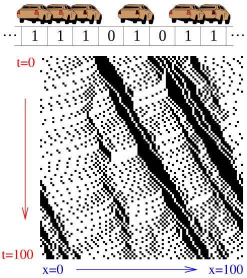
## Macroscopic Models

$$\begin{cases} \rho_t + (\rho u)_x & = 0 \\ (u+h)_t + u(u+h)_x & = \frac{1}{\tau}(U-u) \end{cases}$$




Aggregate/bulk quantities.

## Cellular Models




Easy to implement, fast to compute.

## Micro $\longleftrightarrow$ Macro

- limit #vehicles $\to \infty$ $\rightsquigarrow$ macro
- micro = discretization of macro in Lagrangian variables

## Methodology and Role

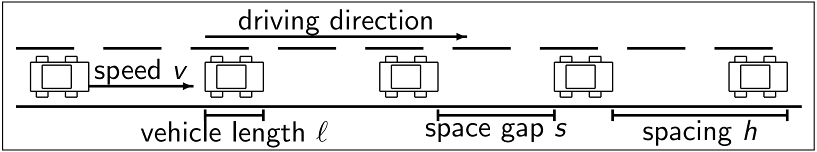
- Natural framework for multiscale phenomena, traveling waves, shocks.
- Real-time state estimation, privacy

## Cellular $\longleftrightarrow$ Macro

- limit #cells $\to \infty$ $\rightsquigarrow$ macro
- cellular = discretization of macro in Eulerian variables

# Equilibrium Traffic Flow

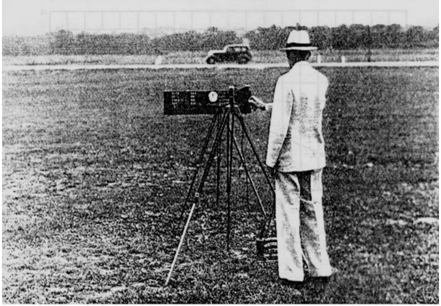
## Uniform traffic flow (here for single lane)




## Fundamental quantities

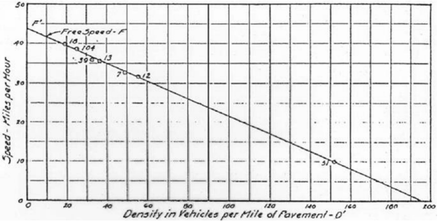
- speed $v$: distance traveled per unit time
- spacing (space headway) $h$: road length per vehicle
- space gap $s = h - \ell$
- density $\rho = 1/h$: number of vehicles per unit road length ($\rho_{\max} = 1/\ell$)
- time headway $h_t = h/v$: time between two vehicles passing fixed position
- flow rate (throughput) $q = 1/h_t = \rho v$: number of vehicles passing fixed position per unit time

## Bruce Greenshields Collecting Data (1933)



[This was only 25 years after the first Ford Model T (1908)]

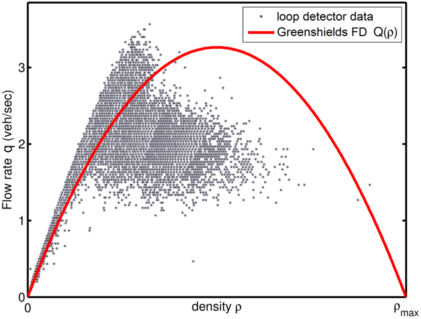
## Postulated Density–Velocity Relationship




## Traffic Flow Theory

- Density $\rho$: #vehicles per unit length of road (at a fixed time)
- Flow rate $q$: #vehicles per unit time (passing a fixed position)
- Bulk velocity: $u = q/\rho$

## Fundamental Diagram of Traffic Flow

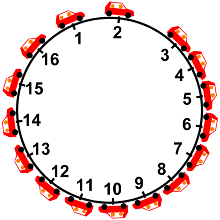



$$q = Q(\rho) \rightsquigarrow v = \frac{1}{\rho} Q(\rho) \rightsquigarrow v = V(s)$$

# Overview

# Microscopic Traffic Models (No Lane-Changing Here)

- Vehicles at positions $\ldots < x_{j-1} < x_j < x_{j+1} < \ldots$
- Vehicle $j$ follows vehicle $j+1$.
- Car-following: veh. $j$ affected only by veh. $j+1$.
- Two types of arrangements:
  - a. Infinite road with one vehicle leading.
  - b. Ring road ($N$ follows 1): proxy for infinite road with periodicity; key advantage: fixed average density.




## Types of Model Dynamics

- First order: $\dot{x}_j = V(s_j)$ with gap $s_j = x_{j+1} - x_j - \ell$.
- with delay: $\dot{x}_j(t) = V(s_j(t-\tau))$ where $\tau > 0$ delay [humans vs. ACC]
- Second order: $\ddot{x}_j = f(s_j, \dot{s}_j, v_j)$. Here $\dot{s}_j = \dot{x}_{j+1} - \dot{x}_j$ velocity difference.
- combinations, high-order, etc.

# Car Following

## Types of Models Dynamics

- First order with delay: $\dot{x}_j(t) = V(s_j(t-\tau))$.
- Second order: $\ddot{x}_j = f(s_j, \dot{s}_j, v_j)$.

## Equilibrium

- Requirement on $V$, $f$: equilibrium means equi-spaced with identical velocities, i.e.: $x_j^{\text{eq}}(t) = (s^{\text{eq}} + \ell)j + v^{\text{eq}}t$, where $v^{\text{eq}} = V(s^{\text{eq}})$ directly (first order) or indirectly via $f(s^{\text{eq}}, 0, v^{\text{eq}}) = 0$ (second order).
- Obtain equilibrium relation $v^{\text{eq}} = V(s^{\text{eq}})$ from fundamental diagram.
- Linearize (infinitesimal perturbation $x_j = x_j^{\text{eq}} + y_j$).
- First order: $\dot{y}_j(t+\tau) = V'(s)(y_{j+1}(t) - y_j(t))$.
  Second order: $\ddot{y}_j = \alpha_1\,(y_{j+1} - y_j) - \alpha_2\,\dot{y}_j + \alpha_3\,\dot{y}_{j+1}$,
  where $\alpha_1 = \frac{\partial f}{\partial s}$, $\alpha_2 = \frac{\partial f}{\partial \dot{s}} - \frac{\partial f}{\partial v}$, $\alpha_3 = \frac{\partial f}{\partial \dot{s}}$ (all evaluated at equilibrium).
- Common sense requires: $V'(s) > 0$, and $\alpha_1 \geq 0$, $\alpha_2 \geq \alpha_3$, $\alpha_3 \geq 0$.

# Car Following: String Stability

## Linearized Dynamics

- First order: $\dot{y}_j(t+\tau) = V'(s)(y_{j+1}(t) - y_j(t))$.
- Second order: $\ddot{y}_j = \alpha_1\,(y_{j+1} - y_j) - \alpha_2\,\dot{y}_j + \alpha_3\,\dot{y}_{j+1}$.

## Frequency Response of Car Following I/O Behavior

- Laplace transform ansatz $y_j(t) = c_j e^{\omega t}$, where $c_j, \omega \in \mathbb{C}$.
- Yields I/O system: $c_j = F(\omega) c_{j+1}$ with transfer function

$$F(\omega) = \left(1 + \tfrac{1}{V'(s^{\text{eq}})}\omega e^{\omega\tau}\right)^{-1} \quad \text{resp.} \quad F(\omega) = \frac{\alpha_1 + \alpha_3\,\omega}{\alpha_1 + \alpha_2\,\omega + \omega^2}\,.$$

- $\mathrm{Re}(\omega)$: temporal growth/decay — $|F|$: growth/decay across vehicles
  $\mathrm{Im}(\omega)$: frequency of oscillation — $\theta(F)$: phase shift across vehicles
- **Def.:** string stability means $|F(\omega)| \leq 1\ \forall \omega \in i\mathbb{R}$. [other $\omega$ irrelevant as $t \to \infty$]
- The models above are string stable iff

$$2\tau V'(s^{\text{eq}}) \leq 1 \qquad \text{resp.} \qquad \alpha_2^2 - \alpha_3^2 - 2\alpha_1 \geq 0\,.$$

# Car Following: Ring Stability

- Ring road: Periodic wrap-around requires $F(\omega)^N = 1$.
  This equation has $2N$ roots.
- **Def.:** ring stability means that all roots $\omega_j$ have $\mathrm{Re}(\omega_j) \leq 0$.
- Require this to hold for any $N$ (infinite road limit):
  Curve $\mathcal{C} = \{z \in \mathbb{C} : |F(z)| = 1\}$ must lie in left half plane
  $\mathbb{C}^- = \{z \in \mathbb{C} : \mathrm{Re}(z) \leq 0\}$.
- **Thm.:** If $F(\omega)$ has no poles in $\mathbb{C}^+$ and $\lim_{\mathrm{Re}(z)\to\infty} |F(z)| = 0$, then:
  string stability $\Longleftrightarrow$ ring stability.

  [**Proof:** $F$ holomorphic in $\mathbb{C}^+$. Hence $\max_{z\in\mathbb{C}^+} |F(z)| = \max_{z\in i\mathbb{R}} |F(z)|$, i.e., it suffices to check ring stability along the imaginary axis.]
- Thm. applies to considered models ($2^{\text{nd}}$ order, because $\alpha_1, \alpha_2 \geq 0$).

## Results

- First order delay, or second order dynamics, can reproduce unstable human driving (phantom jam). We now pursue the latter.
- Nonlinear waves are not captured by this linear instability analysis.

## Intermezzo: ACC Experiment

# Are Commercially Implemented Adaptive Cruise Control Systems String-Stable?

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# Two-Species Car-Following (Humans and AVs)

- Slightly unstable human driver model, i.e. $\alpha_2^2 - \alpha_3^2 - 2\alpha_1 < 0$.
- What changes when a few automated vehicles are added to the flow?
  (that drive slightly differently than humans)
  Can the few AVs stabilize traffic flow, and thus prevent traffic waves?
- Humans: $\ddot{x}_j = f(h_j, \dot{h}_j, v_j)$; AVs: $\ddot{x}_j = g(h_j, \dot{h}_j, v_j)$.
- Have AVs leave same equilibrium spacing as humans. Linearize.
- Humans: $\ddot{y}_j = \alpha_1\,(y_{j+1} - y_j) - \alpha_2\,u_j + \alpha_3\,u_{j+1}$
  AVs: $\ddot{y}_j = \beta_1\,(y_{j+1} - y_j) - \beta_2\,u_j + \beta_3\,u_{j+1}$
- Transfer functions: $F(\omega) = \frac{\alpha_1 + \alpha_3\,\omega}{\alpha_1 + \alpha_2\,\omega + \omega^2}$ and $G(\omega) = \frac{\beta_1 + \beta_3\,\omega}{\beta_1 + \beta_2\,\omega + \omega^2}$.
- Stability criterion with AV penetration rate $\gamma$:

$$\boxed{\text{Curve } \mathcal{C}_\gamma = \{z \in \mathbb{C} \,:\, |F(z)|^{1-\gamma} \cdot |G(z)|^{\gamma} = 1\} \subset \mathbb{C}^-.}$$

Paradox 1: Stability independent of relative ordering of AVs.
That cannot be true in reality.

# Control via a Single Autonomous Vehicle

- Rewrite linearized system as control system
$$\begin{cases} \dot{\mathbf{z}} = \mathbf{M} \cdot \mathbf{z} + \mathbf{b} \cdot \mathbf{u}, \\ \mathbf{u} = \mathbf{F} \cdot \mathbf{C} \cdot \mathbf{z} \,. \end{cases}$$
- Only AV's velocity controlled: $\mathbf{b} = [0, \ldots, 0, 1, 0, \ldots, 0]^T$.
- $\mathbf{C}$: system information known to AV. $\mathbf{F}$ = feedback control matrix.
- **Fact:** The system is always controllable [except for shift invariance], even if the AV knows nothing but the equilibrium velocity.
- For instance: $\beta_1 = \alpha_1$, $\beta_3 = \alpha_3$, and increase $\beta_2$ until system is stabilized.
- Paradox 2: Ring traffic can be stabilized via a single AV, for any $N$. That cannot be true in reality.

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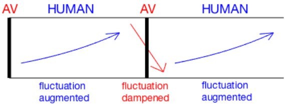
# Resolution of Paradoxes

- Linear stability only captures $t \to \infty$ behavior.
- For transient $t$, a small perturbation may produce a large deviation.
- Instability of human driving: perturbations grow from car to car.
- Stability of coupled system: AV(s) reduce(s) perturbation by more than amplification caused by all humans.
- Just before hitting the AV, perturbation could be amplified a lot.
- With noise, stabilized system may fail to remain close to equilibrium:

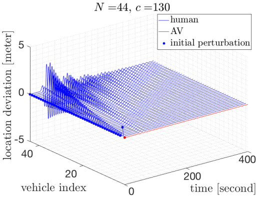
$$du_j = [\alpha_1(y_{j+1} - y_j) - \alpha_2 u_j + \alpha_3 u_{j+1}]dt + s_j dB_t$$

## Amplification and decay of perturbation

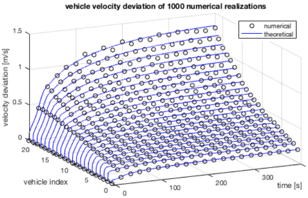



Max. possible: $N \approx 25$.

## System response to single perturbation




## With noise: system's mean deviation



# Overview



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## Continuum Description

Number of vehicles between points $a$ and $b$:

$$m(t) = \int_a^b \rho(x,t)\,\mathrm{d}x$$

Traffic flow rate (= flux) is product of density $\rho$ and vehicle velocity $u$

$$q = \rho u$$

Change of number of vehicles equals inflow $q(a)$ minus outflow $q(b)$

$$\int_a^b \rho_t\,\mathrm{d}x = \frac{d}{dt}m(t) = q(a) - q(b) = -\int_a^b q_x\,\mathrm{d}x$$

Equation holds for any choice of $a$ and $b$, thus

continuity equation $\qquad \rho_t + (\rho u)_x = 0$

## First Order Traffic Model

Postulate $u = U(\rho)$

LWR model: $\rho_t + (\rho U(\rho))_x = 0$

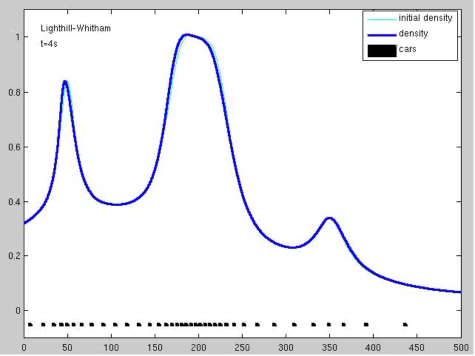
Scalar hyperbolic conservation law

## Second Order Traffic Models

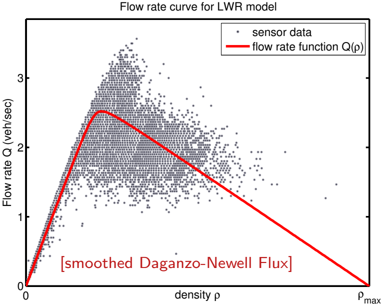
Add a second equation, modeling vehicle acceleration, e.g.:

$$u_t + uu_x = -\frac{p'(\rho)}{\rho}\rho_x + \frac{1}{\tau}(U(\rho) - u)$$

## Evolution of Traffic Density for LWR Model




## Data-Fitted Flow Rate Curve




## Model Quality of LWR

The LWR model quite nicely explains the shape of traffic jams (vehicles run into a shock).

## Shortcomings of LWR

Cannot explain FD spread.

Cannot explain phantom traffic jams (perturbations never grow due to maximum principle).

## Payne-Whitham (PW) Model [Analysis for ARZ Model is Very Similar]

$$\begin{cases} \rho_t + (\rho u)_x &= 0 \\ u_t + u u_x + \frac{1}{\rho} p(\rho)_x &= \frac{1}{\tau}(U(\rho) - u) \end{cases}$$

## Mathematical Structure: System of Balance Laws

$$\underbrace{\begin{pmatrix} \rho \\ u \end{pmatrix}_t + \begin{pmatrix} u & \rho \\ \frac{1}{\rho}\frac{dp}{d\rho} & u \end{pmatrix} \cdot \begin{pmatrix} \rho \\ u \end{pmatrix}_x}_{\text{hyperbolic part}} = \underbrace{\begin{pmatrix} 0 \\ \frac{1}{\tau}(U(\rho) - u) \end{pmatrix}}_{\text{relaxation term}}$$

## Relaxation to Equilibrium

Formally, we can consider the limit $\tau \to 0$.
In this case: $u = U(\rho)$, i.e., the system reduces to the LWR model.

## Important Fact

Solutions of the $2 \times 2$ system converge to solutions of LWR, only if a condition is satisfied $\longrightarrow$ next slide. . .

### System of Balance Laws (e.g., PW Model)

$$\begin{pmatrix}\rho\\ u\end{pmatrix}_t + \begin{pmatrix}u & \rho\\ \frac{1}{\rho}\frac{dp}{d\rho} & u\end{pmatrix}\begin{pmatrix}\rho\\ u\end{pmatrix}_x = \begin{pmatrix}0\\ \frac{1}{\tau}(U(\rho)-u)\end{pmatrix}$$

### Eigenvalues

$$\left\{\begin{array}{l}\lambda_1 = u - c\\ \lambda_2 = u + c\end{array}\right\} \quad c^2 = \frac{dp}{d\rho}$$

### Linear Stability Analysis

(LS) When are constant base state solutions $\rho(x,t)=\tilde\rho$, $u(x,t)=U(\tilde\rho)$ stable (i.e. infinitesimal perturbations do not amplify)?

### Reduced Equation

(RE) When do solutions of the $2\times 2$ system converge (as $\tau\to 0$) to solutions of the reduced equation

$$\rho_t + (\rho U(\rho))_x = 0 \quad ?$$

### Sub-Characteristic Condition

(SCC) $\lambda_1 < \mu < \lambda_2$, where $\mu = (\rho U(\rho))'$

### Theorem [Whitham: Comm. Pure Appl. Math 1959]

(LS) $\Longleftrightarrow$ (RE) $\Longleftrightarrow$ (SCC)

### Example: Stability for PW Model

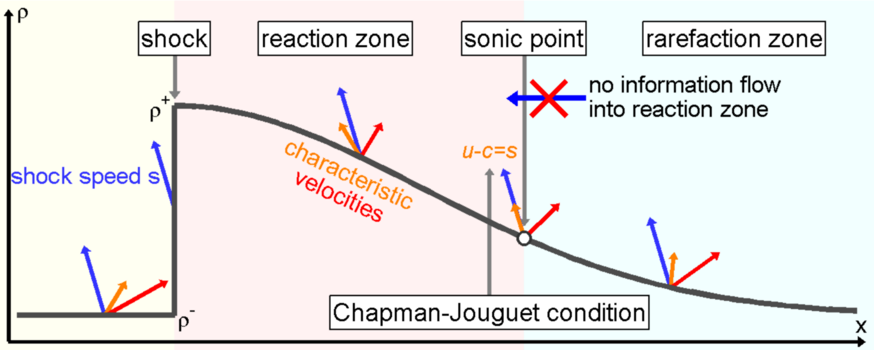
$$\text{(SCC)} \Longleftrightarrow U(\rho)-c(\rho) \le U(\rho)+\rho U'(\rho) \le U(\rho)+c(\rho) \Longleftrightarrow \frac{c(\rho)}{\rho} \ge -U'(\rho).$$

For $p(\rho)=\frac{\beta}{2}\rho^2$ and $U(\rho)=u_\mathrm{m}\left(1-\frac{\rho}{\rho_\mathrm{m}}\right)$: stability iff $\rho<\rho_\mathrm{c}$, where $\rho_\mathrm{c}=\frac{\beta\rho_\mathrm{m}^2}{u_\mathrm{m}^2}$.

Phase transition: If enough vehicles on the road, uniform flow is unstable.

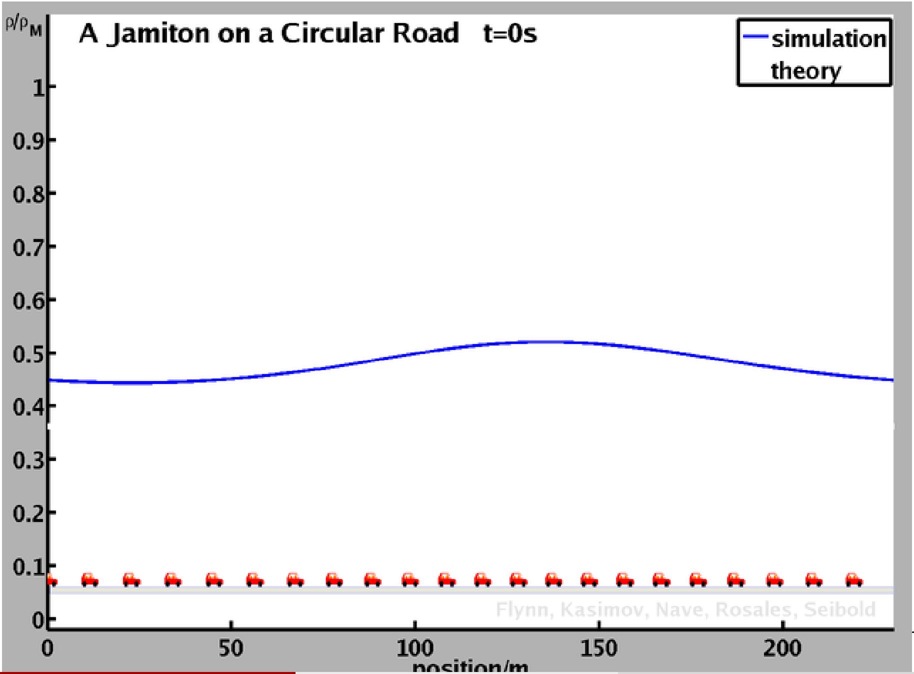
# Traffic Wave = Jamiton = Self-Sustained Detonation Wave

ZND theory [Zel'dovich (1940), von Neumann (1942), Döring (1943)]:
Reaction zone travels unchanged with speed of shock.
Rankine-Hugoniot conditions one condition short (unknown shock speed).
Sonic point is event horizon. It provides missing boundary condition.

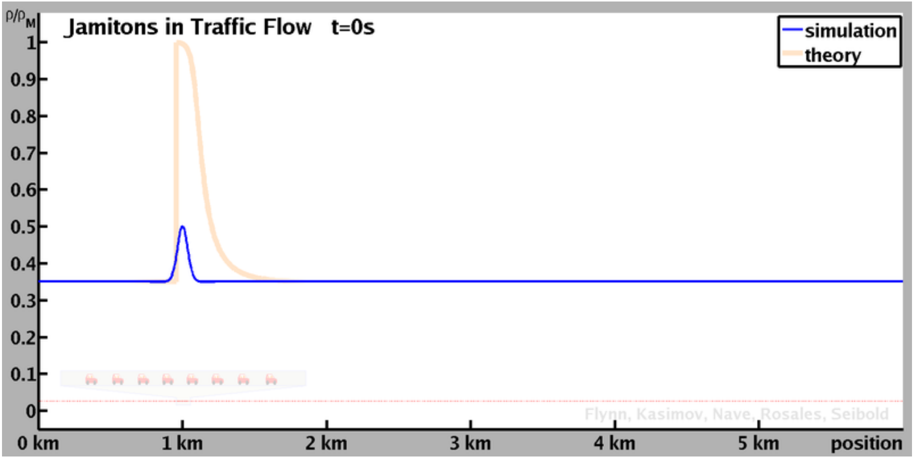



Traveling wave ansatz yields ODE $\quad u' = \dfrac{(u-s)(U(\frac{m}{u-s})-u)}{(u-s)^2-c(\frac{m}{u-s})^2}$ ,

that allows one to explicitly construct the $t \to \infty$ traveling wave solutions.


A Jamiton on a Circular Road t=0s
simulation
theory
ρ/ρM
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
50
100
150
200
position/m
Flynn, Kasimov, Nave, Rosales, Seibold

Infinite road; lead jamiton gives birth to a chain of “jamitinos”.



## Jamiton Fundamental Diagram

For each sonic density $\rho_S$ that violates the SCC: construct maximal jamiton.
$\rightsquigarrow$ Line segment in FD.
Jamitons can explain spread in real FD.

## Emulating Sensors

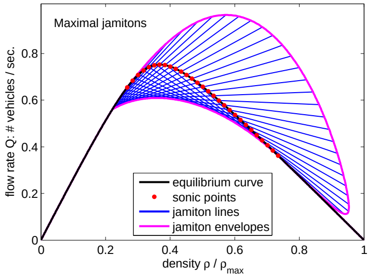
At fixed position, calculate all possible temporal averages of jamiton profiles.

Resulting aggregated jamiton FD is a subset of the maximal jamiton FD.

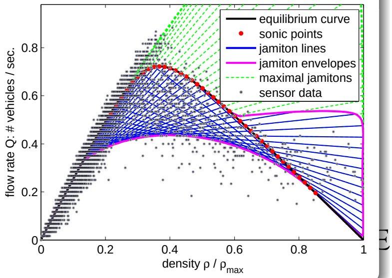
## Good Agreement With Sensor Data

We can reverse-engineer model parameters, such that the aggregated jamiton FD shows a good qualitative agreement with sensor data.

## Jamiton Fundamental Diagram




## Sensor Data with Jamiton FD



# Overview

# Three Scales in Unstable Traffic Flow

- Vehicle scale (10–100m): heterogeneities matter, specific braking patterns matter, collisions could happen
  [Micro models need to resolve this scale.]
- Jamiton scale (500m): systematic wave component of the flow (like waves on and in the ocean)
  [Unstable second-order models need to resolve this scale.]
- Urban scale (10km): effective large scale flow patters

Dream: models for the urban scale, that nevertheless capture the cumulative effect instabilities and waves on flow rate and aggregate fuel consumption, emissions, etc.

Here are some ideas how to systematically derive those.

[Single long highway only. Later can also add: physical road features, network effects (routing), inflow and outflow, spillback from ramps, incidents, etc.]

# Averaged Model

### Second-Order Traffic Models (ARZ) With Instability and Waves

$$\begin{cases} \rho_t + (\rho u)_x & = 0 \\ (u + h(\rho))_t + u(u + h(\rho))_x & = \frac{1}{\tau}(U(\rho) - u) \end{cases}$$

Think of this as the Navier-Stokes equations at high Reynolds number.

### Averaging the Model

$\rho = \bar{\rho} + \tilde{\rho}$, where

$$\bar{\rho}(x,t) = \int_{x-\delta}^{x+\delta} w(z)\rho(z,t)\,\mathrm{d}z$$

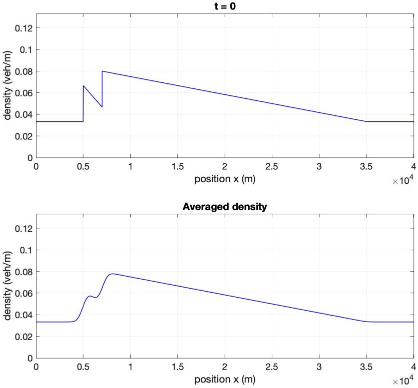
with: jamiton scale $\ll \delta \ll$ urban scale (and $w$ weight function).

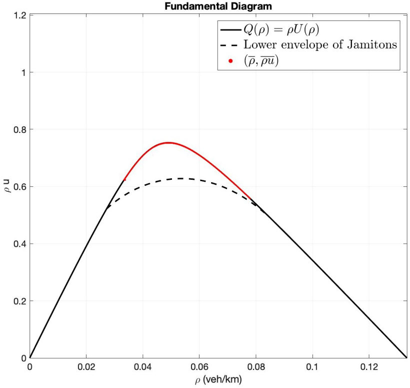
Think of this as Reynolds-averaging the Navier-Stokes equations.

### Key Question

What equation does $\bar{\rho}$ (approximately) satisfy?

# Numerical Investigation



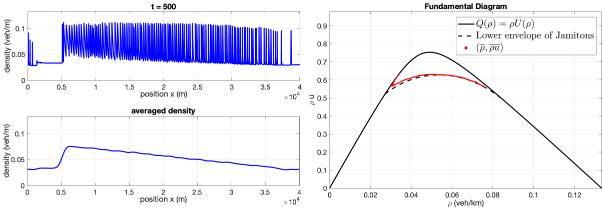




## Result

Averaged solution $(\bar{\rho}, \bar{q})$ describes curve in FD.
With jamitons established, new FD by lower envelope of jamiton FD:

$$\bar{\rho}_t + (\bar{Q}(\bar{\rho}))_x = 0$$

## Jamiton Averaging




## Averaged Models

First order:

$$\bar{\rho}_t + (\bar{Q}(\bar{\rho}))_x = 0$$

Second order:

$$\begin{cases} \bar{\rho}_t + (\bar{Q}(\bar{\rho}, \eta))_x = 0 \\ \eta_t + \frac{\bar{Q}}{\bar{\rho}} \eta_x = -b(\bar{\rho})\eta \end{cases}$$

## Even Better: Add Effective Observables

For any $\bar{\rho}$ (resp. $(\bar{\rho}, \eta)$), there is a unique jamiton. For that jamiton, integrate any nonlinear function $\psi(u, a)$ (fuel consumption, emissions) over jamiton profile to obtain effective function $\Psi(\bar{\rho})$.

## Key Point Again

Rather than obtaining these models and functions empirically, they result from a systematic averaging procedure of the models with instabilities.

# Conclusions

- Dynamic instabilities and traffic waves are fundamental features of (human) traffic flow.
- Humans do not intend to produce those waves. It is hard to change human driving.
- Even automated systems can have instabilities ($\longrightarrow$ ACC experiment).
- Vehicle automation and connectivity will change the flow dynamics. Need models to understand challenges of those heterogeneous human–AI cyber-physical systems, ideally before bad stuff happens.
- At the same time, well-controlled CAVs can, even in low numbers, remove traffic waves.
- This can have quite substantial wealth (40% less fuel consumed) and health (70% less $NO_x$ emissions) consequences.